

### Distributed Rigidity Maintenance with Range-only Sensing

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## Coordination in Multi-agent Systems



General Robotics, Automation, Sensing & Perception

#### **System Requirements**

- `low-level' control
- sensing and communication
- mission objectives
	- local
	- team
- distributed algorithms

#### **What are the** *architectural* **requirements for a multi-agent system?**



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### Coordination in Harsh Environments





Sensors measuring distances, however, are very accurate and independent of any





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#### **"Connectedness" of the sensing and communication network**





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certain "team" objectives and specific sensing/communication capabilities might dictate additional architectural requirements

- formation keeping
- localization







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formation specified by a set of inter-agent distances

agents can measure distance to neighbors

sensor limitations only allow a subset of available measurements



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#### Can the desired formation be maintained using only the available distance measurements?

#### **No!**



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A *minimum* number of distance measurements are required to *uniquely* determine the desired formation!

#### Graph Rigidity



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#### $\Diamond$  Motivation

- $\Diamond$  Graph Rigidity and the Rigidity Eigenvalue
- Distributed Rigidity Maintenance

 $\Diamond$  Outlook



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bar-and-joint frameworks

$$
\left\{\begin{array}{l} \mathcal{G}=(\mathcal{V},\mathcal{E}) \\ p:\mathcal{V}\rightarrow\mathbb{R}^2 \\ \text{mass every vertex to a} \end{array}\right\}.
$$

maps every vertex to a point in the plane



 $(\mathcal{G}, p_0)$   $(\mathcal{G}, p_1)$ 

Two frameworks are *congruent if*

$$
(\mathcal{G},p_0)~(\mathcal{G},p_1)
$$



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Two frameworks are *equivalent if*  $\|p_0(v_i) - p_0(v_j)\| = \|p_1(v_i) - p_1(v_j)\|$  $\forall \{v_i, v_j\} \in \mathcal{E}$  $||p_0(v_i) - p_0(v_j)|| = ||p_1(v_i) - p_1(v_j)||$  $\forall v_i, v_j \in \mathcal{V}$ 



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A framework (*G, p*0) is *globally rigid* if every framework that is equivalent to  $(\mathcal{G}, p_0)$ is congruent to  $(\mathcal{G}, p_0)$ .

frameworks that are both *equivalent* and *congruent* are related by only "trivial" motions



• rotations





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parameterizing frameworks by a variable representing "time" allows to consider "motions" of a framework (*G, p, t*)

A trajectory is *edge consistent* if  $||p(v,t) - p(u,t)||$  is constant for all  $\{v, u\} \in \mathcal{E}$  and all *t*.

edge consistent trajectories generate a family of equivalent frameworks

$$
\{p(u) \in \mathbb{R}^2 \mid ||p(u) - p(v)||_2^2 = \ell_{uv}^2, \forall \{u, v\} \in \mathcal{E}\}
$$
  
\n
$$
\Rightarrow \frac{d}{dt} ||x_u(t) - x_v(t)|| = 0, \forall \{u, v\} \in \mathcal{E}
$$

 $\Rightarrow$   $(\dot{x}_u(t) - \dot{x}_v(t))^T (x_u(t) - x_v(t)) = 0$  *infinitesimal motions* 



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How can we check if a graph is generically or infinitesimally rigid?



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## The Rigidity Matrix



Lemma 1 (Tay1984) *A framework* (*G, p*) *is infinitesimally rigid if and only if*  $\mathbf{rk}[R]=2|\mathcal{V}|-3$ 



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## The Symmetric Rigidity Matrix

The Symmetric Rigidity Matrix

 $\mathcal{R} = R(p)$  ${}^{T}R(p)$ 

$$
\begin{cases}\n\lambda_4 & \text{the Rigidity Eigenvalue} \\
(\lambda_7)\n\end{cases}
$$

a symmetric positive semi-definite matrix with eigenvalues 
$$
\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_{2|\mathcal{V}|}
$$

Theorem 1 *A framework is infinitesimally rigid if* and only if the rigidity eigenvalue is strictly positive; i.e.  $\lambda_4 > 0$ .

$$
\text{proof: } P\mathcal{R}P^T = (I_2 \otimes E(\mathcal{G})) \begin{bmatrix} W_x^2 & W_{xy} \\ W_{xy} & W_y^2 \end{bmatrix} (I_2 \otimes E(\mathcal{G})^T)
$$
\n
$$
\text{weights depend on relative positions } \quad [W_x^2]_{kk} = (p_i^x - p_j^x)^2
$$



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#### Frameworks for Dynamic Environments





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## Weighted Frameworks

When is there a sensing link between agents?



