PASSIVITY, MONOTONICITY, AND NETWORK OPTIMIZATION: NEW PERSPECTIVES FOR NETWORK SYSTEMS ANALYSIS

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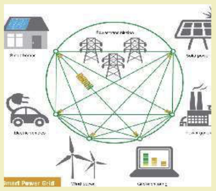
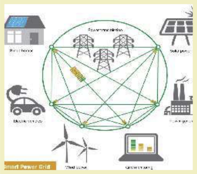
NETWORKED DYNAMIC SYSTEMS



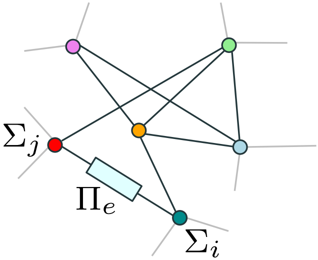
Networks of dynamical systems are one of the enabling technologies of the future.





- how do we analyze these systems
- how do we design these systems

IN THIS TALK...

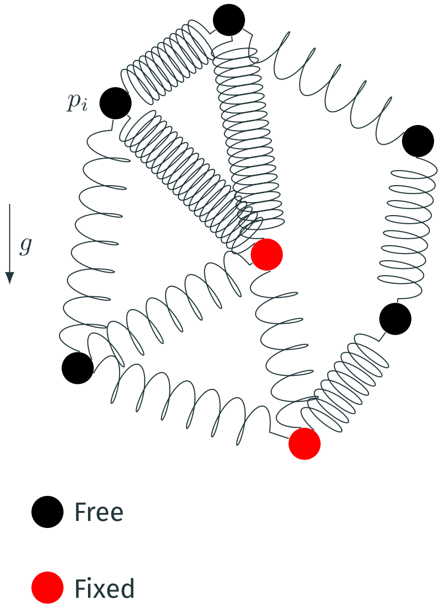


Explore the structure and mechanisms of networked systems to reveal deep connections between properties of dynamical systems and optimization theory.

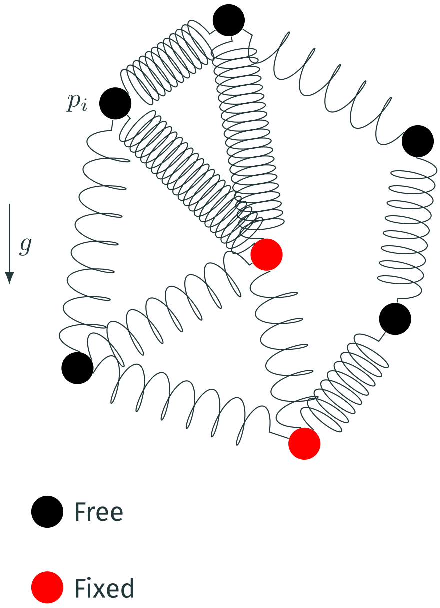
- A general model of diffusively coupled networks
- Characterization of network equilibriums via Network Optimization
- Convergence properties of dynamic networks via passivity theory
- Passivation, monotonization, and equilibrium independent passive short systems

A PHYSICS WARM-UP

A MASS-SPRING NETWORK

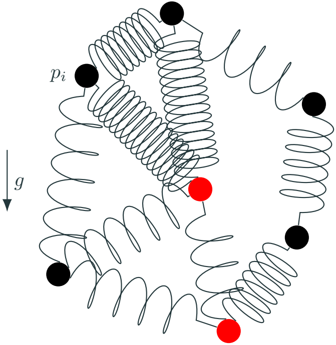


- A fixed network of (linear) springs
- springs connected to masses with position $p_i \in \mathbb{R}^2$ and mass m_i
- r masses have a fixed position (anchors)



- A fixed network of (linear) springs
- springs connected to masses with position $p_i \in \mathbb{R}^2$ and mass m_i
- r masses have a fixed position (anchors)

Determine the positions of the free masses that minimize the total potential energy of the mass-spring network.



- Potential Energy due to gravity

$$m_i g^T p_i$$

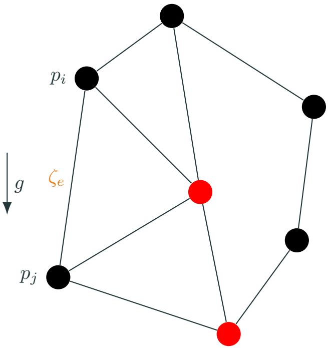
- Elastic Potential Energy of springs

$$\frac{1}{2} k_{ij} (\|p_i - p_j\| - r_{ij})^2$$

an optimization problem (take 1)

$$\min_{p_i} \quad \sum_i m_i g^T p_i + \sum_{i \sim j} \frac{1}{2} k_{ij} (\|p_i - p_j\| - r_{ij})^2$$

$$\texttt{s.t.} p_i = \mathbf{p}_i^*, \; i = 1, \ldots, r \; \texttt{(fixed nodes)}$$



- Potential Energy due to gravity (nodes)

$$m_i g^T p_i, \; i = 1, \dots, n$$

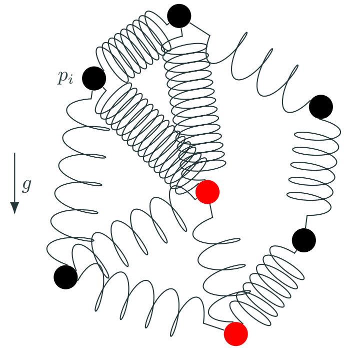
- Elastic Potential Energy of springs (edges)

$$\frac{1}{2} k_e (\|\underbrace{p_i - p_j}_{\zeta_e}\| - r_e)^2, \; e = 1, \dots, m$$

an optimization problem (take 2)

$$\min_{p_i, \zeta_e} \quad \sum_{i=1}^{r} (m_i g^T p_i + \mathbb{I}_{\mathbf{p_i^*}}(p_i)) + \sum_{i=r+1}^{n} m_i g^T p_i + \sum_e \frac{1}{2} k_e (\|\zeta_e\| - r_e)^2$$

$$\texttt{s.t.}\, p_i - p_j = \zeta_e, \; \forall e = (i, j)$$



A Convex Program!

an optimization problem (take 2)

$$\min_{p_i, \zeta_e} \quad \sum_{i}^{r} (m_i g^T p_i + \mathbb{I}_{\mathbf{p}_{\mathbf{i}}^*}(p_i)) + \sum_{i=r+1}^{n} m_i g^T p_i + \sum_{e} \frac{1}{2} k_{ij} (\|\zeta_e\| - r_e)^2$$

$$\texttt{s.t.}\, p_i - p_j = \zeta_e, \; \forall e = (i, j)$$

A MASS-SPRING NETWORK - THE DYNAMICS

- dynamic model for the masses

$$\Sigma_i \, : \begin{cases} \begin{bmatrix} \dot p_i \\ \ddot p_i \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_i \\ \dot p_i \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u_i + m_i g \\ y_i \;\; = \begin{cases} \begin{bmatrix} p_i \\ 0 \end{bmatrix}, & i = 1, \ldots, r \;\; \texttt{(anchors)} \\ \begin{bmatrix} p_i \\ \dot p_i \end{bmatrix}, & i = r+1, \ldots, n \end{cases} \end{cases}$$

- springs couple masses together

$$\Pi_e \, : \begin{cases} u_i & = \sum_{i \sim j} k_{ij} (\|p_i - p_j\| - r_{ij}) \frac{p_j - p_i}{\|p_j - p_i\|} + \\ & \quad\; b_{ij}(\dot p_j - \dot p_i) \\ & = \sum_{i \sim j} \kappa_{ij}(y_i - y_j) \end{cases}$$

A MASS-SPRING NETWORK - THE DYNAMICS

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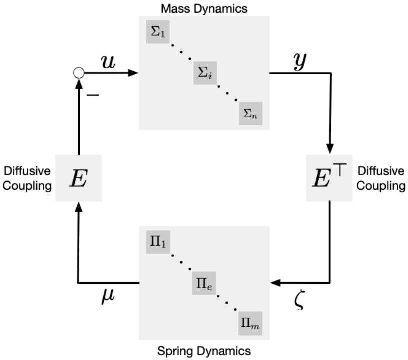
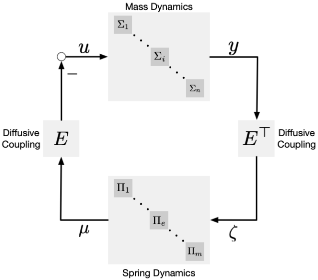


An example of a diffusively coupled network!

