SWARM 2017

SENSOR MODALITIES IN MULTI-ROBOT COORDINATION: CONSTRAINT AND SOLUTIONS

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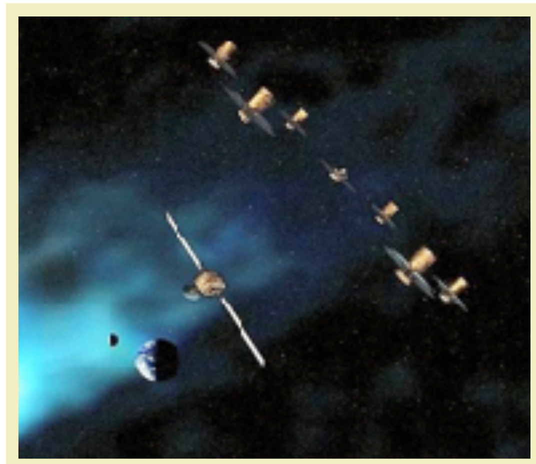
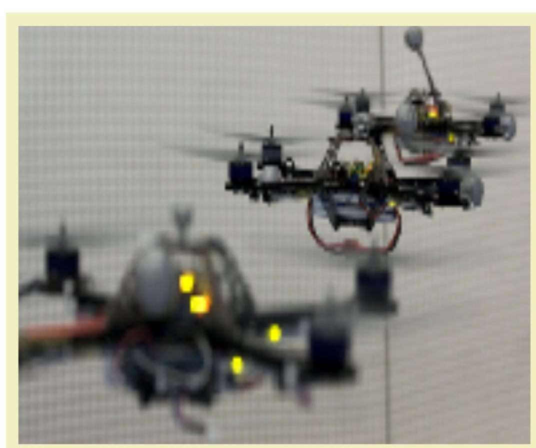
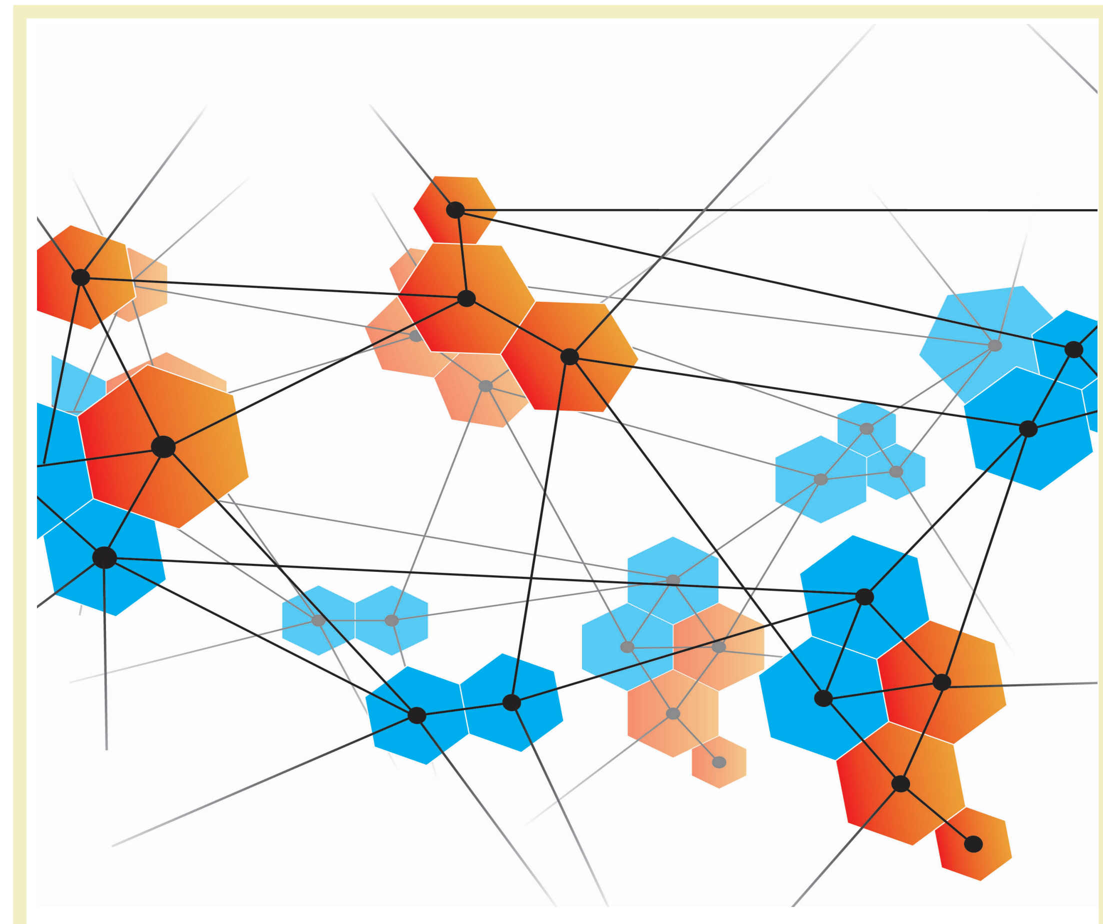
WHAT IS MULTI-ROBOT COORDINATION?

