Economic Dispatch for a Network of Micro-Gas Turbines

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Economic Dispatch for a Network of Micro-Gas Turbines

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Abstract

We propose a novel approach for solving an economic dispatch problem of a power network comprised of multiple micro-gas turbines (MGT) together with a utility. The ED problem is naturally formulated as a mixed-integer nonlinear optimization problem which is known to be computationally difficult to solve. We present a numerically efficient solution to this problem by decomposing the ED algorithm into two parts. The decomposition is enabled with the introduction of a new auxiliary variable, and the subproblems can be solved independently.

The first part of the decomposition, termed the “inner problem”, finds the optimal allocation of output power of each MGT as a function of the total requested power. We show that given MGTs with either convex or concave cost functions, the optimal allocation can be attained analytically. For these two cases we investigate the difference between the two solutions. In the convex case, the solution results in a power allocation that aims at balancing the MGT output power for a given demand. For concave cost functions, the solution requires that operational MGTs operate at their capacity before additional MGTs join to meet demand. This solution can then be considered as a look-up table for the second part of the decomposition, described next.

The second part of the decomposition, termed the “outer problem”, determines the desired total output power from the MGTs by modeling all the generators as a single unit. In this way, the problem reduces to a standard single-unit ED problem, and we show how to solve this efficiently using the celebrated shortest path algorithm. To do so, we consider the MGT network as one aggregated generator. We show how to model this fictive aggregated generator dynamically and economically. The optimal solution of the problem is then translated to an operation schedule for each MGT in the network. This is done using appropriate heuristics for convex or concave generator cost functions based on the analytic solutions developed for the inner problem.

Additionally, within this work, we lay a path for building a steady-state model and an economic model for a specific commercial MGT, the Capstone C65. The models are built based on measurement data that was obtained at the Turbomachinery and Heat Transfer Laboratory at the Technion.

We support our results with a two numerical case studies. In the first case-study our method against a commercial mixed-integer solver (MOSEK) for a simple example. The second case-study simulates a real case scenario, where we perform simulations for

two different demand profiles, a small and a large hotel. The MGTs considered for the simulations are Capstone C65, and the utility cost is modeled according to publicly available electricity tariffs.

Abbreviations and Notations

IEA	:	International Energy Agency
EIA	:	The U.S Energy Information Administration
CAGR	:	Compound Anticipated Growth Rate
MGT	:	Micro-Gas Turbine
CCHP	:	Combined Cooling, Heat and Power
CHP	:	Combined Heat and Power
UC	:	Unit Commitment
ED	:	Economic Dispatch
AGC	:	Automatic Generation Control
SPA	:	Shortest Path Algorithm
MILP	:	Mixed Integer Linear Programming
t	:	time variable
N	:	the number of generators in a network
T	:	the planning horizon
$C_i()$	:	cost function of generator i
$p_i(t)$	:	output power at time t of generator i
$\bar{u}_i(t)$	:	vector of control variables at time t of generator i
J	:	objective function
c^V	:	constant representing the variable generator's costs
c^F	:	constant representing the fixed generator's costs
$y_i(t)$	:	start-up control input of ith generator at time t
$z_i(t)$	:	shut-down control input of ith generator at time t
$P(t)$	:	demand at time t
$SP(t)$	:	secured power at time t
$LP(t)$	:	power loss at time t
$P_{max,i}$	:	maximum output power of generator i
R^U	:	generator's power up-ramping limitation
R^D	:	generator's power down-ramping limitation
R^{SU}	:	generator's power start-up ramping limitation
R^{SD}	:	generator's power shut-down ramping limitation
$D_i(t), E_i$	:	constraint sets of the ith generator
$g_i(), h_i()$	:	constraint function§ of generator i

E	:	edge set of a graph
V	:	node set of a graph
$O()$	:	big O notation - an upper bound on algorithm's complexity
SUP	:	Single Unit ED Problem
ΔT	:	time interval / time step
$c()$	:	number of time steps needed for state transition
$f_i()$	:	ith generator's dynamic transition function
DAG	:	Directed Acyclic Graph
$C_{capstone}$	:	generator's cost function specifically for Capstone C65
C_{GT}	:	MGT's cost function
$P_U(t)$	:	output power of the utility at time t
$C_U()$	:	cost function of the utility
p_k	:	specific output power at state k of a generator
$e()$	:	overall cost of an edge
s	:	number of MGT's discrete-model states
σ	:	the auxiliary variable
δ_p	:	discrete model power step
$\mathcal{F}$	:	a set of feasible solutions
$\mathcal{P}^\star$	:	an optimal solution set
$\mathrm{p}^\star$	:	an optimal solution
AGT	:	fictive aggregated generator
$C_{AGT}()$	:	AGT's cost function
MIQP	:	Mixed Integer Quadratic Programming
FPE	:	Fixed price of Energy
ROI	:	Return Of Investment

Chapter 1

Introduction

In recent years, the rapid advancement in technological capabilities accelerates the ever-growing demand for electrical appliances and gadgets along side overall energy consumption. Commercial industries and households consumes more energy in an increasing rate, and as economic reports present the growth in the energy demand over the years, economic models forecast the growth in energy consumption in the years to come. According to recent publication from BP (*BP p.l.c* formerly known as British Petroleum) one of the largest oil and gas corporations in the world, over a decade from 2009 to 2019 the world energy consumption increased in more than 20% [1]. Figure 1.1 demonstrates the yearly growth since 1994 including the decade 2009 to 2019. Additionally, Figure 1.1 exhibits the mix of main energy sources over the years, and provides a sense of trends in fuel types. Specifically, a growth in consumption from sources such as renewable sources and natural gas.

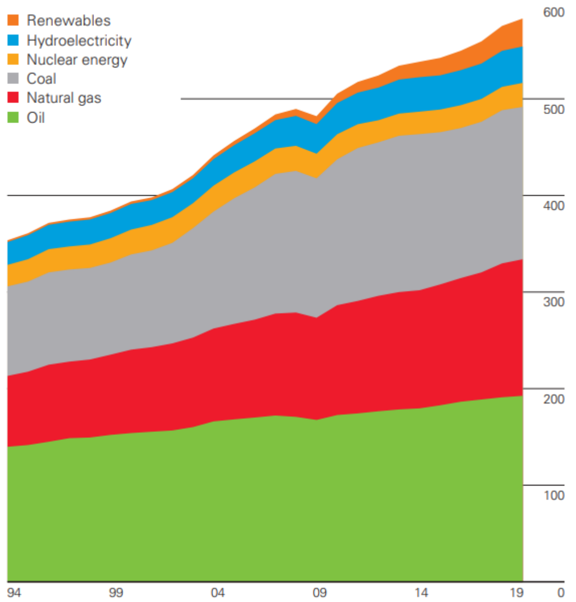


Figure 1.1: Yearly energy consumption over the world in exa-joules (reproduced from [1]).

The near future projections present a fair amount of uncertainty due to the COVID-19 pandemic, yet models expect continue in growth. According to the International Energy Agency (IEA) The resulted growth may vary under different scenarios in about 5%-10% from 2019 to 2030 [4].

As demand for energy keeps growing, in resent years, common perception around the world have a change in concepts regarding how power should be generated from. People are more and more aware of the impact polluting power plants have on our planet, and policy makers are bound to guide and regulate toward less harming and polluting power generation. Hence, the power generation industry is shifting toward cleaner and more efficient power production, gradually increasing the use in natural sustainable fuels. The U.S Energy Information Administration (EIA) reassures this projections published upcoming trends in [2]. Figure 1.2 depicts the overall growth in energy consumption by fuel type. While the energy consumption from renewable fuels constantly increases,

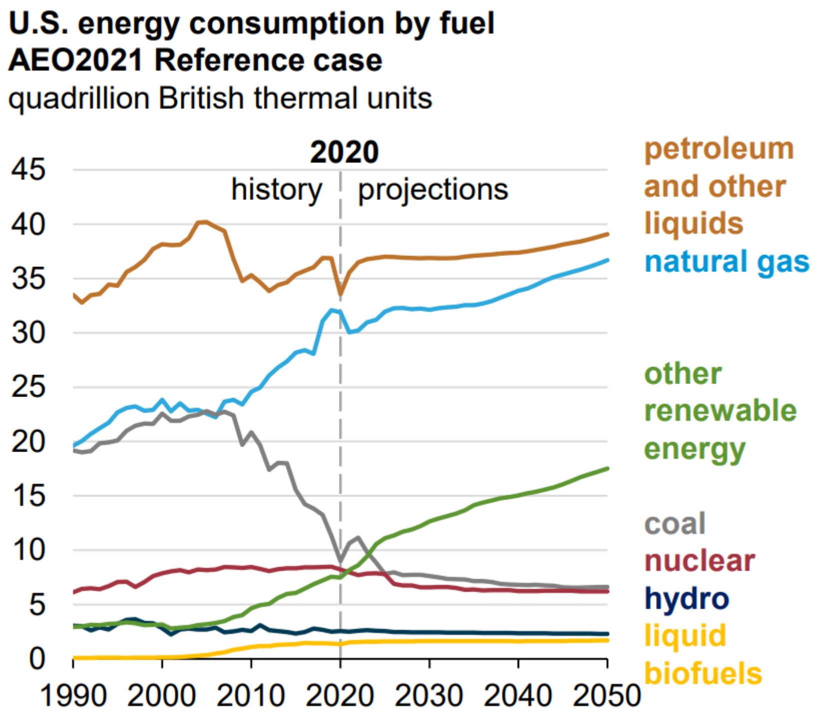


Figure 1.2: Projection of fuel types for energy generation (reproduced from [2]).

the energy produced from natural gas is consumed in the same growing rate. Hence, according to the EIA, from 2020 to 2050 expectations are for almost doubling natural gas consumption in the U.S. Furthermore, according to EIA, specifically for electricity generation, the use in natural gas is relatively consistent from 2020 to 2050, while nuclear and coal is expected to be cut in half. This makes generating power from natural gas a very natural intermediate stage toward clean energy generation. Accordingly, the gas turbine market is expected to grow with compound anticipated growth rate (CAGR) of 8.2% from 2020 to 2026 [5]. Specifically for Micro-Gas Turbines (MGTs), considering output power up to 500 kW, the CAGR forcast to 2027 is 10% [6]. MGTs are a compelling solution for combined cooling, heat and power (CCHP) solution, as MGTs operate in variable speed, relatively compact in size, simple to operate, easy to install and have low maintenance demands. The MGTs are also typically very efficient with low NO_x emission [7]. As the field of power generation moves away from the old centralized generation toward distributed generation, this features makes MGTs a very attractive candidate to be integrated within a power grid as a remote independent supplier or as part of power network.

In contrast to more conventional methods with one power plant, new and cleaner methods accommodate smaller generating units that can be spread around geographically. As such, a need for better and more efficient ways to distribute power is rising. While in the past days the conventional architecture for power distribution networks was comprised of a single utility generating power, recent architectures integrate several generating units as part of a "smart-grid." Hence, moving from a centralized approach to a decentralized one for the distribution of power.

When confronting a power distribution problem in a network, one has to consider power management and economic worthiness. In general terms, the problem is well known as the overall solution of two problems: the *unit commitment* (UC) problem and the *economic dispatch* (ED) problem. Although these two problems are relatively well studied over the years, when considering an integration of decentralized power network there is much room for further innovation. Specifically in this work, innovation in integrating MGTs.

1.1 Literature Review

Economic dispatch problems challenged researchers and engineers from long back. Early works on how to properly divide loads between generating units can be found since the 1920's, with methods such as the "best point load" [8] where units are scheduled starting with the most efficient one and proceeding downwards to the last efficient unit. Methods for solving the ED problem continued to emerge over the following years for different settings and limitations that concerned industry and academics. For example, during the 1940's some breakthroughs were achieved in optimal dispatching. An efficient method for an hour by hour calculation of the transmission losses with changing load was presented in 1943 [9]. Later within this decade, a new approach to economically divide power was proposed, using a network analyzer to asses the transmission losses of the network [10].

Since the 1950's, as technology progressed, ED problems were solved with the aid of digital computers [11]. Numerical techniques for calculating load-flow (low loss transmission) via digital computers continued to evolve in the 1960's and 1970's as outlined in the survey paper by Stott [12]. In his paper, Stott covers techniques related to methods such as the y-matrix, the z-matrix and the Newton-Rapshon. Happ, in his survey paper from 1977[13], overviews the preceding overall progress within the field of economic dispatch. Happ classifies the type of ED problems into *single area*, where the main idea is to properly divide the loads between the available generators ,*multi-area* where power transmission losses are also considered within the problem, *valve-point loading* problems, considering increasing costs of generators, and *optimal load flow* to minimize fuel consumption (i.e costs) considering network characteristics. In their review from 1990 [14], Chowdhury and Rahrman gather some of the work that was done regrading ED problems between the years 1977 to 1988. Within this work, Chowdhury and Rahrman classifies the ED problems into four categories: optimal power flow, economic dispatch in relation to AGC (Automatic Generation Control), dynamic dispatch, and economic dispatch with non-conventional generation sources.

In dynamic dispatch, in addition to the costs related to producing the output power, the ED optimization problem considers the dynamics of the power network (e.g changing loads and demands) regarding future planning horizons. Ross and kim developed dynamic programming techniques for such short-term dynamic economic dispatch, where the optimization considers future load changes [15]. Throughout the 1980's and the 1990's additional work was done applying dynamic programming methods with regards to ED and UC. Three different approaches for the dynamic programming method were compared in [16]. Van den Bosch and Honderd [17] suggested a decomposition of the UC problem into sub-problems with reduced complexity. In [18] a dynamic programming method is utilized to solve a UC problem for two-state generating units (i.e., on/off), and the generating units are classified beforehand according to their similar characteristics. An attempt to reduce execution time with dynamic programming method utilizing a variable window size according to change in demands was done in [19]. In [20] the ED problem includes the transmission losses and solved with dynamic programming. Another method of dynamic programming while performing de-commitment of generating units until achieving the lowest possible cost is in [21]. This method is then compared to two other optimization methods - the Lagrangian relaxation and the sequential unit commitment. Recently, a distributed calculation dynamic programming was presented for smart grids integration [22]. Another method for distributed generation with micro-grids based on dynamic programming method presented in [23], considering several stochastic uncertainties while formulating the ED problem.

For ED and UC problems where MGTs are part of the power network, there is some work done in resent years. Rist et al. [24] provided a detailed model of an MGT, and integrated the model within an ED optimization model under a CHP demand. The optimization problem was then solved by transforming the problem to that of a shortest path problem over a directed acyclic graph. In [25] a gas turbine was considered in a CHP setting to produce at the lowest cost with a neural-network optimization approach. Nemati et al. [26] introduced two methods for solving the UC and ED optimization problem with a genetic algorithm and an enhanced MILP algorithm. Both methods are simulated in a micro-grid scenario including six photo-voltaic units, one wind turbine, two micro gas turbines, one fuel cell and two diesel generators. A UC optimization was performed recently for micro-grid with MILP optimization method for three combustion generators and one MGT [27]. Finally, some ED solving robust methods consider realistic settings, where the demand or costs can not be known deterministically [28, 29, 30].

1.2 Thesis Contributions

In this work we consider an ED problem with a power network comprised of several MGTs and a utility. We present a computationally efficient method for solving the multi-agent (i.e., MGTs) ED optimization problem. The novel method dramatically reduces complexity and computational run-time by decomposing the optimization problem into two sub-problems. Our results focus particularly on cost functions that are either convex or concave. Finally, we implement and validate our method in a realistic case study including the use of a detailed MGT model based on physical measurements. The main contributions of this work are:

i) a decomposition method to dramatically reduce the computational complexity and the run-time of the multi-agent ED problem;

ii) development of micro-gas turbine generator model amenable for use in a UC/ED optimization framework;

iii) simulation results for realistic scenarios based on data providing demand profiles and electricity tariffs.

1.3 Thesis Outline

Chapter 1 of this thesis presents the motivation for this work, including background and literature review. Within Chapter 2, we lay out the basics of UC and ED optimization problems, including definitions and notations. Additionally, in Chapter 2 we present several known methods for solving the UC and ED optimization problems. In Chapter 3 we define the ED optimization problem with a single MGT. In this chapter we also model an actual MGT, the Capstone C65, based on real experimental measurements. We discretize the model to obtain with a graph describing the dynamics, and explain how the single MGT case is solved with the celebrated shortest path algorithm. Chapter 4 consists most of the main novelties of this work. In this chapter we describe and define the ED problem with multiple MGTs in the power network. Within this chapter we include additional methods for analytical optimal solutions to further reduce computational complexity. Finally, we discuss how our method for efficient solutions dramatically reduce the computational complexity of the multiple MGTs ED problem.

In Chapter 5 we present simulation results for several case studies. These case studies include a comparison of run-time performance between our method to a commercial solver. Additionally, we simulate the overall method model, with the Capstone C65 model integrated, in a “real” case scenario. The results substantiates our novel decomposition and analytical methods. Chapter 6 presents the conclusion of this work and future work to be done.

Chapter 2

The Unit Commitment and Economic Dispatch Problems

Optimization problems are widely encountered in scientific disciplines such as engineering and computer science, but also in fields such as economics and applied mathematics. One common type of optimization problem is the minimization problem where the goal is to minimize an objective function possibly subjected to constraints [31, 32, 33]. The *unit commitment* (UC) and *economic dispatch* (ED) problems may be described as a minimization problem of a given objective function, namely a cost function. For example, a goal might be to minimize monetary costs of power and heat distribution considering the costs under CHP operation of generators. Another objective of UC and ED problems may be to minimize environmental pollution of power generation, by minimizing the emission of ecologically harming gases. In these problems the objective is to minimize this cost, subject to technical, physical, or other types of constraints.

Within this chapter, we present an exposition to the UC and ED problems, establishing the needed settings for this work. We first describe the UC problem in Section 2.1 and in Section 2.2 we define the ED problem. In the final Section 2.3 we discuss about known methods of solving the UC and ED problems.

