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Rigidity Theory for Multi-Robot Coordination: Architectural Needs and Implementation Challenges

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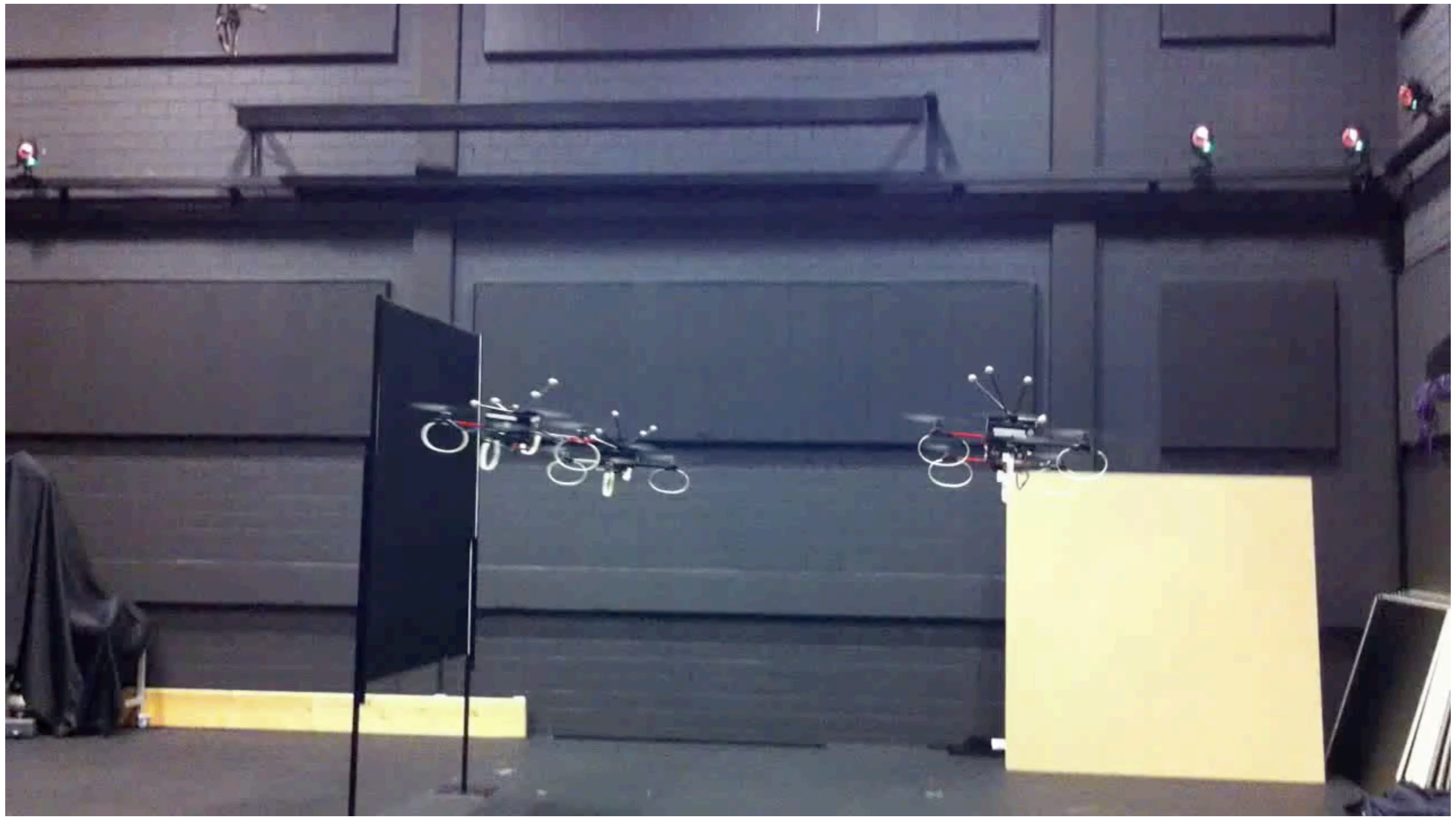
IROS 2014 - Workshop on Taxonomies of Interconnected Systems: Topology in Distributed Robotics

Challenges in Multi-Robot Systems



Solutions to coordination problems in multi-robot systems are *highly* dependent on the sensing and communication mediums available!

