Network Feedback Passivation of Passivity-Short Multi-Agent Systems

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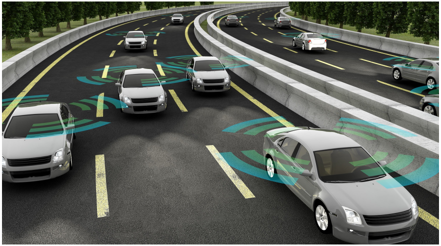
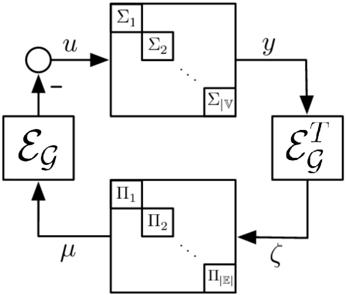
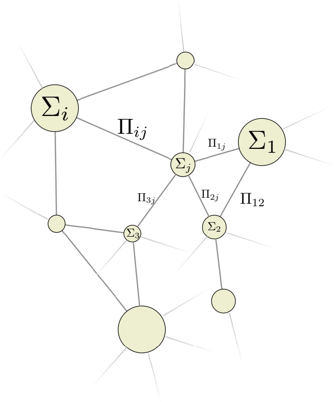
Multi-Agent Systems



Diffusively Coupled Networks

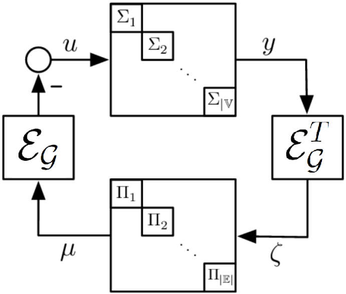




- Σ_i are nonlinear dynamical systems representing the agents.
- Π_e are nonlinear dynamical system representing the edge controllers.
- Can be used to model neural networks, vehicle networks, and networks of oscillators, among others.

Diffusively Coupled Networks

The output of a network with passive agents and controllers converges.



Convex opt. problem defined on $\mathcal{G}$

$$\min_{\mathrm{y},\zeta} \sum_{i \text{ vertex}} K_i^\star(\mathrm{y}_i) + \sum_{e \text{ edge}} \Gamma_e(\zeta_e)$$

$$\text{s.t.} \quad \mathcal{E}_\mathcal{G}^T \mathrm{y} = \zeta$$

Many systems in practice are not passive:

- Generators (always generate energy) [Harvey, 2016];
- Dynamics of robot systems from tourqe to position [Babu, 2018];
- Power-system network (turbine-governor dynamics) [Trip, 2018];

How to extend the network optimization framework when passivity does not hold?

Steady-State Relations

For the closed loop to reach a steady-state, each agent and controller must reach a steady-state.

Definition (Bürger et al.,2014)

The collection of all steady-state input-output pairs of system is called the *steady-state input-output relation*.

- A steady-state relation can be seen as a set-valued function. Given a steady-state input u and a steady-state output y, define:

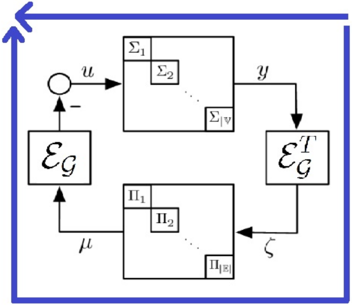
$$k(\mathrm{u}) = \{\mathrm{y} : \ (\mathrm{u}, \mathrm{y}) \in k\}$$
$$k^{-1}(\mathrm{y}) = \{\mathrm{u} : \ (\mathrm{u}, \mathrm{y}) \in k\}$$

- Let k_i be the relations for the agents Σ_i, γ_e be the relations for the controllers Π_e, and let k, γ be the stacked relations.

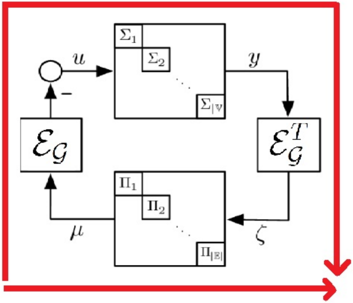
Steady-State Equations

- Let $\mathrm{u}, \mathrm{y}, \zeta, \mu$ be a steady-state of the closed-loop system. The consistency of the steady-states yields the following "equations":

$$0 \in k^{-1}(\mathrm{y}) + \mathcal{E}_{\mathcal{G}}\gamma(\mathcal{E}_{\mathcal{G}}^T \mathrm{y})$$