2.1 The UC Problem

The essence of a UC problem is to meet a given consumer demand for power with multiple generators. The demand can be for electrical power, heat, or both, and may change over a specific time period. The generators satisfy the demand under a CHP operation if needed. The explicit goal of the UC optimization problem is to try and meet the demand, while also minimizing the costs associated to the production of power and heat. Cost minimization is done by scheduling the generators' operation (i.e., between on and off states). The UC optimization problem is subjected to given constraints and limitations. In this section we define the UC problem starting with the cost function, proceeding to an overview of constraints types, and finally formulate the optimization model of the UC problem.

2.1.1 The UC Cost Function

The cost function represents the overall costs of producing electrical power or heat by the generators. For example, in order to produce a specific amount of electrical power, the generator will need to consume fuel at a sufficient rate. Hence, the cost function of the generator relates between the amount of power that was produced, to how much fuel was needed during the production of that power. Moreover, the cost function may incorporate the cost of the generator dynamics such as start-up, shut-down or any other transitions between generator states.

The notation of the cost function at time t is $C(p(t), \bar{u}(t))$, where $p(t)$ is the output electrical power of the generator, and $\bar{u}(t)$ is a vector of control variables. We consider the output power as a dynamical state, as it represents the overall generator state to produce a specific output power. Following this notion, the control variable is a control signal applied to achieve generator's transition between states evolving in time. As there are multiple generators, i.e., N generators, we index the cost function of each generator with it's state and control such that $C_i(p_i(t), \bar{u}_i(t)), \forall i = 1, \ldots, N$. The consumer demand in the UC problem is defined for a specific time-frame called the *planning horizon*, and denoted as T. The UC optimization then schedules the generators over discrete time instances within the planning horizon for each $t = 0, \ldots, T$. The overall cost function of the UC problem is the accumulated cost of the generating units over the planning horizon, and can be expressed as

$$J = \sum_{t=1}^{T} \sum_{i=1}^{N} C_i(p_i(t), \bar{u}_i(t)). \tag{2.1}$$

The cost functions C_i of each generating unit may be convex, concave, or non-convex, non-concave functions, depending on the generator's characteristics.

Example 2.1 (Linear Cost Functions)**.** *One may consider, for example, a generator with a linear cost function i.e., a linear relation between operational states, control inputs and costs. Following this direction, the expression for the cost function can be elaborated explicitly with fixed and variable costs such that*

$$C(p(t), u(t)) = c^V p(t) + c^F u(t). \tag{2.2}$$

The constant $c^V p(t)$ is defined considering the generator characteristics, e.g. fuel consumption rate, which effects how much money invested in the production of a specific amount of electrical power. Each amount of output power has its own cost and hence the variable cost. Additionally, we consider a fixed cost expressed as $c^F u(t)$. in this example, there is a single control input represented by a binary control variable $u(t) \in \{0, 1\}$. Therefore if the generator is operating $u(t) = 1$, and there is a fixed added cost defined *by the constant* c^F.

As previously mentioned, the UC problem defines which generating units are operational at each time instance within the planning horizon. Therefore, it's paramount to consider start-up and shut-down costs within the cost function. In order to formulate the start-up and shut-down costs, we introduce two new binary control variables $y(t)$ and $z(t)$, which represents control signals for start-up and shut-down, respectively. If for example $y(t) = 1$, a start-up command was issued to the generator. Following this notion, the control vector of the generator is $\bar{u}(t) = [u(t), y(t), z(t)]^\top$.

2.1.2 Constraints

There can be several types of constraints in the setting of a UC problem. These include the main goal of meeting the consumer demand, to additional constraints that may arise from transmission network topology, physical limitations of the generating units or any other type of limitation and restriction depicting the nature of the UC problem. Here we present several common constraints found in the UC problem.

power balance

The first constraint represents the goal, for which the aggregated output power from all the generating units must meet the consumer demand. We define the aggregate consumer demand with the time dependent function $P(t)$, and formalize this constraint mathematically as

$$\sum_{i=1}^{N} p_i(t) = P(t),\; t = 1, \dots, T. \tag{2.3}$$

Thus, at each time t the total produced power must equal the requested demand.

In some cases there may be a need in securing power overhead from the network, or consider power loss over the grid. We denote secured power as SP and loss power to be compensated as LP. The power balance in (2.3) is updated with these two additional considerations such that

$$\sum_{i=1}^{N} p_i(t) = P(t) + SP(t) + LP(t),\; t = 1, \dots, T. \tag{2.4}$$

power bounds

As a "real world" electro-mechanical element, the generating units have physical limitations. Such limitations include the maximum output power that is possible from a generating unit, denoted as $P_{max,i}$. Additionally, we define the control-state $u_i(t)$ to represent only 'on' and 'off' operational status of a generating unit; that is we model

$u_i(t)$ as the binary signal $u_i(t) \in \{0,1\}$. The constraint formulation is given by

$$0 \leq p_i(t) \leq P_{max,i} u_i(t), \quad \forall t = 1, \ldots, T. \tag{2.5}$$

Constraint (2.5) states that if the generating unit i is working, it's output power is bounded by P_{max} otherwise the output power is confined to be 0.

ramping limitations

Another physical limitation of a generating unit is the speed that it can change its power output owing to the response time associated with large electro-mechanical machines. Such constraints are termed *ramp-rate* constraints and can be formulated as

$$p(t) - p(t-1) \leq R^U u(t-1), \quad \forall t = 1, \ldots, T. \tag{2.6}$$

The change in output power $p(t)$ between two time instances $(t, t-1)$ is represented as the difference in the left-hand side of inequality (2.6). The right-hand side limits the difference of output power with a known physical up-ramping limitation denoted as R^U. The right-hand side also assures that the generator is operating before the state transition occurs because in this case the start-up situation is not considered.

Similarly, by denoting the down-ramping limitation as R^D, the expression for the formalized constraint is

$$p(t-1) - p(t) \leq R^D u(t), \quad \forall t = 1, \ldots, T. \tag{2.7}$$

Note the switch between elements position in the left-hand side of inequality (2.7) with respect to (2.6), as this is an output decreasing case $p(t-1) > p(t)$.

Next we impose a bound on the generators output power when initiating a start-up command. We denote the allowed output power increment at start-up as R^{SU}. Incorporating this start-up limitation in (2.6) yields the next expression:

$$p(t) - p(t-1) \leq R^U u(t-1) + R^{SU} y(t), \quad \forall t = 1, \ldots, T. \tag{2.8}$$

Note that $y(t) \in \{0,1\}$ is a binary variable.

Similarly, we incorporate into the down-ramping limitation expressed in (2.7), a binary variable $z(t) \in \{0,1\}$ which represents a shut-down command,

$$p(t-1) - p(t) \leq R^D u(t-1) + R^{SD} z(t), \quad \forall t = 1, \ldots, T. \tag{2.9}$$

logical limitations

The relations between the control variables of the generator $\bar{u}(t) = [u(t), y(t), z(t)]$ can be logically expressed as

$$\begin{aligned} & y(t) - z(t) = u(t) - u(t-1), \quad \forall t = 1, \ldots, T \\ & y(t) + z(t) \leq 1, \qquad\qquad\qquad\quad \forall t = 1, \ldots, T. \end{aligned} \tag{2.10}$$

The equality in expression (2.10) restrict the relation between the previous (i.e., at $t-1$) and current control signals. If, for example, $\{u(t) = 0, u(t-1) = 0\}$ the unit does not changes it's off state. If $\{u(t) = 1, u(t-1) = 0\}$, hence the generator operation state is changed from 'off' to 'on', then a start-up command is given. The inequality in (2.10), states that the generating unit can only be starting-up or shutting-down but never both in a single time instance.

2.1.3 UC Optimization Model

Based on the preceding formulations for the cost function and constraints in Sections 2.1.1 and 2.1.2, we present the optimization model of the UC problem:

$$\begin{aligned} \min_{p_i(t), \bar{u}_i(t)} \quad & \sum_{t=1}^{T} \sum_{i=1}^{N} C_i(p_i(t), \bar{u}_i(t)) \\ \text{subject to:} \quad & \sum_{i=1}^{N} p_i(t) = P(t) && \forall t = 1, \ldots, T, \ i = 1, 2, \ldots, N \\ & g_i(p_i(t), \bar{u}_i(t), t) \in D_i(t), && \forall t = 1, \ldots, T, \ i = 1, 2, \ldots, N \\ & 0 \leq p_i(t) \leq P_{max,i} \cdot u_i(t), && \forall t = 1, \ldots, T, \ i = 1, 2, \ldots, N \\ & \bar{u}_i(t) \in \{0,1\} \times \{0,1\} \times \{0,1\}, && \forall t = 1, , ..., T, \ i = 1, 2, \ldots, N. \end{aligned} \tag{2.11}$$

The optimization model expressed in (2.11) describes the minimization of the cost function subjected to a set of constraints. The first constraint is the power balance constraint in which we ensure that the demand is satisfied. The second constraint represents the ramping-rates, logical limitations and start-up/shut-down limitations, namely the overall dynamical constraints of the generators. For simplicity, we lump the dynamical constraints into an abstract constraint set noted as $D_i(t)$ with the map $g()$. The third constraint is the power bounds of each generator. The last constraint frames the control variables to be of binary values.

2.2 The ED Problem

Similarly to the UC problem, the ED optimization problem strives to satisfy a given demand with a network of multiple generators. The main difference is that the ED problem considers generators that are already committed, as determined by the solution of the UC problem. Therefore, The ED problem determines the power allocation of each committed generator with minimum cost for each time instance t.

A set of constraints is also considered within the ED problem settings. This set of constraints considers additional aspects of the generators, e.g. the power capacity of transmission lines. The ED optimization model formulation is

$$\begin{aligned} \min_{p_i} \quad & \sum_{i=1}^{N} C_i(p_i(t), \bar{u}_i(t)) && (2.12) \\ \text{subject to} \quad & \sum_{i=1}^{N} u_i(t) p_i(t) = P(t), \quad i = 1, \ldots, N \\ & h_i(p_i(t), \bar{u}_i(t)) \in E_i, \quad i = 1, \ldots, N \\ & 0 \le p_i(t) \le P_{max,i}, \quad i = 1, \ldots, N. \end{aligned}$$

Note that in the formulated ED optimization model (2.12), we state the time variable t, although the ED problem is static. We do so to emphasize that the problem must be solved for each time instance t within a given planning horizon T. Also, the power balance constraint is considered only for the operational generators, that is for generators where $u_i(t) = 1$.

The ED optimization model expressed in (2.12) has a set of additional abstract constraints E_i. The power output is mapped into the constraints set via the functions $h_i(p_i, \bar{u}_i)$. These constraints may represent technical limitations of the generating units and additional technical limitations of the power distribution network as mentioned before. As the limitations of the power distribution network are outside the scope of this work, we will not explore this further.

If we consider both the UC and ED problems, we get a combined overall solution, providing an on-off schedule and commitment level for each generator. Following this direction we can consider the UC and ED problem as one optimization problem which is solved simultaneously [34]. Since in this work we solve the UC and ED as a single optimization problem, we refer to it simply as one comprehensive economic dispatch problem, i.e., the ED problem.

2.3 Solution Methods for the ED Problem

Observing the optimization model for the ED problem, it is clear that some variables are binary (e.g. the variables in the control vector $\bar{u}(t)$ at (2.11)). The formulation of the ED optimization model allows for some or all variables to be integers. These types of optimization problems with integer variables are known as *mixed integer programming* (MIP) problems. There are several approaches to solve the ED problem as an MIP problem. We now outline some of these methods.

One of the most basic methods is the *exhaustive enumeration*, in which optimal solutions are obtained by enumerating all the possible solutions and search for the best one. As the number of potential solutions are exponential in the number of variables, this method is often not computationally feasible. An ED problem with N binary variables, for example, has 2^N enumerations [35, 36]. With N variables taking values from the set $\{\alpha_1, \alpha_2, \ldots, \alpha_s\}$, we get s^N enumerations, where the problem size grows exponentially with N. Hence utilizing the exhaustive enumeration method typically result in a very long run-times, and thus this is not a very popular approach for solving ED problems. In [37] there is a comparison between three methods to solve an ED problem. One of the methods is a *brute force* method, which actually solves by exhaustive enumeration. The other two types are the known *Newton method* and the *merit order loading* method. The Newton method solves the ED problem utilizing Taylor's expansion on the gradient of the Lagrangian of the constrained optimization problem. Then, output power values of each generating unit is obtained in iterative manner until reaching wanted tolerance. The merit order method basically first brings online the generating units with the lowest costs, i.e schedule the generating units from the cheapest to the most costly. In particular, the method applied in the paper, exploits the fact that the specified generating units has decreasing incremental cost as the output power increases. Thus starting at the cheapest generator at capacity and continue adding generating units while meeting the power balance constraint.

Another method is the *branch and bound* ($B\&B$). In this method we utilize a search strategy enumerating all the possible solution by branching out at decision points, and search for lower and upper bounds on the optimal solution in order to evade unnecessary calculations (i.e., pruning rules). In their survey, Lawler and Wood [38] present the $B\&B$ method to use within applications such as integer programming, nonlinear programming and more, also comparing calculations efficiency and run-time. Furthermore, [38] also presents the relation of the $B\&B$ method to the *dynamic programming* method that will be discussed later in this section. In a more recent work, Morrison et al. [39], surveys recent development of the three main components of $B\&B$ algorithms, that is the search strategy, the branching strategy and the pruning rules. In [40] the $B\&B$ method is embedded to solve an economic dispatch problem. As the settings of the problem in [40] results in a complex mixed integer quadratic programming (MIQP) problem under a quadratic constraints, the proposed $B\&B$ algorithm solves the problem accurately, and faster with respect to other methods presented. In [41], an additional approach to solving a unit commitment problem with a $B\&B$ algorithm is presented.

Dynamic programming is a well studied method for solving optimization problems. A survey on the dynamic programming method was performed by Larson [42], categorising the techniques into - procedures for obtaining complete feedback control solution, procedures for finding optimal control sequence from a single initial state, procedures for infinite-stage problems and additional category of procedures which do not fit to the previous three categories. The utilization of the dynamic programming method to solve ED and UC problems in particular, is mentioned excessively within the literature review Section 1.1 of this work.

As a branch of the dynamic programming method, the shortest path algorithm solve optimization problems quickly and efficiently. Specifically with the known Dijkstra's algorithm, calculations end up in almost linear computational complexity given by $O(|E|+|V|\log|V|)$, where $|E|$ and $|V|$ represents the number of edges and vertices in the model graph, respectively [43]. In particular for solving the ED and UC optimization problems the shortest path algorithm was utilized in [24, 44, 45].

In [46] there is a comparison between the *linear relaxation* method and general MIP methods specifically for solving a UC problem. A new method known as *firefly algorithm* is used to solve an ED problem in [47]. Heuristic optimization technique based on *harmony search* is presented to solve ED problem in [48]. Additional heuristic optimization techniques are utilised to solve the ED problem. Some of them are the *genetic algorithm* [49], *simulated annealing* [50] and *practical swarm optimization* [51].

The ED optimization, as a constrained mixed-integer problem, is hard to solve. In fact, it is classified as NP-hard in the general case [52]. The methods mentioned in this section ,aim to find solutions in a reasonable computational run-time and effort. Our approach in the thesis is to employ the SPA, and via a decomposition of the problem find additional methods to reduce the complexity of the problem.

Chapter 3

Economic Dispatch with a Single MGT

After describing and defining the ED optimization problem in detail, and presenting the benefits and the motivation of exploiting natural gas via MGTs within the preceding chapters, in this chapter we present the ED problem with an MGT as a generating unit. We first define the single unit ED problem, following with a general discrete model for the MGT. Based on the general model and measurements data, we develop state model with additional economic model for a real MGT. We then formulate the single unit optimization model for the single unit ED problem. Finally we discuss about graph representation and the SPA application for solving the single unit problem. This chapter is based on the previous work done in [24], where the ED problem considered the MGT under CHP operation.

3.1 Single Unit Problem

We now consider an ED problem with a single MGT and a utility as the power generating units to satisfy the demand. This ED problem is defined to be a single unit ED problem (SUP) (see Figure 3.1). A key element in the single unit problem is the MGT.

Figure 3.1: Illustration of a single-unit problem.

As mentioned in Chapter 1, the MGT may generate both electrical power and heat, and therefore considered as a CHP generating unit. Hence, the overall ED problem minimizes the costs by schedule and defines the output power and heat produced by the MGT and the utility. A reasonable assumption, for example, will be that the production of power and heat by the MGT is cheaper than the utility, therefore the utility will supply only in the case that the MGT is at capacity and can not meet demands. For simplicity, as the main ideas and notions are kept, throughout this work we will focus on the economic dispatch specifically for electrical power distribution.

3.2 MGT Model

In this section we provide background on the MGT discrete-time state model in general, a specific discrete model for a real MGT, and a cost model for that real MGT.

3.2.1 MGT Discrete-Time State Model

In order to compute the optimal solution of the single unit problem, a valid model of an MGT is needed. The overall dynamics of the MGT are of a continuous nature, but we derive a discretized model for computational reasons. Furthermore, as we are interested only in the steady states of the MGT the discritization is performed accordingly, and we neglect transient aspects such as settling time. How this is done precisely will be later demonstrated when we model the real MGT.

The states of the MGT are modeled as $p(t)$ representing the generator steady-states that produce an explicit output power as described previously in Chapter 2. The dynamics of the MGT can therefore be expressed generally as

$$p(t + c(p(t), \bar{u}(t)) \cdot \Delta T) = f(p(t), \bar{u}(t)). \tag{3.1}$$

The transition function f depends on the current electrical output power and the control signals. The transitions yield changes in the electrical output power, namely transition to the next state, expressed as $p(t + c(p(t), \bar{u}(t)) \cdot \Delta T)$. The size of a time interval ΔT is fixed over the entire planning horizon, whereas $c(p(t), \bar{u}(t))$ determines the overall number of time steps that is needed for state transition, stated explicitly as a function of the current state and control. For example, if $c(p(t), \bar{u}(t)) = 1$ then the model considers one time step for state transition from $p(t)$ to $f(p(t), \bar{u}(t)$. If $c(p(t), \bar{u}) = 2$ then two time steps are needed to transition to the next state.