WHAT IS MULTI-ROBOT COORDINATION?
(AGENT)



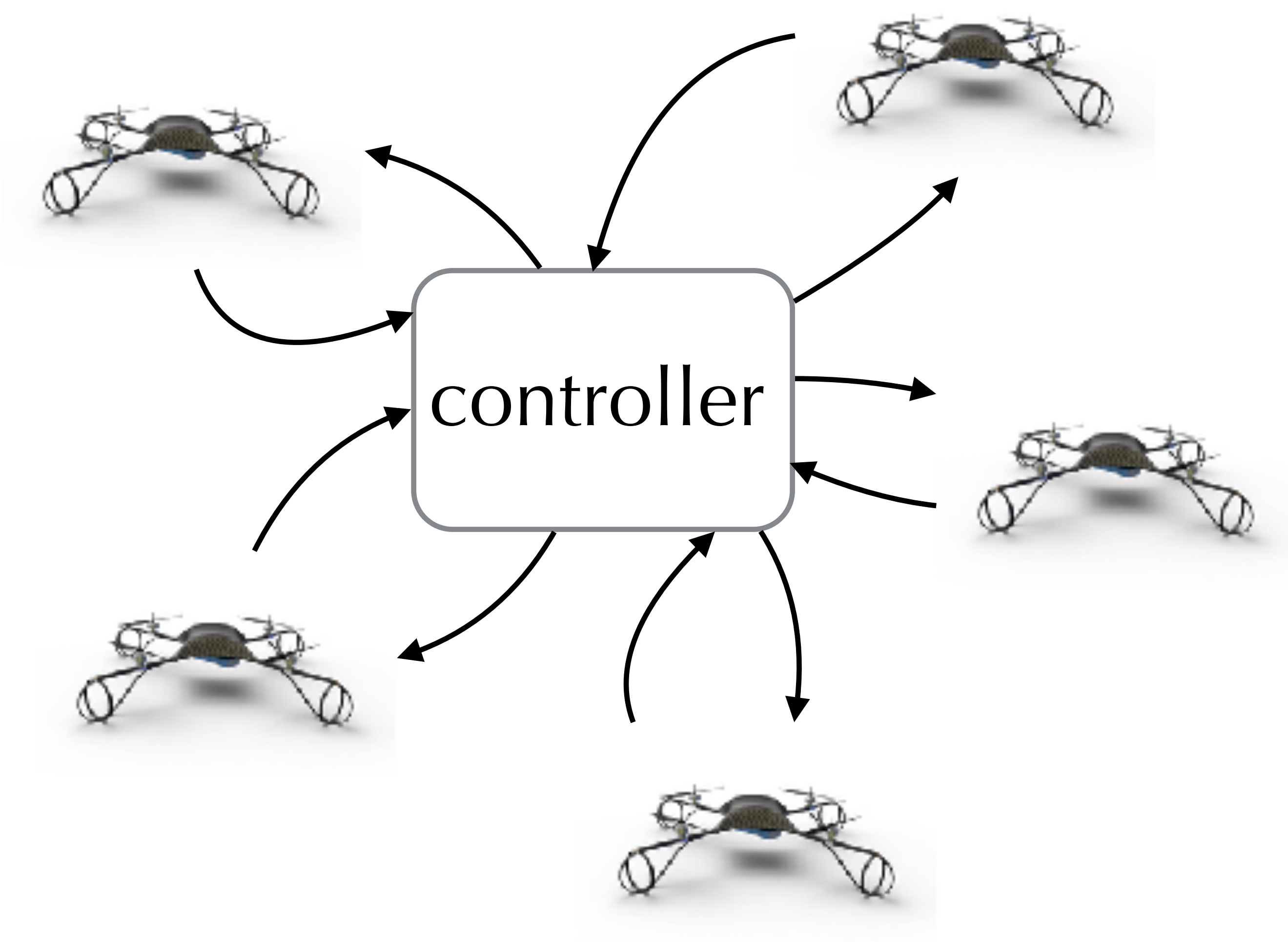




NETWORKS OF DYNAMICAL SYSTEMS ARE ONE OF THE ENABLING TECHNOLOGIES OF THE FUTURE

HOW DO WE CONTROL MULTI-ROBOT SYSTEMS?