selection criteria depends on mission requirements, cost, environment…

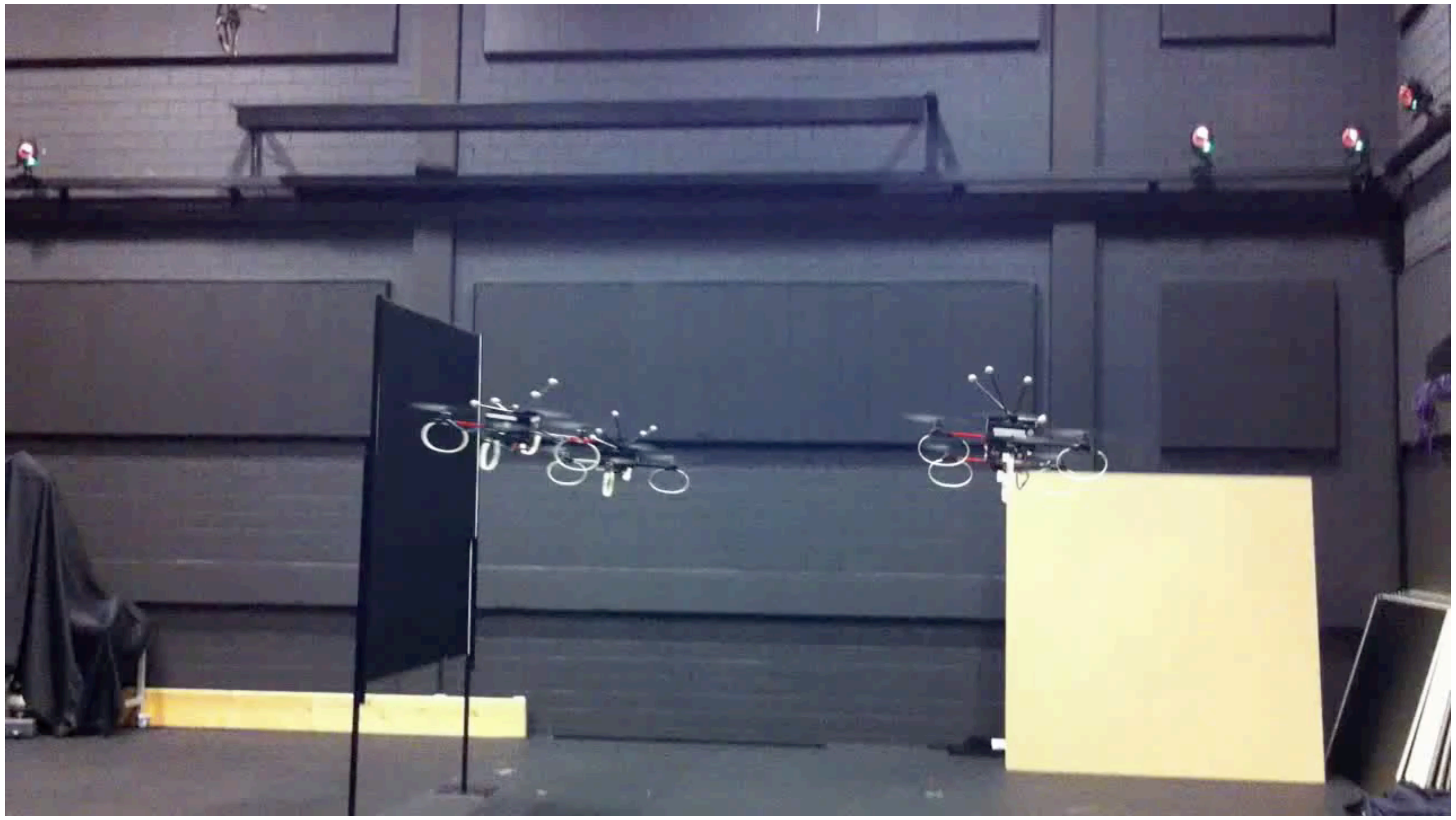
<u>Sensing</u>

- GPS
- Relative Position Sensing
- Range Sensing
- Bearing Sensing

<u>Communication</u>

- Internet
- Radio
- Sonar
- MANet

Challenges in Multi-Robot Systems



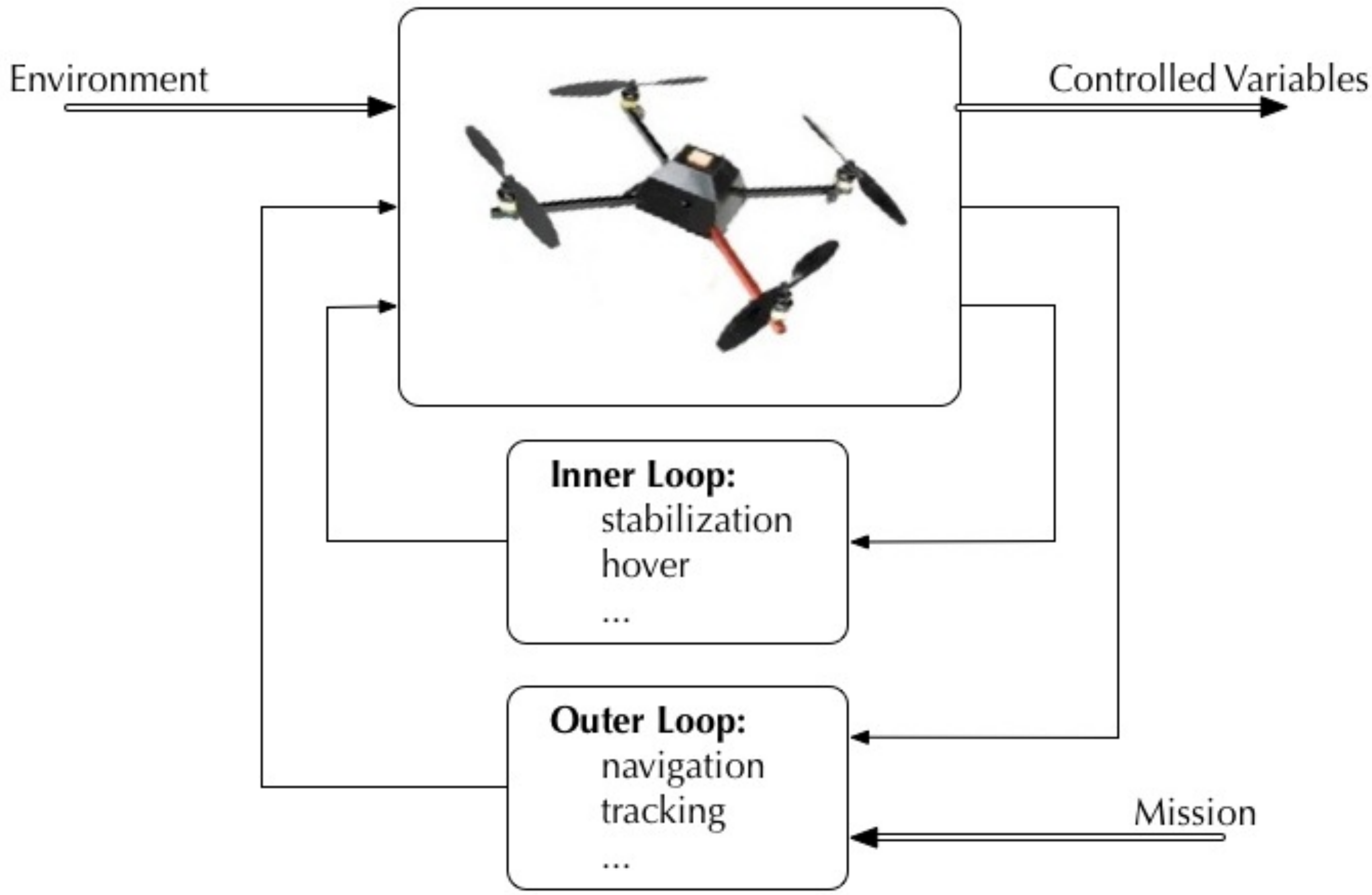
Solutions to coordination problems in multi-robot systems are *highly* dependent on the sensing and communication mediums available!

selection criteria depends on mission requirements, cost, environment…

Are there *architectural features* of a multi-agent system that are independent of any particular mission or hardware capabilities?

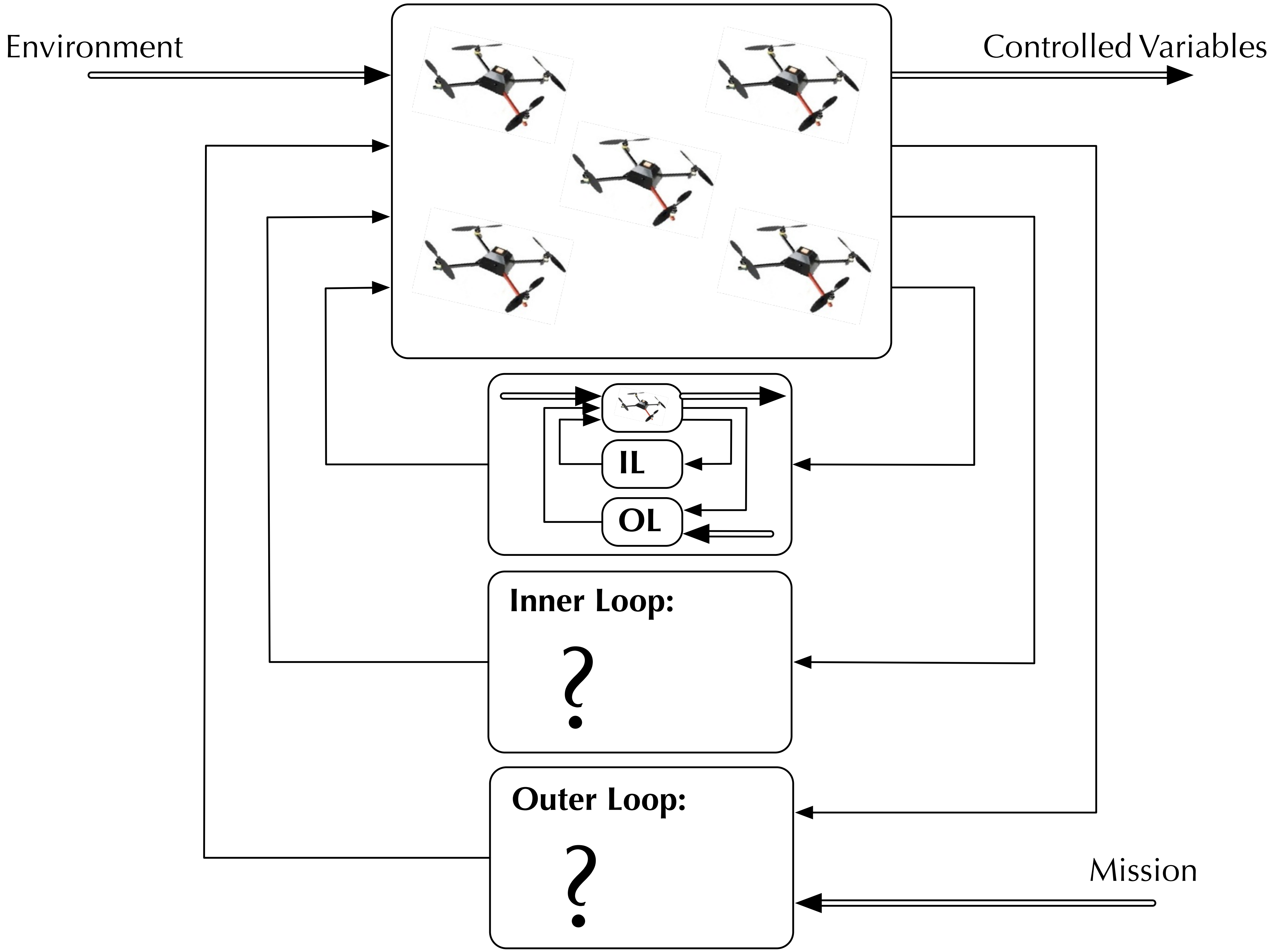
Towards a Multi-Robot Control Architecture