In this work we integrate and model a specific MGT, the Capstone C65 generator. An engine module of the capstone C65 with it's main elements is presented in Figure 3.2. Additional information about Capstone C65 can be found at [53].

The modeling technique of the MGT is based on real experimental measurements. The measurement data was obtained at the Turbomachinery and Heat Transfer Laboratory at the Technion, during monitored operation of a Capstone C65. To model the dynamics, we distinguish between specific operational events of the MGT such as

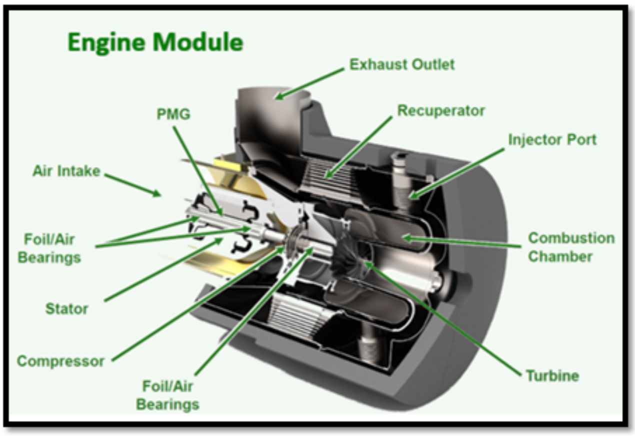


Figure 3.2: A Capstone C65 engine module.

start-up, shut-down and any changes in the output power. These events will be later directly linked to the model via MGT operational command, namely the control signal considered at the model's control vector $\bar{u}$. To determine an appropriate sample time for the model, we consider how long the generator takes to reach a steady state following different state transition commands.

The measured data shown in Figure 3.3, depicts the rotations per minute (RPM), and output power performance in kilo-Watts of a Capstone C65 over a time period of 1,800 seconds. The start-up command was given after 100 seconds, initiating the

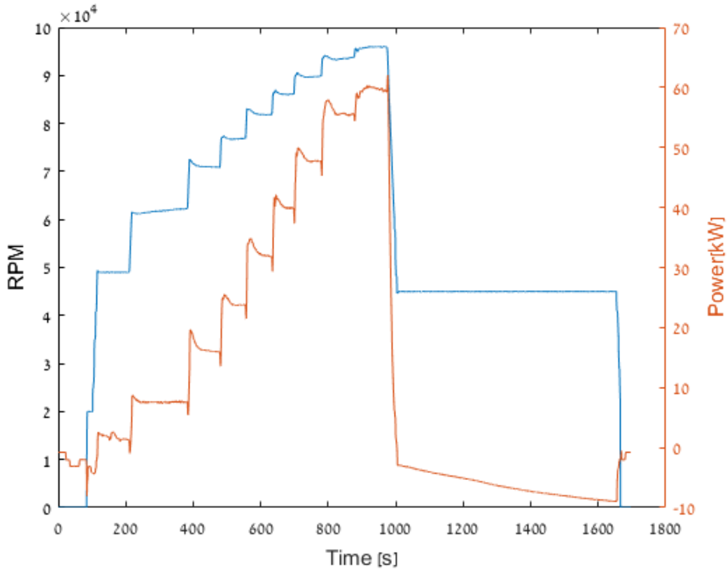


Figure 3.3: Performance of a Capstone C6 micro-gas turbine. Data obtained from the Turbomachinery and Heat Transfer Laboratory at the Technion.

operation of the MGT, following by additional 100 seconds for settling time.

After 200 seconds, the input command was changed for the first time during operation, increasing the output power to about 10kW. The process of increasing output power was repeated seven more times, until reaching an electrical output power of about 60kW. Notice that the response of the generator to step changes in the power demand, shows a transient response that includes some overshoot and settling time of roughly 70 seconds.

At time instance $t = 1000$ seconds, a shut-down command was given and a settling-time taking place for additional 680 seconds. Note that for the modeling we consider down-ramping time equal to up-ramping time, although we did not actually run an experiment to verify this.

At this point we can accurately assess the timing of each dynamical event. The fixed time-step for the discretization of the steady-state MGT model was chosen to be $\Delta T = 100$ seconds, based on the measurements. The transition times of each state transition is gathered in Table 3.1, including the value of $c(p(t), \bar{u}(t))$.

state transition	transition time (for $\Delta T = 100$[sec])	$c(p(t), \bar{u}(t))$
start-up	ΔT	1
shut-down	$7\Delta T$	7
single up-ramping	ΔT	1
single down-ramping	ΔT	1

Table 3.1: State transitions and their assessed time duration.

For the sake of complete MGT dynamical modeling, we also consider the constraint of power bound limitations. In this aspect we deduce from the measured data that the MGT capacity is $p_s = 60kW$. Additionally, the model's discrete steps of available output power is spaced with a constant value δ_p that can be chosen based on any needed criteria, such that $p(t) \in \{p_1 = 0, p_2 = \delta_p, p_3 = 2\delta_p, \ldots, p_s = 60\text{kW}\}$. Note that based on the measured data for specific output power values, we may interpolate additional values for the model.

MGT Model with a State-Transition Graph

The discrete model of the MGT can be represented using a graph, specifically an acyclic directed graph (DAG) [54]. The graph nodes represents the MGT's states, namely the possible output power at each time instance t. The graph edges describe the allowable state transitions of the MGT.

In Figure 3.4 we present the discretized MGT model derived previously for the Capstone C65 over a DAG. Shut-down sequence described in a period of $7\Delta T$ with an appropriate edge from any operational state, e.g. from node state p_2 at time instance $(k-1)\Delta T$ to node state $p_1 = 0$ at time instance $(k+6)\Delta T$. Note that in order to not over-detail the graph in Figure 3.4, some edges that represents valid MGT transitions were omitted.

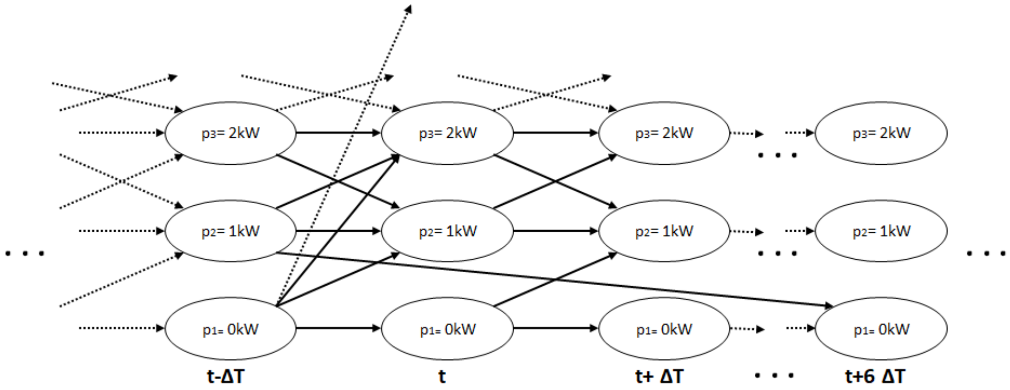


Figure 3.4: MGT discrete model dynamics over directed acyclic graph.

3.2.2 MGT Cost Modeling

As previously discussed in Chapter 2, the cost function represents the operational costs of an MGT, and depends on the output power and state transitions. To develop a complete model for the Capstone C65, we obtain data from an additional set of measurements. The measured data links between a given value of output power (kW) to the fuel consumption rate (liters per second) as presented in Figure 3.5. For modeling

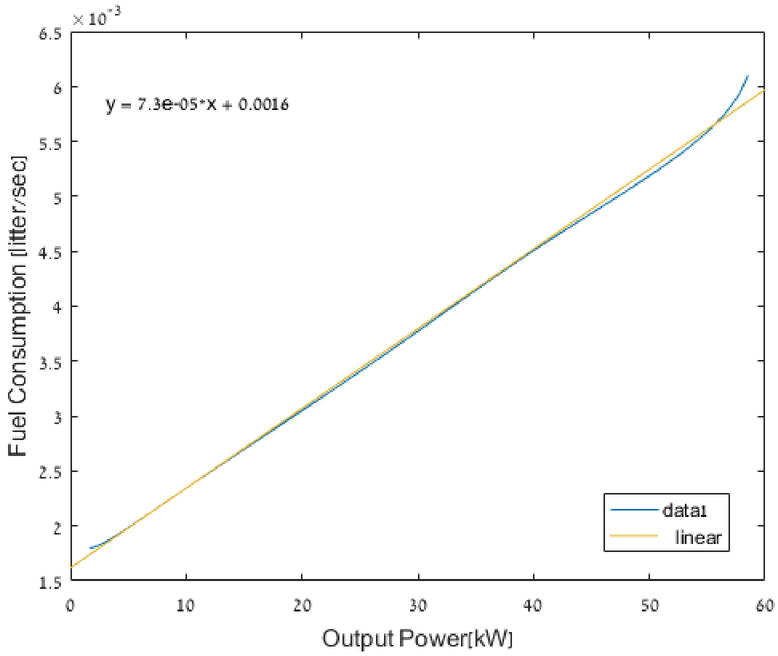


Figure 3.5: Fuel consumption as a function of output power for the Capstone C65 MGT.

purposes we utilise the function fitting application in MATLAB, and result with an *approximated cost function* for the Capstone C65. Moreover, when the MGT is off-line it does not consume fuel. Therefore, the cost function is adjusted to reflect this by splitting it into two cost function. We define the cost function of the Capstone C65 $C_{capstone}$ to be the sum of two cost functions C_1 and C_2, that is

$$C_{capstone}(p(t), \bar{u}(t), t) = C_1(p(t), \bar{u}(t)) + C_2(p(t), \bar{u}(t)). \tag{3.2}$$

The cost function C_1 is the formulation of the costs during start-up and shut-down such that

$$C_1(p(t), \bar{u}(t)) = 3.75(y(t) + z(t)). \tag{3.3}$$

During start-up and shut-down events the cost is associated only with the MGT aging. Following the model presented in [24], we use the static cost of \$3.75 for star-up/shut-down events. Thus the value of $C_1(p(t), \bar{u}(t))$ is 0 whenever $y = z = 0$, and \$3.75 otherwise.

The cost function C_2 represents the cost when the MGT is operating,

$$C_2(p(t), \bar{u}(t)) = 2.7 \times 10^{-4}(7.3 \times 10^{-5} \cdot p(t)) \cdot \Delta T \cdot u(t). \tag{3.4}$$

In this work, we consider a gas price coefficient of \$0.27/1000liter based on data from [55]. Multiplying this coefficient with the approximated cost function leads to a cost function in USD per second ($\$/sec$). Additional multiplication by the time interval ΔT, given in seconds, results in (3.4) which is in USD. Note that the cost function now is not described with general notation as C_{GT} but rather $C_{capstone}$ specifically for the Capstone C65.

MGT costs over a DAG

The cost function together with the graph modeling of the dynamics are combined to produce a weighted DAG, where each edge is given a weight corresponding to the cost of operating the generator. To do so, we utilize a cost averaging method. We first consider the costs related to each output power as described previously for the cost function $C_{capstone}$. We calculate the appropriate edge weight according to

$$\bar{C}_{capstone}(t, t + c\Delta T) = c\Delta T \frac{C_{capstone}(p(t)) + C_{capstone}(p(t + c\Delta T))}{2}, \tag{3.5}$$

where $c := c(p(t), \bar{u}(t))$ for notational convenience. As the allowed dynamic transitions are represented with an edge, all of the operational costs are covered within the model graph.

3.3 SUP Optimization Model

The overall optimization model of the single unit problem is again sumarized below,

$$\min_{p(t),\bar{u}(t),P_U(t)} \quad \sum_{t=1}^{T}[C_{GT}(p(t),\bar{u}(t)) + C_U(P_U(t))] \tag{3.6}$$

subject to

$$p(t + c(p(t),\bar{u}(t)) \cdot \Delta T) = f(p(t),\bar{u}(t))$$
$$P(t) = p(t) + P_U(t)$$
$$p(t) \in \{\mathrm{p}_1 = 0, \mathrm{p}_2, \ldots, \mathrm{p}_s\}$$
$$P_U(t) \geq 0, \quad \forall t = 1, ..., T.$$

The cost function expressed in the optimization model (3.6) is comprised of a general MGT cost function $C_{GT}(p(t),\bar{u}(t))$ and the utility's cost function $C_U(P_U(t))$. Note that the MGT costs are a function of the MGT's output power $p(t)$ and a specific set of control signals expressed in the control vector $\bar{u}(t)$. The control vector abstractly represents control signals given to the MGT such as start-up, shut-down, up/down-ramp etc.

The utility is a power generating unit assumed to have unlimited electrical power for distribution. Additionally, the model expressed at (3.6) can be modified to consider the case where the MGT sells power back to the utility. This is done by removing the non-negativity constraint $P_U(t) \geq 0$, thus $P_U(t) < 0$ corresponds to the case where the utility purchases power instead of selling. The objective function is updated accordingly to support different costs for the case of buying power from the utility, and the case of selling power back to the utility.

The single-unit optimization problem is subjected to four constraints. The first constraint represents the MGT dynamics as discussed in Section 3.2. The second constraint is the power balance constraint, which requires the output of the generating units (MGT and utility) to meet the consumer demand, denoted as $P(t)$. The demand is a function of time, representing all the consumers in the network as one accumulated need for electrical power. The third constraint restricts the output power of the MGT to a set discrete power steps i.e., of integer values, as discussed in Section 3.2. Finally the forth constraint subjects the electrical power which is contributed from the utility to be restricted to non-negative values.

3.4 Solving the Single Unit Problem Using the Shortest Path Algorithm

The optimization model for the single unit problem expressed in (3.6), can be solved efficiently with the celebrated shortest path algorithm (SPA). We already embedded the MGT costs within the model graph in Section 3.2.2.

In order to cover the optimization for the single unit problem, and solve with SPA, we consider also the costs of the utility to each edge weight of the model graph. We do it by first calculating the averaged output power of the MGT over the interval $[t,\, c\Delta T)$, with

$$\bar{p}(t, t + c\Delta T) = c\Delta T \frac{p(t) + p(t + c\Delta T)}{2}. \tag{3.7}$$

Next we calculate the cost of purchasing power from the utility at time t according to

$$C_U(t) = C_U(P(t) - p(t)). \tag{3.8}$$

The averaged utility cost over the interval $[t,\, c\Delta T)$ is therefore

$$\bar{C}_U(t, t + c\Delta T) = c\Delta T \frac{C_U(t) + C_U(t + c\Delta T)}{2}. \tag{3.9}$$

Finally, the overall cost that is assigned to the edge is

$$e(p(t), p(t + c\Delta T)) = \bar{C}_{GT}(t, t + c\Delta T) + \bar{C}_U(t, t + c\Delta T). \tag{3.10}$$

Note that the graph is built in such manner that it's nodes and edges cover all of the states and transitions that evolve throughout the planning horizon. As such, there may be, in some time instances for some demands, cases where the demand is lower then the available output power states which are represented by graph nodes. In these cases, since the optimization model expressed at (3.1) is subjected to the constraint $P_U(t) \geq 0$ for all $t = 1, \ldots, T$ and thus restricting sell back to the utility, the edges connecting to these states are ignored and trimmed out form the graph at the specified time instances.

Additionally, classic shortest path algorithms designate two nodes, the initial node and terminal node, between which the path should be determined. In this problem setting, these nodes have no physical meaning, and we allow transitions from the initial node to any MGT state, and any MGT state to the terminal node. We next assign cost to these edges,

$$\begin{aligned} e(\text{init}, p(1))) &= 0.5 \cdot (C_{GT}(1) + C_U(1)) \\ e(p(T), \text{term})) &= 0.5 \cdot (C_{GT}(T) + C_U(T)), \end{aligned} \tag{3.11}$$

where 'init' is the initial node, and 'term' is the termination node. The costs of the generator and utility at the $t = 1$ are $C_{GT}(1)$ and $C_U(1)$ respectively. Similarly, the costs of the generator and utility at the $t = T$ are $C_{GT}(T)$ and $C_U(T)$. Note that the costs here described in general formation for any state at $t = 1$ and $t = T$. Furthermore, these costs are designed to account for the averaging cost assignment across the other edges. In this way, the shortest path return the true cost of the generation schedule. The following example demonstrates how costs are assigned in a simple single-unit problem.

Example 3.1. *Consider a single unit problem with an MGT containing two states such that* $p \in \{0, 1\}$*. The MGT dynamics are given and presented accordingly in Figure 3.6. The cost function of the MGT is expressed as* $C_{GT}(p) = p$*. The cost function of the utility is* $C_U(p_U) = 10 \cdot p_U$*, where* p_U *is the output power of the utility. The demand given for the planning horizon* $T = 3$*, such that*

$$P(t) = \begin{cases} 0, & t = 1 \\ 1, & t = 2 \\ 2, & t = 3. \end{cases}$$

The cost of each transition is presented as edge weight in Figure 3.6, where init is the initiation node, and term is the termination node. The MGT cost is calculated according to (3.5) *and the overall cost is calculated according to* (3.10)*.*

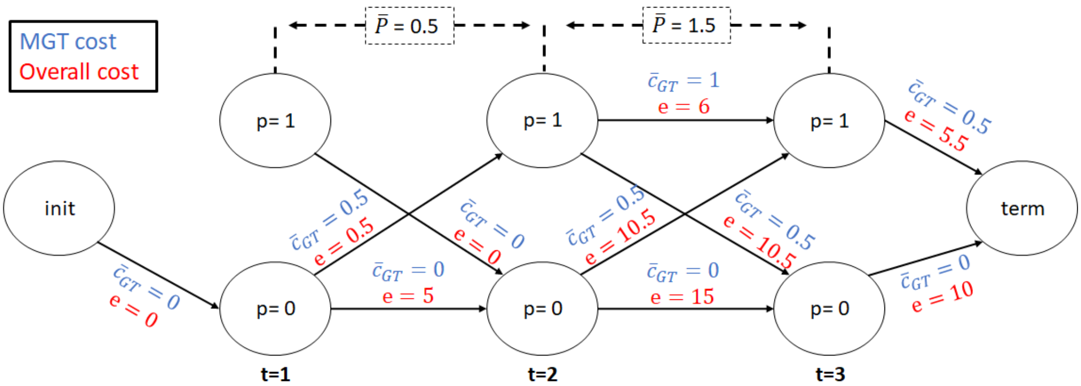


Figure 3.6: A single unit problem graph illustration for Example 3.1.