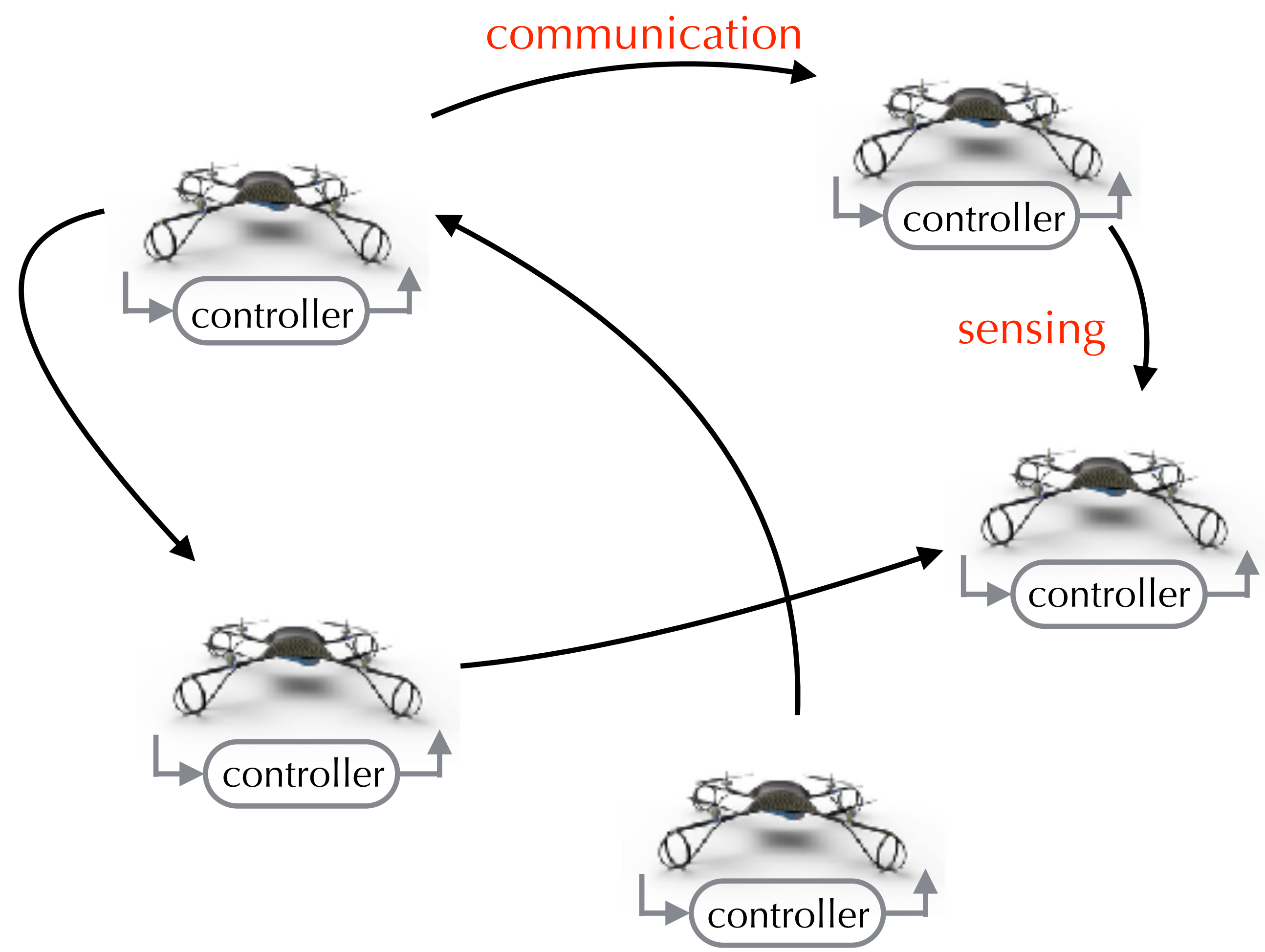
centralized approach



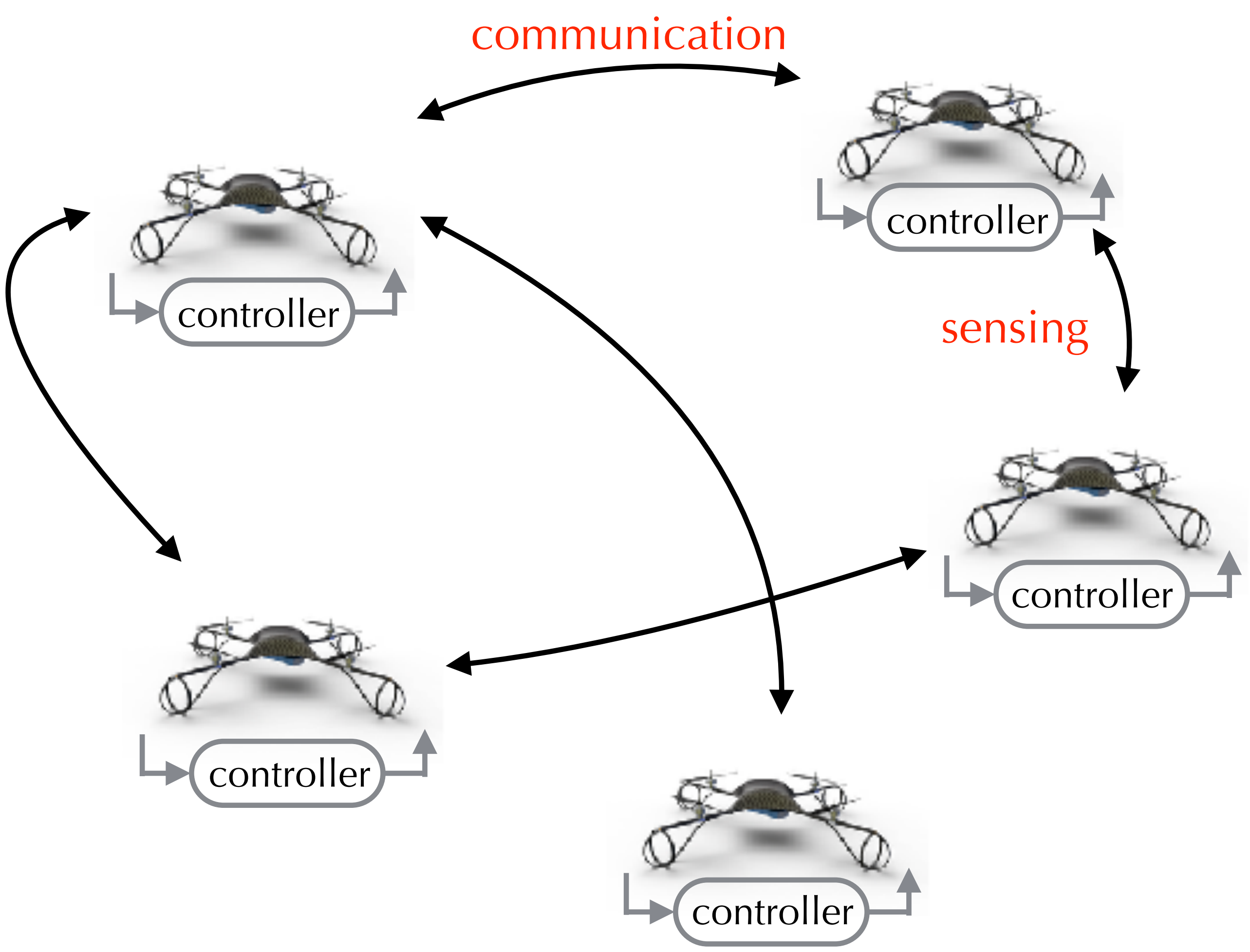
not scalable