## Weighted Frameworks



$$
R(p,\mathcal{W})=W(\mathcal{G},p)R(p)
$$

 $R(p, \mathcal{W}) = W(\mathcal{G}, p)R(p)$   $\mathcal{R} = R(p, \mathcal{W})^T R(p, \mathcal{W})$ 

Corollary 1 *A weighted framework* (*G, p, W*) *is infinitesimally rigid if and only if the weighted rigidity eigenvalue is strictly positive.*



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- $\Diamond$  Motivation
- $\Diamond$  Graph Rigidity and the Rigidity Eigenvalue
- Distributed Rigidity Maintenance
- $\Diamond$  Outlook



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### Rigidity Maintenance



When *relative sensing* is used, rigidity becomes an important *architectural requirement* for a multi-agent system

 $\Rightarrow$ to achieve higher level objectives (i.e. formation control, localization), the rigidity property must be maintained *dynamically*



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## The Rigidity Potential

How can rigidity be maintained with only local information?



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## The Rigidity Potential

How can rigidity be maintained with only local information?

**Key observation**: Gradient of rigidity eigenvalue has a distributed structure!



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# Distributed Rigidity Maintenance

#### a distributed implementation requires

- estimation of a common inertial frame
- estimation of the rigidity eigenvalue and eigenvector





### Estimation of a Common Frame

Agents do not have access to relative positions, only distance



rigidity of formation can be used for each agent to estimate relative position to a *common point*



• one agent endowed with *special ability*

 $p_c$ 

- able to measure *relative position* w.r.t to two agents
- all other agents only measure distances



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#### Estimation of a Common Frame of our problem. Consistently with our notation, we define  $\sim$ d = *p*  $P$  *p*  $P$  *<i>n*  $P$  *n*  $P$  *n* also denote by `*ij* the measured distance k*p<sup>j</sup> pi*k, as

 $\hat{\mathbf{a}}$ 



*•* Properties of error function<br> **•** non-perative and convex function

- non-negative and convex function<br>• *F* 0 if and only if estimated distance
- $\Phi$  *•*  $=$  0 ii<sub>i</sub><sub>c</sub> • **= 0** if and only if estimated distances equal measured distances

\*based on approach of Calafiore *et al.,* 2010.



*pc*

hop communication channels, two neighboring agents *i* and *j*

can then build an estimate *p*ˆ*j,c p*ˆ*i,c* of their actual relative

 $p$  in a component  $p$  in a component  $p$  in a component  $p$ 

 $p_c$  the roboth sets, etc.

• "special" agent becomes the

(moving) point each agent is trying

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to estimate the relative position of

#### Estimation of a Common Frame  $\blacksquare$ mation of a common Frame property, one can restate  $\alpha$  restate (15) in matrix form as  $\alpha$  restate (15) in matrix form as  $\alpha$

**First-Order Gradient Descent** ˙  $p$  st-Order Gradient Descent

$$
\hat{\hat{p}} = -\frac{\partial e}{\partial \hat{p}} = -\mathcal{R}(\hat{p})\hat{p} + R(\hat{p})\ell + \Delta^c
$$

**Proposition** If the framework is (infinitesimally) *rigid then the vector of true values*  $p - (\mathbb{1} \otimes p_c) =$  $\left[ (p_1 - p_c)^T \cdots (p_{|\mathcal{V}|} - p_c)^T \right]^T$  *is an isolated local minimizer of*  $e(\hat{p})$ *. Therefore, there exists an*  $\epsilon > 0$  *such that, for all initial conditions satisfying*  $\|\hat{p}(0) - p - (\mathbb{1} \otimes p_c)\| < \epsilon$ , *the estimation*  $\hat{p}$  *converges to*  $p - (\mathbb{1} \otimes p_c)$ *.* 

*<sup>c</sup>* <sup>2</sup> <sup>R</sup>*|E|* contains the remaining terms of the right-hand-side

 $*$ proof based on Krick *et al.,* 2009

we employ a continuous-time variation of the algorithm that  $\mathcal{C}$ 

will compute the smallest non-zero eigenvalue and eigenvector

of the symmetric rigidity matrix.

following iteration,



*pc*

**of the set of the set o** 

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### Estimation of Rigidity Eigenvalue





## Estimation of Rigidity Eigenvalue

 $\Rightarrow$ 

recall...