- System Equilibrium

$$\begin{cases} 0 &= \dot{p}_i \\ 0 &= m_i g + \sum_{i\sim j} k_{ij}(\|p_i - p_j\| - r_{ij})\frac{p_j - p_i}{\|p_j - p_i\|} \end{cases}$$

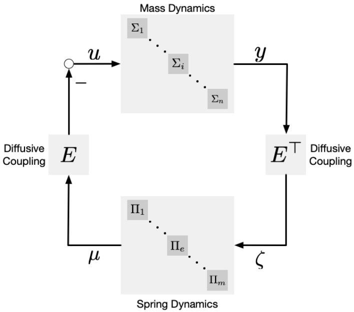


Minimum Total Potential Energy Principle (MTPE)

Equilibrium configurations extremize the total potential energy. Stable equilibriums correspond to minimizers of the total potential energy.

Dynamics

- Diffusively Coupled Network

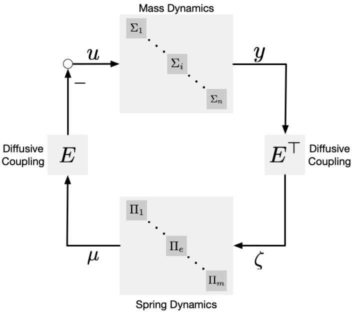


- Dissipasivity Theory

$$V(x) = \frac{1}{2}\sum_i \|\dot{p}_i\|^2 + \frac{1}{2}\sum_{i\sim j} k_{ij}\|p_i - p_j\|_2^2$$

Dynamics

- Diffusively Coupled Network



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$$V(x) = \frac{1}{2}\sum_i \|\dot{p}_i\|^2 + \frac{1}{2}\sum_{i\sim j} k_{ij}\|p_i - p_j\|_2^2$$

Optimization

- Convex Optimization

$$\min_{p_i,\zeta_e} \quad J(p,\zeta)$$
$$\texttt{s.t.}\, p_i - p_j = \zeta_e, \forall\, e = (i,j)$$

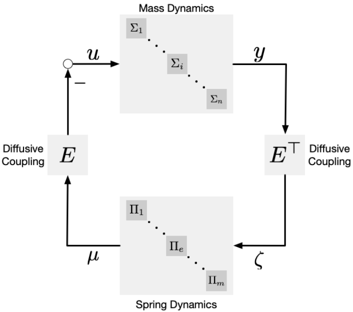
- Optimality Conditions

$$0 \in \partial J(p,\zeta)$$

LESSONS AND TOOLS

Dynamics

- Diffusively Coupled Network



- Dissipasivity Theory

$$V(x) = \frac{1}{2}\sum_i \|\dot{p}_i\|^2 + \frac{1}{2}\sum_{i\sim j} k_{ij}\|p_i - p_j\|_2^2$$

Optimization

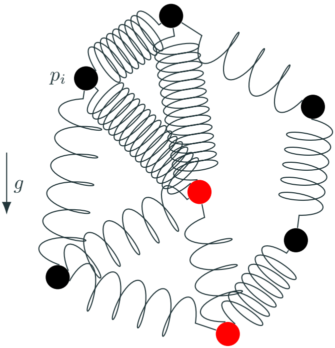
- Convex Optimization

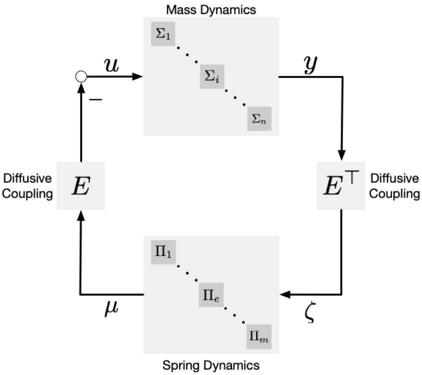
$$\min_{p_i,\zeta_e} \quad J(p,\zeta)$$
$$\texttt{s.t.}\, p_i - p_j = \zeta_e, \forall\, e = (i,j)$$

- Optimality Conditions

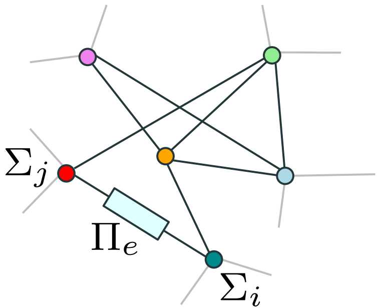
MTPE Principle ensures that the dynamics of the diffusively coupled network solve the optimization problem, and vice versa.

- What class of systems can be "solved" by examining a related optimization problem?
- What class of optimization problems can be be "solved" by a dynamical system?



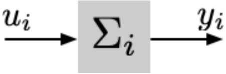


DIFFUSIVELY COUPLED NETWORKS



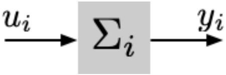
A network system is comprised of dynamical systems that interact with eachother over an information exchange network (a graph).

Agent dynamics:



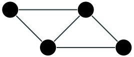
$$\Sigma_i : \begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = h_i(x_i, u_i) \end{cases}$$

Agent dynamics:



$$\Sigma_i : \begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = h_i(x_i, u_i) \end{cases}$$

Information Exchange Network:



$$E = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

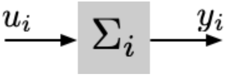
$$\mathcal{G} = (\mathbb{V}, \mathbb{E})$$

$$[E]_{ij} = \begin{cases} \pm 1 & (i,j) \in \mathbb{E} \\ 0 & \texttt{o.w.} \end{cases}$$

$$E^\top \mathbf{1} = 0$$

A NETWORK MODEL

Agent dynamics:



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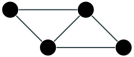

$$\mathcal{G} = (\mathbb{V}, \mathbb{E})$$

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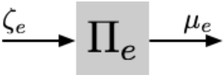
$$[E]_{ij} = \begin{cases} \pm 1 & (i,j) \in \mathbb{E} \\ 0 & \texttt{o.w.} \end{cases}$$

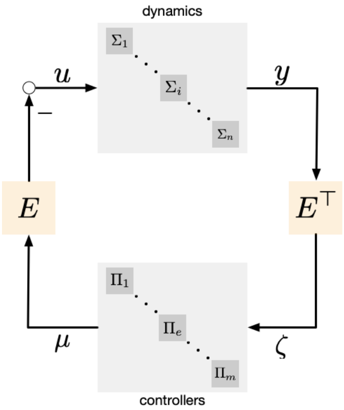
$$E^\top \mathbf{1} = 0$$

Controller dynamics:



$$\Pi_e : \begin{cases} \dot{\eta}_e = \phi_e(\eta_e, \zeta_e) \\ \mu_e = \psi_e(\eta_e, \zeta_e) \end{cases}$$

DIFFUSIVE COUPLING



$$(\Sigma, \Pi, \mathcal{G})$$

- Consensus Dynamics

$$\dot{x}_i = -\sum_{i\sim j} w_{ij}(x_j - x_i)$$

- Kumamoto Model

$$\dot{\theta}_i = -k\sum_{i\sim j} \sin(\theta_i - \theta_j)$$

- Traffic Dynamics

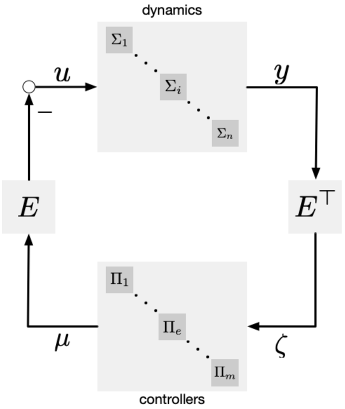
$$\dot{v}_i = \kappa_i\left(V_i^0 - v_i + V_i^1 \sum_{i\sim j} \tanh(p_j - p_i)\right)$$

- Neural Network

$$C\dot{V}_i = f(V_i, h_i) + \sum_{i\sim j} g_{ij}(V_j - V_i)$$

$$\dot{h}_i = g(V_i, h_i)$$

STEADY-STATE NETWORK SOLUTIONS



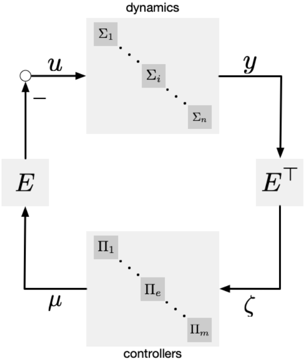
What properties must the systems Σ_i and Π_e possess such that $(\Sigma, \Pi, \mathcal{G})$ admits and converges to a steady-state solution?

$$u(t) \to \mathrm{u}$$
$$y(t) \to \mathrm{y}$$
$$\zeta(t) \to \zeta$$
$$\mu(t) \to \mu$$

- Consensus: $\mathrm{y} = \alpha \mathbf{1}$ ($\zeta = 0$)
- Formation: $\zeta \neq 0$ constant

All signals converge to a constant steady-state

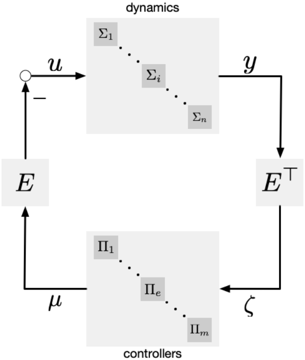
NETWORK OPTIMIZATION MEETS PASSIVITY THEORY



Assumption 1

Each agent Σ_i and controller Π_e admit forced steady-state solutions.