not robust

decentralized/distributed approach

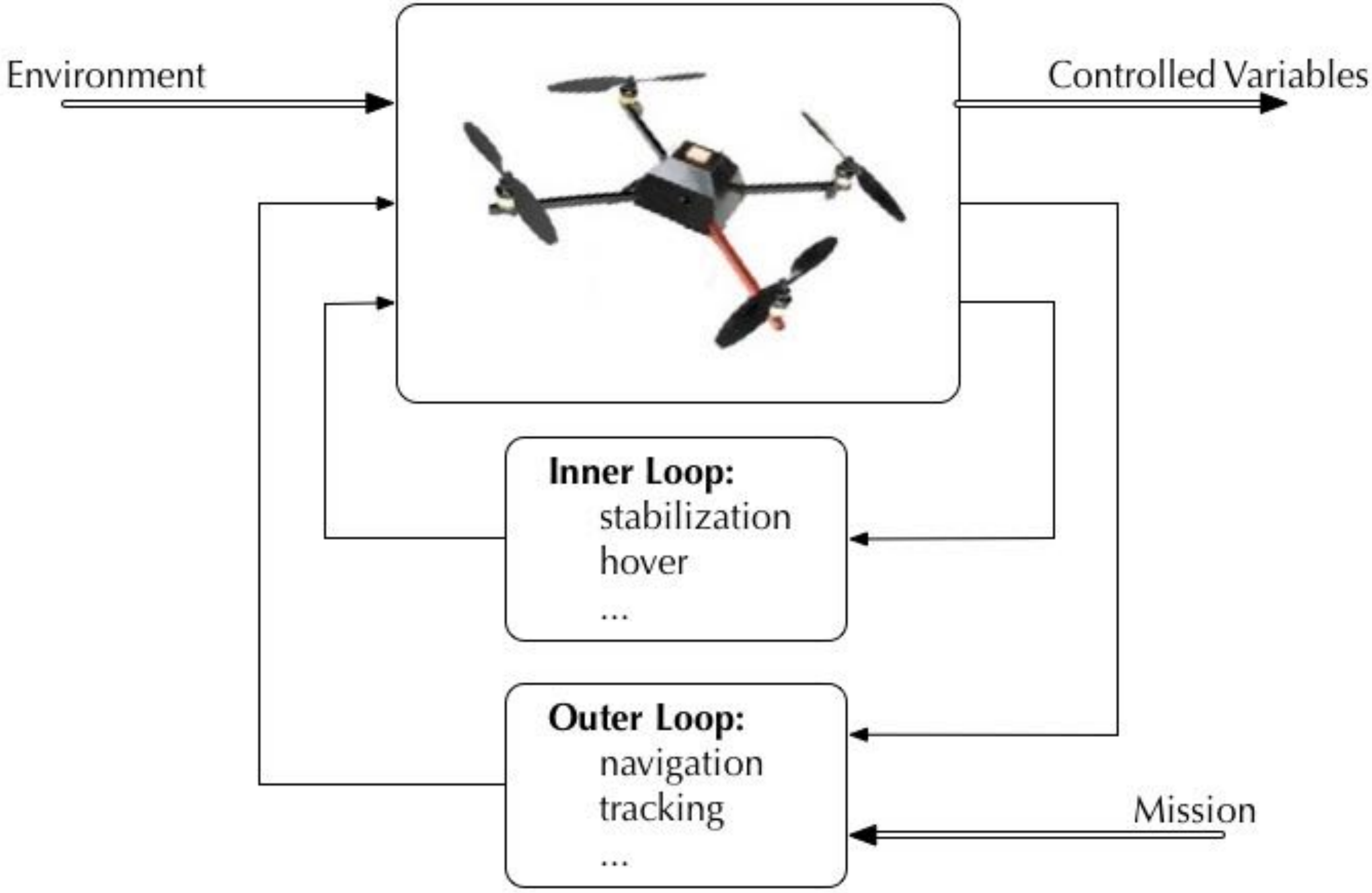


HOW DO WE CONTROL MULTI-ROBOT SYSTEMS?