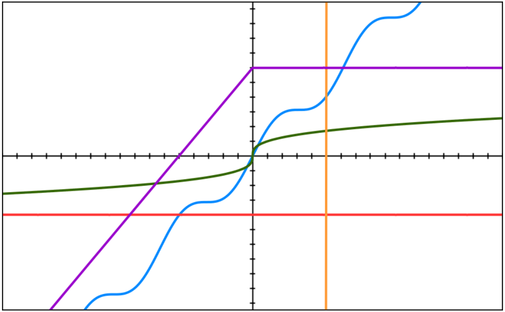


$$0 \in \gamma^{-1}(\mu) - \mathcal{E}_{\mathcal{G}}^T k(-\mathcal{E}_{\mathcal{G}}\mu)$$



How to ensure the existence of a solution to the consistency equations?

The Role of Maximal Monotonicity



Theorem

Suppose all the relations k_i, γ_e are maximally monotone. Then both consistency "equations" have a solution. In other words, there is a vector y *such that* $0 \in k^{-1}(\mathrm{y}) + \mathcal{E}_{\mathcal{G}}\gamma(\mathcal{E}_{\mathcal{G}}^T \mathrm{y})$*, and a vector* μ *such that* $0 \in \gamma^{-1}(\mu) - \mathcal{E}_{\mathcal{G}}^T k(-\mathcal{E}_{\mathcal{G}}\mu)$.

- Thus, we demand that k_i and γ_e are maximally monotone.

Integral Functions of Maximal Monotonic Relations

Rockafellar's Theorem (Rockafellar,1969)

A relation is maximally monotone if and only if it is the subgradient of some convex function.

- Let $K_i, K_i^\star, \Gamma_e, \Gamma_e^\star$ be integral functions of $k_i, k_i^{-1}, \gamma_e, \gamma_e^{-1}$.
- Subgraident is a generalized form of the gradient. If k_i is smooth then $\nabla K_i = k_i$
- Let $K = \sum_i K_i$ and $\Gamma = \sum_e \Gamma_e$.
- In calculus, minimizing a function F can be done by solving the equation $\nabla F = 0$. We do the opposite.

$0 \in k^{-1}(\mathrm{y}) + \mathcal{E}_\mathcal{G}\gamma(\mathcal{E}_\mathcal{G}^T \mathrm{y})$	$0 \in \gamma^{-1}(\mu) - \mathcal{E}_\mathcal{G}^T k(-\mathcal{E}_\mathcal{G}\mu)$
$\min_{\mathrm{y},\zeta} \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e)$ $s.t. \quad \mathcal{E}_\mathcal{G}^T \mathrm{y} = \zeta$	$\min_{\mathrm{u},\mu} \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e)$ $s.t. \quad \mathrm{u} + \mathcal{E}_\mathcal{G}\mu = 0.$

MEIP

The discussion above motivates the following refinement of passivity[1]

Definition (MEIP)

A SISO system is called *(output-strictly) maximal monotone equilibrium-independent passive* (MEIP) if:

1. The system is (output-strictly) passive with respect to any steady-state input-output pair.
2. The steady-state input-output relation is maximally-monotone.

Many SISO systems are MEIP:

- Port-Hamiltonian systems;
- Reaction-diffusion systems;
- Gradient-descent systems;
- Single integrators.

[1] M. Burger, D.Zelazo and F. Allgower, "Duality and network theory in passivity-based cooperative control", Automatica, vol. 50, no. 8, pp, 2051–2061, 2014.

Analysis Theorem of MEIP Multi-Agent Systems

Theorem (Bürger, Zelazo and Allgöwer, 2014)

Consider the closed loop system, and suppose all agents Σ_i are output-strictly MEIP and all edge controllers Π_e are MEIP.
Then the signals $u(t), y(t), \zeta(t)$ and $\mu(t)$ converge to constants $\hat{u}, \hat{y}, \hat{\zeta}$ and $\hat{\mu}$ which are optimal solutions to the problems (OFP) and (OPP):

(OPP)	**(OFP)**
$\min_{\mathrm{y},\zeta} \quad \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e)$	$\min_{\mathrm{u},\upmu} \quad \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\upmu_e)$
$s.t. \quad \mathcal{E}_\mathcal{G}^T \mathrm{y} = \zeta$	$s.t. \quad \mathrm{u} + \mathcal{E}_\mathcal{G}\upmu = 0.$