The graph size of the single unit problem is sT. The graph size is linearly related to the number of sates from the MGT's discrete model, and the number of discrete time-slots comprising the planing horizon T. This ED optimization problem, as a MIP optimization problem, is of high computational complexity as discussed in Subsection 2.3. Nevertheless, this problem can be solved rather quickly even for large size graphs by applying the SPA [24].

Chapter 4

ED with Multiple MGTs

The multi-unit ED problem solves the optimization problem for an electrical power distribution network that is comprised of the demand, utility, and a network of N MGTs (see Figure 4.1). Integrating more MGTs to the power network enables meeting

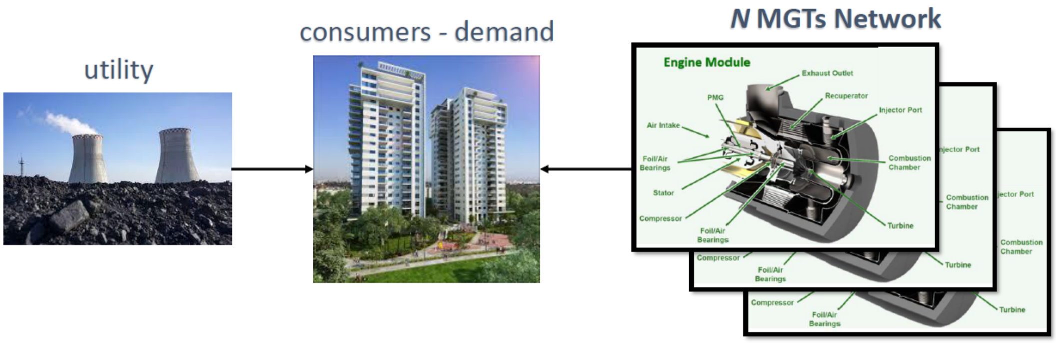


Figure 4.1: Illustration of a multi-unit problem.

higher consumer demand at a lower overall cost. Furthermore the power network is more resilient to malfunctions and uncertainties. Additionally, considering a network of MGTs is a major step toward an independent sub-networks of power suppliers. Alas, the multi-unit ED problem, i.e., with N MGTs, is proven to be very hard to solve, as the problem size explodes exponentially with N (to be discussed within this chapter). This leads to a very complex problem computationally.

In this chapter we present our approach for dramatically reducing the computational complexity of the multi-unit ED problem. We first define the multi-unit ED optimization model, then we introduce an auxiliary variable and we utilise this variable to decompose the problem into two independent sub-problems. Finally we present how this method reduces the problem complexity and simplifies the needed calculations to solve the problem.

4.1 Optimization Model for the Multi-Unit Problem

Consider a multi-unit ED problem with a network of N MGTs and the utility. The goal is to meet the consumer demands with the minimum possible cost over a given planning horizon T. The formulated optimization model of the multi-unit problem is expressed as

$$\begin{aligned}
&\min_{p_i(t),\bar{u}_i(t),P_U(t)} \sum_{t=1}^{T}\sum_{i=1}^{N} C_{GT,i}(p_i(t),\bar{u}_i(t)) + C_U(P_U(t)) && && (4.1)\\
&\text{subject to} \quad \sum_{i=1}^{N} p_i(t) + P_U(t) = P(t), && \forall t = 1,...,T &&\\
&\qquad p_i(t+c\Delta T) = f_i(p_i(t),\bar{u}_i(t)), && \forall t = 1,...,T, \quad \forall i = 1,2,...,N &&\\
&\qquad p_i(t) \in \{\mathrm{p}_{1,i} = 0,...,\mathrm{p}_{s,i}\}, && \forall t = 1,...,T, \quad \forall i = 1,2,...,N &&\\
&\qquad P_U(t) \geq 0, && \forall t = 1,...,T &&
\end{aligned}$$

The objective function of the multi-unit problem is comprised of each MGT cost function $C_{GT,i}(p_i(t),\bar{u}_i(t))$, and the cost function of the utility $C_U(P_U(t))$.

Note that each MGT has it's own output power state variable $p_i(t)$, control vector $\bar{u}_i(t)$, and transition function describing its dynamics, f_i.

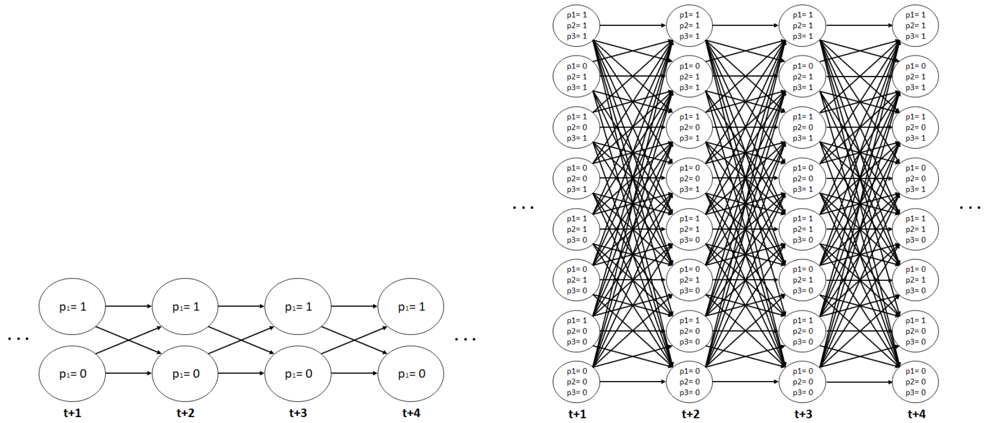
The minimization problem is subjected to the power balance constraint to meet the given demand $P(t)$ as before, this time considering all the generating units within the power network. The MGT dynamics are modeled similarly to the single-unit problem in Section (3.1). The utility output power is restricted to be non-negative as in the single unit case.

4.2 Multi-Unit Complexity

In Chapter 3, we described the single unit case, where the graph representing the dynamics of the MGT has sT nodes, where s is the number of MGT states and T is the planning horizon.

In the multi-unit problem we have s states for each MGT (we assume at this point that all the MGTs are identical). The combination of all possible states at one time instance is therefore s^N. When considering the planing horizon the graph has $s^N T$ nodes, showing an exponential increase. We illustrate this idea in the next example.

Example 4.1. *Consider a single unit problem with MGT that has a discrete model with two output power states (i.e., $s = 2$), such that $p \in \{0,1\}$. The MGT dynamics are a simple one step transition evolving over one time interval (i.e., $\Delta T = 1$ and $c = 1$). Now consider a multi-unit problem with $N = 3$ identical MGTs with the same model as previous setup. Figure 4.2 visualises the dramatic difference between the graph size of a single-unit problem and the multi-unit problem. In the single-unit problem,*



(a) $N = 1, s = 2$ (b) $N = 3, s = 2$

Figure 4.2: Graph sizes for two cases.

the number of nodes presented in Figure 4.2a is $2T$. *A relatively small increase in the amount of generators in the power network results in a graph with* 2^3T *nodes, as demonstrated in Figure 4.2b. Furthermore, in the single unit case there are a total of* $4(T-1)$ *edges where in the multi-unit case there are* $64(T-1)$ *edges.*

As the graph size, and therefore the problem size, increases exponentially with N, even if one utilizes the SPA approach as described in the preceding chapter, the problem may be prohibitively large to solve.

4.3 A Decomposition Approach

In order to simplify the complexity of the multi-unit problem, we present a method of decomposing the overall multi-unit problem into two sub-problems. The two sub-problems are named the *inner* problem and the *outer* problem. Each one of the sub-problems is solved independently, and a combined solution is an optimal solution for the overall original multi-unit problem with N MGTs. The *inner* problem is an ED problem which determines the allocation of output power from each MGT, given a desired aggregated power as a demand.

The *outer* problem is as a special case of a single unit problem as described in 3.1, but instead of an MGT we incorporate an aggregated generating unit. The generating unit represents the MGTs network, where we then can assign the appropriate costs to each output power state according to the *inner* problem solution.

The multi-unit problem decomposition process starts with the observation that

$$P_U(t) = P(t) - \sum_{i=1}^{N} p_i(t).$$

In this direction, we introduce the auxiliary variable $\sigma(t)$ as,

$$\sigma(t) = \sum_{i=1}^{N} p_i(t). \tag{4.2}$$

Plugging the auxiliary variable into the power balance constraint we get

$$P_U(t) = P(t) - \sigma(t), \quad \forall t = 1, ..., T.$$

We proceed in this direction and incorporate the auxiliary variable in the ED multi-unit model.

4.3.1 Incorporate the Auxiliary Variable

Considering the optimization model in (4.1), we express $P_U(t)$ with the auxiliary variable $\sigma(t)$ and the demand $P(t)$ as in (4.2) and rewrite (4.1) as

$$\begin{aligned}
&\min_{p_i(t),\bar{u}_i(t),\sigma(t)} \sum_{t=1}^{T} \sum_{i=1}^{N} C_{GT,i}(p_i(t), \bar{u}_i(t)) + C_U(P(t) - \sigma(t)) && && (4.3)\\
&\text{subject to} \quad \sigma(t) - \sum_{i=1}^{N} p_i(t) = 0, && \forall t = 1, ..., T \\
&\qquad p_i(t + c\Delta T) = f_i(p_i(t), \bar{u}_i(t)), && \forall t = 1, ..., T, \quad \forall i = 1, ..., N \\
&\qquad p_i(t) \in \{\mathrm{p}_{1,i}, ..., \mathrm{p}_{s,i}\}, && \forall t = 1, ..., T, \quad \forall i = 1, ..., N \\
&\qquad P(t) - \sigma(t) \geq 0, && \forall t = 1, ..., T.
\end{aligned}$$

The cost function of the utility C_U is now dependent on the demand and the auxiliary variable. Additionally, the constraint stating that the power purchased from the utility is non-negative is also expressed in terms of the demand and the auxiliary variable. The minimization is now performed over the optimization variables $p_i(t), \bar{u}_i(t)$, and $\sigma(t)$.

We observe in the optimization model (4.3) that the overall cost function has two parts which are comprised of different variables. The first part is the sum of the N MGTs costs as a function of the output power $p_i(t)$ and the control vector $\bar{u}_i(t)$ of each MGT. The second part is the utility cost function which depends on the demand and the auxiliary variable. Theses two parts of the objective function are coupled only by the constraint, that the sum of the generator outputs must equal the aggregated output.

At this point we separate the minimization by performing it over $\sigma(t), \bar{u}_i(t)$ and

$p_i(t)$ independently

$$\min_{\sigma(t)} \min_{p_i(t),\bar{u}_i(t)} \sum_{t=1}^{T}\sum_{i=1}^{N} C_{GT,i}(p_i(t),\bar{u}_i(t)) + C_U(P(t)-\sigma(t)) \tag{4.4}$$

$$\begin{aligned} \text{subject to} \quad & \sigma(t) = \sum_{i=1}^{N} p_i(t), && \forall t = 1,...,T \\ & p_i(t+c\Delta T) = f_i(p_i(t),\bar{u}_i(t)), && \forall t = 1,...,T, \quad \forall i = 1,...,N \\ & p_i(t) \in \{\mathrm{p}_{1,i},...,\mathrm{p}_{s,i}\}, && \forall t = 1,...,T, \quad \forall i = 1,...,N \\ & P(t) - \sigma(t) \geq 0, && \forall t = 1,...,T. \end{aligned}$$

Note that when neglecting time, each generator has a cost associated with a given output power. In this direction, we can consider solving the following problem: for a given aggregate power demand $\sigma(t)$, what is the optimal allocation of power for each generator $p_i(t)$ such that $\sum_i p_i(t) = \sigma(t)$. This problem is precisely the economic dispatch problem presented in Section 2.2. As we will show in Section 4.3.2, this problem can be solved analytically. This then allows us to consider a classic unit commitment problem for a fictitious generator with output power variable $\sigma(t)$, presented in Section 4.3.3. This separation is reflected in (4.4) by noting that the cost function $C_U(P(t) - \sigma(t))$ does not involve the variable $p_i(t)$, enabling such a separation.

4.3.2 The Inner Problem

The goal of the *inner* optimization problem is to minimize the operation costs of the power network comprised only of the MGTs (i.e., without the utility). The optimization is achieved by allocating the MGTs output power economically, to meet some fixed value of the auxiliary variable. Hence, the auxiliary variable is given to the inner problem as time-independent "external" parameter σ, and the optimization problem is static. To get the optimal allocation of output power from the MGTs network we minimize the objective function over p_i. The *inner* optimization model is

$$\begin{aligned} \min_{p_i} \quad & \sum_{i=1}^{N} C_{GT,i}(p_i) \\ \text{subject to} \quad & \sum_{i=1}^{N} p_i = \sigma \\ & p_i \in \{\mathrm{p}_{1,i} = 0,...,\mathrm{p}_{s,i}\}. \end{aligned} \tag{4.5}$$

The time-independent *inner* problem yields a stand-alone optimal solution for the MGTs power network given a specific σ, regardless of the planning horizon which was imposed within the original multi-unit problem. As we are interested in the optimal solution of the MGTs power network for any possible value of σ, we perform the *inner* optimization formulated in (4.5) repeatedly for each possible σ.

The set of optimal solutions for each σ is kept, and may now serve as a "look-up table" to be forwarded to the next step of solving the *outer* problem as will be discussed in Section 4.3.3.

We now provide an analytic solution for (4.5) for the case of *convex* or *concave* cost functions.

Convex Cost Functions

The next theorem shows that when considering an optimization problem as modeled in (4.5) for the case of convex cost functions, there is an optimal solution that can be determined analytically.

Theorem 4.1. *Consider the following optimization problem,*

$$\begin{aligned} \min_{p_i} \quad & \sum_{i=1}^{N} C(p_i) \\ \textit{subject to} \quad & \sum_{i=1}^{N} p_i = \sigma \\ & p_i \in \{\mathrm{p}_1 = 0, \mathrm{p}_2, \ldots, \mathrm{p}_s\}, \end{aligned}$$

where $C : \mathbb{R} \to \mathbb{R}$ *is a strictly convex function, and* $\mathrm{p}_{i+1} = \mathrm{p}_i + \delta_p$ *for* $i = 1, \ldots, s-1$ *and some* $\delta_p > 0$*. Let* $\mathcal{F}$ *denote the set of feasible solutions. The optimal solution set* $\mathcal{P}^\star \subset \mathcal{F}$ *is such that any solution* $\mathrm{p}^\star \in \mathcal{P}^\star$*, has the structure* $\mathrm{p}_i^\star \in \{Q, Q+\delta_p\}$ *for* $i = 1, \ldots, N$*, with* $Q = \lfloor \frac{\sigma}{N} \rfloor$*. In particular, for* $r = (\sigma \mod N)$ *and* $w = r/\delta_p$*, then*

$$p_i^\star = \begin{cases} \mathrm{Q} + \delta_p, & i = 1, 2, \ldots, w \\ \mathrm{Q}, & i = w+1, \ldots N. \end{cases}$$

Proof. We start by choosing without loss of generality some feasible solution $x \in \mathcal{F}$. We need to show that if at least one $x_i \notin \{Q, Q+\delta_p\}$, then there is another solution $y \in \mathcal{F}$ such that $\sum_{i=1}^{N} C(y_i) < \sum_{i=1}^{N} C(x_i)$. Namely, the solution y is cheaper than the solution x. This then would show that all optimal solutions should have the prescribed structure.

Consider, without loss of generality, a feasible solution $x \in \mathcal{F}$ with $x_i \leq Q - \delta_p$ for some i. Consequently, because of the constraint $\sum_{i=1}^{N} x_i = \sigma$, there must also be some entry j satisfying $x_j \geq Q + \delta_p$. We now construct a new solution $y \in \mathcal{F}$ as

$$y_k = \begin{cases} x_k, & k \neq i, j \\ x_i + \delta_p, & k = i \\ x_j - \delta_p, & k = j. \end{cases}$$

It is straightforward to verify that $\sum_{i=1}^{N} y_k = \sigma$ holds, i.e., y_k is a feasible solution. It now remains to verify that $\sum_{i=1}^{N} C(y_i) \le \sum_{i=1}^{N} C(x_i)$.

From the construction of the solution y, we obtain the difference in the cost of the two candidate solutions,

$$\sum_{i=1}^{N} C(x_i) - \sum_{i=1}^{N} C(y_i) = C(x_i) + C(x_j) - C(x_i + \delta_p) - C(x_j - \delta_p). \tag{4.6}$$

Furthermore, it follows that

$$x_i < x_i + \delta_p \le x_j - \delta_p < x_j.$$

From the chordal slope lemma (see Appendix A) for strictly convex functions we have

$$\frac{C(x_i + \delta_p) - C(x_i)}{x_i + \delta_p - x_i} < \frac{C(x_j) - C(x_j - \delta_p)}{x_j - (x_j - \delta_p)}, \tag{4.7}$$

which simplifies to

$$C(x_i + \delta_p) + C(x_j - \delta_p) < C(x_j) + C(x_i). \tag{4.8}$$

Hence, the expression in (4.6) is greater than zero, and the cost of solution y is cheaper than the cost of solution x. The proof can follow generally for $x_i \ge Q + 2\delta_p$. Thus, the optimal solution must have the structure proposed in the theorem.