A framework is infinitesimally rigid if and only if the rigidity eigenvalue is positive

$$
\xi_i = -\frac{\partial V_\lambda}{\partial \lambda_4} \left( \frac{\partial \lambda_4}{\partial p_i} \right)
$$

requires all agents to have knowledge of rigidity eigenvalue and eigenvector

strategy

<sup>1</sup> **Power Iteration Method**

algorithm for estimating the *dominant eigenvalue* of a matrix  $x_{k+1} =$  $Ax_k$  $||Ax_k||$ 

#### **Distributed Implementation** use of dynamic consensus filters



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#### Power Iteration Method Dowar Horotion Mathod es diversited. The internet is the internet of the largest eigenvector engagement of the largest eigenvector e

Rigidity eigenvalue is *not* the dominant eigenvalue of symmetric rigidity matrix  $\overline{\phantom{a}}$ Rigidity eigenvalue is *not* the dominant vector and eigenvalue the responses the results

 $IM[T] = span[\mathcal{N}(\mathcal{R})]$ power iteration on "deflated" matrix  $\tilde{\mathcal{R}} = I - TT^T - \alpha \mathcal{R}$ 

recall...

$$
PRPT = (I2 \otimes E(G)) \begin{bmatrix} W_x & W_{xy} \\ W_{xy} & W_y \end{bmatrix} (I_2 \otimes E(G)T)
$$
  

$$
\mathcal{N}(R) = \text{span} \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} p^y - p_c^y 1 \\ p_c^x 1 - p^x \end{bmatrix} \right\}
$$



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#### that the power iteration applied to the matrix *<sup>R</sup>*˜ will compute er Iteration Method The continuous-time counterpart of the power iteration Power Iteration Method converge to the rigidity eigenvector. We present the main terms of the main terms of the main terms of the main

continuous-time centralized power iteration method

$$
\dot{\mathbf{v}}(t) = -\left(k_1 TT^T + k_2 \mathcal{R} + k_3 \left(\frac{\hat{\mathbf{v}}(t)^T \hat{\mathbf{v}}(t)}{3|\mathcal{V}|} - 1\right) I\right) \hat{\mathbf{v}}(t)
$$

where v ˆ is the *estimate* of the rigidity eigenvector, and the **Theorem** Assume that the symmetric rigidity matrix R has distinct non-zero eigenvalues, and let v denote the rigidity  $eigenvector$ . Then for any initial condition  $\hat{\mathbf{v}}(t_0) \in \mathbb{R}^{3|\mathcal{V}|}$ such that  $\mathbf{v}^T \hat{\mathbf{v}}(t_0) \neq 0$ , the trajectories of (17) converge sustem (19) de system de la strategie  $\lim_{t\to\infty} \hat{\mathbf{v}}(t) = \gamma \mathbf{v}$  for  $\gamma \in \mathbb{R}$ , if and only if the gains  $k_1, k_2$ *dependent inducer conditions. eigenvector. Then for any initial condition* v  $\mathcal R$  the symmetric  $\mathcal R$  $\int$  $|\mathcal{V}| = |\mathcal{V}|$  $rac{1}{2}$  $\mathcal{S}^{\mathcal{C}}$  $e_{\cdot},$  $k_2$  in this sub-section for  $k_3$ the rigidity estimator that overcomes these difficulties, in particular by leveraging the results of Section IV. In the *to the subspace spanned by the rigidity eigenvector, i.e., and k*<sup>3</sup> *satisfy the following conditions:*

1) 
$$
k_1, k_2, k_3 > 0,
$$
  
2)  $k_1 > k_2 \lambda_7,$ 

3)  $k_3 > k_2 \lambda_7$ .  $\lambda$ <sub>7</sub>,  $\lambda$ <sub>7</sub>,  $\lambda$ <sub>2</sub>,  $\lambda$ <sub>7</sub>,  $\lambda$ <sub>2</sub>,  $\lambda$ <sub>2</sub>,  $\lambda$ <sub>2</sub>,  $\lambda$ <sup>2</sup>,  $\lambda$ <sup></sup> *and k*<sup>3</sup> *satisfy the following conditions:*

 $\frac{1}{2010}$ particular structure of the symmetric rigidity matrix. \*adapted from Yang *et al.,* 2010