Network Signal	Optimization Variable
Agents' Output $y_i(t)$	y_i
Network Controllers Input $\zeta_e(t)$	ζ_e
Network Controllers Output $\mu_e(t)$	$\upmu_e$
Agents' Input $u_i(t)$	u_i

Passive-Short Systems

We focus on output passive-short systems:

Definition

Let Υ be a SISO dynamical system with steady-state pair (u_0, y_0). We say that Υ is *output passive-short* w.r.t. (u_0, y_0) if there's a storage function S and $\rho < 0$, so that for any input $u(t)$ we have:

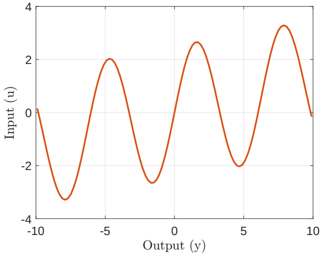
$$\frac{d}{dt}S(x(t)) \leq (y(t) - y_0)(u(t) - u_0) - \rho(y(t) - y_0)^2.$$

Definition

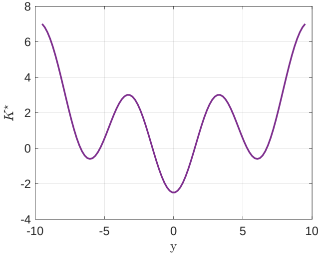
Let Υ be a SISO dynamical system. We say that Υ is *equilibrium-independent output passive-short* (EI-OPS) if there is some $\rho < 0$ such that the system is output-passive short with parameter ρ with respect to all equilibria.

Failure of the Network Optimization Framework for Passive-Short Systems

- Consider a network of agents of the form $\dot{x}_i = -\nabla U(x_i) + u_i, y_i = x_i$.
- Take $U(x_i) = 2.5(1 - \cos(x_i)) + 0.1x_i^2$. The agents are not MEIP, but rather EI-OPS with $\rho = -2.4$



k_i^{-1}

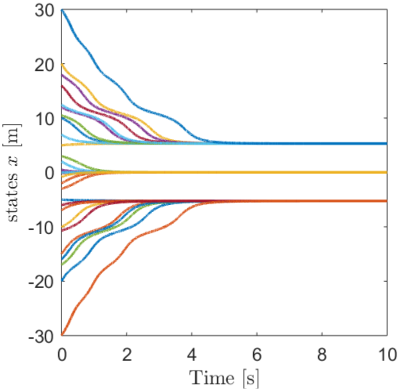


$K_i^\star$

- Take controllers as static gains of size 1, so $\Gamma(\zeta) = 0.5\zeta^2$.
- The minimum of (OPP) is achieved at $\mathrm{y} = \zeta = 0$.

Failure of the Network Optimization Framework for Passive-Short Systems

- The closed-loop system was run. The trajectory can be seen below.



- **The closed-loop system converges to a value other than the minimizer of (OPP)**
- This happens due to the nonconvexity of the function K.

Agent-Based Convexification and Passivation

- Idea - Try to convexify (OPP) by adding a Tikhonov term $\sum_i \frac{1}{2}\beta_i y_i^2$ for some $\beta_i > 0$.
- The problem (OPP) transforms into:

$$\begin{aligned} \min_{\mathrm{y},\zeta} \quad & \sum_i (K_i^\star(\mathrm{y}_i) + \frac{1}{2}\beta_i \mathrm{y}_i^2) + \sum_e \Gamma_e(\zeta_e) \\ \text{s.t.} \quad & \mathcal{E}_\mathcal{G}^T \mathrm{y} = \zeta \end{aligned} \qquad \text{(ROPP)}$$

- We denote the agents' regularized integral functions $\Lambda_i^\star(\mathrm{y}_i) = K_i^\star(\mathrm{y}_i) + \frac{1}{2}\beta_i \mathrm{y}_i^2$
- How can we interpret $\Lambda_i^\star$?