Finally, note also that all solutions in the set $\mathcal{P}^\star$ can be obtained from each other by a simple permutation. Thus all solutions generated by the method outlined in this proof will converge to an optimal solution. ■

Remark. Note that for the case $\sigma < N$ it follows that $Q = 0$. This shows that the optimal solution includes generators that produce no power.

Theorem 4.1 gives the exact characterization of how optimal solutions should look like for any given demand, and therefore can be done offline. Furthermore, the given solution in the convex case aims to balance the generated power across all the MGTs.

Example 4.2. *In this example we present the commitment and output power allocation of the MGTs power network within the context of Theorem 4.1. We show the results for different values of the auxiliary variable σ. Consider five ($N = 5$) identical MGTs with the convex cost function $C(p) = p^2$ such that the overall cost is*

$$C(p) = \sum_{i=1}^{N} p_i^2. \tag{4.9}$$

Each one of the five MGTs has four operational states, i.e., $p_i \in \{0, 1, 2, 3\}$. We consider the problem for different values of σ as $\sigma_i, \quad i = 1, 2, \ldots, 16$. We consider the aggregate demand $\sigma(t)$ to follow the profile shown in Figure 4.3.

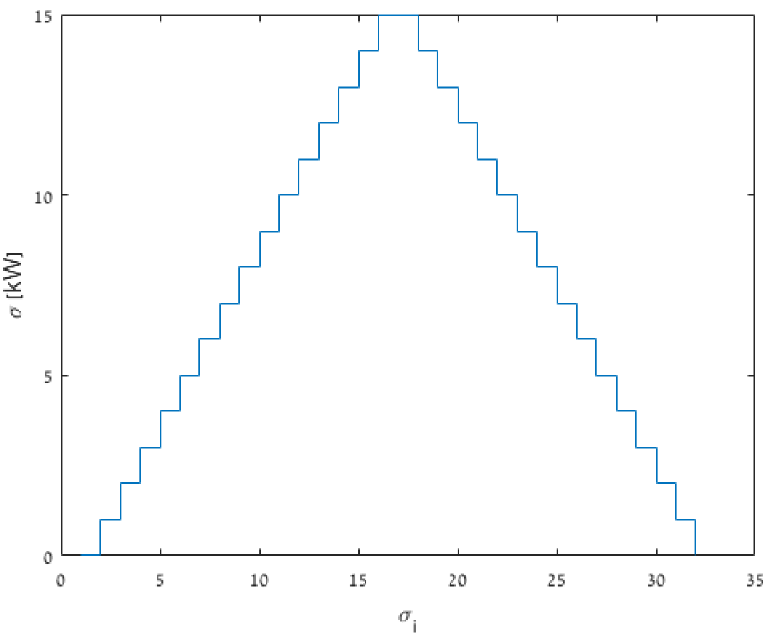


Figure 4.3: Demand σ as a function of time.

The output power of the MGTs for each value of $\sigma(t)$ were analytically determined according to Theorem 4.1, and the results are presented graphically in Figure 4.4. The figure shows that indeed generators distribute the generation of power in a balanced fashion.

Concave Cost Functions

As was shown for the convex cost function case, there is an explicit structure for optimal solutions when considering concave cost functions.

Theorem 4.2. *Consider the following optimization problem,*

$$\begin{aligned} \min_{p_i} \quad & \sum_{i=1}^{N} C(p_i) \\ \textit{subject to} \quad & \sum_{i=1}^{N} p_i = \sigma \\ & p_i \in \{\mathrm{p}_1 = 0, \mathrm{p}_2, ..., \mathrm{p}_s\} \end{aligned}$$

where $C : \mathbb{R} \to \mathbb{R}$ is a strictly concave function, and $p_{i+1} = p_i + \delta_p$ for $i = 1, \ldots, s-1$ and some $\delta_p > 0$. Let $\mathcal{F}$ denote the set of feasible solutions. The optimal solution set $\mathcal{P}^\star \subset \mathcal{F}$ is such that any solution $\mathrm{p}^\star \in \mathcal{P}^\star$, has the structure $\mathrm{p}_i^\star \in \{0, r, \mathrm{p}_s\}$ for

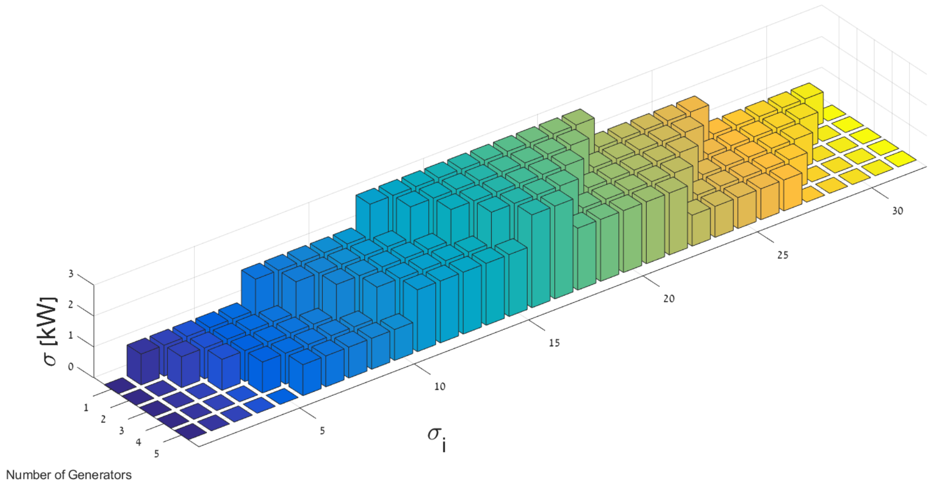


Figure 4.4: MGTs allocation for changing σ with convex cost function.

$i = 1, \ldots, N$, *with* $Q = \lfloor \frac{\sigma}{\mathrm{p}_s} \rfloor$. *In particular, for* $r = (\sigma \mod \mathrm{p}_s)$ *then*

$$p_i^\star = \begin{cases} \mathrm{p}_s, & i = 1, 2, \ldots, Q \\ \mathrm{r}, & i = Q + 1 \\ 0, & i = Q + 2, \ldots, N. \end{cases}$$

Proof. We start by choosing without loss of generality some feasible solution $x \in \mathcal{F}$. We need to show that if both $x_i, x_j \notin \{0, p_s\}$. Then there is another solution $y \in \mathcal{F}$ such that $\sum_{i=1}^N C(y_i) < \sum_{i=1}^N C(x_i)$. Namely, the solution y is cheaper than the solution x. This then would show that all optimal solutions should have the prescribed structure.

Consider, without loss of generality, a feasible solution $x \in \mathcal{F}$ such that $x_i \le x_j$, with $x_j \le p_s - \delta_p$ for some j and $x_i \ge \delta_p$ for some i. Consequently, because of the constraint $\sum_{i=1}^N x_i = \sigma$, x_i and x_j must balance each other. We now construct a new solution $y \in \mathcal{F}$ as

$$y_k = \begin{cases} x_k, & k \neq i, j \\ x_j + \delta_p, & k = j \\ x_i - \delta_p, & k = i. \end{cases}$$

It is straightforward to verify that $\sum_{i=1}^N y_k = \sigma$ holds, i.e., y_k is a feasible solution. It now remains to verify that $\sum_{i=1}^N C(y_i) \le \sum_{i=1}^N C(x_i)$.

$$\sum_{i=1}^N C(x_i) - \sum_{i=1}^N C(y_i) = C(x_i) + C(x_j) - C(x_j + \delta_p) - C(x_i - \delta_p). \tag{4.10}$$

Furthermore, it follows that

$$x_i - \delta_p < x_i \leq x_j < x_j + \delta_p.$$

From the chordal slope lemma (see Appendix A) for strictly concave functions we have

$$\frac{C(x_i) - C(x_i - \delta_p)}{x_i - (x_i - \delta_p)} > \frac{C(x_j + \delta_p) - C(x_j)}{x_j + \delta_p - x_j}, \tag{4.11}$$

which simplifies to

$$C(x_j) + C(x_i) > C(x_i + \delta_p) + C(x_j - \delta_p). \tag{4.12}$$

Hence, the expression in (4.10) is greater than zero, and the cost of solution y is cheaper than the cost of solution x.

The proof can follow generally for $x_j \leq p_s - n\delta_p$ and $x_i \geq n\delta_p$ for an integer n in the same manner. Thus, the optimal solution must have the structure proposed in the theorem.

Finally, note also that all solutions in the set $\mathcal{P}^\star$ can be obtained from each other by a simple permutation. Thus all solutions generated by the method outlined in this proof will converge to an optimal solution. ■

In the concave case, we get from Theorem 4.2 the exact characterization of how optimal solutions should look like for any given demand. Similarly to the convex case, this can be performed offline. The given solution prioritizes the maximization of output power from each MGT before bringing new ones online.

Example 4.3. *Similar to the convex case example, in this example we present the output power allocation of the MGTs power network within the context of Theorem 4.2. We show the results for different values of the auxiliary variable σ. Consider five ($N = 5$) identical MGTs with the concave cost functions $C(p) = -p^2 + 10p$, such that the overall cost is*

$$C(p_i) = \sum_{i=1}^{N}(-p_i^2 + 10p_i). \tag{4.13}$$

Each one of the five MGTs has four operational states, i.e., $p_i \in \{0, 1, 2, 3\}$. The values for σ_i are considered with $i = 1, \ldots, 16$. The $\sigma(t)$ profile is given as before, as depicted in Figure 4.3.

The output power of the MGTs for each value of $\sigma(t)$ were analytically determined according to Theorem 4.2, and the results are presented graphically in Figure 4.5. The figure shows that the output power from the generators is not spread evenly over the generators as in the convex case. Rather, each generator first reaches its output capacity before introducing a new generator to meet the aggregate demand.

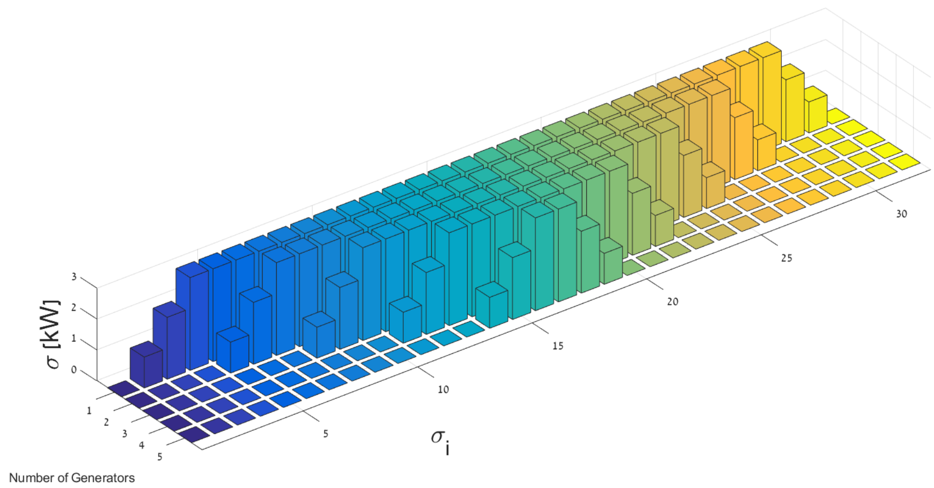


Figure 4.5: MGTs allocation for changing σ with concave cost function.

4.3.3 The Outer Problem

The role of the outer problem is to determine a schedule for the auxiliary variable $\sigma(t)$, representing the total aggregated power of all the generators. To do so, we approach the outer problem similarly to the single-unit problem, considering the MGTs power network as a single aggregated power generating entity (i.e., a new generator). In this direction, we need to model the new generator, noted as AGT, and integrate it into the optimization model. The model of the aggregated generator is built to consider an aggregated dynamics derived from the MGTs. The optimization model for the outer problem is

$$\begin{aligned} \min_{\sigma(t),\bar{u}_{AGT}(t)} \quad & \sum_{i=1}^{T}[C_{AGT}(\sigma(t),\bar{u}_{AGT}(t)) + C_U(P(t)-\sigma(t)] \qquad (4.14) \\ \text{subject to} \quad & \sigma(t+c\Delta T) = f_{AGT}(\sigma(t),\bar{u}_{AGT}(t)), \quad \forall t = 1,2,\ldots,T \\ & \sigma(t) \in \{\sigma_1,\sigma_2,\ldots,\sigma_{max}\}, \quad \forall t = 1,2,\ldots,T \\ & \sigma(t) \leq P(t), \quad \forall t = 1,2,\ldots,T. \end{aligned}$$

Note that the optimization model given in (4.14) is similar to the single-unit model in (3.6), but does not explicitly minimizes the power purchased from the utility, $P_U(t)$, as it coupled within the power balance constraint to σ. The cost of each σ is optimized independently within the inner problem, and can be fed to the outer optimization model as a look up table.

We will now further develop the aggregated generator model for the scenario of identical MGTs, specifically for the Capstone C65 model.

aggregated generator model

In Section 3.2.1 we modeled the Capstone C65 MGT. We use this model to derive a model for the aggregated generator with output power $\sigma(t)$.

- **power bounds**
  The minimal aggregated power is zero in the case where there are no MGTs online. For a power network of N Capstone C65 MGTs, with maximal output power of each generator is p_s, so the maximum aggregated output power is $N \cdot \mathrm{p}_s$. The power steps in the model are generally defined with increments δ_p, so there are a total of $(N \cdot \mathrm{p}_s)/\delta_p$ available states of output power from the aggregated generator. Note that p_s/δ_p is exactly the number of output power states of each MGT which was defined as s. The total number of output power states is therefore sN such that

$$\sigma(t) \in \{\sigma_1 = 0, \sigma_2, \ldots, \sigma_{sN}\}, \quad \forall t = 1, \ldots, T, \tag{4.15}$$

  with $\sigma_{i+1} = \sigma_i + \delta_p$, $\sigma_1 = 0$ and $\sigma_{sN} = N\mathrm{p}_s$.

- **dynamics**
  The dynamics of the aggregated generator is governed by the characteristics of the MGTs that comprise the power network, and expressed with the transition function f_{AGT},

$$\sigma(t + c(\sigma, \bar{u}_{AGT}(t)) \cdot \Delta T) = f_{AGT}(\sigma(t)(t), \bar{u}_{AGT}(t)). \tag{4.16}$$

  Here, $\bar{u}_{AGT}(t) \in \{0,1\} \times \{0,1\} \times \{0,1\}$ represents the control signal for the fictitious generator. The variable has the same representation as described in Section 2.1.2, corresponding to on/off, startup and shutdown i.e, variables $\{u, y, z\}$ respectively. As the AGT is comprised from N MGTs, the dynamics must consider the state-transition times accordingly. For example, the AGT is considered to be on for $\sigma(t) > 0$, however, transitioning from $\sigma(t) = \sigma_i$ to σ_{i+1} may require bringing a new generator online. The function $c(\sigma(t), \bar{u}_{AGT})$ must therefore be chosen to represent these transitions. In Theorems 1 and 2, we saw that all the solutions within the optimal set $p^\star \in \mathcal{P}^\star$, can be obtained with simple permutations. Hence, we must choose for the AGT model only one $p^\star$ for each σ. In this direction we develop a heuristic mapping the schedule for $\sigma(t)$ to a schedule for each individual generator. As the MGT's are indexed with $i = 1, \ldots, N$, we choose to add or remove MGTs from operational mode in ascending or descending manner. That is, if MGTs $i = 1, \ldots, k$ are operational at time t in order to generate $\sigma(t)$, and an additional MGT is needed to produce $\sigma(t+1)$, then MGT $i = k+1$ must be online by time $t+1$. Similarly, if the MGTs $i = 1, \ldots, k$ are operational at time t, and in order to produce $\sigma(t+1)$ we need one less MGT, we shut down the kth MGT.

We present two heuristics for the generator dynamics, one corresponding to the convex case (Theorem 4.1) at Algorithm 4.1, and one for the concave case (Theorem 4.2) at Algorithm 4.2. That is, we map the fictitious AGT transition maps to a schedule for the actual MGTs, based on the heuristics for each case.