control architecture for a *single* quadrotor



Towards a Multi-Robot Control Architecture

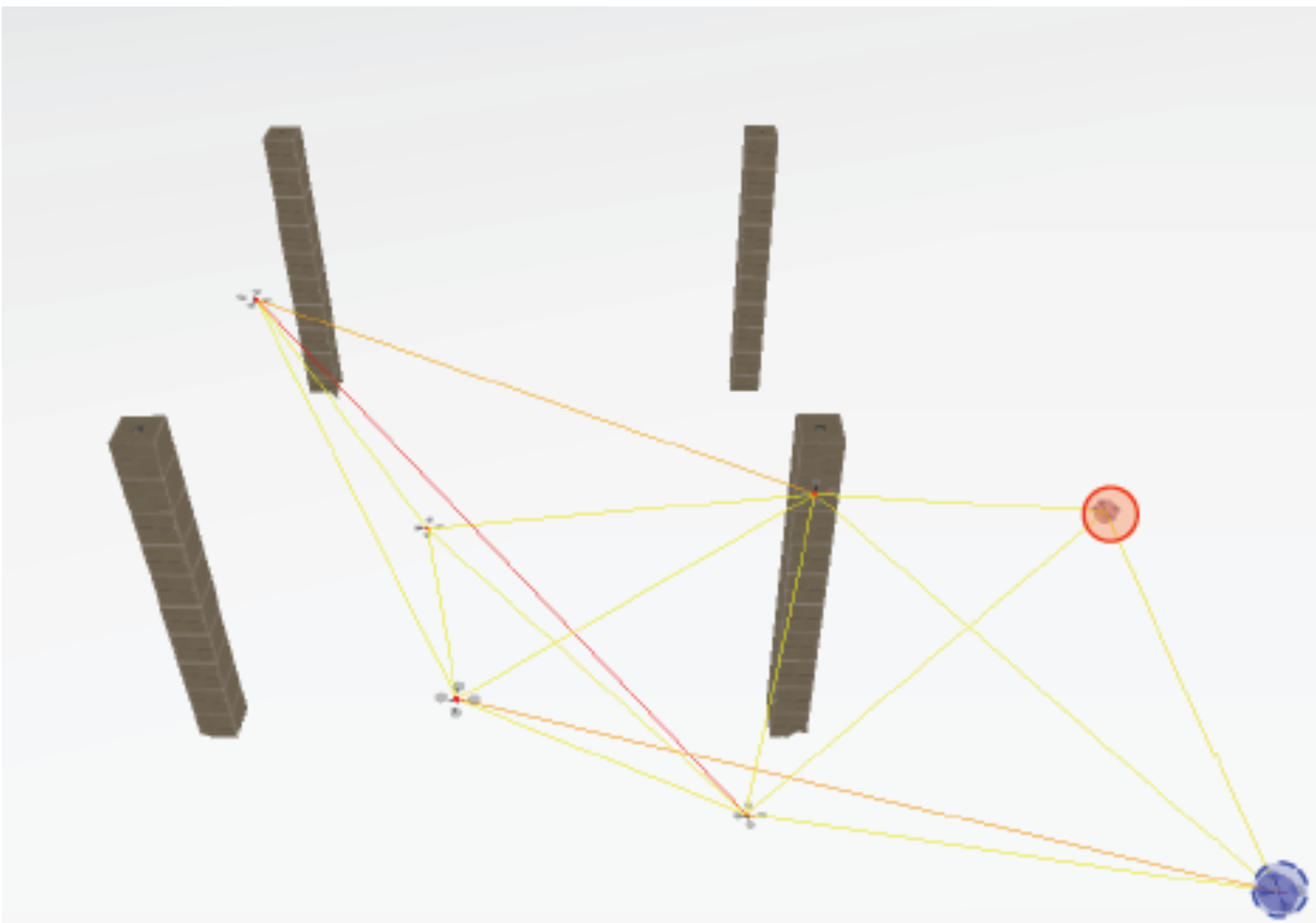
what is the architecture for a *multi-robot* system?



Towards a Multi-Robot Control Architecture

what is the architecture for a *multi-robot* system?

Connectivity

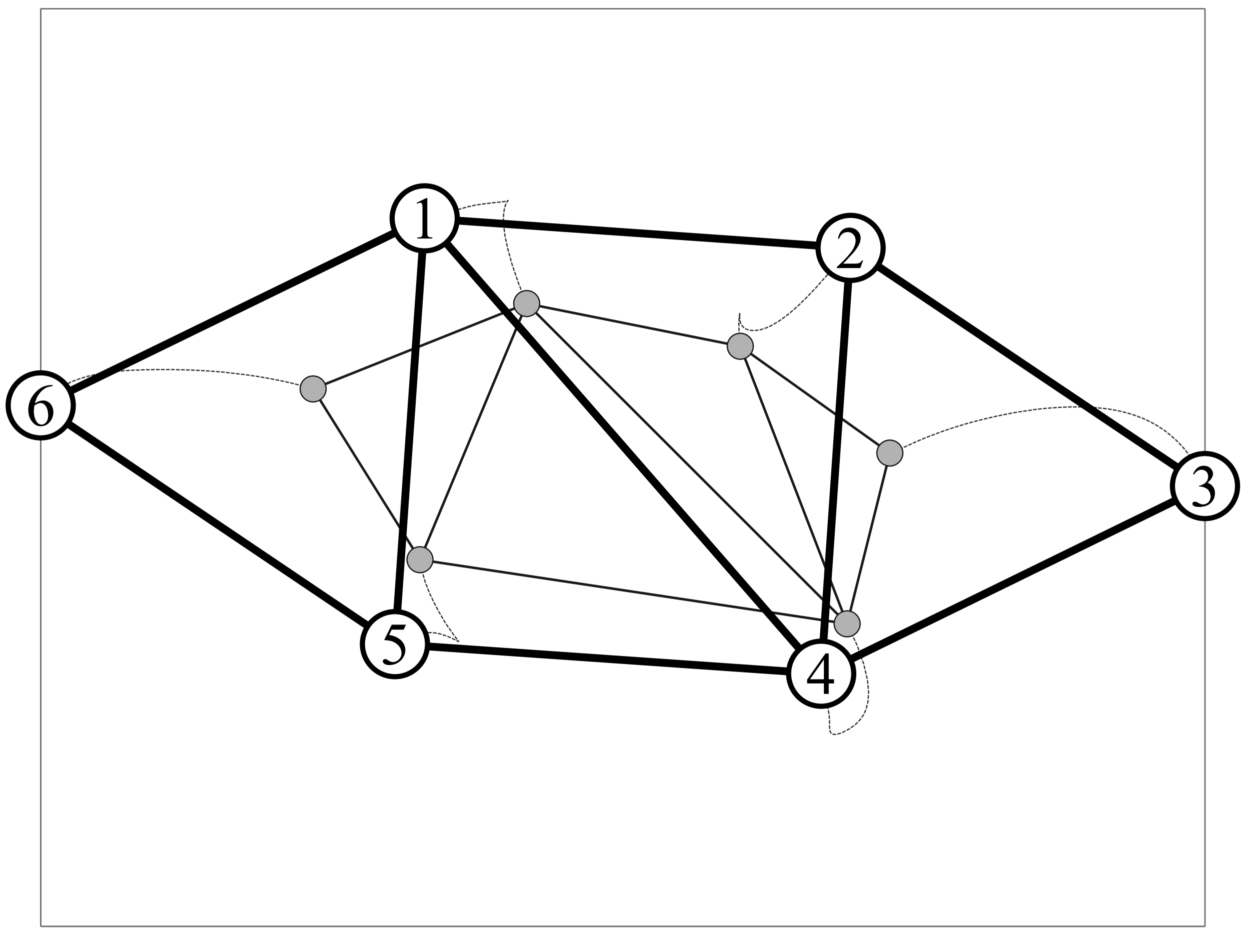


Ji and Egerstedt, 2007
Dimarogonas and Kyriakopoulos, 2008
Yang *et al.*, 2010
Robuffo Giordano *et al.*, 2013