STEADY-STATE INPUT-OUTPUT MAPS

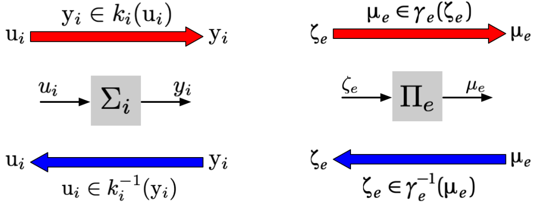


Assumption 1

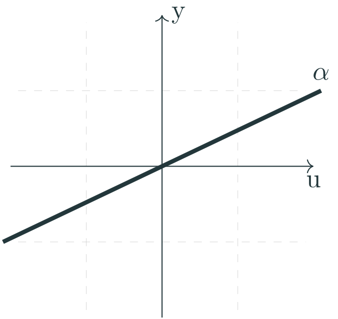
Each agent Σ_i and controller Π_e admit forced steady-state solutions.

Input-Output Maps

The *steady-state input-output map* $k : \mathcal{U} \to \mathcal{Y}$ associated with Σ is the set consisting of all steady-state input-output pairs (u, y) of the system.



$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$\Rightarrow k(\mathrm{u}) = \{\mathrm{y} \mid \underbrace{(-CA^{-1}B + D)}_{\alpha}\,\mathrm{u}\}$$

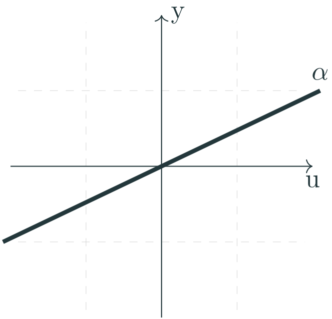


SISO and stable linear system

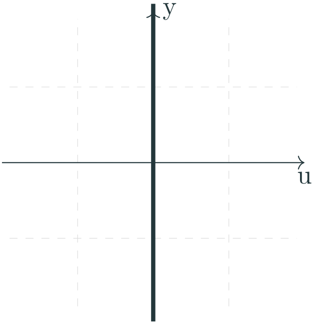
INPUT-OUTPUT RELATIONS

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$\Rightarrow k(\mathrm{u}) = \{\mathrm{y} \mid \underbrace{(-CA^{-1}B + D)}_{\alpha}\,\mathrm{u}\}$$

$$\dot{x} = u$$
$$y = x$$
$$\Rightarrow k = \{(0, \mathrm{y}),\ \mathrm{y} \in \mathbb{R}\}$$



SISO and stable linear system



simple integrator

NETWORK CONSISTENCY EQUATIONS

The network interconnection imposes constraints on allowable steady-states

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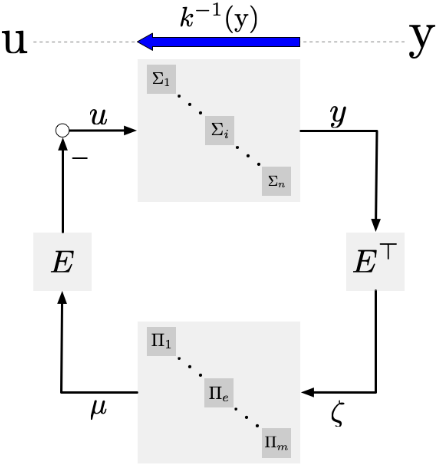
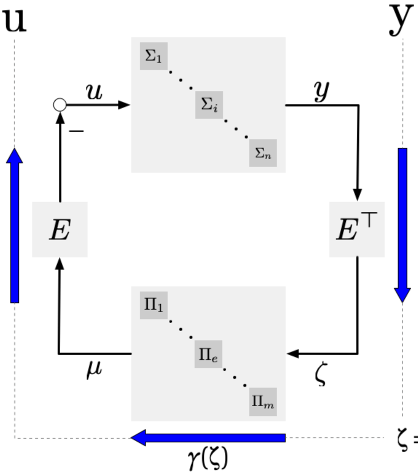


$$\mathrm{u} \in k^{-1}(\mathrm{y})$$

NETWORK CONSISTENCY EQUATIONS

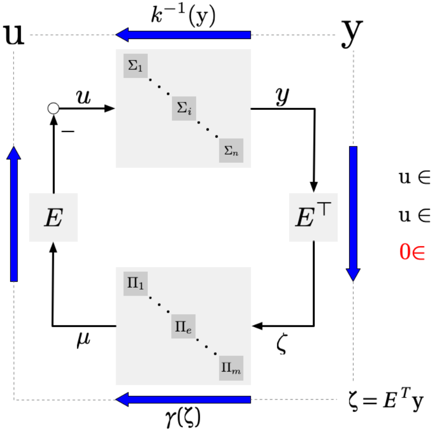
The network interconnection imposes constraints on allowable steady-states



$$\mathrm{u} \in -E\gamma(E^T \mathrm{y})$$

$$\zeta = E^T \mathrm{y}$$

The network interconnection imposes constraints on allowable steady-states

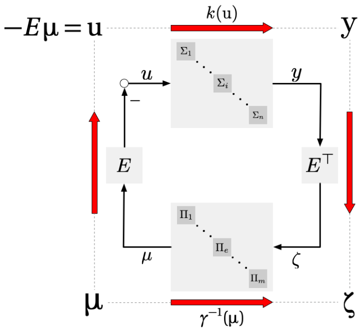


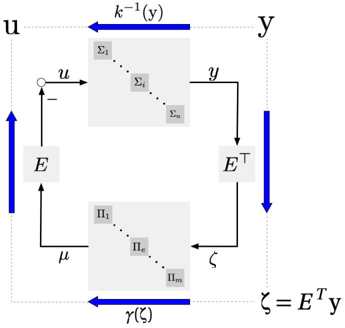
$$\mathrm{u} \in k^{-1}(\mathrm{y})$$

$$\mathrm{u} \in -E\gamma(E^T\mathrm{y})$$

$$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T\mathrm{y})$$

The network interconnection imposes constraints on allowable steady-states





$\zeta \in \gamma^{-1}(\mu)$

$\zeta \in E^T k(-E\mu)$

$0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$

$u \in k^{-1}(y)$

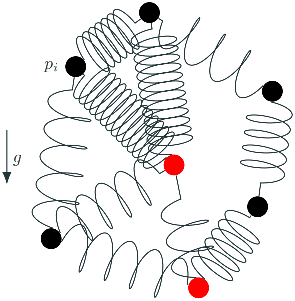
$u \in -E\gamma(E^T y)$

$0 \in k^{-1}(y) + E\gamma(E^T y)$

The network system $(\Sigma, \Pi, \mathcal{G})$ admits a steady-state if and only if there exists a solution to the system of non-linear inclusions

$$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T \mathrm{y})$$
$$0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$$

- When do solutions exist?
- How do we find them?

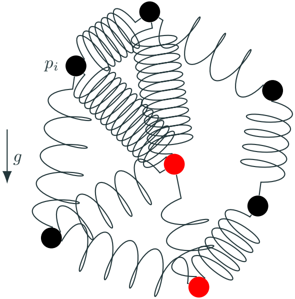


A Convex Program!

Minimum Total Potential Energy Problem

$$\min_{p_i,\zeta_e} \quad \sum_{i}^{r}(m_i g^T p_i + \mathbb{I}_{\mathbf{p_i^*}}(p_i)) + \sum_{i=r+1}^{n} m_i g^T p_i + \sum_{e} \frac{1}{2} k_{ij} (\|\zeta_e\| - r_e)^2$$

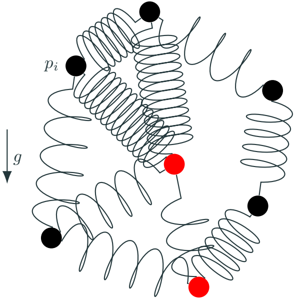
$$\texttt{s.t.}\, p_i - p_j = \zeta_e, \; \forall e = (i,j)$$



A Convex Program!