Algorithm 4.1 MGT Scheduling Heurisitc: Convex Case

Require: N ▷ The number of MGTs
Require: T ▷ Planning horizon
Require: δ_p ▷ AGT power increment
Require: $\sigma(t) \quad \forall t = 1, \dots T$
  for $t = 1 : T$ **do**
    $Q(t) = \lfloor \frac{\sigma(t)}{N} \rfloor$
    $r(t) \leftarrow (\sigma(t) \mod N)$
    $w(t) \leftarrow r(t)/\delta_p$
    if $Q(t) = 0 \ \& \ w(t) < N$ **then**
      $k(t) \leftarrow w(t)$ ▷ $k(t)$ is the number of operational MGTs at t
    else
      $k(t) \leftarrow N$
    end if
    if $k(t) > 0$ **then**
      $u_i(t) \leftarrow 1, \quad \forall i = 1, 2, \dots, k(t)$
      $u_i(t) \leftarrow 0, \quad \forall i = k(t) + 1, k(t) + 2, \dots, N$
    else
      $u_i(t) \leftarrow 0, \quad \forall i = 1, 2, \dots, N$
    end if
    if $k(t + 1) > k(t)$ **then** ▷ MGTs start-up
      $y_i(t) \leftarrow 1, \quad \forall i = k(t) + 1, k(t) + 2 \dots, k(t + 1)$
    else if $k(t + 1) < k(t)$ **then** ▷ MGTs shut-down
      $z_i(t) \leftarrow 1, \quad \forall i = k(t), k(t) - 1 \dots, k(t + 1) + 1$
    else
      $y_i(t) \leftarrow 0, \quad \forall i = 1, 2 \dots, N$
      $z_i(t) \leftarrow 0, \quad \forall i = 1, 2 \dots, N$
    end if
    $p_i(t) \leftarrow Q(t) + \delta_p, \quad \forall i = 1, 2 \dots, k(t)$ ▷ Next MGTs state
    $p_i(t) \leftarrow Q(t), \quad \forall i = k(t) + 1, k(t) + 2, \dots, N$
  end for

Example 4.4. *In this example we illustrate AGT's transitions that evolve with time over a DAG, while considering the heuristics impacts. In particular we emphasize the difference between two cases of power networks, one that comprised of MGTs with a convex and one comprised of MGTs with a concave cost function.*

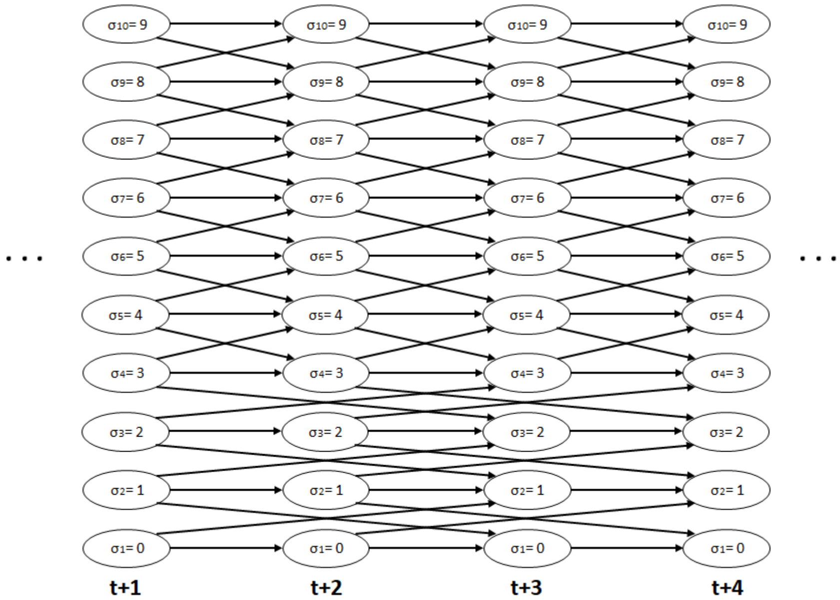
Consider a power network of $N = 3$ identical MGTs, with output power states $p_i \in \{0, 1, 2, 3\}$ for $i = 1, 2, 3$ and a discrete state model as described in Table 4.1. Then the AGT model of this network has output power states $\sigma \in \{0, 1, 2, \dots, 9\}$.

The Figure 4.6a demonstrates the transitions of the AGT model when the cost

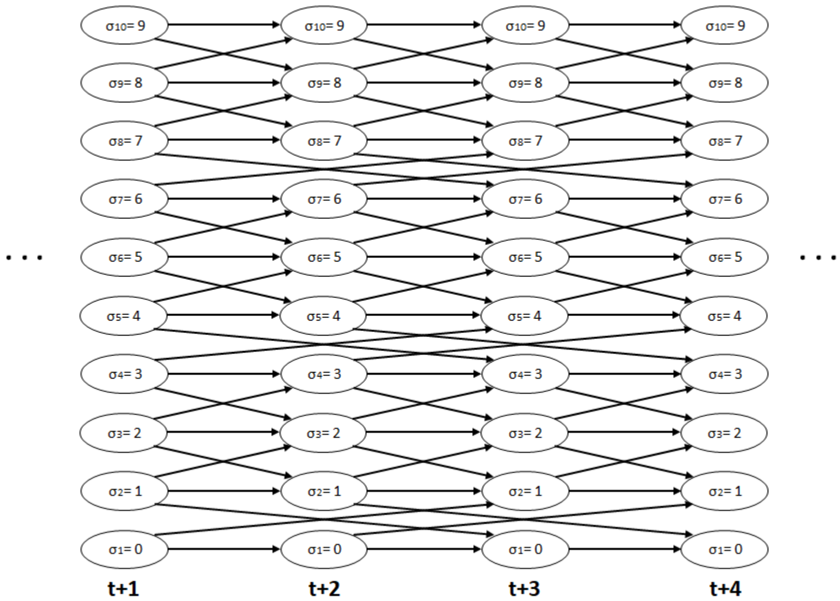
Algorithm 4.2 MGT Scheduling Heurisitc: Concave Case

Require: N ▷ The number of MGTs
Require: T ▷ Planning horizon
Require: δ_p ▷ AGT power increment
Require: p_s ▷ MGT's maximum output power
Require: $\sigma(t) \quad \forall t = 1, \dots T$

- **for** $t = 1 : T$ **do**
  - $Q(t) = \lfloor \frac{\sigma(t)}{p_s} \rfloor$
  - $r(t) \leftarrow (\sigma(t) \mod p_s)$
  - **if** $r(t) = 0$ **then**
    - $k(t) \leftarrow Q(t)$ ▷ $k(t)$ is the number of operational MGTs at t
  - **else**
    - $k(t) \leftarrow Q(t) + 1$
  - **end if**
  - **if** $k(t) > 0$ **then**
    - $u_i(t) \leftarrow 1, \quad \forall i = 1, 2, \dots, k(t)$
    - $u_i(t) \leftarrow 0, \quad \forall i = k(t) + 1, k(t) + 2, \dots, N$
  - **else**
    - $u_i(t) \leftarrow 0, \quad \forall i = 1, 2, \dots, N$
  - **end if**
  - **if** $k(t + 1) > k(t)$ **then** ▷ MGTs start-up
    - $y_i(t) \leftarrow 1, \quad \forall i = k(t) + 1, k(t) + 2 \dots, k(t + 1)$
  - **else if** $k(t + 1) < k(t)$ **then** ▷ MGTs shut-down
    - $z_i(t) \leftarrow 1, \quad \forall i = k(t), k(t) - 1 \dots, k(t + 1) + 1$
  - **else**
    - $y_i(t) \leftarrow 0, \quad \forall i = 1, 2 \dots, N$
    - $z_i(t) \leftarrow 0, \quad \forall i = 1, 2 \dots, N$
  - **end if**
  - $p_i(t) \leftarrow p_s, \quad \forall i = 1, 2 \dots, Q(t)$ ▷ Next MGTs state
  - $p_i(t) \leftarrow r(t), \quad i = Q(t) + 1$
  - $p_i(t) \leftarrow 0, \quad \forall i = Q(t) + 2, \dots, N$
- **end for**



(a) MGTs network with convex cost function.



(b) MGTs network with concave cost function.

Figure 4.6: An AGT discrete model over DAG with heuristic considerations.

state transition	duration time [sec]	$c(p(t), \bar{u}(t))$
start-up	200	2
shut-down	200	2
single up-ramping	100	1
single down-ramping	100	1

Table 4.1: State transitions and their assessed time duration.

function of the MGTs is convex. As expected, when starting from the non-operational state σ_1 towards higher output values, we encounter sequential start-ups of the individual MGTs until σ_4. A total of three start-ups as the number of MGTs that comprise the AGT. According to the heuristic, we have $y_1 \leftarrow 1$ between σ_1 to σ_2, followed by $y_2 \leftarrow 1$ between σ_2 to σ_3 and so on. This is due to the balancing nature of the optimal solution stated in Theorem 4.1. This is also true for the shut-down sequence. The concave case is presented in Figure 4.6b. In this case, when moving up starting at σ_1 we observe the prioritizing of maximum output power from each MGT. The heuristics here are such that we have $y_1 \leftarrow 1$ following by reaching MGT capacity. Only then, from σ_4 to σ_5 we have additional start-up $y_2 \leftarrow 1$. Again, when moving down-wards between σs, shut-down sequence follows the same logic.

- **cost modeling**

  The optimal cost of each σ is given from the *inner* problem as $C_{AGT}(\sigma)$, where the costs are gathered together as a look-up table. We use these costs to build a cost function for the AGT which depends on the power output and the transitions costs. The transition costs are formulated as

$$\bar{C}_{AGT}(t, t + c\Delta T) = c\Delta T \frac{C_{AGT}(\sigma(t)) + C_{AGT}(\sigma(t + c\Delta T))}{2}. \quad (4.17)$$

  As the AGT model represents a cluster of N MGTs, there are N possible events of MGT start-up and N possible events of MGT shut-down. The associated costs of this events are considered as additional cost of the appropriate AGT's edges, according to how the heuristic works.

The final stage for solving the *outer* problem is to build the appropriate weighted DAG and solve with SPA. The cost of purchasing power from the utility at time t is calculated according to

$$C_U(t) = C_U(P(t) - \sigma(t)). \quad (4.18)$$

The averaged utility cost over the an interval is

$$\bar{C}_U(t, t + c\Delta T) = c\Delta T \frac{C_U(t) + C_U(t + c\Delta T)}{2}. \quad (4.19)$$

Then the overall cost assigned to the edge weight is

$$e(\sigma(t), \sigma(t + c\Delta T)) = \bar{C}_{AGT}(t, t + c\Delta T) + \bar{C}_U(t, t + c\Delta T). \tag{4.20}$$

4.3.4 Decomposition Method Summary and Complexity

In Figure 4.7 we present a summary for utilizing the decomposition method. Consid-

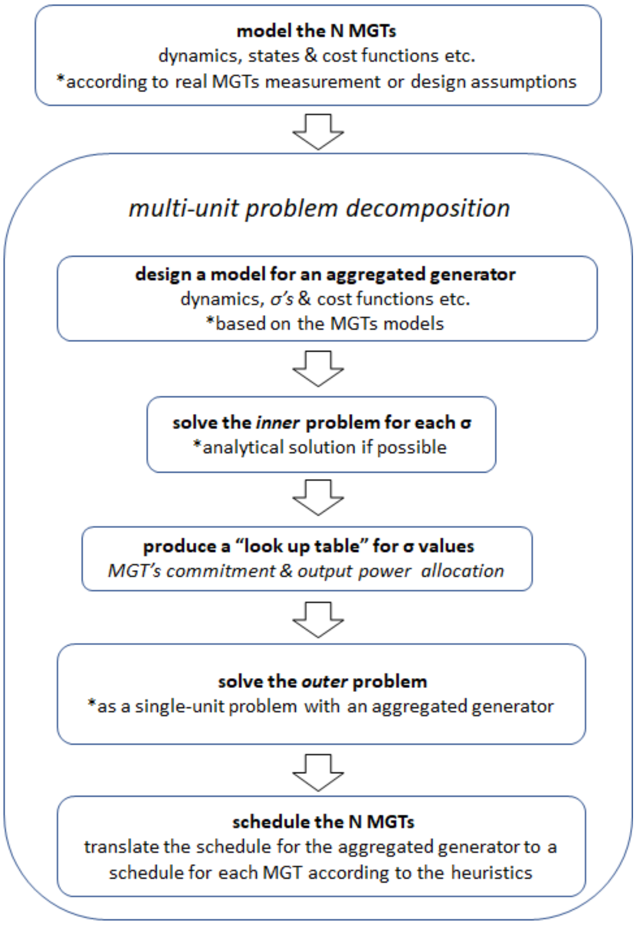


Figure 4.7: Step by step decomposition of the multi-unit problem.

ering an electrical power network with N different MGTs, then one of them (or some of them) has the maximal number states expressed as $p_s/\delta_p = s$.

In order to form an upper bound on the computational complexity, we consider a multi-unit problem where all of the N MGTs are with s states. In Section 4.2 we discussed the exponential growth of the graph size $s^N T$ and the resulted problem complexity. Using the decomposition method, we compute the values of each σ by performing the *inner* optimization $(N \cdot s)$ times and then storing the results. Furthermore, specifically for the convex and concave cost functions we have the solution analytically without any computation at all.

The *outer* problem graph can be modeled with a graph on $(N \cdot s) \cdot T$ nodes, and can be solved as a single-unit problem with the shortest path algorithm. The overall computational complexity is $O(|E| + |V| \log |V|)$, where we remind that $|E|$ and $|V|$ represents the number of edges and vertices in the model graph, respectively.

By decomposing the original multi-unit problem with $O(s^N \cdot T)$ exponential complexity, we actually solve two optimization problem with substantially lower complexity. This enables finding the optimal solution to a large multi-unit ED problem within acceptable calculation converge-time.

Chapter 5

Results and Discussion

To evaluate the performance and efficiency of the decomposition method for the ED optimization problem with multiple MGTs, we implement and simulate it with MATLAB for several different scenarios. All of the simulations were performed with Intel Core i3-6100 CPU @3.70 GHz.

5.1 Comparison of Decomposition Method with Commercial Solver

In this section we compare the simulation run-time of a multi-unit ED problem using two different solution methods:

- our proposed decomposition method;
- Mixed Integer Quadratic Programming (MIQP) performed using a commercial solver (MOSEK).

We simulated both methods with different number of MGTs in the power network.

Consider a multi-unit problem with N identical MGTs. Each MGT has three states such that $p_i \in \{0, 1, 2\}$ for all $i = 1, \ldots N$. All of the state-transitions require one time step (i.e., $\Delta T = 1$). This includes start-up and shut-down transitions. The cost function of an MGT is expressed as

$$C_{GT}(p_i(t)) = p_i^2(t).$$

The planning horizon is taken to be $T = 5$. There are no explicit additional MGT costs such as start-up costs, shut-down costs etc. As the MGT cost function is a convex function, the analytical cost expression, as given in Theorem 4.1, is embedded within the simulation.

Number of MGTs	MIQP		Decomposition	
	Run-Time[sec]	Total Cost	Run-Time[sec]	Total Cost
$N = 2$	2.48	16	0.23	16
$N = 3$	2.66	12	0.23	12
$N = 4$	5.21	10	0.23	10
$N = 5$	184.35	10	0.23	10

Table 5.1: Comparing the run-time and total cost between methods: decomposition vs MIQP.

The demand in this multi-unit ED problem is

$$P(t) = \begin{cases} 0, & t = 1 \\ 1, & t = 2 \\ 2, & t = 3 \\ 3, & t = 4 \\ 4, & t = 5. \end{cases}$$

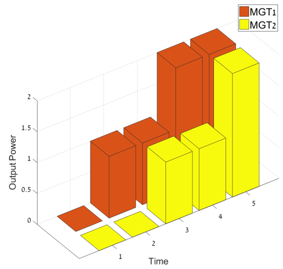
The cost function of the utility is $C_U(p_U) = 10 \cdot p_U(t)$, where $p_U(t)$ is the output power of the utility.

In Table 5.1 we gathered the run-time and the total cost results for both methods, where each simulation was performed for different number of MGTs. With the decomposition method, the simulation run-time for $N = 2, 3, 4, 5$ MGTs take less than quarter of a second. The simulation run-time of the MIQP method is 10 times slower with two and three MGTs. With four MGTs the run-time is doubled. The simulation of five MGTs explodes, where the run-time is 800 times slower than the decomposition method. Increasing N any further results in unreasonable or not-converging run-time when simulating the MIQP method. The total cost in both methods is the same. The results in Table 5.1 demonstrate the efficiency of the decomposition method, and it's superiority with regards to the MIQP method.

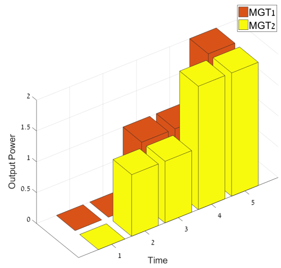
In Figure 5.1 we present the schedule and output power allocation of MGTs, corresponding with the simulations results in Table 5.1. The scheduling and power allocation is the same in both methods i.e., optimal solutions within the boundaries of MGTs permutation. Note that the application of the decomposition method heuristics take place and visualized in the graphs. That is, MGTs are brought online in an organized manner starting from MGT#1, then MGT#2 and so on.

5.2 A Real Case Scenario

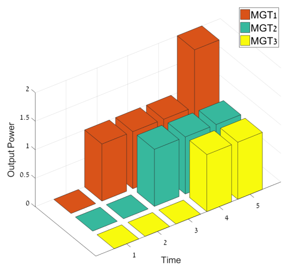
In this section we simulate a multi-unit problem in a real life scenario settings. Therefore, the simulation considers models of real elements as described in Section 5.2.1. The simulation of the real case scenario is solved with the decomposition method discussed



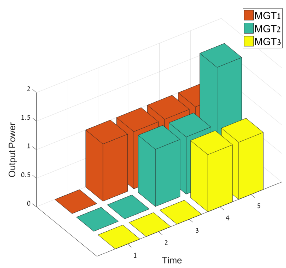
(a) Decomposition with $N = 2$.



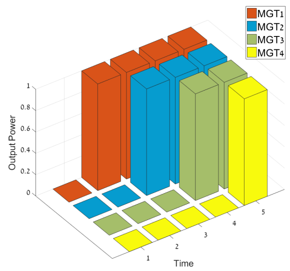
(b) MIPQ with $N = 2$.



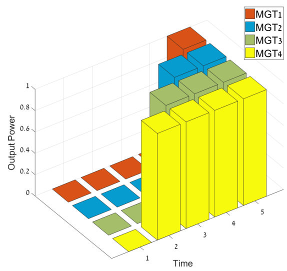
(c) Decomposition with $N = 3$.



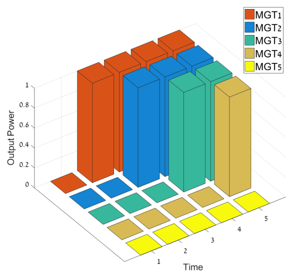
(d) MIPQ with $N = 3$.



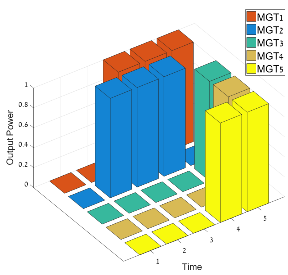
(e) Decomposition with $N = 4$.



(f) MIPQ with $N = 4$.



(g) Decomposition with $N = 5$.