Theorem (Jain, S., Zelazo, LCSS 2018)

Consider the augmented agent $\tilde{\Sigma}_i$ achieved by considering an output-feedback $u_i = v_i - \beta_i y_i$ for the i-th agent Σ_i. Then $\tilde{\Sigma}_i$ has an integral function, and it equal to $\Lambda_i^\star(\mathrm{y}_i)$

Agent-Based Convexification and Passivation

Theorem (Jain, S., Zelazo, LCSS 2018)

Consider a diffusively-coupled network with EI-OPS agents and MEIP controllers. Let $\rho_1, \cdots, \rho_n$ be the agent's shortage-of-passivity parameters. If $\beta_i > |\rho_i|$ for $i = 1, \cdots, n$, then (ROPP) is convex.
Moreover, the augmented closed-loop system, with the augmented agents and original controllers, globally asymptotically converges, and its steady-state is the minimizer of (ROPP)

$$\begin{aligned} \min_{\mathrm{y},\zeta} \quad & \sum_i (K_i^\star(\mathrm{y}_i) + \frac{1}{2}\beta_i \mathrm{y}_i^2) + \sum_e \Gamma_e(\zeta_e) \\ \text{s.t.} \quad & \mathcal{E}_\mathcal{G}^T \mathrm{y} = \zeta \end{aligned} \tag{ROPP}$$

Agent-Based Convexification and Passivation

- The Tikhonov regularization term $\sum_i \beta_i \mathrm{y}_i^2$ for (OPP) resulted in the classical output-feedback passivizing term $u_i = v_i - \beta_i y_i$.
- This regularization term can't always be applied
  - Some agents might not be able to sense their output y_i in a global framework, but only relative outputs $y_i - y_j$.
  - Some agents might not be amenable, and will not implement said feedback (e.g. in open networks).

Can we find another regularization term that yields a network-based feedback term?

Network Convexification and Passivation

- Idea - Try to convexify (OPP) by adding a *network* Tikhonov term $\sum_e \frac{1}{2}\beta_e\zeta_e^2$ for some $\beta_i > 0$.
- The problem (OPP) transforms into:

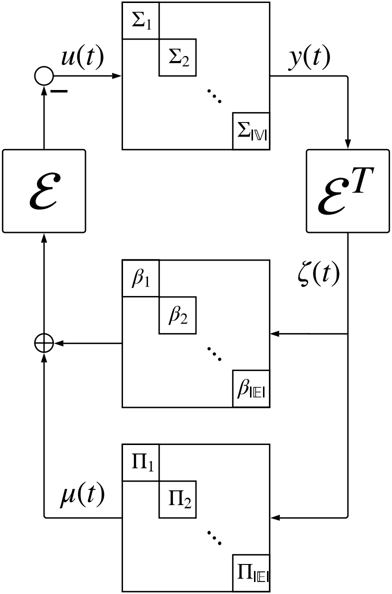
$$\begin{aligned} \min_{\mathrm{y},\zeta} \quad & \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \frac{1}{2}\beta_e\zeta_e^2 + \sum_e \Gamma_e(\zeta_e) \\ \text{s.t.} \quad & \mathcal{E}_\mathcal{G}^T \mathrm{y} = \zeta \end{aligned} \qquad \text{(NROPP)}$$

- We consider the function $\Lambda_N^\star(\mathrm{y}) = \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \frac{1}{2}\beta_e(\mathcal{E}_\mathcal{G}^T \mathrm{y})_e^2$.
- How can we interpret $\Lambda_N^\star$?

Theorem

Consider the augmented agents $\tilde{\Sigma}$ achieved by considering a network-feedback $u = v - \mathcal{E}_\mathcal{G}\mathrm{diag}(\beta)\mathcal{E}_\mathcal{G}^T y$. $\tilde{\Sigma}$ is a MIMO system with input-output steady-state relation λ_N, and $\Lambda_N^\star$ is the integral function of λ_N^{-1}.

Network Convexification and Passivation



Network Convexification and Passivation

- Can we choose the gains β_e-s so that $\Lambda_N^\star$ is convex?

Theorem

Suppose the graph $\mathcal{G}$ is connected. Let $\bar{\rho}$ be the average of the output-passivity indices $\rho_1, \cdots, \rho_N$ of the agents $\Sigma_1, \cdots, \Sigma_N$. If $\bar{\rho} > 0$, then there exists gains β_e so that $\Lambda_N^\star$ is strictly convex. In that case the system $\tilde{\Sigma}$ is passive with respect to all equilibria.