Minimum Total Potential Energy Problem

$$\min_{p_i,\zeta_e} \quad \sum_i J_i(p_i) + \sum_e \Gamma_e(\zeta_e)$$
$$\texttt{s.t.}\, E^T p = \zeta$$



A Convex Program!

Minimum Total Potential Energy Problem

$$\min_p \quad J(p) + \Gamma(E^T p)$$

First-order Optimality Condition:

$$0 \in \partial J(p) + E\partial\Gamma(E^T p)$$

SOLUTION OF NETWORK EQUATIONS

The network system $(\Sigma, \Pi, \mathcal{G})$ admits a steady-state if and only if there exists a solution to the system of non-linear inclusions

$$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T \mathrm{y})$$
$$0 \in \gamma^{-1}(\mathrm{\mu}) - E^T k(-E\mathrm{\mu})$$

RECALL First-order Optimality Condition:

$$0 \in \partial J(p) + E\partial\Gamma(E^T p)$$

Network equations are the first-order optimality conditions of a corresponding optimization problem!

The network system $(\Sigma, \Pi, \mathcal{G})$ admits a steady-state if and only if there exists a solution to the system of non-linear inclusions

$$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T \mathrm{y})$$
$$0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$$

RECALL First-order Optimality Condition:

$$0 \in \partial J(p) + E\partial\Gamma(E^T p)$$

Network equations are the first-order optimality conditions of a corresponding optimization problem!

What is it?

Definition

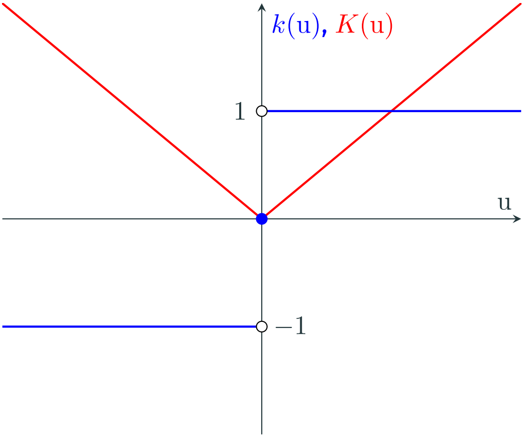
Let k_i be the input-output relation for system Σ_i. Define the function $K_i : \mathbb{R} \to \mathbb{R}$ such that $\partial K_i(\mathrm{u}_i) = k_i(\mathrm{u}_i)$ and $K = \sum_i K_i$. The function K is called the *cost function* associated with the system Σ_i.

Similarly,

$$\partial K_i^\star(\mathrm{y}_i) = k_i^{-1}(\mathrm{y}_i),\ K^\star = \sum_i K_i^\star$$

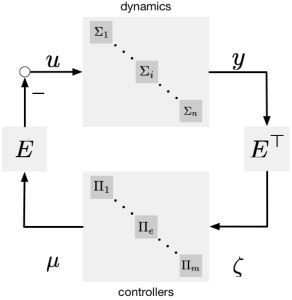
$$\partial \Gamma_e(\zeta_e) = \gamma_e(\zeta_e),\ \Gamma = \sum_e \Gamma_e$$

$$\partial \Gamma_e^\star(\mu_e) = \gamma_e^{-1}(\mu_e)\ \Gamma^\star = \sum_e \Gamma_e^\star$$



$K(\mathrm{u}) = |\mathrm{u}|$

$\mathrm{y} = k(\mathrm{u}) = \mathrm{sgn}(\mathrm{u})$



Steady-state values $\mathrm{u}, \mathrm{y}, \zeta$ and μ are the solutions of the following pair of optimization problems[1]:

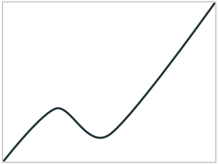
$$\min_{\mathrm{y},\zeta} \quad \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e) \qquad \text{s.t.} \quad E^T\mathrm{y} = \zeta.$$

$$\min_{\mathrm{u},\mu} \quad \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e) \qquad \text{s.t.} \quad \mathrm{u} = -E\mu.$$

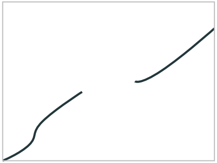
First-order Optimality Condition	First-order Optimality Condition
$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T\mathrm{y})$	$0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$

[1] [Bürger, Z, Allgower, 2014]

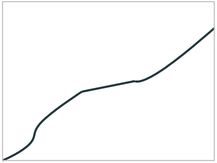
MONOTONE MAPS AND CONVEXITY



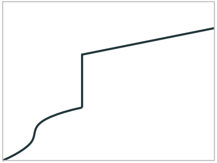
Not Monotone



Monotone but not maximal



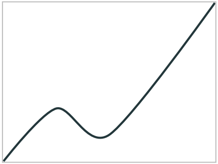
Maximal monotone function



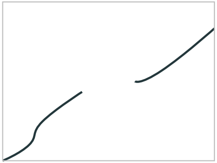
Maximal monotone relation

A relation on $\mathbb{R}$ is monotone
if they are non-decreasing curves in $\mathbb{R}^2$

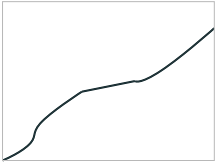
MONOTONE MAPS AND CONVEXITY



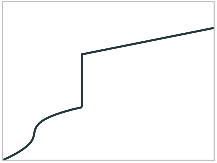
Not Monotone



Monotone but not maximal



Maximal monotone function

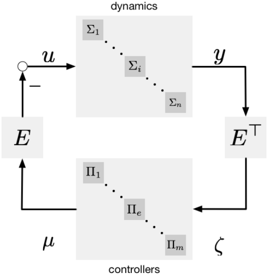


Maximal monotone relation

Theorem

The subdifferentials of convex functions on $\mathbb{R}$ are maximally monotone relations from $\mathbb{R}$ to $\mathbb{R}$.[a]

[a] [R. T. Rockafellar, Convex Analysis. Princeton University Press, 1997]

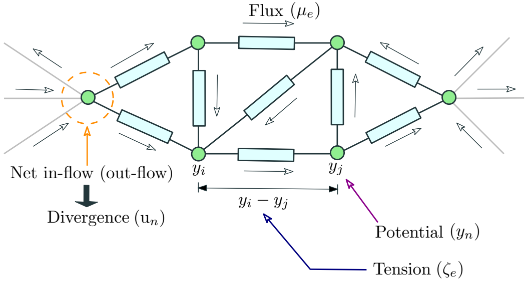


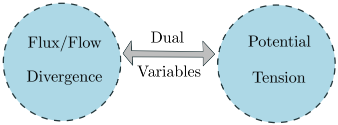
Theorem[1]

If the input-output maps k_i and γ_e are maximally monotone, then the steady-state values $\mathrm{u}, \mathrm{y}, \zeta$ and μ are the solutions of the following pair of convex dual optimization problems:

Optimal Flow Problem (OFP)	**Optimal Potential Problem (OPP)**
$\min_{\mathrm{y},\zeta} \quad \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e)$	$\min_{\mathrm{u},\mu} \quad \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e)$
$s.t. \quad E^T \mathrm{y} = \zeta.$	$s.t. \quad \mathrm{u} = -E\mu.$

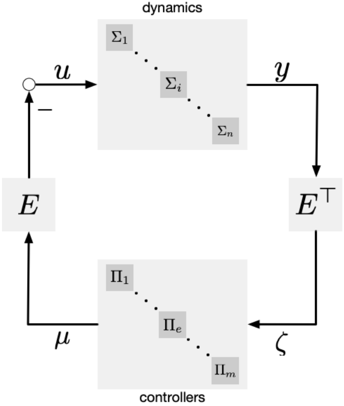
[1] [Bürger, Z, Allgower, 2014]





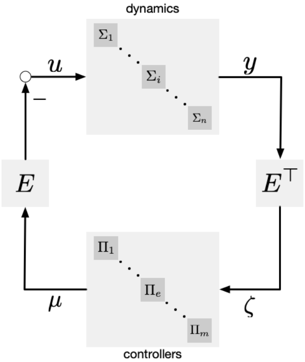
Optimal Flow Problem[1]	**Optimal Potential Problem**[1]
$\min_{u,\mu} \sum_{n=1}^{\|\mathcal{V}\|} C_n^{\text{div}}(u_n) + \sum_{e=1}^{\|\mathcal{E}\|} C_e^{\text{flux}}(\mu_e)$	$\min_{y,\zeta} \sum_{n=1}^{\|\mathcal{V}\|} C_n^{\text{pot}}(y_n) + \sum_{e=1}^{\|\mathcal{E}\|} C_e^{\text{ten}}(\zeta_e)$
$s.t. \quad u + E\mu = 0.$	$s.t. \quad E^T y = \zeta.$

[1] [R. T. Rockafellar, Network Flows and Monotropic Optmizations. John Wiley and Sons, Inc., 1984]



Diffusively coupled dynamic networks can be associated to static network optimization problems!