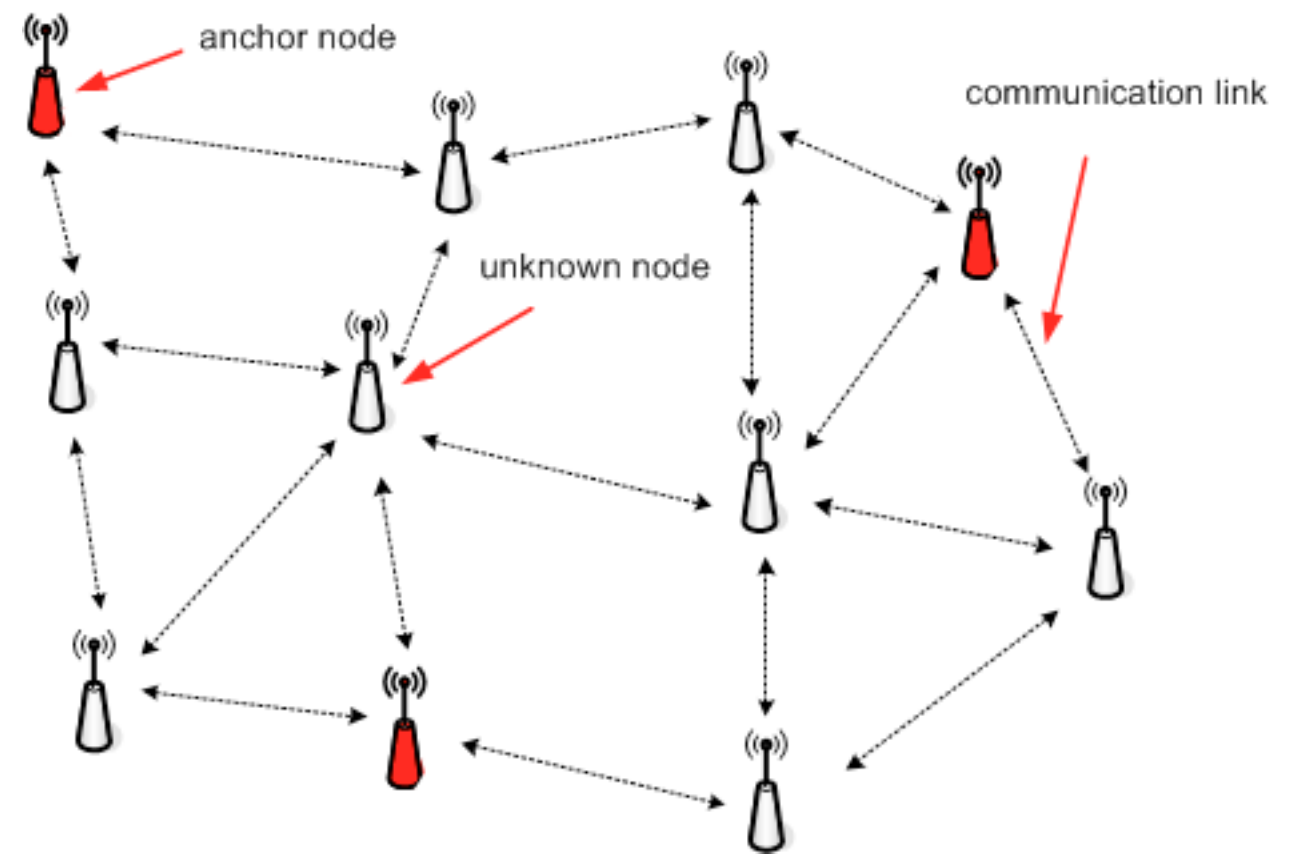
Towards a Multi-Robot Control Architecture

is connectivity sufficient for higher-level objectives?

formation control



localization

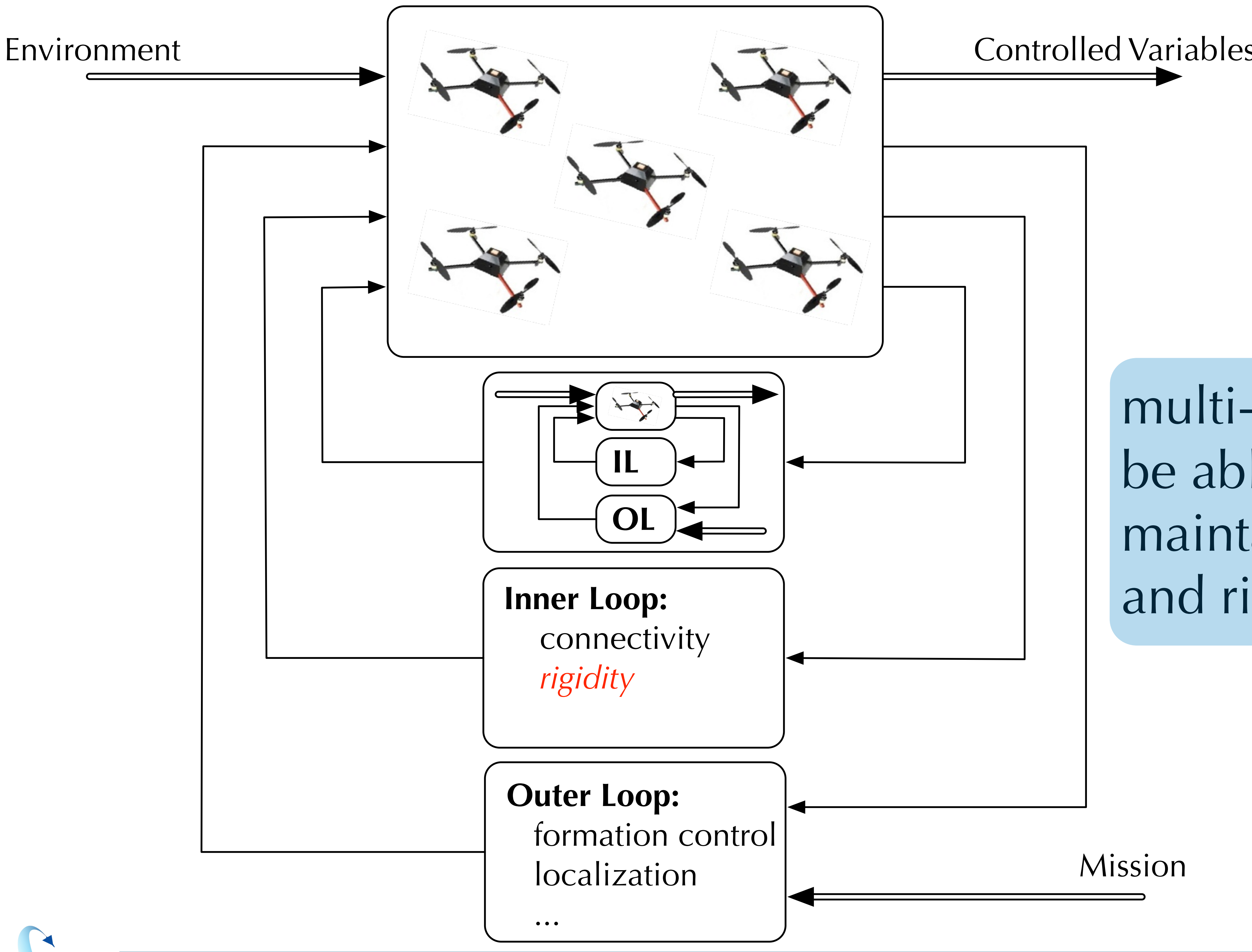


http://www.commsys.isy.liu.se/en/research

Rigidity Theory provides the correct framework to address many multi-agent mission objectives

Towards a Multi-Robot Control Architecture

what is the architecture for a *multi-robot* system?



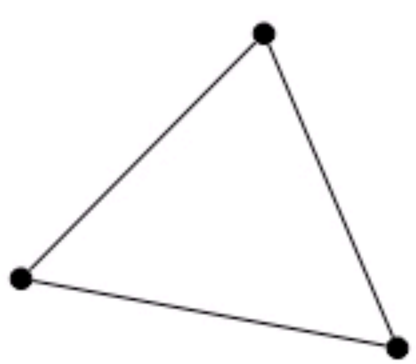
multi-robot systems must be able to *dynamically* maintain the connectivity and rigidity of the team

Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

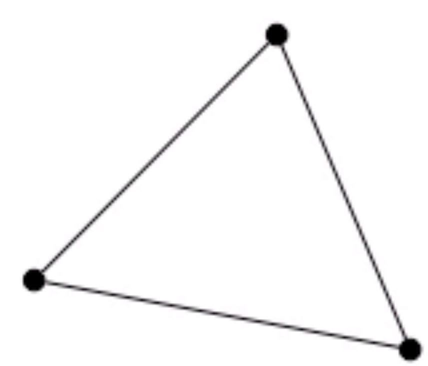
Distance Rigidity

- maintain distance pairs
- rigid body rotations and translations



Parallel Rigidity

- maintain angles (shape)
- rigid body translations and dilations

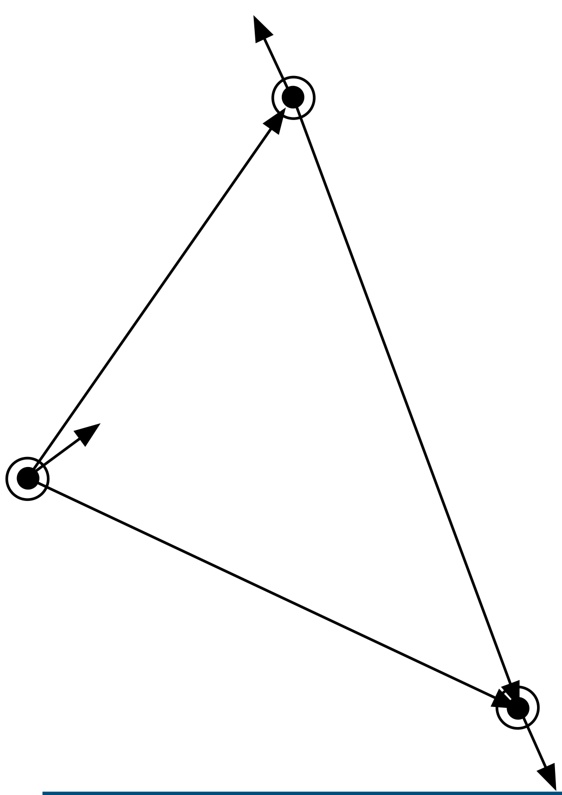


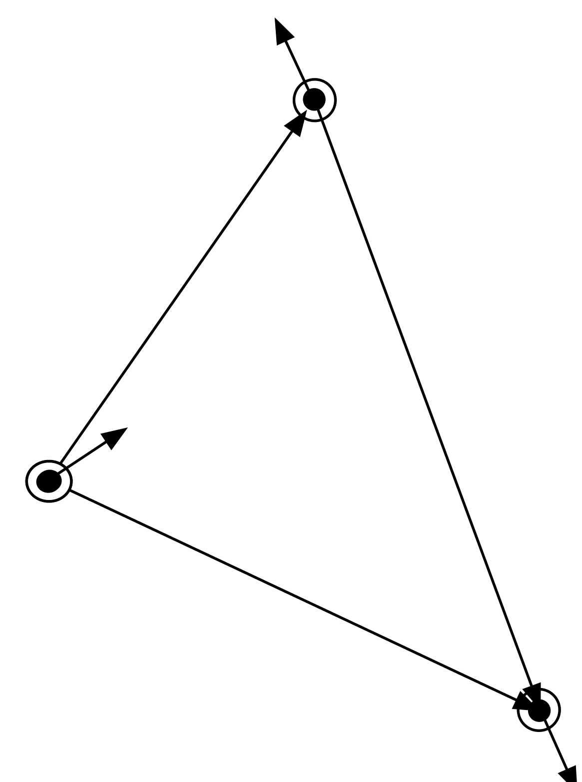
Infinitesimal Motions in SE(2)

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

SE(2) Rigidity

- maintain bearings in *local* frame
- rigid body rotations and translations + coordinated rotations





Rigidity Theory

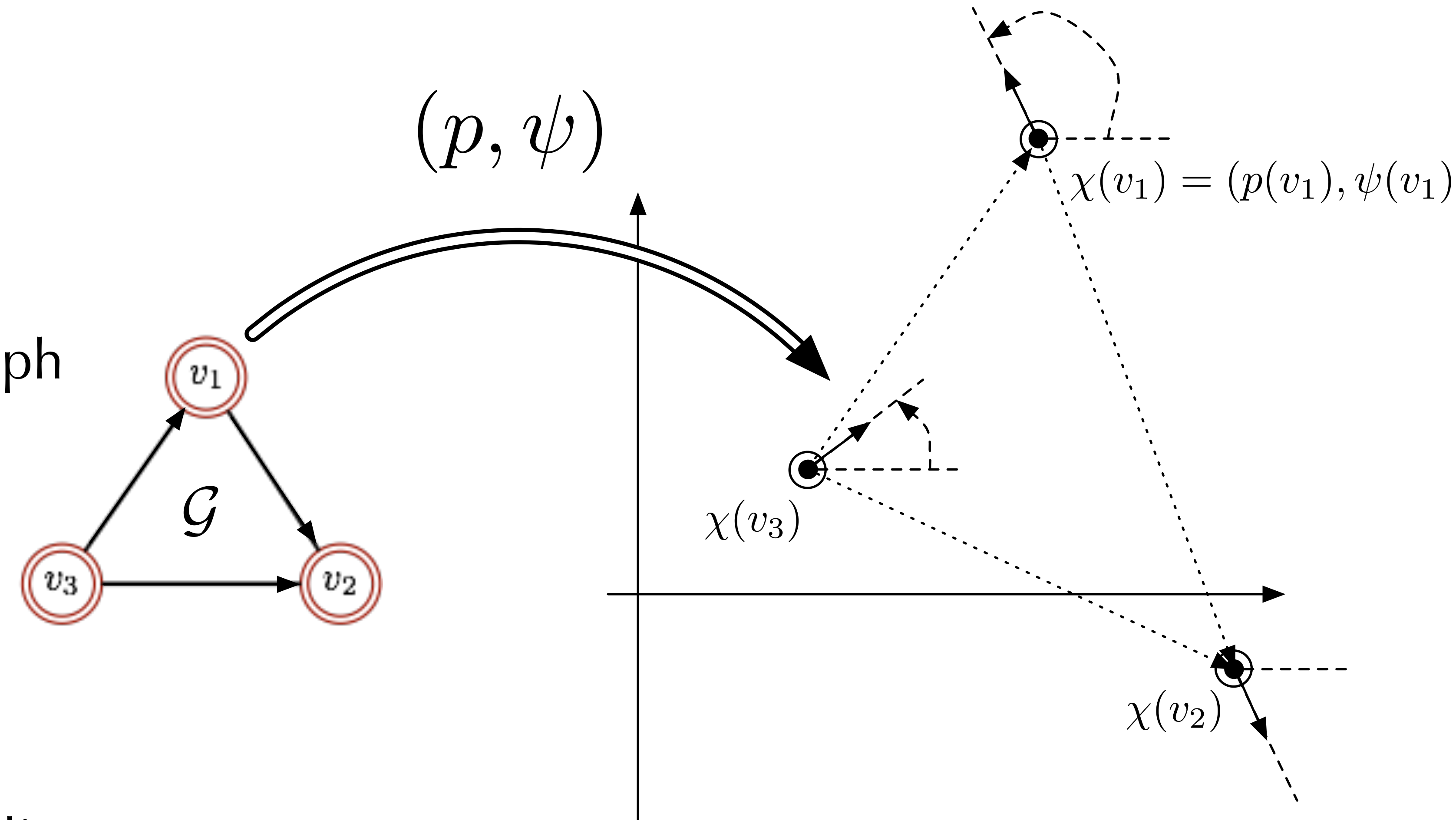
bar-and-joint frameworks in SE(2)

$$(\mathcal{G}, p, \psi)$$

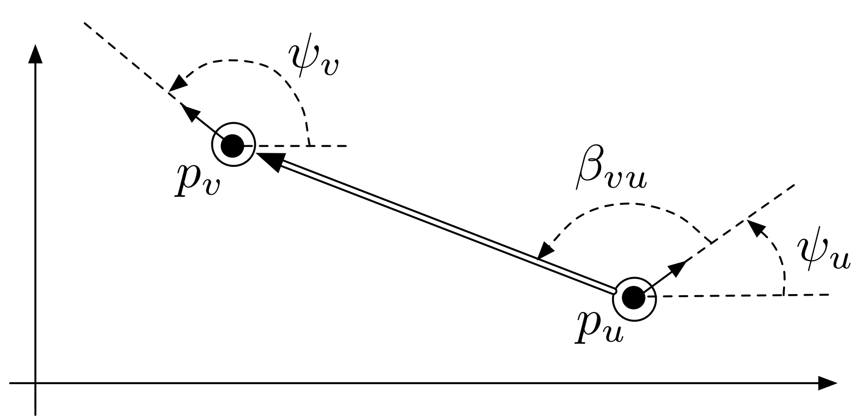
$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ a *directed* graph

$p : \mathcal{V} \to \mathbb{R}^2$

$\psi : \mathcal{V} \to \mathcal{S}^1$



a directed edge indicates availability of relative bearing measurement



stacked vector of entire framework

$$\chi_p = p(\mathcal{V}) \in \mathbb{R}^{2|\mathcal{V}|}$$

$$\chi_\psi = \psi(\mathcal{V}) \in \mathcal{S}^{1^{|\mathcal{V}|}}$$

Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility of structures formed by rigid bodies connected by flexible linkages or hinges.