- Actually, we can choose equal gains of size $\mathbf{b} + \epsilon$, where

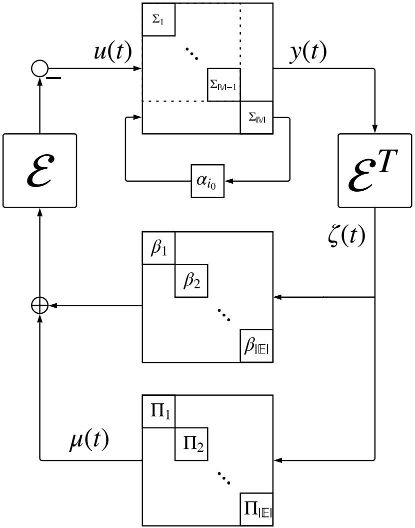
$$\mathbf{b} = \frac{\lambda_{max}(\bar{\rho}^{-1}\mathcal{E}_\mathcal{G}^T \mathrm{diag}(\rho)^2 \mathcal{E}_\mathcal{G} - \mathcal{E}_\mathcal{G}^T \mathrm{diag}(\rho)\mathcal{E}_\mathcal{G})}{\lambda_2(\mathcal{G})^2}$$

Network Convexification and Passivation

Theorem

Consider a diffusively-coupled network with EI-OPS agents and MEIP controllers. Let $\rho_1, \cdots, \rho_n$ be the agent's shortage-of-passivity parameters, and let $\bar{\rho}$ be their average. If $\bar{\rho} > 0$ and for all edges e, $\beta_e > \mathbf{b}$, then (NROPP) is convex.
Moreover, the augmented closed-loop system, with the augmented agents and original controllers, globally asymptotically converges, and its steady-state is the minimizer of (NROPP)

$$\begin{aligned} \min_{\mathrm{y},\zeta} \quad & \sum_i K_i^\star(\mathrm{y}_i) + \sum_e (\Gamma_e(\zeta_e) + \frac{1}{2}\beta_e \zeta_e^2) \\ \text{s.t.} \quad & \mathcal{E}_\mathcal{G}^T \mathrm{y} = \zeta \end{aligned} \tag{NROPP}$$



What to do when $\bar{\rho} \leq 0$?
Add another Tikhonov term $\sum_{i=1}^{n} \alpha_i \mathrm{y}_i^2$.

Only a small subset of the nodes need to sense their own output and be amenable to the network designer.

Example: Vehicle Network

- Consider a network of 100 vehicles trying to coordinate their velocity
- The dynamics of the velocity x_i of the i-th agent evolves as

$$\dot{x}_i = \kappa_i(-x_i + V_0^i + V_1^i u_i)$$

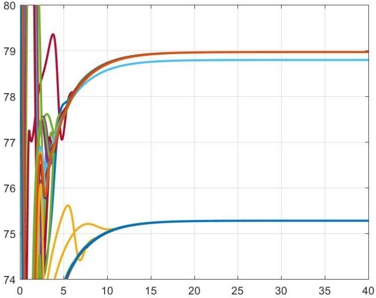
where $u_i = \sum_{j \sim i} \tanh(p_j - p_i)$

- The system is EI-OPS with $\rho_i = \kappa_i$. $\kappa_i < 0$ corresponds to drowsy driving.
- (OPP) is written as:

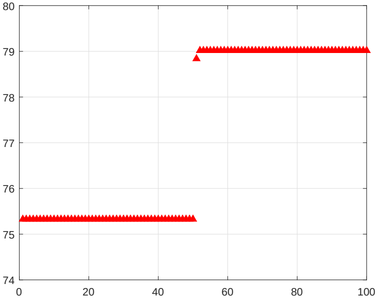
$$\begin{aligned} \min_{\mathrm{y},\zeta} \quad & \sum_i \frac{1}{2\,V_i^1}(\mathrm{y}_i - V_i^0)^2 + \sum_e \frac{1}{2}|\zeta_e| \\ \text{s.t.} \quad & \mathcal{E}_{\mathcal{G}}^T \mathrm{y} = \zeta \end{aligned}$$

We implement the network-only regularization technique with $\beta_e = \mathbf{b} + \epsilon$.

Example: Vehicle Network



(a) Vehicles' trajectories under network-only regularization.



(b) Asymptotic behaviour predicted by (NROPP).

Conclusions

- Network optimization is a powerful tool that appears naturally in multi-agent systems.
- For non-passive agents, the network optimization framework might fail to predict the true steady-state limit.
- For EI-OPS agents, regularizing (OPP) results in a passivizing feedback, validating the network optimization framework.
- One can use network-based regularization terms to help get network-based passivation.
- **How to choose the self-regulating nodes to get small gains?**

Acknowledgements

GIF German-Israeli Foundation for Scientific Research and Development

designs

IAAC Israeli Association for Automatic Control