Monotone steady-state maps $\Leftrightarrow$ Network Duality



Assumption 1

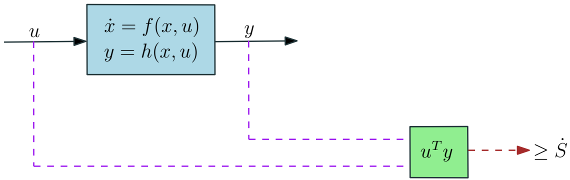
Each agent Σ_i and controller Π_e admit forced steady-state solutions.

Assumption 2

The input-output maps of each agent, k_i, and controller, γ_e, are maximally monotone.

Under what conditions does the network actually *converge* to these steady states?

PASSIVITY FOR DYNAMICAL SYSTEMS



$$\boxed{\begin{aligned} 0 &= f(0,0) \\ 0 &= h(0,0) \end{aligned}}$$

Definition [Khalil 2002]

A system is passive if there exists a C^1 storage function $S(x)$ such that

$$u^T y \geq \dot{S} = \frac{\partial S}{\partial x} f(x,u), \quad \forall (x,u) \in \mathbb{R}^n \times \mathbb{R}^p$$

Moreover, it is said to be

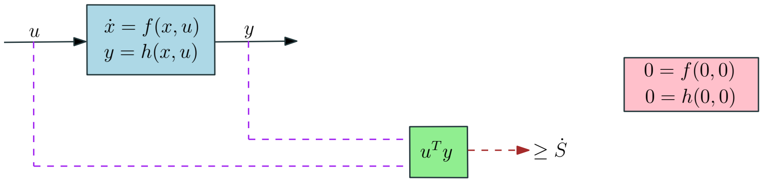
- Input-strictly passive if $\dot{S} \leq u^T y - u^T \phi(u)$ and $u^T \phi(u) > 0, \forall u \neq 0$
- Output-strictly passive if $\dot{S} \leq u^T y - y^T \rho(y)$ and $y^T \rho(y) > 0, \forall y \neq 0$

Definition

Let Σ be a SISO system with a constant input-output steady-state pair (u, y). The system is said to be *input-output* (ρ, ν)*-passive* wrt (u, y) if there exists a storage function $S(x)$ and numbers $\rho, \nu \in \mathbb{R}$, such that $\rho\nu < 1/4$ and

$$\dot{S} = \frac{\partial S}{\partial x} f(x, u) \leq (y - \mathrm{y})(u - \mathrm{u}) - \rho(y - \mathrm{y})^2 - \nu(u - \mathrm{u})^2,$$

for any trajectory u, y.



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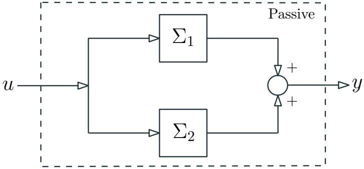
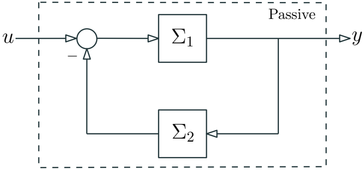
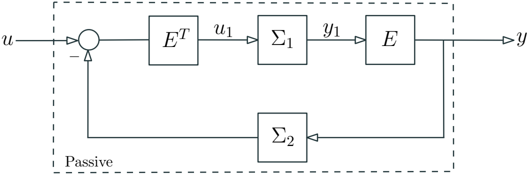
for any trajectory u, y.

- $\rho = \nu = 0 \Rightarrow$ **passivity**
- $\rho, \nu > 0 \Rightarrow$ **strict input/output passivity**
- $\rho, \nu < 0 \Rightarrow$ **passive short**

- Parallel Interconnection
- Negative Feedback Interconnection
- Symmetric Interconnection







Theorem[1]

Consider the network system $(\Sigma, \Pi, \mathcal{G})$ comprised of SISO agents and controllers. Suppose that there are vectors $\mathrm{u_i}, \mathrm{y_i}, \zeta_e$ and μ_e such that

i) the systems Σ_i are output strictly-passive with respect to $\mathrm{u_i}$ and $\mathrm{y_i}$;
ii) the systems Π_e are passive with respect to ζ_e and μ_e;
iii) the vectors $\mathrm{u}, \mathrm{y}, \zeta$ and μ satisfy $\mathrm{u} = -\mathcal{E}\mu$ and $\zeta = \mathcal{E}^T \mathrm{y}$.

Then the output vector $y(t)$ converges to y as $t \to \infty$.

1 [Arcak, 2007], [Bürger, Z, Allgower, 2014]

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Then the output vector $y(t)$ converges to y as $t \to \infty$.

- requires passivity w.r.t. to specific equilibrium configuration

[1] [Arcak, 2007], [Bürger, Z, Allgower, 2014]

EIP[1]

A SISO system $\Sigma : u \mapsto y$ is said to be *equilibrium-independent input-output* (ρ, ν)*-passive* if it is input-output (ρ, ν)-passive with respect to any equilibrium $(\mathrm{u}, k(\mathrm{u}))$.

EIP systems ($\rho, \nu \geq 0$) have monotone steady-state input-output maps!

$$\dot{S} \leq (y - \mathrm{y})^T (u - \mathrm{u}) \implies k \text{ monotonically increasing function}$$

1 [G.H. Hines et al., 2011], [M. Sharf, A. Jain, Z., 2020]

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A SISO system $\Sigma : u \mapsto y$ is said to be *equilibrium-independent input-output* (ρ, ν)*-passive* if it is input-output (ρ, ν)-passive with respect to any equilibrium $(\mathrm{u}, k(\mathrm{u}))$.

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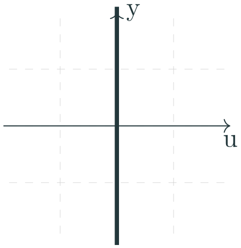


$$\dot{x}(t) = u(t), y(t) = x(t)$$

- Passive with respect to $\mathcal{U} = \{0\}$ and any output value $\mathrm{y} \in \mathbb{R}$ with storage function $S(x) = \frac{1}{2}(x - \mathrm{y})^2$.
- The equilibrium input-output map $k = \{(0, \mathrm{y}) : \mathrm{y} \in \mathbb{R}\}$ is not a single valued function and hence the integrator is **NOT** *EIP*.

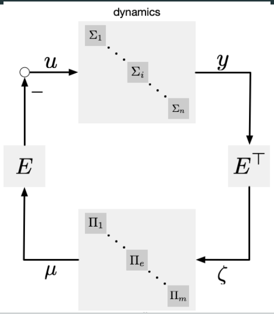
[1] [G.H. Hines et al., 2011], [M. Sharf, A. Jain, Z., 2020]

MEIP[1]

A dynamical SISO system Σ is *maximal equilibrium independent* passive if the following conditions hold:

- The system Σ is passive with respect to any steady-state $(\mathrm{u}, \mathrm{y}) \in k$.
- The relation k is maximally monotone.

[1][M. Bürger et al., 2014]



Assumption 1

Each agent Σ_i and controller Π_e admit forced steady-state solutions.

Assumption 2

The agent dynamics Σ_i are output-strictly MEIP and the controllers are MEIP.

Theorem[1]

Assume Assumptions 1 and 2 hold. Then the signals $u(t), y(t), \zeta(t), \mu(t)$ converge to the solutions of the following pair of convex dual optimization problems:

Optimal Flow Problem (OFP)	Optimal Potential Problem (OPP)
$\min_{\mathrm{y},\zeta} \quad \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e)$	$\min_{\mathrm{u},\upmu} \quad \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\upmu_e)$
$s.t. \quad E^T\mathrm{y} = \zeta.$	$s.t. \quad \mathrm{u} = -E\upmu.$

[1] [Bürger, Z, Allgower, 2014]

NEW PERSPECTIVES ON PASSIVATION

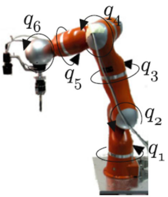
MONOTONICITY AND ITS ROLE IN SYSTEMS THEORY

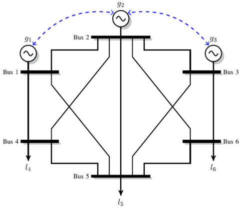
Systems (Passivity) ⟷ **Monotonicity (I/O Relations)** ⟷ **Optimization (Convexity)**

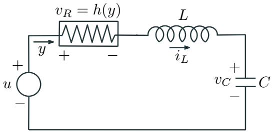
What else can we say about MEIP systems?