Distance Rigidity

Rigidity Matrix

$$R(p)\xi = 0$$

Parallel Rigidity

Parallel Rigidity Matrix

$$R_{||}(p)\xi = 0$$

SE(2) Rigidity

SE(2) Rigidity Matrix

$$\underbrace{\left[\begin{array}{cc} D_{\mathcal{G}}^{-1}(\chi_p)R_{||}(\chi_p) & \overline{E}(\mathcal{G}) \end{array}\right]}_{\mathcal{B}_{\mathcal{G}}(\chi(\mathcal{V}))}\zeta = 0$$

Theorem

A framework is infinitesimally (distance, parallel) rigid if and only if the rank of the rigidity matrix is $2|\mathcal{V}| - 3$

A framework is SE(2) infinitesimally rigid if and only if the rank of the rigidity matrix is $3|\mathcal{V}| - 4$

Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility of structures formed by rigid bodies connected by flexible linkages or hinges.

Distance Rigidity

distance formation control

$$\dot{p}_i = \sum_{j\sim i} \left(\|p_i - p_j\|^2 - d_{ij}^2\right)(p_j - p_i)$$

- control requires distances and relative positions
- distance-only control requires estimation of relative positions

[Krick2007, Anderson2008, Dimarogonas2008, Dörfler2010]

Parallel Rigidity

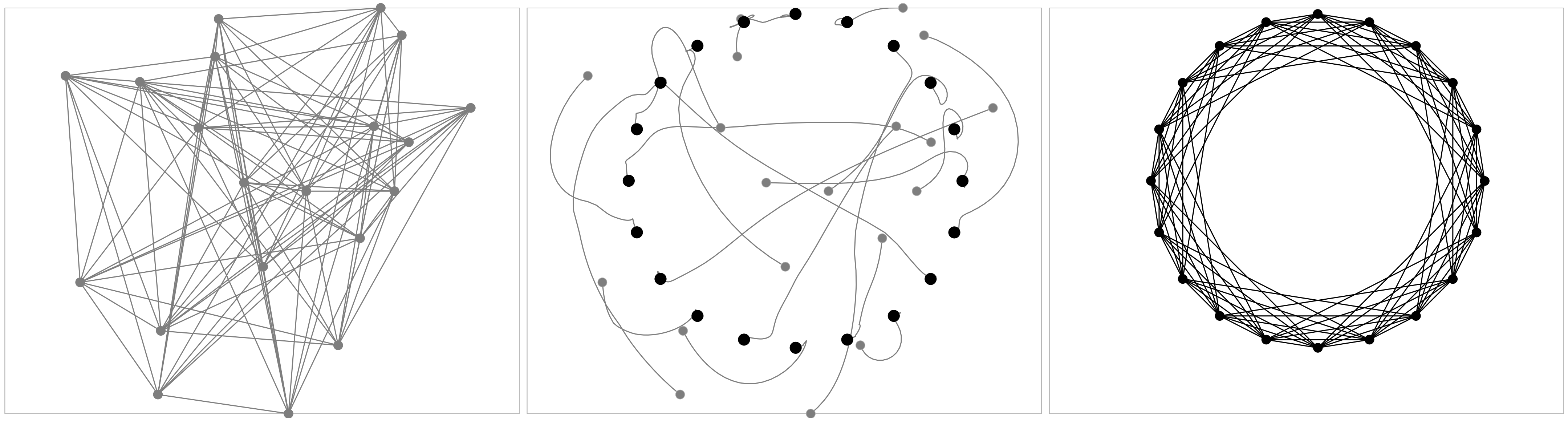
bearing formation control

$$\dot{p}_i = -\sum_{j\sim i} \frac{1}{\|p_i - p_j\|}\left(I_2 - \frac{(p_j - p_i)(p_j - p_i)^T}{\|p_i - p_j\|^2}\right) g_{ij}^*$$

- control requires bearings and distances
- bearing-only control modification (almost global stability)

[Zhao and Zelazo, TAC2014 (submitted)]

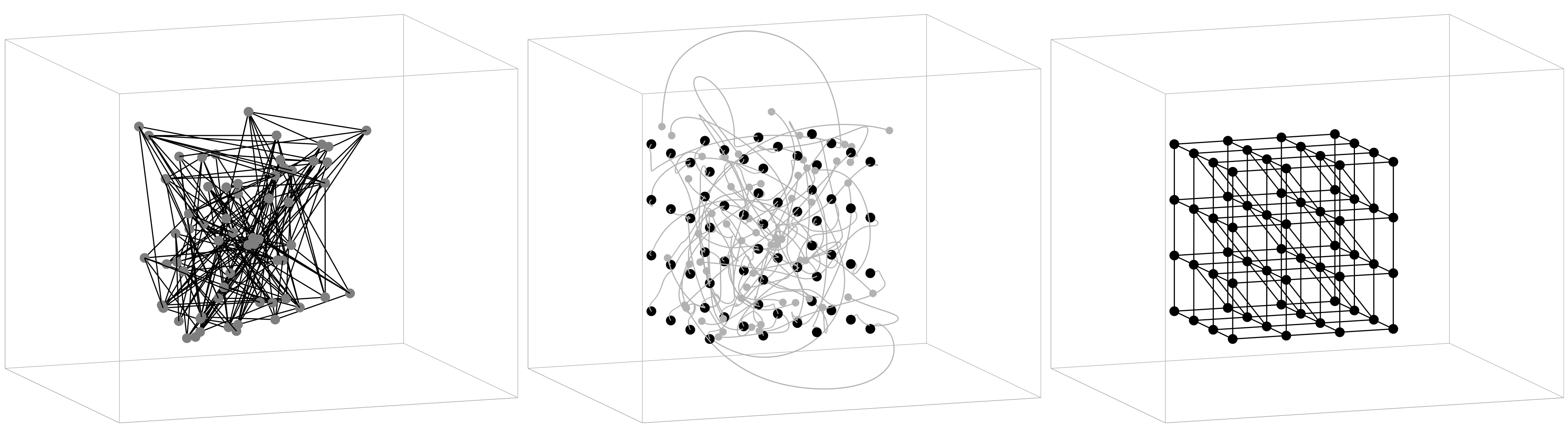
Formation Control: Bearing-Constrained Formations



(a) Randomly generated initial formation

(b) Agent trajectory

(c) Final formation



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Towards a Multi-Robot Control Architecture