*preventing a successful convergence of the estimation of* v*.*

matrix  $\blacksquare$ 

 $\mathbf{I}$ 

 $\vert$ 

 $\vert$ 

 $\vert$ 

the *PI average consensus filter* [33] to distributedly compute

 $\vert$ 

 $\mathbf{I}_{\mathbf{r}}$ 

 $\overline{\phantom{a}}$ 

 $\vert$ 

estimator state (i.e., the quantity v

same spirit as the solution proposed in [28], we make use of

the *PI average consensus filter* [33] to distributedly compute



**הפקולתה להנדסת אוירונוטיקה וחלל** 1) *k*1*, k*2*, k*<sup>3</sup> *>* 0*,*  $\bf{F} \rm{aculty}$  of  $\bf{A} \rm{erospace}$   $\bf{Engineering}$ *Furthermore, for any choice of constants k*1*, k*2*, k*<sup>3</sup> *>* 0*, the* Faculty of Aerospace Engineering

**Tokyo Tech, September 13, 2013, Tokyo, Japan**  $3, 2013,$  $\mathbf{u}$  in distributed structure (i.e., the symmetric rigidity matrix  $\mathbf{u}$  $\frac{1}{\sqrt{2}}$  $\mathfrak{g}_{\mathfrak{p}}$ 

#### Distributed Power Iterat stability and tune the speed of the filter. An analysis of the  $stri$ hitad Dawar Itar is in the control of the set of the representation of how the PI consensus filters are embedded the eigenvector associated with the rigidity eigenvalue.<br>The rigidity eigenvalue associated with the rigidity eigenvalue.7 The rigidity eigenvalue associated with the  $\blacksquare$  Power Iterate continuous-time is the power iteration in the power in the power in the power is the power in th algorithm now takes the form  $\mathcal{O}(\mathcal{E})$  and  $\mathcal{E}(\mathcal{E})$  and  $\mathcal{E}(\mathcal{E})$  $f(x)$   $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$   $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ the *consensus protocol* can be used to distributedly compute the average of a set of numbers [5]. The speed at which the



$$
\dot{\hat{\mathbf{v}}}(t) = -\left(k_1TT^T + \underbrace{k_2\mathcal{R}}_{1}\right) + \underbrace{k_3\left(\frac{\hat{\mathbf{v}}(t)^T\hat{\mathbf{v}}}{3|\mathcal{V}|} \right)^{p^{z,c}(t)}}_{1}
$$

consensus protocol can compute this value is a function the

1) symmetric rigidity matrix is a  $PRP^T = (I_2 \otimes E)\mathcal{G}$ "naturally" distributed operator  $\overline{f}$  $P\mathcal{R}P^T = (I_2 \otimes E(\mathcal{G}))$ "naturally" distributed operator. We present here to the main series of the main series o  $\mathcal{P}(\mathcal{C})$  $\frac{1}{1}$ sensing links between and integration and  $\frac{1}{1}$  $T_{\text{max}}$  and the disc of a potential

$$
\text{Q} \left( \frac{\hat{\mathbf{v}}(t)^T \hat{\mathbf{v}}(t)}{3|\mathcal{V}|} - 1 \right) \hat{\mathbf{v}}(t) = (Avg(\hat{\mathbf{v}}(t) \circ \hat{\mathbf{v}}(t)) - 1) \hat{\mathbf{v}}(t) \quad \text{average of a vector can be distributedly computed using consensus algorithm*}
$$

*filteration for esting consensus algorithm<sup>\*</sup>* reference frame (i.e., the quantity *T T <sup>T</sup>* , see Theorem II.16  $\mathbf{r}$ *i* a vector can be distributedly asing conscrisus argontumi

This quantity can therefore be distributed using the distributed using the distributed using the distributed u<br>This computed using the distributed using the distributed using the distributed using the distributed using th PI Consensus Filter *of agents in the ensemble, and thus is a scalable solution.* **Theorem V.1. ISBN V.1. ISBN V.1. Assume that the symmetric right vertex is also vertex of the symmetric rigidity matrix**  $R$  **and**  $R$  **and**  $R$  **are**  $R$  **and**  $R$  **a** employ the following *PI average consensus filter* proposed in PI-Consensus Filter [Freeman et al. 2006]

$$
\begin{bmatrix}\n\dot{z}(t) \\
\dot{w}(t)\n\end{bmatrix} = \begin{bmatrix}\n-\gamma I - K_P L(\mathcal{G}(t)) & K_I L(\mathcal{G}(t)) \\
-K_I L(\mathcal{G}(t)) & 0\n\end{bmatrix} \begin{bmatrix}\nz(t) \\
w(t)\n\end{bmatrix} \quad \text{dynamic consensus filter}
$$
\n
$$
+ \begin{bmatrix}\n\gamma I \\
0\n\end{bmatrix} u(t) \quad \text{triangle gains}
$$
\n
$$
y(t) = \begin{bmatrix}\nI & 0\n\end{bmatrix} \begin{bmatrix}\nz(t) \\
w(t)\n\end{bmatrix}.
$$
\n
$$
[x(t)]
$$