(h) MIPQ with $N = 5$.

Figure 5.1: MGTs scheduling and output power allocation for different number of MGTs. The schedules are the same within a permeation of the MGT labels.

in details in Section 4.3.

5.2.1 Setting-Up the Problem

Demand

In the real case scenario we consider two consumers:

- small hotel - area of $4,013m^2$
- large hotel - area of $11,345m^2$

The demand is the electrical power in kilo-watts, needed by each of the hotels over a one day period. The data is taken from the information published by the U.S DOE [3]. The data of the changing demand is shown in Figure 5.2. for both hotels.

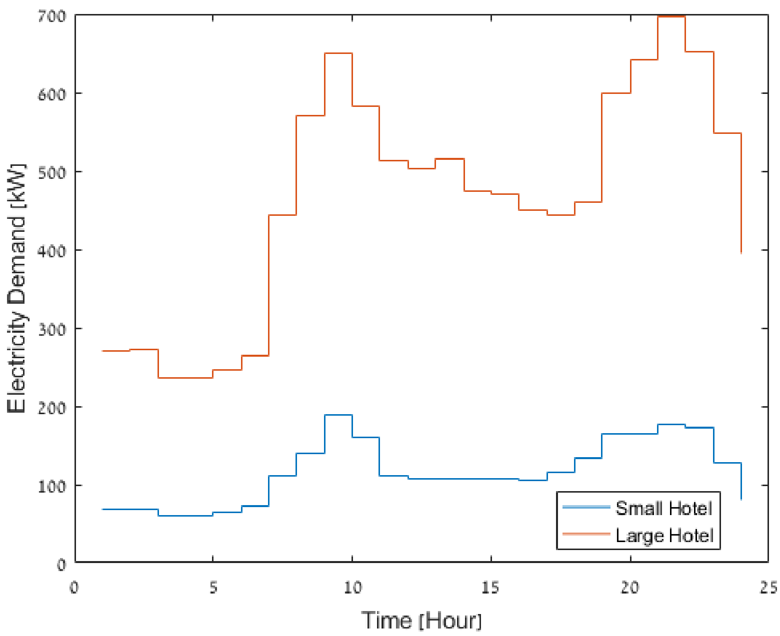


Figure 5.2: Demand over a single day (1/1/2004) for a small and large hotel [3].

MGT Model

We integrate the MGT model of the Capstone C65 as developed in Section 3.2. As the cost function of the Capstone C65 is linear it may be considered as convex or concave case for the heuristics and analytical calculations. Therefore, for the simulations in this section we embed analytical calculations for both convex and concave cost functions as shown in Theorems 4.1 and 4.2. The results of the two cases will later be compared and discussed.

Utility Costs - Electricity Tariffs

The utility is considered as an unlimited source of electrical power, nevertheless, purchasing electrical power cost money. In this work the electricity tariffs are modeled similarly to what is done in [24]. There are three cost components: fixed cost of service (FC), fixed pricing of energy (FPE) and demand charging (DC). The DC is a changing value according to different components such as peak hour pricing and seasonal pricing. In this work we consider the FPE as part of the simulation. The power purchased from the utility is $P_U(t) = P(t) - \sum_{i=1}^{N} p_i(t)$ for all $t = 1, 2, \ldots, T$, and the cost function defining the cost of electricity is

$$g(P_U(t)) = A \cdot \Delta T \cdot P_U(t). \tag{5.1}$$

Here, A is the energy charge such that for small hotel $A = 0.0273\$/kWh$, and for big hotel $A = 0.0291\$/kWh$.

5.2.2 Simulations Results

We simulate the multi-unit problem for this setting over several variations:

- MGT networks with different sizes considering to a **small** hotel demand. We consider both convex and concave heuristics.
- MGT networks with different sizes considering a **large** hotel demand. We consider both convex and concave heuristics.
- Selling-back to the utility. Considering two MGTs, a utility and a demand of a small hotel with convex heuristics.

In Table 5.2 we gathered the simulation results of the real case scenario with $N = 2, 3, 4, 5$ MGTs. We compare two elements: the simulation run-time and the total cost. We compare the elements with regards to both convex and concave heuristics. The run-time increases with N, as expected when the problem is getting bigger. The run-time performance is similar i.e., differs in up to several hundredths of a second, when comparing cases with the same number of MGTs.

In all the solutions, excluding the small hotel concave case, the total cost is in inverse relationship with the number of MGTs. This is because when considering the real costs, producing power with MGTs is cheaper than purchasing power from the utility (for the tariffs used in this example). Thus, the simulations results strengthen our motivation to incorporate MGTs into power networks. In particular, if we were to only purchase from the utility to meet small hotel demands, the total cost will be \$75.78.

Within the optimal solutions, MGTs are turned on and off according to the heuristics as extensively discussed in the preceding chapter. The total cost is impacted

# MGTs	Convex		Concave		Concave without SU&SD costs	
	Run-Time [sec]	Cost[$]	Run-Time [sec]	Cost[$]	Run-Time [sec]	Cost[$]
$N = 2$	0.46	9.67	0.39	13.88	0.38	9.67
$N = 3$	0.66	0.44	0.67	13.88	0.68	0.44
$N = 4$	1.06	0.19	1.08	13.88	1.13	0.19
$N = 5$	1.73	0.19	1.72	13.88	1.73	0.19

(a) Small hotel.

# MGTs	Convex		Concave	
	Run-Time[sec]	Cost[$]	Run-Time[sec]	Cost[$]
$N = 2$	0.43	225.53	0.39	225.53
$N = 3$	0.65	186.32	0.68	186.32
$N = 4$	1.10	147.33	1.09	147.33
$N = 5$	1.98	115.42	1.71	115.42

(b) Large hotel.

Table 5.2: Comparison of run-time, number of start-ups and shut-downs and total cost between convex and concave cases, while meeting demands.

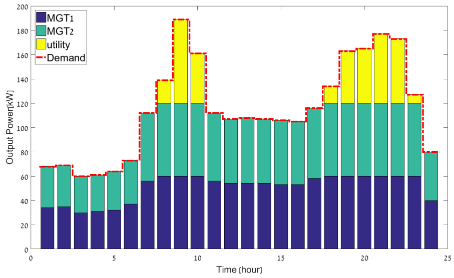
accordingly, as these operations have additional cost defined in Section 3.2.2. Moreover, since the SPA looks for the cheapest paths, it naturally avoids paths with higher costs. This is well demonstrated in Table 5.2a, where the concave case solutions are much more expensive than the convex case. In fact, due to the costs of MGTs start-up and shut-down events, the optimal solution is fixed for all N variations, where excess power procurement from the utility is preferred over additional MGTs. This is regardless of how many available MGTs are in the power network. Even-though, the optimal solution includes two operational MGTs. That is, the incorporation of the MGTs into the power network yields overall cost reduction.

To emphasize the notion of start-up and shut-down costs, we add the most right extension to Table 5.2. In the extension we bring the solutions of the concave cases without MGTs start-up and shut-down costs (i.e., the cost is zero). As expected, with linear cost functions of MGTs, the solutions are now identical to the convex case solutions with the same N.

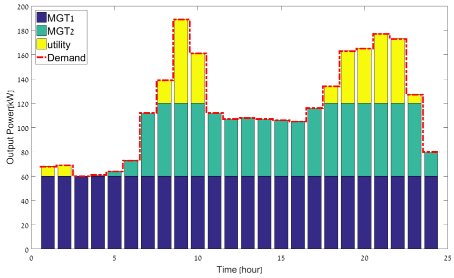
This also holds for the large hotel results, demonstrated in Table 5.2b. Furthermore, there is a dramatic difference in total costs between the small and large hotels. Since the demand from the large hotel is much higher, we have to purchase more power from the utility once the MGTs network reaches capacity. Additionally, the large hotel results are identical for convex and concave cases, therefore, based on the superiority of the convex case in the small hotel demand, one might prefer convex heuristics. However, this may not be necessarily true in general. In this real-case scenario examples we addressed one day snap shot of demands, and simulations consider MGTs that are already operational. In particular, the construction of the shortest path graph allows for the initial power generation state (at $t = 1$) to be 'ON' without incurring any associated start-up cost. We could potentially require that all generators must start in an off state, leading to a more fair comparison between the convex and concave heuristics.

The schedule and output power allocation of the generating units is visualized for each simulation result included in Table 5.2. In Figure 5.3 we present for the small hotel and in Figure 5.5 for the large hotel. Note that networks performs as anticipated according to the heuristics. In Figure 5.4 we compare the small hotel convex cases with the concave case without start-up and shut-down cost (i.e., Table 5.2a extension).

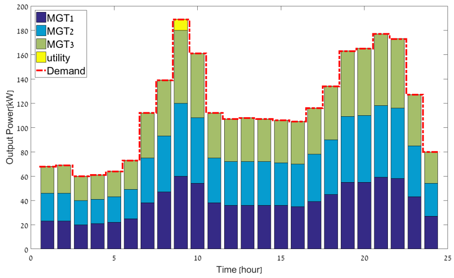
The total cost when considering two MGTs with convex heuristics, and a utility to meet small hotel demand is \$9.67. If we allow the MGTs power network to sell power back to the utility, with identical tariff as for buying power, we actually end up with a revenue of 2.63 USD. The output power allocation of the generating units is presented in Figure 5.6, next to the solution without sell-back for the conveniences of comparison. As expected, the MGTs perform at capacity throughout the planning horizon, in order to reduce the network costs as much as possible.



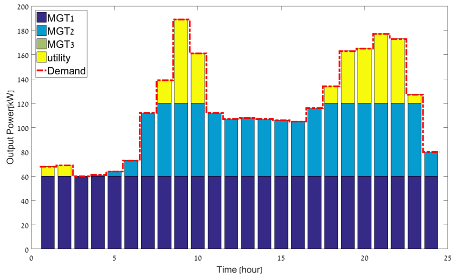
(a) Convex case with $N = 2$.



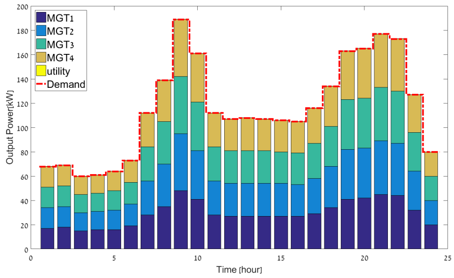
(b) Concave case with $N = 2$.



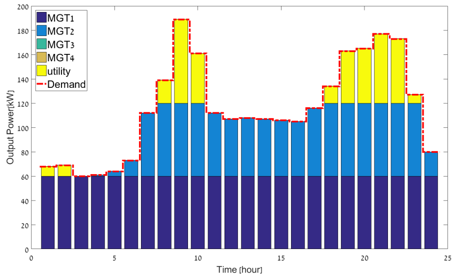
(c) Convex case with with $N = 3$.



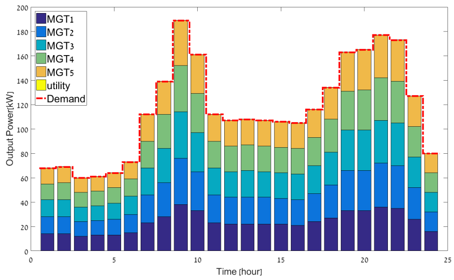
(d) Concave case with $N = 3$.



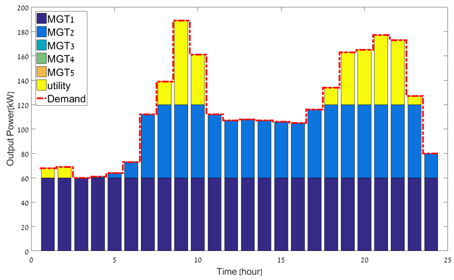
(e) Convex case with $N = 4$.



(f) Concave case with $N = 4$.

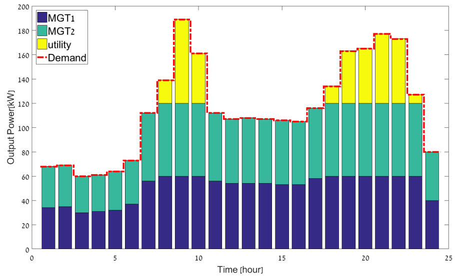


(g) Convex case with $N = 5$.

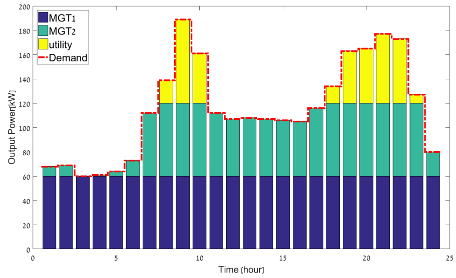


(h) Concave case with $N = 5$.

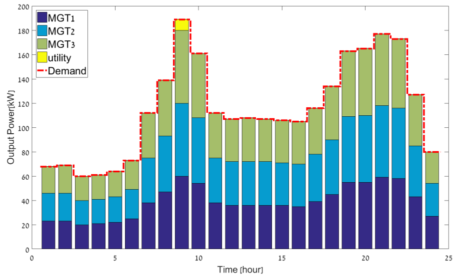
Figure 5.3: Schedule and allocation of generating units to meet small hotel demand.



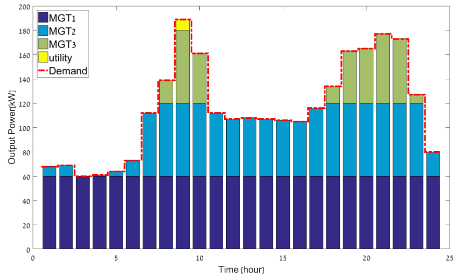
(a) Convex case with $N = 2$.



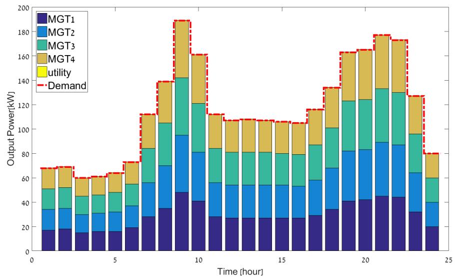
(b) Concave case w/o SU&SD costs; $N = 2$.



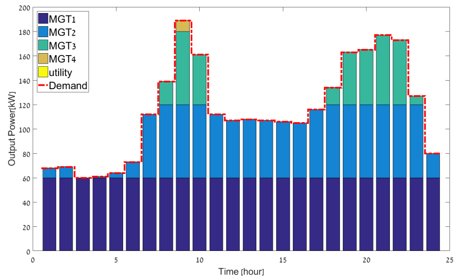
(c) Convex case with with $N = 3$.



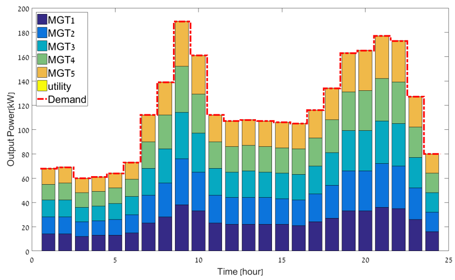
(d) Concave case w/o SU&SD costs; $N = 3$.



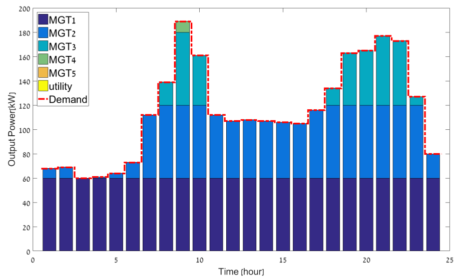
(e) Convex case with $N = 4$.



(f) Concave case w/o SU&SD costs; $N = 4$.

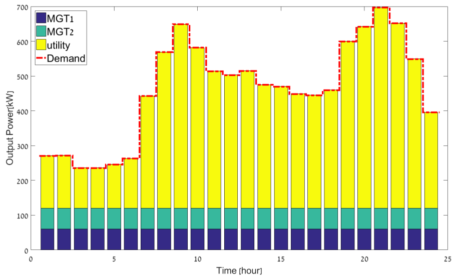


(g) Convex case with $N = 5$.

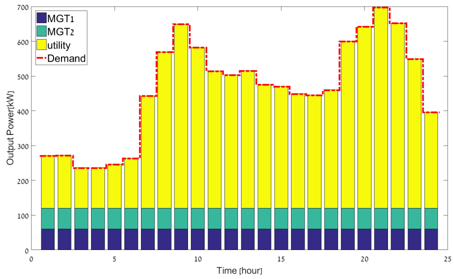


(h) Concave case w/o SU&SD costs; $N = 5$.

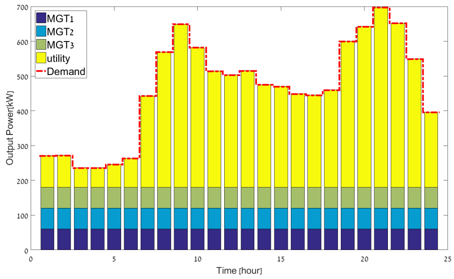
Figure 5.4: Schedule and allocation of generating units to meet small hotel demand.



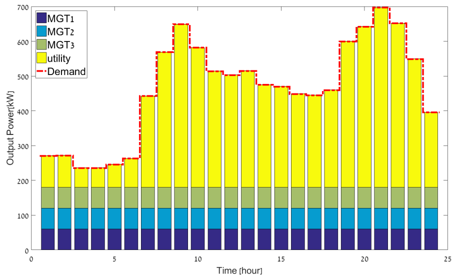
(a) Convex case with $N = 2$.



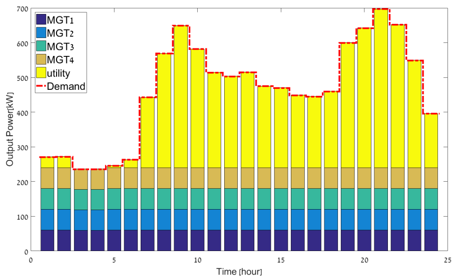
(b) Concave case with $N = 2$.