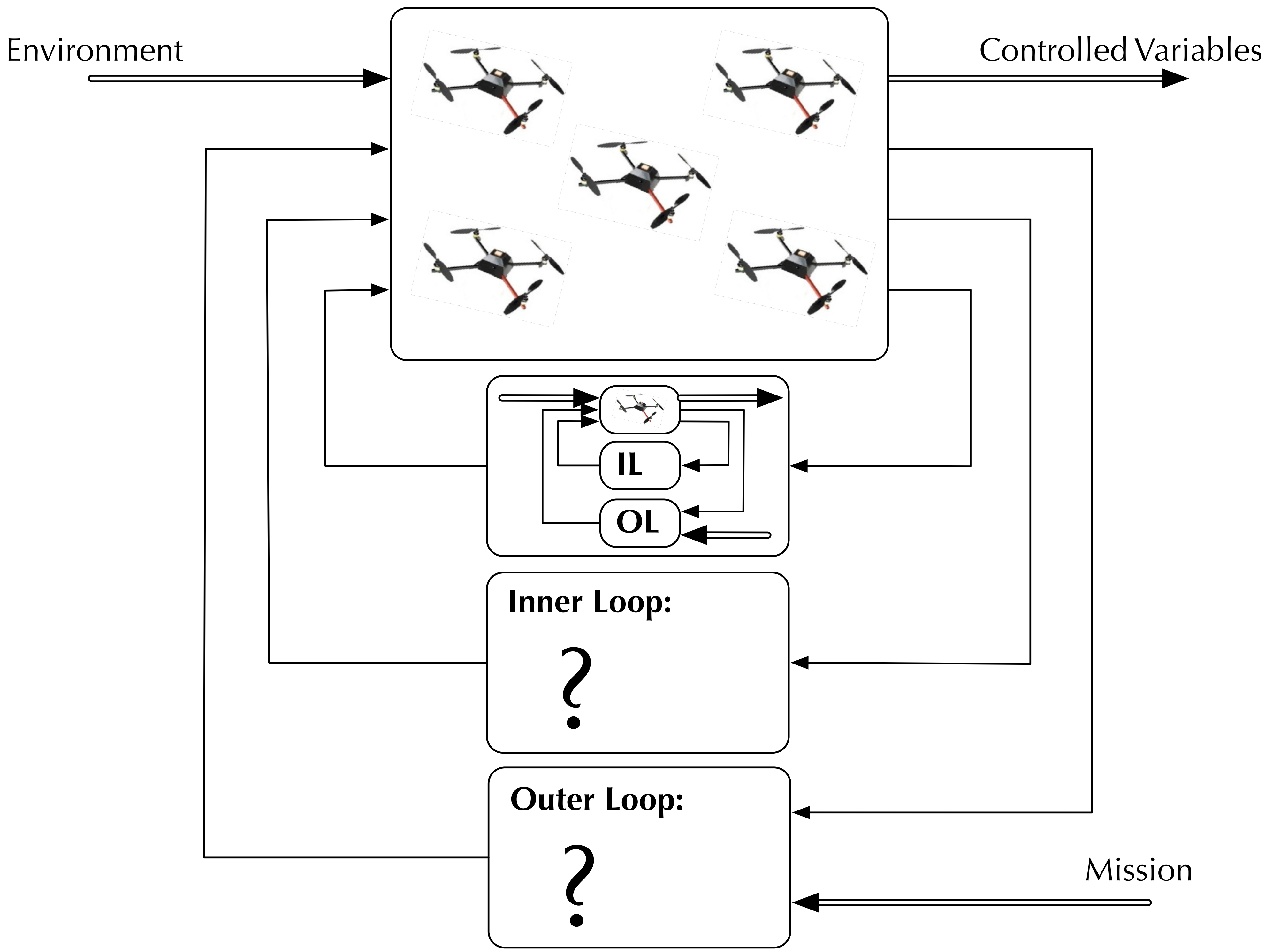


What is the control architecture?

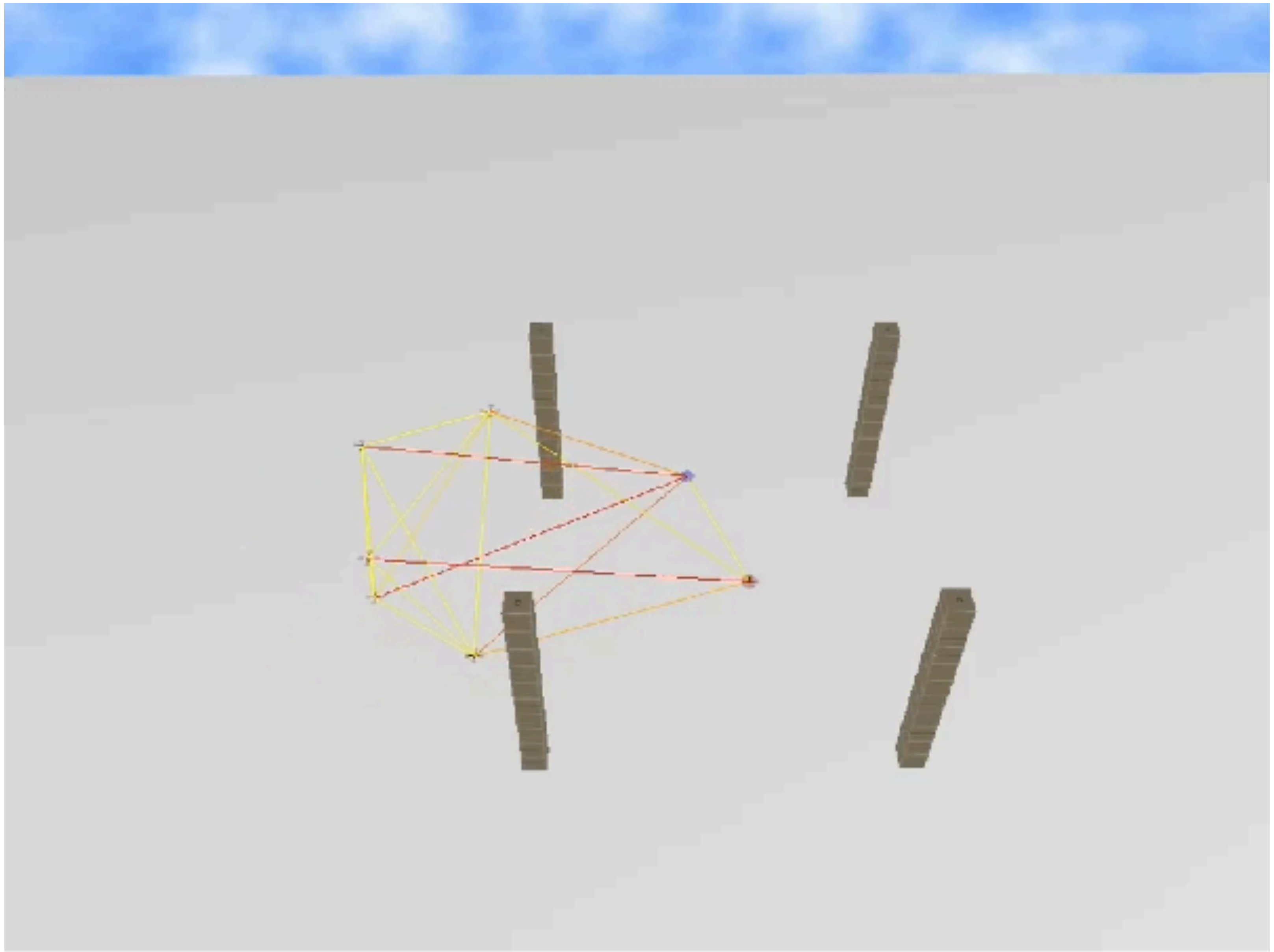
1 ROBOT


Environment
Controlled Variables
Inner Loop:
stabilization
hover
...
Outer Loop:
navigation
tracking
...
Mission