 $\begin{array}{c} \n r \n \end{array}$  $\frac{1}{2}$  dynamic conconcus filters • dynamic consensus filter

*is the order of the overall filter is* independent *of the number*

1 v2

 $\frac{1}{\sqrt{2}}$ 

propose in this sub-section a distributed implementation for

particular by leveraging the results of Section IV. In the

 $\frac{1}{2}$ 

the *PI average consensus filter* [33] to distributedly compute

 $\mathcal{F}_{\mathcal{F}}$ 

aynamic consensus<br>tunable gains • tunable gains

in (20).

 $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  $\sum_{i=1}^n$ • tracks average of timevarying signal



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#### the eigenvector associated with the rigidity eigenvalue.<br>The rigidity eigenvalue associated with the rigidity eigenvalue.7 The rigidity eigenvalue associated with the buted Power Iteration algorithm now takes the form  $\mathcal{O}(\mathcal{E})$  and  $\mathcal{E}(\mathcal{E})$  and  $\mathcal{E}(\mathcal{E})$ Distributed Power Iteration

$$
\dot{\mathbf{v}}(t) = -\left(\mathbf{k}_1 T T^T + \mathbf{k}_2 \mathbf{R} + \mathbf{k}_3 \left(\frac{\hat{\mathbf{v}}(t)^T \hat{\mathbf{v}}(t)}{3|\mathcal{V}|} - 1\right) I\right) \hat{\mathbf{v}}(t)
$$





 $\sim$   $\sim$ 

הפקולתה להנדסת אוירונוטיקה וחלל *and k*<sup>3</sup> *satisfy the following conditions:*

Faculty of Aerospace Engineering  $\mathcal{C}$  **dynamics dynamics dynamics** consensus protocol intervals and  $\mathcal{C}$  $\mathbf{r}$  racuity of  $\mathbf{A}\mathbf{e}$ . Fig. 5. Block diagram showing PI consensus filters in calculation of  $\mathbb{R}^n$  c of Aerospace Engineering

#### Di<br>I  $\bigcap$  $\overline{U}$

V.2). Additionally, the underlying network is also dynamic as

where v ˆ is the *estimate* of the rigidity eigenvector, and the **Corollary V.4.** Let  $\overline{\mathbf{v}}_i^2(t)$  denote the output of the PI consensus | filter for estimating the quantity  $Avg(\hat{v}(t) \circ \hat{v}(t))$  for agent *i*. Then agent *i*'s estimate of the rigidity eigenvalue,  $\hat{\lambda}_7^i$ , can be  $\alpha$  btained as the proof methodologies are the same for the same *obtained as*

$$
\hat{\lambda}_7^i = \frac{k_3}{k_2} \left( 1 - \overline{\mathbf{v}}_i^2(t) \right).
$$

Theorem V.1. *Assume that the symmetric rigidity matrix R*



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## Putting it all together...

#### **power iteration**

$$
\dot{\mathbf{v}}_{i}^{x} = -k_{1}|\mathcal{V}| \left( \overline{\mathbf{v}}_{i}^{x} + z_{i}^{xy}(t) \hat{p}_{i,c}^{y} + z_{i}^{xz} \hat{p}_{i,c}^{z}(t) \right) - k_{2} \sum_{j \in \mathcal{N}_{i}(t)} W_{ij} \left( \hat{\mathbf{v}}_{i}^{x}(t) - \hat{\mathbf{v}}_{j}^{x} \right) - k_{3} \left( \overline{\mathbf{v}}_{i}^{x} - 1 \right) \hat{\mathbf{v}}_{i}^{x}
$$
\nframe estimation

\n
$$
\dot{\hat{p}}_{i,c} = \sum_{j \in \mathcal{N}_{i}(t)} (\|\hat{p}_{j,c} - \hat{p}_{i,c}\|^{2} - \ell_{ij}^{2}) (\hat{p}_{j,c} - \hat{p}_{i,c}) - \delta_{ii,c} \hat{p}_{i,c} - \delta_{ii} \left( \hat{p}_{i,c} - (p_{i} - p_{i,c}) \right) - \delta_{i\kappa} \left( \hat{p}_{\kappa,c} - (p_{\kappa} - p_{i,c}) \right)
$$
\nframe estimation