In practice, systems are usually passivity-short (or non-passive)!

- Generator (always generates energy) [R. Harvey , 2016]
- Oscillating systems with small or nonexistent damping [R. Harvey, 2017]
- Dynamics of robot system from torque to position [D. Babu, 2018]
- Power-system network (turbine-governor dynamics) [S. Trip, 2018]
- Electrical circuits with nonlinear components
- More general as include non-minimum phase systems and systems with relative degree greater than 1 [Z. Qu, 2014]





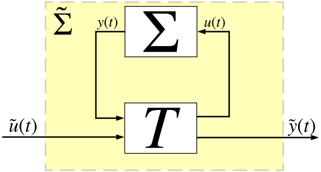


$h(\cdot) \in [\sigma, \infty]$ with $\sigma < 0$

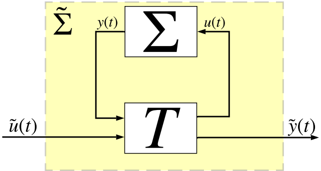
Passive short systems can destroy
the developed network optimization framework!

System Type	Relations	Integral Function
MEIP	k, k^{-1} max. monotone	$K(\mathrm{u}), K^\star(\mathrm{y})$ are convex
Input PS	k is not monotone	$K(\mathrm{u})$ is non-convex
Output PS	k^{-1} is not monotone	$K^\star(\mathrm{y})$ is non-convex
Input-output PS	k, k^{-1} are not monotone	May not exist

Optimal Flow Problem (OFP)	Optimal Potential Problem (OPP)
$\min_{\mathrm{y},\zeta} \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e)$	$\min_{\mathrm{u},\mu} \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e)$
$s.t.\quad E^T\mathrm{y} = \zeta.$	$s.t.\quad \mathrm{u} = -E\mu.$



For a passive-short system $\Sigma : u \mapsto y$, we aim to find a map T such that the closed-loop system $\tilde{\Sigma} : \tilde{u} \mapsto \tilde{y}$ is passive. This is known as feedback passivation.



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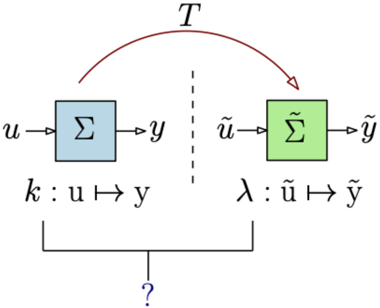


how does feedback passivation affect the steady-state input/output maps?

an example

$$\begin{aligned} \dot{x} &= -x + \sqrt[3]{x} + u \\ y &= \sqrt[3]{x} \end{aligned}$$

$$\bar{\mathrm{u}} = k^{-1}(\bar{\mathrm{y}}) = \bar{\mathrm{y}}^3 - \bar{\mathrm{y}}$$

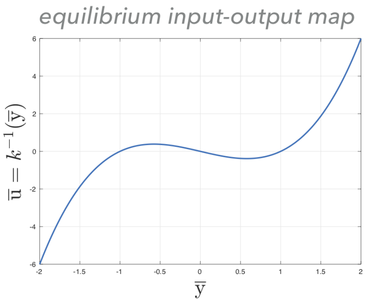
not a monotone input-output relation!

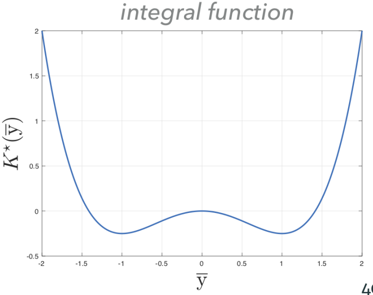
System is output passivity-short

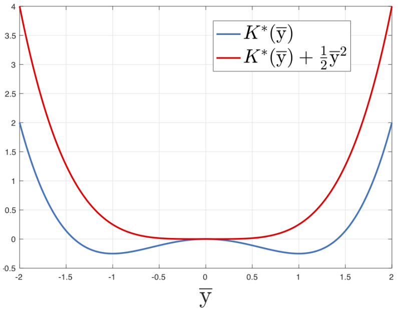
$$S(x) = \frac{3}{4}x^{4/3} - \bar{\mathrm{y}}x + \frac{1}{4}\bar{\mathrm{y}}$$

$$\dot{S} \le (y - \bar{y})(u - \bar{u}) + (y - \bar{y})^2$$

(passivity index $\rho = -1$)







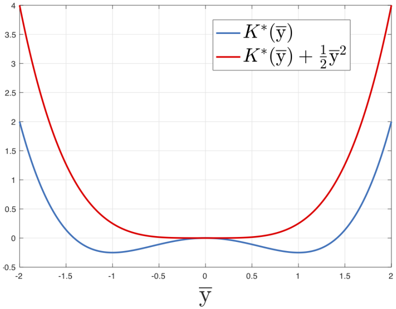
what is the system interpretation of a "convexified" integral function?

$$K^\star(\overline{\mathrm{y}}) = \frac{1}{4}\overline{\mathrm{y}}^4 - \frac{1}{2}\overline{\mathrm{y}}^2$$

$$\tilde{K}^\star(\overline{\mathrm{y}}) = K^\star(\overline{\mathrm{y}}) + \frac{1}{2}\overline{\mathrm{y}}^2$$

(Tikhonov regularization term)

AN EXAMPLE



what is the system interpretation of a "convexified" integral function?

$$K^\star(\bar{y}) = \frac{1}{4}\bar{y}^4 - \frac{1}{2}\bar{y}^2$$

$$\tilde{K}^\star(\bar{y}) = K^\star(\bar{y}) + \frac{1}{2}\bar{y}^2$$

(Tikhonov regularization term)

$$\begin{aligned}\partial\tilde{K}^\star(\bar{y}) &= \partial K^\star(\bar{y}) + \bar{y}\\ \tilde{k}^{-1}(\bar{y}) &= k^{-1}(\bar{y}) + \bar{y}\\ &= \bar{y}^3 - \bar{y} + \bar{y} = \bar{y}^3\end{aligned}$$

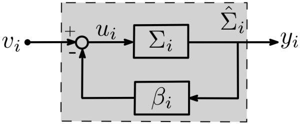
a monotone function!

what system yields this steady-state I/O map?

$$\dot{x} = -x + \sqrt[3]{x} - \underbrace{\sqrt[3]{y}}_{u} + v = -x + v$$

$$y = \sqrt[3]{x}$$

AN EXAMPLE



regularization is realized by output feedback!

$$u = v - y$$
$$\Rightarrow \dot{x} = -x + v$$
$$\Rightarrow \overline{\mathrm{v}} = \tilde{k}^{-1}(\overline{\mathrm{y}}) = \overline{\mathrm{y}}^3$$

(maximally monotone!)