MULTI-ROBOT SYSTEM

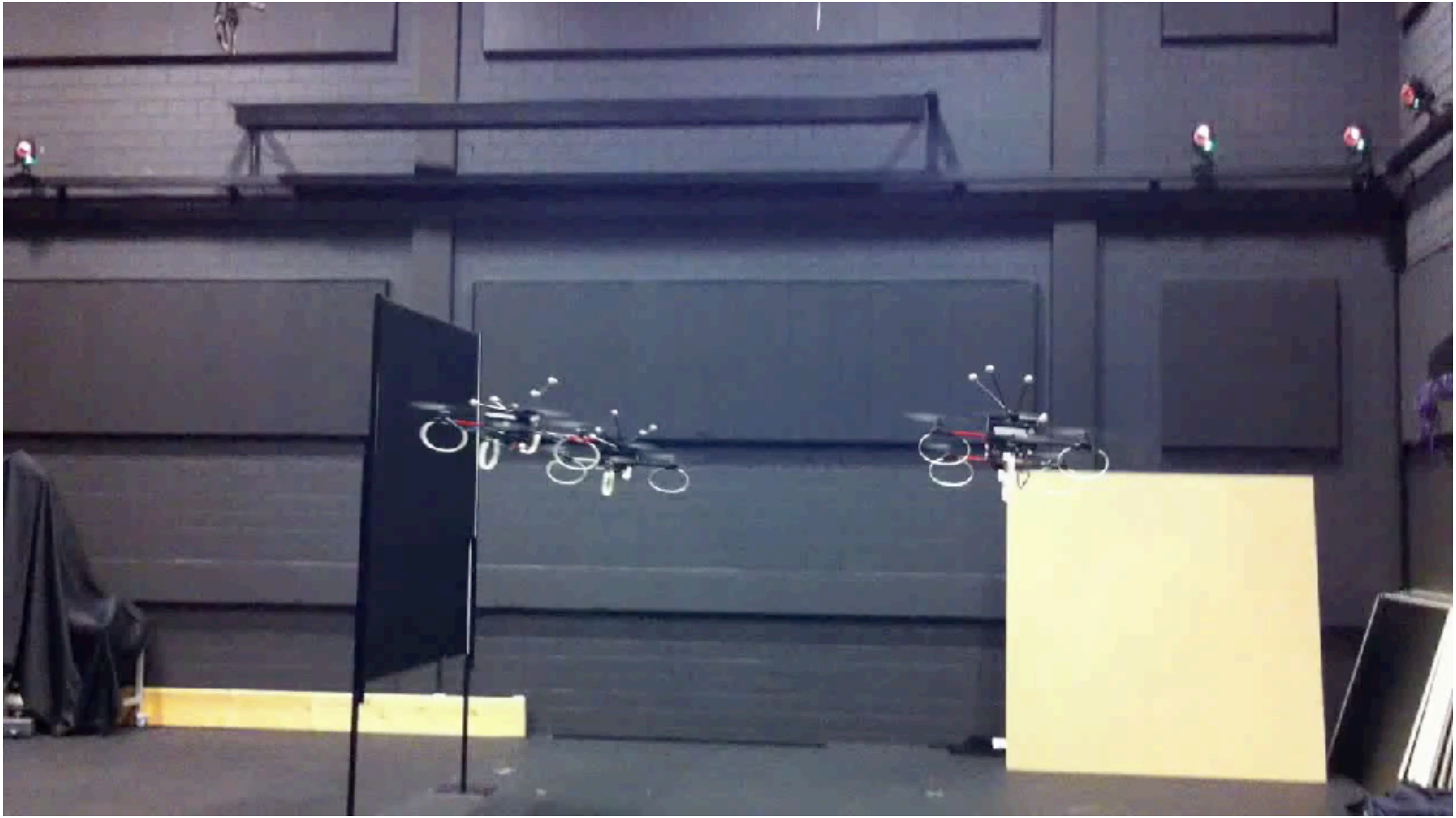


WHAT IS THE ARCHITECTURE OF A MULTI-ROBOT SYSTEM?