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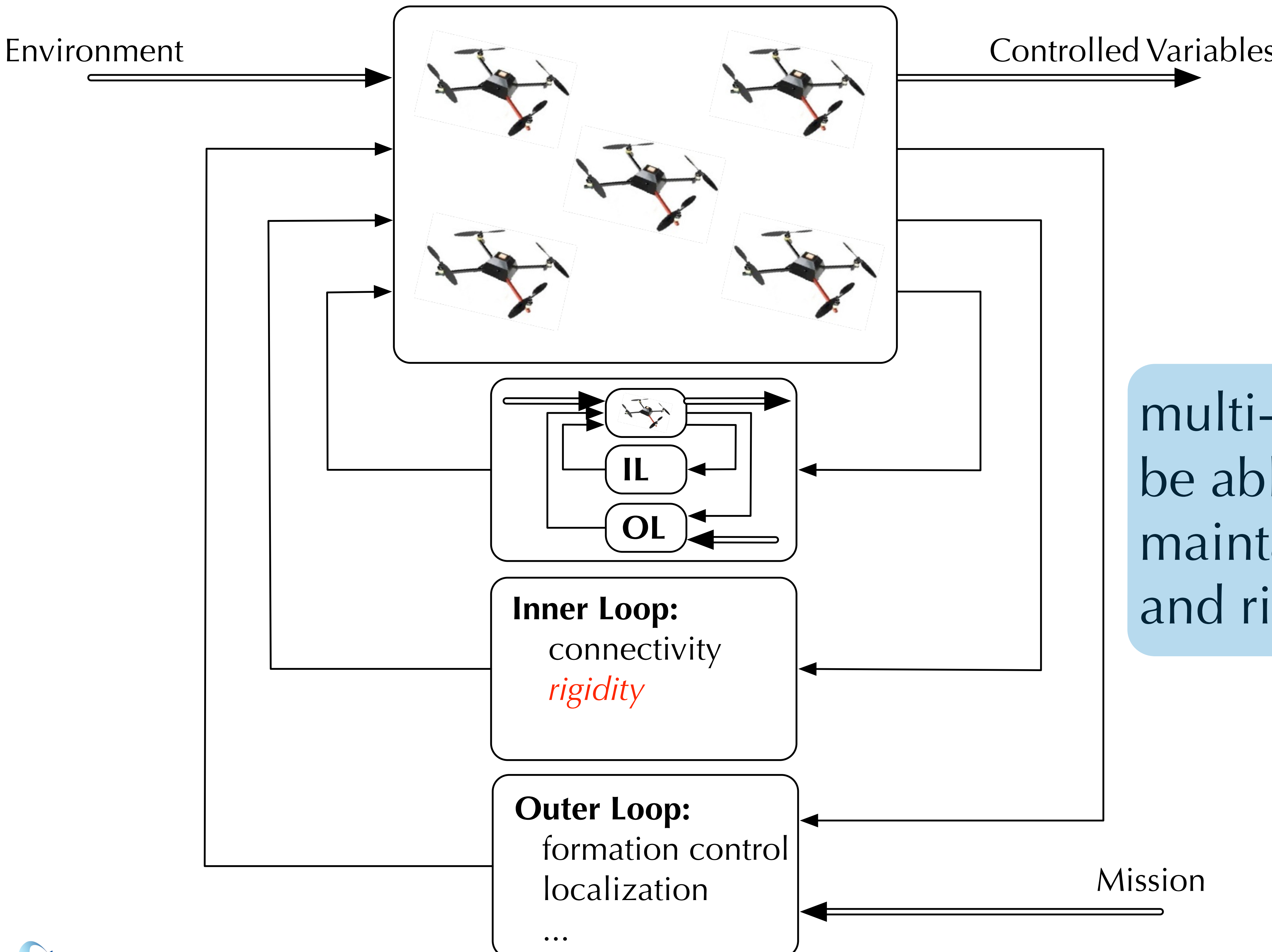


multi-robot systems must be able to *dynamically* maintain the connectivity and rigidity of the team

Rigidity Maintenance

Theorem

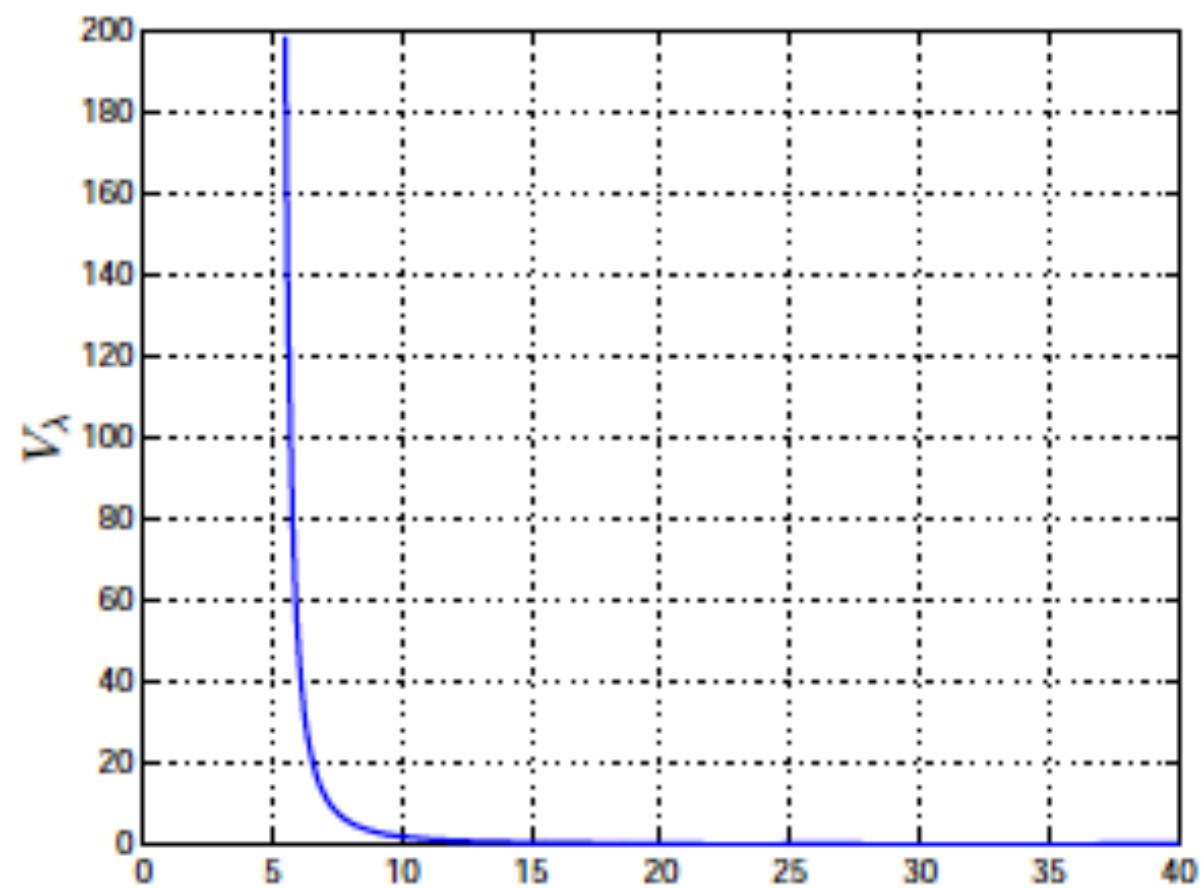
A framework is infinitesimally (distance, parallel) rigid if and only if the *rigidity eigenvalue* is strictly positive.

$$\mathcal{R} = R(p)^T R(p) \quad \mathcal{N}(\mathcal{R}) = \{\text{trivial infinitesimal motions}\}$$

Rigidity Maintenance

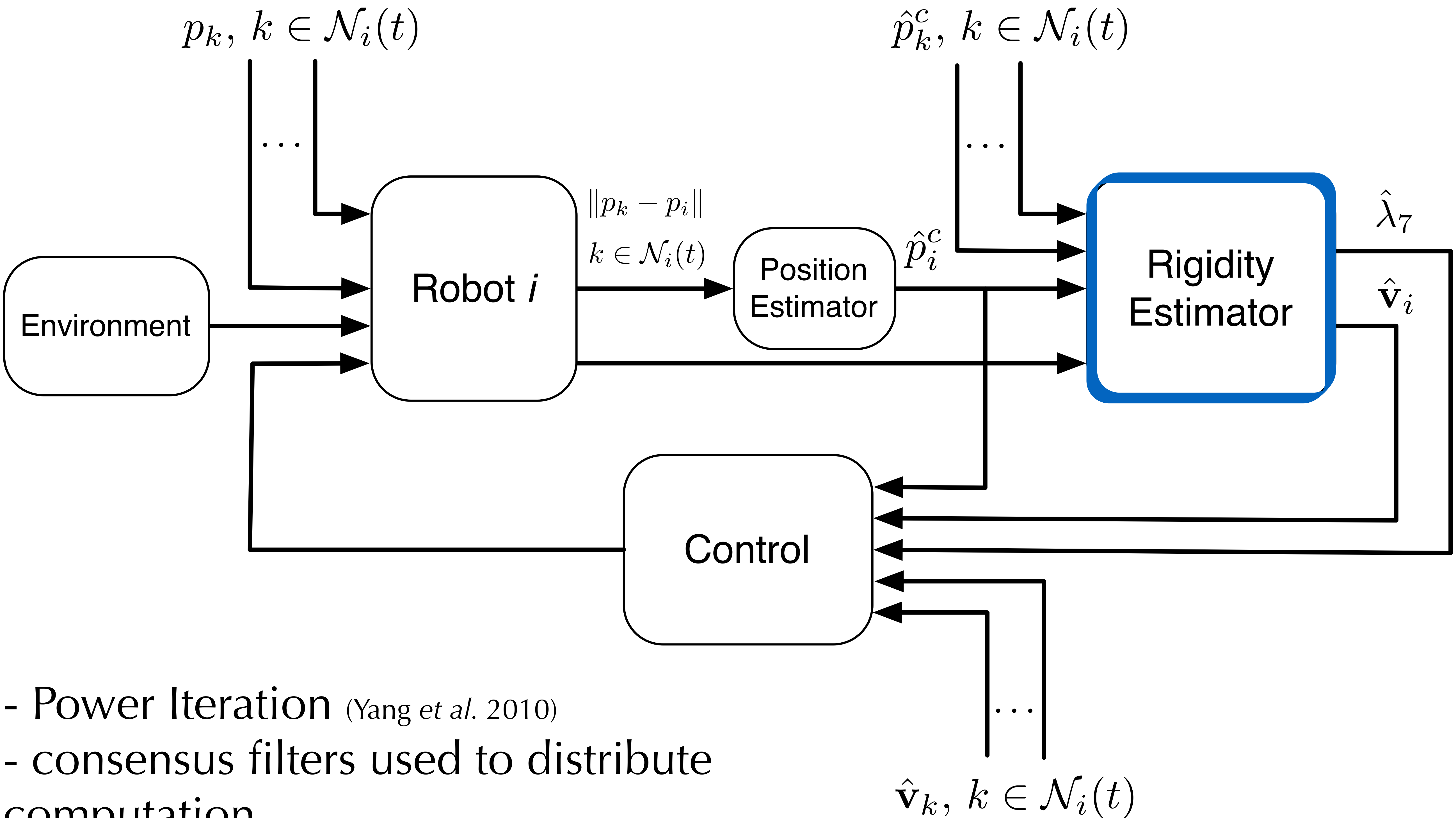
Design a control law to minimize a scalar potential function related to the rigidity eigenvalue