Theorem[1]

Consider the passive-short SISO dynamical system $\Sigma : u \mapsto y$ with I/O steady-state map k and output passivity index $\rho < 0$. Then for any $\beta > |\rho|$, the feedback

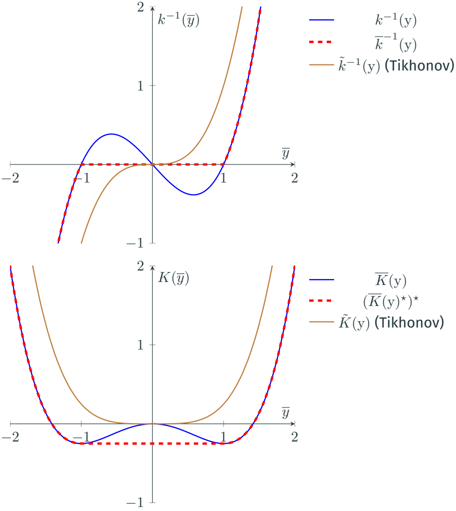
$$u = v - \beta y$$

renders the system $\tilde{\Sigma} : v \mapsto y$ output-strictly maximally monotone EIP with steady-state input map $\tilde{k}$ satisfying

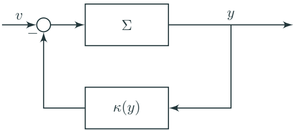
$$\tilde{k}^{-1}(\overline{\mathrm{y}}) = k^{-1}(\overline{\mathrm{y}}) + \beta\overline{\mathrm{y}}.$$

[1] [Jain, Sharf, Z, 2018]

MONOTONIZATION AND CONVEXIFICATION



A “better” convexification leads to different feedback passivation!



the feedback

$$\kappa(y) = \begin{cases} 0, & |x| = |y^3| > 1 \\ y^3 - y, & |x| = |y^3| \le 1 \end{cases}$$

the closed-loop

$$\dot{x} = \begin{cases} -x + \sqrt[3]{x} + v, & |x| > 1 \\ v, & |x| \le 1 \end{cases}$$

$$y = \sqrt[3]{x}.$$

Is it possible to find a linear transformation $T : (\mathrm{u}, \mathrm{y}) \mapsto (\tilde{\mathrm{u}}, \tilde{\mathrm{y}})$ for a non-monotone I/O map $k : \mathrm{u} \mapsto \mathrm{y}$ such that $\tilde{k} : \tilde{\mathrm{u}} \mapsto \tilde{\mathrm{y}}$ is monotone?

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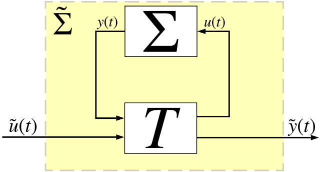

For a passive-short system $\Sigma : u \mapsto y$, we aim to find a map T such that the closed-loop system $\tilde{\Sigma} : \tilde{u} \mapsto \tilde{y}$ is passive. This is known as feedback passivation.

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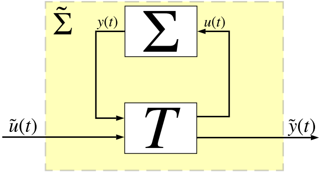

For a passive-short system $\Sigma : u \mapsto y$, we aim to find a map T such that the closed-loop system $\tilde{\Sigma} : \tilde{u} \mapsto \tilde{y}$ is passive. This is known as feedback passivation.

Are these T maps the same?

A GEOMETRIC APPROACH

For an EI-IOP(ρ, ν) system, for any two points $(\mathrm{u}_1, \mathrm{y}_1), (\mathrm{u}_2, \mathrm{y}_2) \in k$, the following inequality holds:

$$0 \leq -\rho(\mathrm{y}_1 - \mathrm{y}_2)^2 + (\mathrm{u}_1 - \mathrm{u}_2)(\mathrm{y}_1 - \mathrm{y}_2) - \nu(\mathrm{u}_1 - \mathrm{u}_2)^2.$$

Projective Quadratic Inequalities and EI-IOP

A *projective quadratic inequality (PQI)* is an inequality with variables $\xi, \chi \in \mathbb{R}$ of the form

$$0 \leq a\xi^2 + b\xi\chi + c\chi^2 = F(\xi, \chi),$$

for some numbers a, b, c, not all zero. The inequality is called *non-trivial* if $b^2 - 4ac > 0$. The associated solution set $\mathcal{A}$ of the PQI is the set of all points $(\xi, \chi) \in \mathbb{R}^2$ satisfying the inequality.

- passivity inequality is a PQI: $\xi = \mathrm{u}_1 - \mathrm{u}_2$, $\chi = \mathrm{y}_1 - \mathrm{y}_2$
- monotonicity is a PQI: $0 \leq (\mathrm{u}_1 - \mathrm{u}_2)(\mathrm{y}_1 - \mathrm{y}_2)$ with $a = c = 0$ and $b = 1$

$$0 \leq a\xi^2 + b\xi\chi + c\chi^2 = F(\xi, \chi)$$

A Recap:

- $F(\mathrm{u}_1 - \mathrm{u}_2, \mathrm{y}_1 - \mathrm{y}_2) \geq 0$ is a PQI for a EI-IOP(ρ, ν) system

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A Recap:

- $F(\mathrm{u}_1 - \mathrm{u}_2, \mathrm{y}_1 - \mathrm{y}_2) \ge 0$ is a PQI for a EI-IOP(ρ, ν) system
- For the linear map $T : (\mathrm{u}, \mathrm{y}) \mapsto (\tilde{\mathrm{u}}, \tilde{\mathrm{y}})$,

$$F(\tilde{\mathrm{u}}_1 - \tilde{\mathrm{u}}_2, \tilde{\mathrm{y}}_1 - \tilde{\mathrm{y}}_2) \ge 0$$

  is also a PQI for a EI-IOP($\tilde{\rho}, \tilde{\nu}$) system

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- $F(\tilde{\mathrm{u}}_1 - \tilde{\mathrm{u}}_2, \tilde{\mathrm{y}}_1 - \tilde{\mathrm{y}}_2) = (\tilde{\mathrm{u}}_1 - \tilde{\mathrm{u}}_2)(\tilde{\mathrm{y}}_1 - \tilde{\mathrm{y}}_2)$ corresponds to monotonicity

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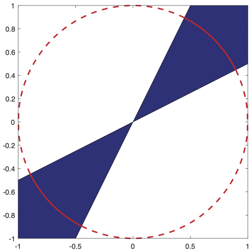
Study the effect of the map T on the solution sets of the PQIs, $T(\mathcal{A})$

The solution set of any non-trivial PQI is a symmetric double-cone. Moreover, any symmetric double-cone is the solution set of some non-trivial PQI.



Theorem[1]

Let (ξ_1, χ_1), (ξ_2, χ_2) be non-colinear solutions of $a_1\xi^2 + \xi\chi + c_1\chi^2 = 0$, and $(\tilde{\xi}_1, \tilde{\chi}_1)$,$(\tilde{\xi}_2, \tilde{\chi}_2)$ be non-colinear solutions of $a_2\xi^2 + \xi\chi + c_2\chi^2 = 0$. Define

$$T_1 = \begin{bmatrix} \tilde{\xi}_1 & \tilde{\xi}_2 \\ \tilde{\chi}_1 & \tilde{\chi}_2 \end{bmatrix} \begin{bmatrix} \xi_1 & \xi_2 \\ \chi_1 & \chi_2 \end{bmatrix}^{-1}, T_2 = \begin{bmatrix} \tilde{\xi}_1 & -\tilde{\xi}_2 \\ \tilde{\chi}_1 & -\tilde{\chi}_2 \end{bmatrix} \begin{bmatrix} \xi_1 & \xi_2 \\ \chi_1 & \chi_2 \end{bmatrix}^{-1}.$$

Then one of T_1, T_2 transforms the PQI $a_1\xi^2 + \xi\chi + c_1\chi^2 \geq 0$ to the PQI $\tau a_2\xi^2 + \tau\xi\chi + \tau c_2\chi^2 \geq 0$ for some $\tau > 0$.

[1] [Sharf, Jain, Z, 2021]

EXAMPLE

Consider the system

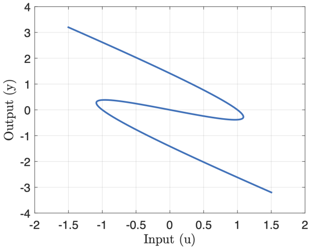
$$\Sigma : \dot{x} = -\sqrt[3]{x} + .5x + .5u,\ y = .5x - .5u$$

Using $S(x) = \frac{1}{6}(x - \mathrm{x})^2$ we have

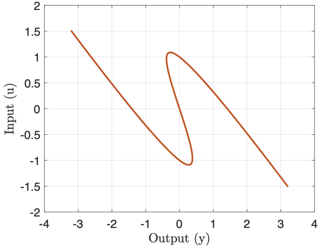
$$\dot{S}(x) \leq (u - \mathrm{u})(y - \mathrm{y}) + \frac{1}{3}(u - \mathrm{u})^2 + \frac{2}{3}(y - \mathrm{y})^2$$

System is EI-IOP(ρ, ν) with
$\rho = -2/3,\ \nu = -1/3$

Passive-short system with non-monotone input-output relations (not even a function!)



(a) k.