\n
$$
\begin{cases}\n\dot{\overline{\mathbf{v}}}_{i}^{x} = \gamma \left( \hat{\mathbf{v}}_{i}^{x} - \overline{\mathbf{v}}_{i}^{x} \right) - K_{P} \sum_{j \in \mathcal{N}_{i}} \left( \overline{\mathbf{v}}_{i}^{x} - \overline{\mathbf{v}}_{j}^{x} \right) + K_{I} \sum_{j \in \mathcal{N}_{i}(t)} \left( \overline{w}_{i}^{2x} - \overline{w}_{j}^{2x} \right) - \mathbf{P} \mathbf{I} \mathbf{-consensus} \mathbf{I} \right] \\
\dot{\overline{w}}_{i}^{2x} = -K_{I} \sum_{j \in \mathcal{N}_{i}(t)} \left( \overline{\mathbf{v}}_{i}^{2x} - \overline{\mathbf{v}}_{j}^{2x} \right) - K_{P} \sum_{j \in \mathcal{N}_{i}(t)} \left( \overline{w}_{i}^{2x} - \overline{\mathbf{v}}_{j}^{2x} \right) + K_{I} \sum_{j \in \mathcal{N}_{i}(t)} \left( \overline{w}_{i}^{2x} - \overline
$$

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#### (31) 1972년 1월 1일 - 대한민국의 대<br>1972년 12월 1일 - 대한민국의 i.  $\overline{\phantom{a}}$  (32) Rigidity Maintenance Controller



use output of rigidity estimator in control

$$
\xi_i^x = -\frac{\partial V(\hat{\lambda}_7^i)}{\partial \lambda_7} \sum_{j \in \mathcal{N}_i} W_{ij} \left( 2(\hat{p}_{i,c}^x - \hat{p}_{j,c}^x) (\hat{\mathbf{v}}_i^x - \hat{\mathbf{v}}_j^x)^2 +
$$
  
2(\hat{p}\_{i,c}^y - \hat{p}\_{j,c}^y) (\hat{\mathbf{v}}\_i^x - \hat{\mathbf{v}}\_j^x) (\hat{\mathbf{v}}\_i^y - \hat{\mathbf{v}}\_j^y) + 2(\hat{p}\_{i,c}^z - \hat{p}\_{j,c}^z) (\hat{\mathbf{v}}\_i^x - \hat{\mathbf{v}}\_j^x) (\hat{\mathbf{v}}\_i^z - \hat{\mathbf{v}}\_j^z) \right) +  
\n\frac{\partial W\_{ij}}{\partial p\_i^x} \hat{S}\_{ij},



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#### **Decentralized Rigidity Maintenance Control** with Range-only Measurements for **Multi-Robot Systems**

Daniel Zelazo, **Technion**, Israel

Antonio Franchi and Heinrich H. Bülthoff, Max Planck Institute for Biological Cybernetics, Germany

Paolo Robuffo Giordano, CNRS at Irisa, France



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## Some unresolved points....

- Power Iteration method assumes *distinct eigenvalues* 
	- proposed scheme can not guarantee that rigidity eigenvalue is unique - can lead to undesirable behaviors
- Formal stability proof for interconnection of all filters is missing
	- *ad hoc* implementation
	- *engineering art* to ensure each filter converges fast enough
	- alternative to power iteration method
- Need to relax requirement for "special agent"



## Outlook

#### Rigidity is an important architectural requirement for multi-agent systems!

- "bearing" rigidity
- full distributed implementations
- formation specification and trajectory tracking
- optimality
- rigidity matroids
- sub-modular optimization
- sensor fusion and localization

 $\bullet$  ...

$$
f(X \cup \{x\}) - f(X) \ge f(Y \cup \{x\}) - f(Y) \quad \bigwedge
$$

j

 $i$  k



*with edges* (vi, vk) *and* (v<sup>j</sup> , vk) *to the graph* G*. The* G *is infinitesimally rigid if and*





**הפקולתה להנדסת אוירונוטיקה וחלל Faculty of Aerospace Engineering** While very simple, this procedure does not indicate which nodes not indicate which nodes, if there are many  $\alpha$ 

**Tokyo Tech, September 13, 2013,** possible nodes to attach to, to connect to. We now proceed how this can be ac-**Engineering Tokyo, Japan** 

#### Acknowledgements

#### どもありがとうございます!

Questions?





Dr. Paolo Robuffo Giordano **Dr. Antonio Franchi** 







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