$$\xi_i = -\frac{\partial V_\lambda}{\partial \lambda_4}\left(\frac{\partial \lambda_4}{\partial p_i}\right)$$



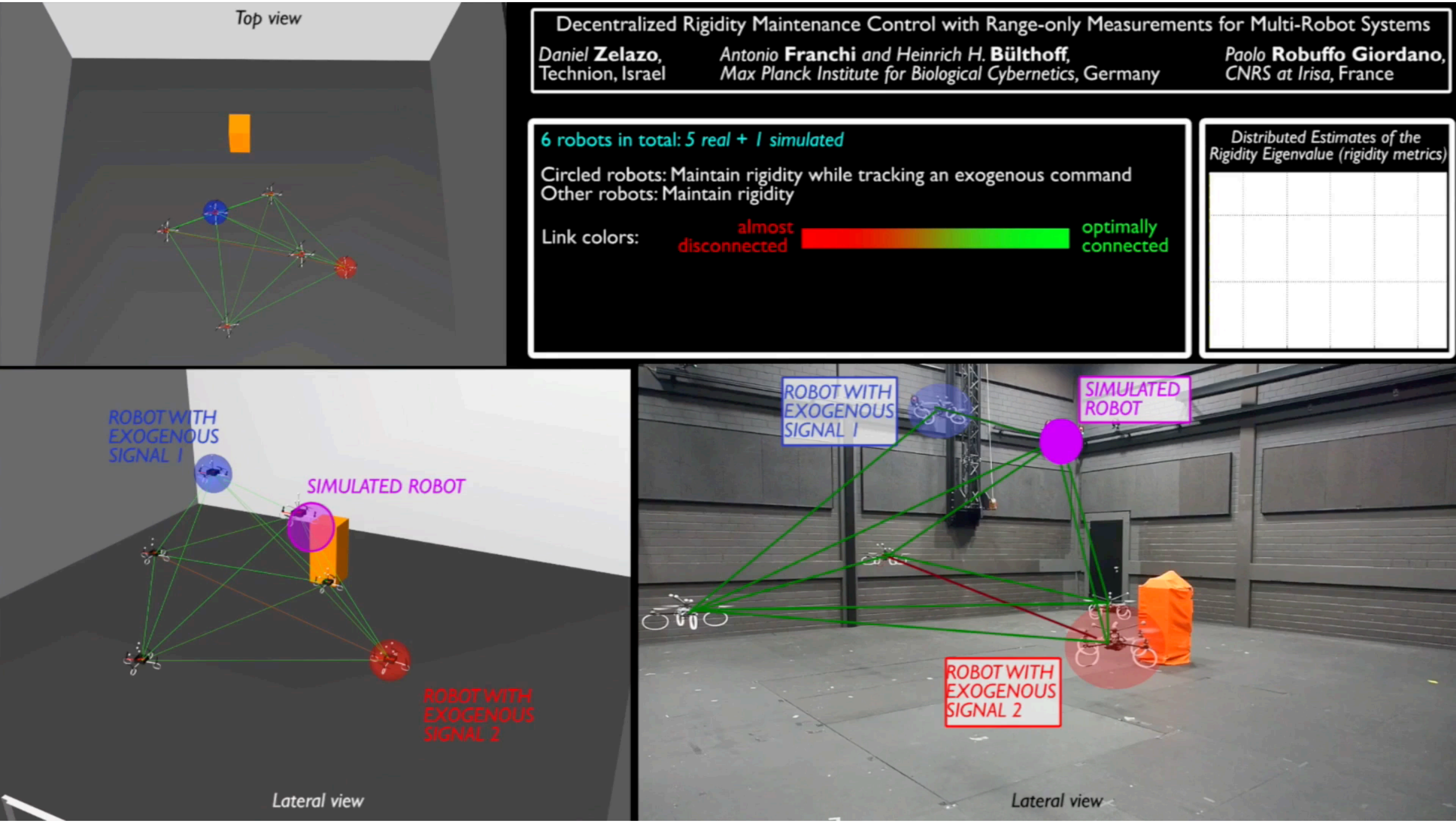
rigidity eigenvalue

Rigidity Maintenance



- Power Iteration (Yang *et al*. 2010)
- consensus filters used to distribute computation

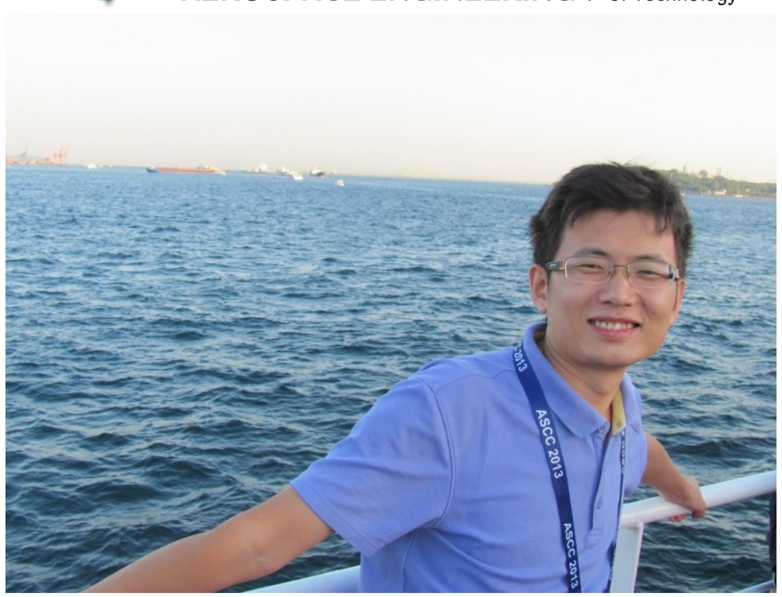
Rigidity Maintenance



Conclusions and Outlook

- coordination methods for multi-agent systems depend on sensing and communication mediums
- *rigidity theory* is a powerful framework for handling high-level multi-agent objectives under different sensing and communication constraints
- *rigidity maintenance* is an important “inner-loop” for multi-robot systems

Acknowledgements



Dr. Shiyu Zhao

Dr. Paolo Robuffo Giordano

Dr. Antonio Franchi

Questions?

References

Rigidity Maintenance

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Rigidity Theory in SE(2)

D. Zelazo, A. Franchi, and P. Robuffo Giordano, "Rigidity Theory in SE(2) for Unscaled Relative Position Estimation using only Bearing Measurements," European Control Conference, Strasbourgh, France, June 2014.

Bearing-Only Formation Control

S. Zhao and D. Zelazo, "Bearing Rigidity and Almost Global Bearing-Only Formation Stabilization," IEEE Transactions on Automatic Control, 2014 (submitted, pre-print on arXiv)