(b) k^{-1}

Consider the system

$$\Sigma : \dot{x} = -\sqrt[3]{x} + .5x + .5u,\ y = .5x - .5u$$

Using $S(x) = \frac{1}{6}(x - \mathrm{x})^2$ we have

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System is EI-IOP(ρ, ν) with $\rho = -2/3,\ \nu = -1/3$

Corresponding PQI:

$$0 \leq \frac{1}{3}\xi^2 + \xi\chi + \frac{2}{3}\chi^2$$

Find a linear map T that monotonizes the input-output relations, i.e., leads to the PQI

$$\tilde{\xi}\tilde{\chi} \geq 0$$

non-colinear solutions to PQI

$$\tilde{\xi}\tilde{\chi} = 0$$

$$\begin{bmatrix} \tilde{\xi}_1 \\ \tilde{\chi}_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \tilde{\xi}_2 \\ \tilde{\chi}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

non-colinear solutions to original PQI

$$0 = \frac{1}{3}\xi^2 + \xi\chi + \frac{2}{3}\chi^2$$

$$\begin{bmatrix} \xi_1 \\ \chi_1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} \xi_2 \\ \chi_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The map

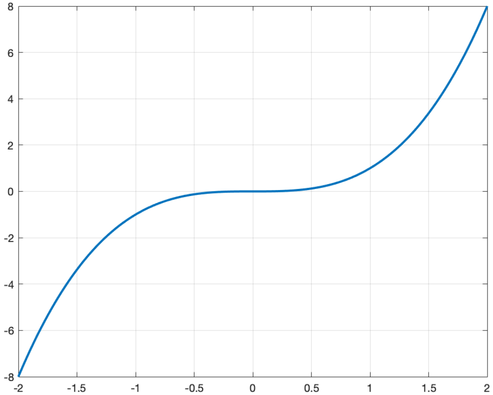
$$T_1 = \begin{bmatrix} \tilde{\xi}_1 & \tilde{\xi}_2 \\ \tilde{\chi}_1 & \tilde{\chi}_2 \end{bmatrix} \begin{bmatrix} \xi_1 & \xi_2 \\ \chi_1 & \chi_2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

can be used to monotonize the relation! Indeed, for $(\xi, \chi) = T^{-1}(\tilde{\xi}, \tilde{\chi})$

$$\begin{aligned} 0 \leq& \frac{1}{3}\xi^2 + \xi\chi + \frac{2}{3}\chi^2 \\ =& \frac{1}{3}(2\tilde{\xi} - \tilde{\chi})^2 + (2\tilde{\xi} - \tilde{\chi})(-\tilde{\xi} + \tilde{\chi}) + \frac{2}{3}(-\tilde{\xi} + \tilde{\chi})^2 = \frac{1}{3}\tilde{\xi}\tilde{\chi}, \end{aligned}$$

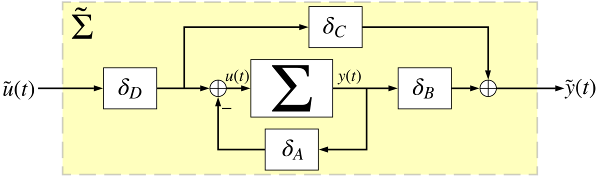
Steady-state input-output maps under T_1,

$$\begin{bmatrix} \tilde{u} \\ \tilde{y} \end{bmatrix} = T_1 \begin{bmatrix} u \\ y \end{bmatrix}$$



Theorem[1]

Let Σ be EI-IOP(ρ, ν). If the map T monotizes the input-output relation k, then it passivizes the system Σ.



$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \underbrace{\begin{bmatrix} \delta_D & 0 \\ 0 & 1 \end{bmatrix}}_{L_D} \underbrace{\begin{bmatrix} 1 & 0 \\ \delta_C & 1 \end{bmatrix}}_{L_C} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \delta_B \end{bmatrix}}_{L_B} \underbrace{\begin{bmatrix} 1 & \delta_A \\ 0 & 1 \end{bmatrix}}_{L_A},$$

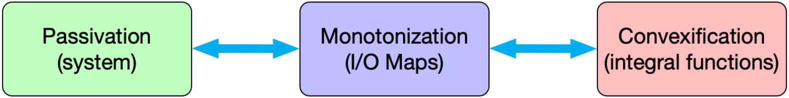
[1] [Sharf, Jain, Z, 2020]

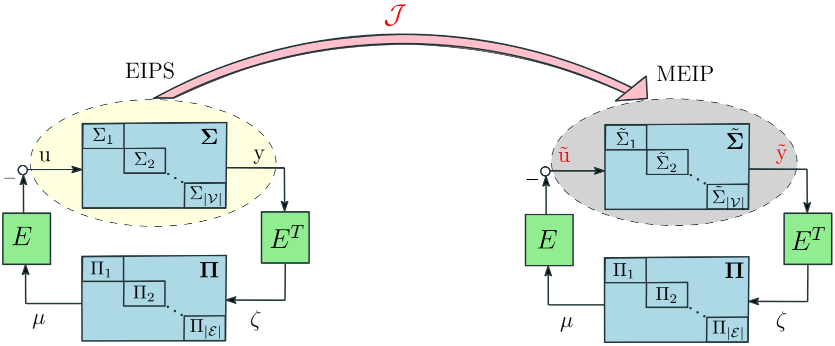
Elementary Transformation	Relation between I/O of Σ and $\tilde{\Sigma}$	Effect on Steady-State Relations	Realization	Effect on Integral Functions
$L_A = \begin{bmatrix} 1 & \delta_A \\ 0 & 1 \end{bmatrix}$	$\tilde{u} = u + \delta_A y$ $\tilde{y} = y$	$\lambda_A^{-1}(\tilde{\mathrm{y}}) = k^{-1}(\tilde{\mathrm{y}}) + \delta_A \tilde{\mathrm{y}}$	output-feedback	$\Lambda^\star(\mathrm{y}) = K^\star(\mathrm{y}) + \frac{1}{2}\delta_A \mathrm{y}^2$
$L_B = \begin{bmatrix} 1 & 0 \\ 0 & \delta_B \end{bmatrix}$	$\tilde{u} = u$ $\tilde{y} = \delta_B y$	$\lambda_B(\mathrm{u}) = \delta_B k(\mathrm{u})$ or $\lambda_B^{-1}(\tilde{\mathrm{y}}) = k^{-1}(\frac{1}{\delta_B}\tilde{\mathrm{y}})$	post-gain	$\Lambda^\star(\mathrm{y}) = \frac{1}{\delta_B} K^\star(\frac{1}{\delta_B}\mathrm{y})$ or $\Lambda(\mathrm{u}) = \delta_B K(\mathrm{u})$
$L_C = \begin{bmatrix} 1 & 0 \\ \delta_C & 1 \end{bmatrix}$	$\tilde{u} = u$ $\tilde{y} = y + \delta_C u$	$\lambda_C(\tilde{\mathrm{u}}) = k(\tilde{\mathrm{u}}) + \delta_C \tilde{\mathrm{u}}$	input-feedthrough	$\Lambda(\mathrm{u}) = K(\mathrm{u}) + \frac{1}{2}\delta_C \mathrm{u}^2$
$L_D = \begin{bmatrix} \delta_D & 0 \\ 0 & 1 \end{bmatrix}$	$\tilde{u} = \delta_D u$ $\tilde{y} = y$	$\lambda_D^{-1}(\mathrm{y}) = \delta_D k^{-1}(\mathrm{y})$ or $\lambda_D(\tilde{\mathrm{u}}) = k(\frac{1}{\delta_D}\tilde{\mathrm{u}})$	pre-gain	$\Lambda^\star(\mathrm{y}) = \delta_D K^\star(\mathrm{y})$ or $\Lambda(\mathrm{u}) = \frac{1}{\delta_D} K(\frac{1}{\delta_D}\mathrm{u})$

EI−IOP(ρ, ν) | Non-monotone | Non-existent

$$
\begin{tikzcd}
\Sigma \arrow[r] \arrow[d, "T"] & k \arrow[r, dashed] \arrow[d, "T"] & K \arrow[d, "T"] \\
\tilde{\Sigma} \arrow[r] & \lambda \arrow[r] & \Lambda
\end{tikzcd}
$$

MEIP | Maximally monotone | Convex





- Without loss of generality assume that the systems at nodes are EIPS (applicable if some of the systems are EIPS)
- Loop Transformation results in a pair of regularized network optimization problems

$$\mathcal{J} = \mathrm{diag}(T_i)$$

CONCLUDING REMARKS

A MONOTONE VIEW

Systems (Passivity) $\longleftrightarrow$ **Monotonicity (I/O Relations)** $\longleftrightarrow$ **Optimization (Convexity)**

New perspectives on networks and passivity

- networks of EIP agents can be understood through solutions of a pair of static dual optimization problems
- passivity and monotonicity of input-output maps are essential
- passivation means monotonization - monotonization means convexification

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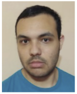

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QUESTIONS?

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