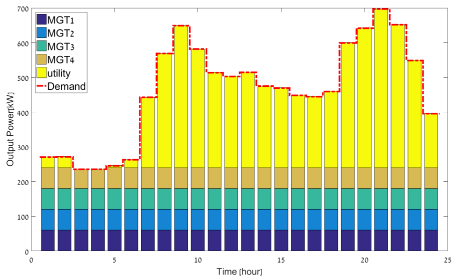
(c) Convex case with with $N = 3$.



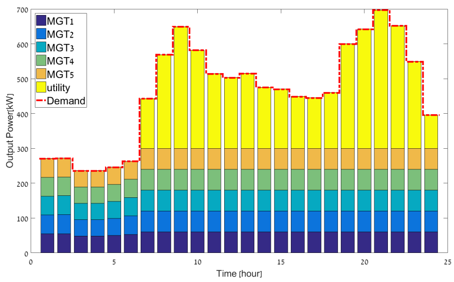
(d) Concave case with $N = 3$.



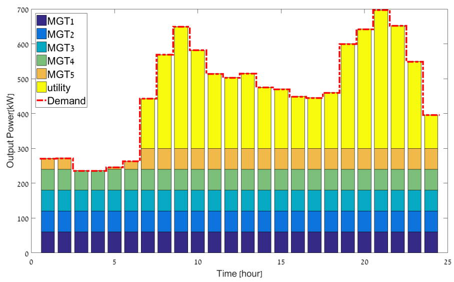
(e) Convex case with $N = 4$.



(f) Concave case with $N = 4$.

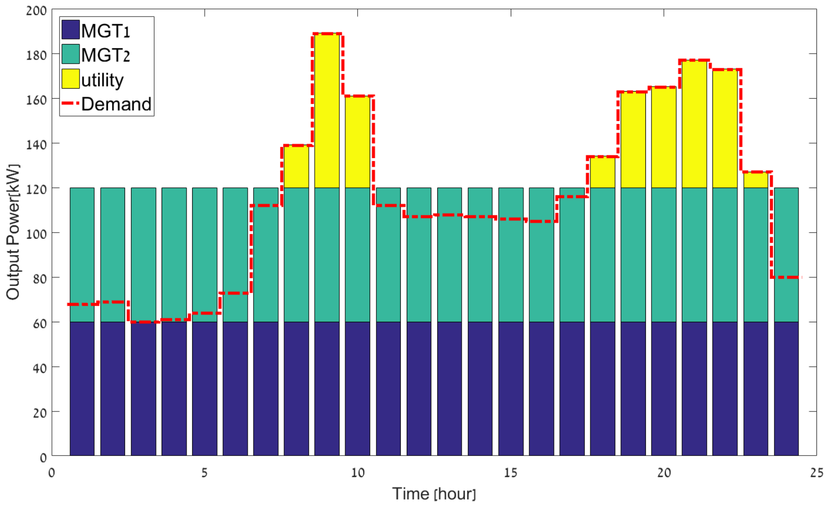


(g) Convex case with $N = 5$.

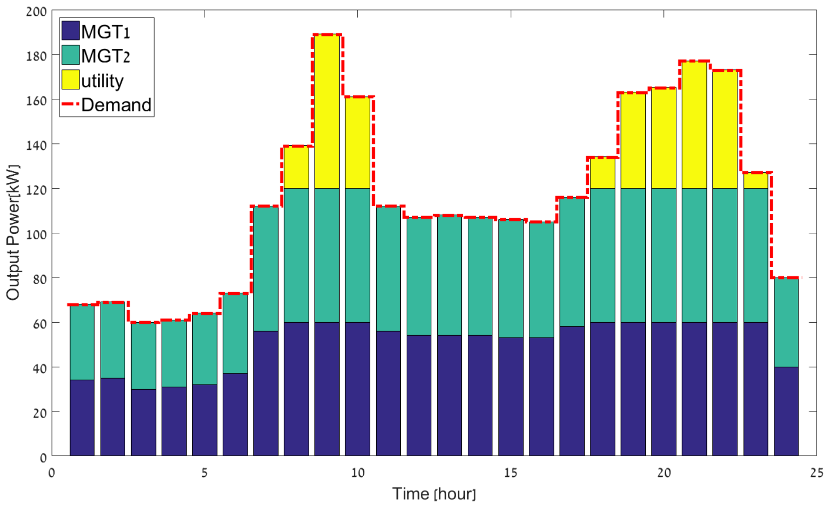


(h) Concave case with $N = 5$.

Figure 5.5: Schedule and allocation of generating units to meet large hotel demand.



(a) With sell-back.



(b) Without sell-back.

Figure 5.6: Output power allocation of two MGTs and a utility to meet small hotel demand.

Chapter 6

Conclusion and Open Questions

In this work we developed a novel approach for solving the multi-unit ED problem efficiently. We began in Chapter 1 with motivation for producing power with MGTs that consumes natural gas as fuel, and a literature review. The literature review briefed from the challenges and solution dealt by academia and industry from the early years of working on ED and UC problems, up-to recent date methods and techniques. Specific references on ED problems and solution methods with MGTs are also included.

We then followed with background for the ED and UC optimization problems, including definitions and mathematical formulations in Chapter 2. In this chapter we also discussed some solution methods for the ED optimization problem, and their associated challenges in terms of computational complexity.

In Chapter 3 we lay more specific foundations to our work, presenting with adequate definitions and expressions, formulating the optimization model for the ED single unit problem. We showed how to model a real MGT for integration within the optimization model. The modeling resulted with a discrete-state model for MGT dynamics and costs. We also showed how the single-unit problem can be solved efficiently with the construction of appropriately designed graph model (i.e. DAG) and employing the SPA.

Chapter 4 started with the definition for the multi-unit ED problem. Once formalizing the problem, we discussed about the problem size and complexity, having also a comparison to the single-unit problem. Next, we presented the main part of our work. The decomposition method was presented and developed based on the introduction of an auxiliary variable. The multi-unit problem was decomposed into two independent problems - the inner problem and the outer problem. The inner problem is an optimization problem that yields the optimal allocation of output power from the MGTs. That is, for a given value of auxiliary variable, the inner problem returns an optimal solution that contains the output power of each MGT. From which we also get the cost associated to the specific value of auxiliary variable. We specifically proved that the inner problem can be solved analytically for convex and concave cost functions. This enlightenment dramatically reduces the problem complexity. The outer problem solves a single unit problem, while considering the MGTs power network as one aggregated generator. To do so, we modeled that fictive aggregated generator based on the inner problem results. The schedule generated by the outer problem is translated to schedules for each MGT according to the heuristics developed in Section 4.3.3. By solving the decomposed optimization problem, we actually reduced the overall size of the original ED multi-unit problem. We concluded this chapter with a discussion about the benefits of the decomposition method, with regards to efficient solution and superior computational run-time.

We backed our work findings with numerical simulations at Chapter 5. By comparing simulation results of the decomposition method with other method performed by a commercial solver, it becomes clear how much our new approach out-performs the contestant when considering run-time. In fact, in some cases the simulations of the other method did not converge to a solution within reasonable time. We then set up a real case scenario including real-life characteristics for problem elements such as demand, utility and MGTs. This was a concluding simulation in which all points of this work came together, and the insights that stem throughout the work could be clearly observed from the overall simulation results. Furthermore, the simulation results demonstrates the economic benefits of integrating MGTs into a distributed power network.

This new approach, being so efficient, has the potential to greatly reduce complications and turmoils when planning power supply network. Future work in this direction includes exploring new real case scenarios with different types of generating units. A CHP environment also needs to be dealt with, as MGTs are capable to also meet heat demands. The integration of MGTs into the power grid has the potential for great monetary savings (or even profit), therefore we propose to perform a full economic analysis including aspects of return on investments (ROI). Based on the new method presented in this work, a full experiment with several MGTs in an actual power network is also suggested. Finally, further comparison and development with real-time algorithms is needed, to further strengthen the utilization of the decomposition method when considering a multi-unit problems.

Appendix A

Chordal Slope Lemma

When dealing with convex functions one of the classical insights is the relation between the function and the chords that connect points on the function. This relation is well defined with the *chordal slope lemma.*

Lemma A.0.1 (Chordal Slope Lemma)**.** *Let $a_1 \leq a_2$ and $b_1 \leq b_2$ be real numbers, and assume $a_1 < b_1, a_2 < b_2$.*

i) If the map $f : [a_1, b_2] \to \mathbb{R}$ is convex, then

$$\frac{f(b_1) - f(a_1)}{b_1 - a_1} \leq \frac{f(b_2) - f(a_2)}{b_2 - a_2}.$$

ii) If the map $f : [a_1, b_2] \to \mathbb{R}$ is concave, then

$$\frac{f(b_1) - f(a_1)}{b_1 - a_1} \geq \frac{f(b_2) - f(a_2)}{b_2 - a_2}.$$

Furthermore, if f is strictly convex (concave), then the inequalities above are strict, unless $a_1 = a_2$ and $b_1 = b_2$.

Lemma A.0.1 is given here without a proof which can be obtained by utilizing *Jensen's Inequality* [56]. A graphical illustration for the convex case is provided in Figure A.1.

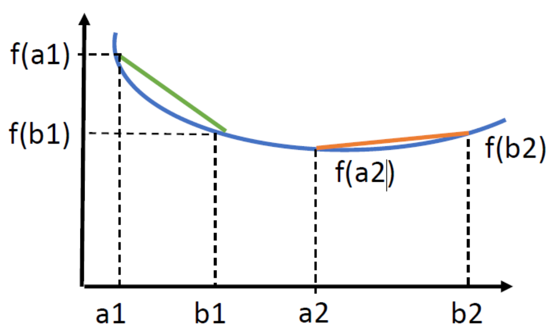


Figure A.1: Illustration of the chordal slope lemma for a convex function.

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שלכל דרישה פיזיבילית מוצמדת העלות המינימאלית המתאימה למענה הספציפי של רשת הטורבינות. יתרה מזאת, אנו מראים כיצד עבור פונקציות עלות מסוימות ניתן לפתור את הבעיה אנליטית באופן מידי.

תת-הבעיה השנייה קובעת מה ההספק המיטבי הנדרש מרשת המיקרו-טורבינות, כאשר כעת ניתן להתייחס לרשת זו כאל ספק (גנרטור) בודד. כלומר, ספק בודד שחלק ממאפייניו נקבעים על סמך הגנרטורים שמרכיבים אותו, וחלקם מתקבלים מפתרון תת-הבעיה הראשונה. בשיטה זו, הבעיה מצטמצמת לבעיית ED סטנדרטית, ללא ריבוי משתתפים, ואנו מציגים פתרון אופטימאלי בעזרת אלגוריתם ידוע למציאת המסלול הקצר ביותר בבעיות אופטימיזציה (ידוע גם כאלגוריתם דייקסטרה). על מנת שנוכל להשתמש באלגוריתם זה, נבנה מודל גרפי מתאים שמייצג את בעיית ה-ED. המודל הגרפי מכיל הן את ההיבטים הדינאמיים, והן את ההיבטים הכלכליים של הבעיה. הפתרון האופטימאלי מתורגם להפעלה אופטימאלית של המיקרו-טורבינות ברשת בכפוף להחלטות יוריסטיות, שאופן קביעתן פותח גם הוא במסגרת עבודה זו.

בנוסף, במסגרת העבודה אנו מתווים דרך לבניית מודל מצב דינאמי ומודל כלכלי של מיקרו-טורבינת גז, וממממשים את המתווה עבור מיקרו-טורבינה מסוג ספציפי. מודל המיקרו-טורבינה נבנה על סמך מדידות שבוצעו במעבדה לטורבו-מכונות ומעבר חום בטכניון.

הגישה החדשנית מקטינה באופן משמעותי את זמן הריצה הנדרש לחישובים כדי לפתור את בעיית האופטימיזציה המורכבת. אנו תומכים בתוצאות המוצגות בעבודה זו עם פיתוחים מתמטיים וסימולציות.

תקציר

בשעה שבתעשייה, בעסקים, ובחיי היומיום, אנו מסתמכים יותר ויותר על טכנולוגיות חדשות ומוצרי חשמל ואלקטרוניקה חדישים, דיווחים שונים ומאמרים כלכליים מציגים גדילה משמעותית בדרישה לאנרגיה חשמלית. כתוצאה מכך, עולה גם הצורך בספקי אנרגיה נוספים, בדרך כלל קומפקטיים, מפוזרים גיאוגרפית, שמחוברים לרשת החשמל באופן "חכם". לאור זאת, השיטה הריכוזית המקובלת להפצת חשמל משנה פניה לעבר גישה מפוזרת ומבוזרת יותר. בהתאם להתפתחות זו, נוצרים אתגרים טכנולוגיים חדשים במטרה לחבר את כל ספקי החשמל לרשת, ולנהל אותם באופן מיטבי ויעיל בהיבט כמות החשמל שהם מספקים, תזמון ההפעלה שלהם ובראיית אילוצים כלכליים שיש להתחשב בהם.

ניהול מיטבי של ספקים ברשת החשמל מתורגם באופן טבעי לבעיית אופטימיזציה הידועה כבעיית ההפצה החסכונית (ED) . באופן כללי את בעיית הED יש לפתור בהינתן דרישה לאנרגיה (הספק חשמלי, חום וכו'). דרישה זאת הינה פונקציה של זמן ומטרת הפתרון לבעיה היא לענות על הדרישה הזו כך שהעלויות יהיו מינימליות (כספיות, זיהום אוויר וכו'). את המענה יש לתת בכפוף לאוסף האילוצים שמתאר את מאפייני הבעיה כגון: מאפיינים אלקטרו-מכאניים של ספקים וההספק המקסימלי שניתן להפיק מהם, מגבלות והפסדים על קווי אספקה וכיוצא בזה. לבעיית הED נבנה מודל מתמטי מתאים, והעלויות עצמן מבוטאות כפונקציית עלות שנדרש למזער את ערכה כתלות במשתני המודל.

בעבודה זו אנו מציעים גישה חדשנית לפתרון בעיית ED עם מספר משתתפים. באופן ספציפי עבור רשת חשמל שמורכבת ממספר מיקרו-טורבינות גז וספק חשמל מרכזי בודד (כחברת חשמל), כאשר מיקרו-טורבינת גז אחת, בהגדרתה, מסוגלת לספק עצמאית עד כ-500 קילוואט. בעיית ED במקרה זה נבנית כבעיית אופטימיזציה לא לינארית אשר מכילה גם משתנים בדידים. מודל האופטימיזציה שנבנה עבור הבעיה מכיל בתוכו את האילוצים האופייניים לבעיה, כגון מגבלות על הספק המרכזי הבודד, והמאפיינים השונים של מיקרו-טורבינות. מאפייני המיקרו-טורבינות מיוצגים בעזרת מודל מצב דינאמי מתאים (בדיד).

בעיות מסוג זה ידועות במורכבותן הרבה ובקושי הכרוך במציאת פתרון עבורן. אנו מציגים פתרון חדשני שהינו יעיל נומרית (חישובית), בכך שאנו מפרקים את בעיית הED לשתי תת-בעיות אופטימיזציה עצמאיות. פירוק הבעיה המקורית מתאפשר בזכות הגדרת משתנה ייעודי חדש.

תת-הבעיה הראשונה מאפשרת פתרון אנליטי לאופן האלוקציה (הקצאה) המיטבי של הספקי היציאה, בהתחשב בכל אחת מהמיקרו- טורבינה שברשת. כלומר, בהינתן דרישה פיזיבילית (שמקיימת את תנאי הבעיה) של הספק חשמלי לתת-הבעיה, הפתרון האופטימאלי יגדיר את הספק היציאה של כל מיקרו-טורבינה. באופן זה מתקבל פתרון אופטימאלי, שמייצג הספק יציאה אחוד מרשת המיקרו-טורבינות כמענה לכל דרישה שמשתנה בזמן. הפתרונות האופטימאליים נשמרים כטבלת ערכים, כך

המחקר בוצע בהנחייתם של פרופסור דניאל זלזו ופרופסור בני צ'וקורל, בפקולטה להנדסת אווירונאוטיקה וחלל.

תודות

ברצוני להודות מקרב הלב למנחה שלי, פרופסור דן זלזו, על הלימוד וההנחיה לאורך כל השלבים השונים במהלך עבודת התזה. זו הייתה חוויה לימודית משמעותית עבורי, ולך יש חלק מרכזי בה. בנוסף, תודה רבה למנחה שלי פרופסור בני צ'וקורל, על תמיכתו ותרומתו לעבודה. לדוקטור מיאל שרף, עזרתך לא תסולא בפז, תודה על הרעיונות החכמים ועצותיך המועילות. תודה למיכאל פלמן (סטודנט לדוקטורט) על הסיוע והמידע המועיל. תודה מעומק ליבי ליהודית גרינברג היקרה, בלעדייך היה הרבה יותר קשה להשלים עבודה זו. תודה לדוקטור אלכסיי דיסקין על סקירת העבודה וההערות בשלבי הכתיבה הסופיים.

תודה מעומק הלב למשפחתי על התמיכה, העידוד והנתינה לאורך הדרך. הוריי היקרים, עדנה ומנחם, אחיי יובל ועומרי פלג, חמותי וחמי, מזל ויאיר חייק, וגיסיי, שרון ורואי מלינגר, משה, הדר וישי חייק.

לסיכום, תודתי ואהבתי העמוקה לאשתי הילה. הילה, את שותפתי היקרה לדרך ולא היה ניתן להשלים את העבודה הזו ללא תמיכתך וטיפולך המסור בילדינו נגה, אסף ויונתן.

הפצה חסכונית (ED) לרשת של מערכות מיקרו טורבינות גז

חיבור על מחקר

לשם מילוי חלקי של הדרישות לקבלת התואר
מגיסטר למדעים

נתנאל פלג

הוגש לסנט הטכניון — מכון טכנולוגי לישראל
שבט התשפ"ב חיפה ינואר 2022

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נתנאל פלג