CONNECTIVITY

Ji and Egerstedt, 2007
Dimarogonas and Kyriakopoulos, 2008
Yang *et al.*, 2010
Robuffo Giordano *et al.*, 2013

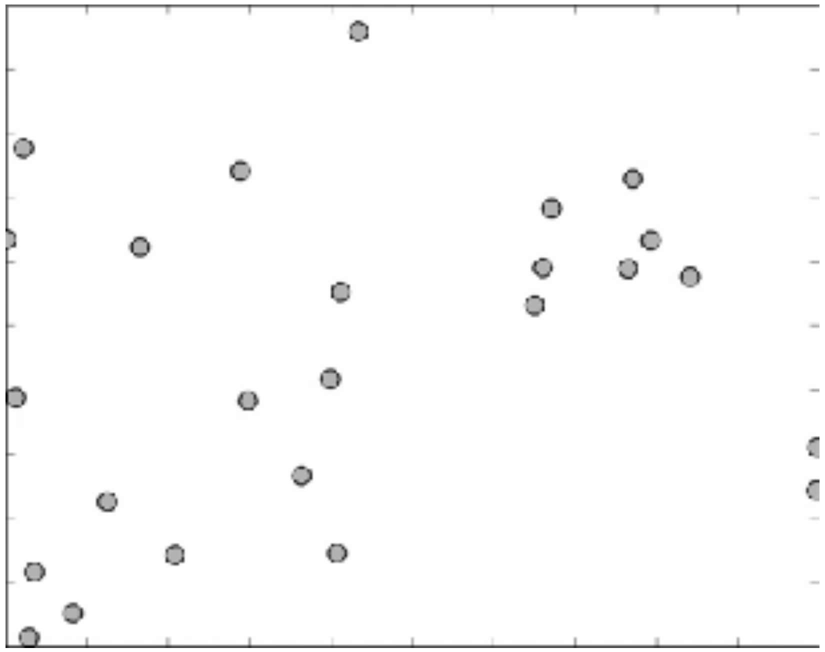


Courtesy of P. Robuffo Giordano and A. Franchi

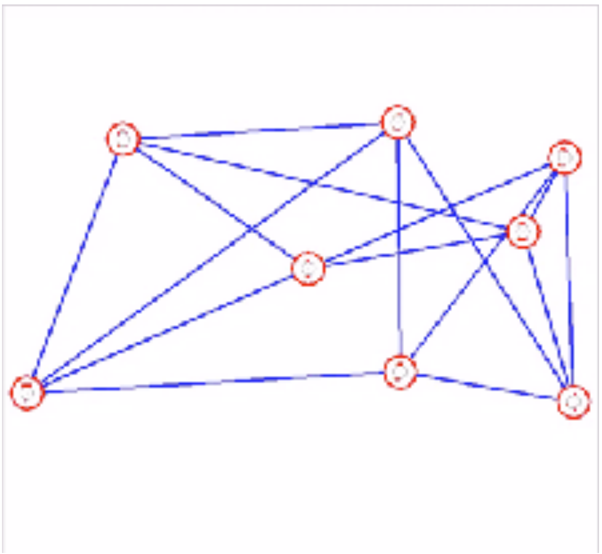
Solutions to coordination problems in multi-robot systems are highly dependent on the sensing and communication mediums available!

COORDINATION OBJECTIVES

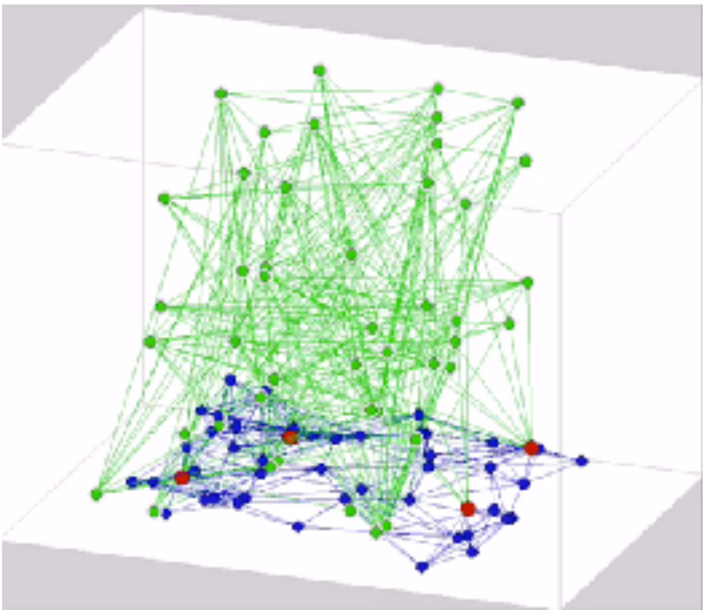
rendezvous



formation control



localization

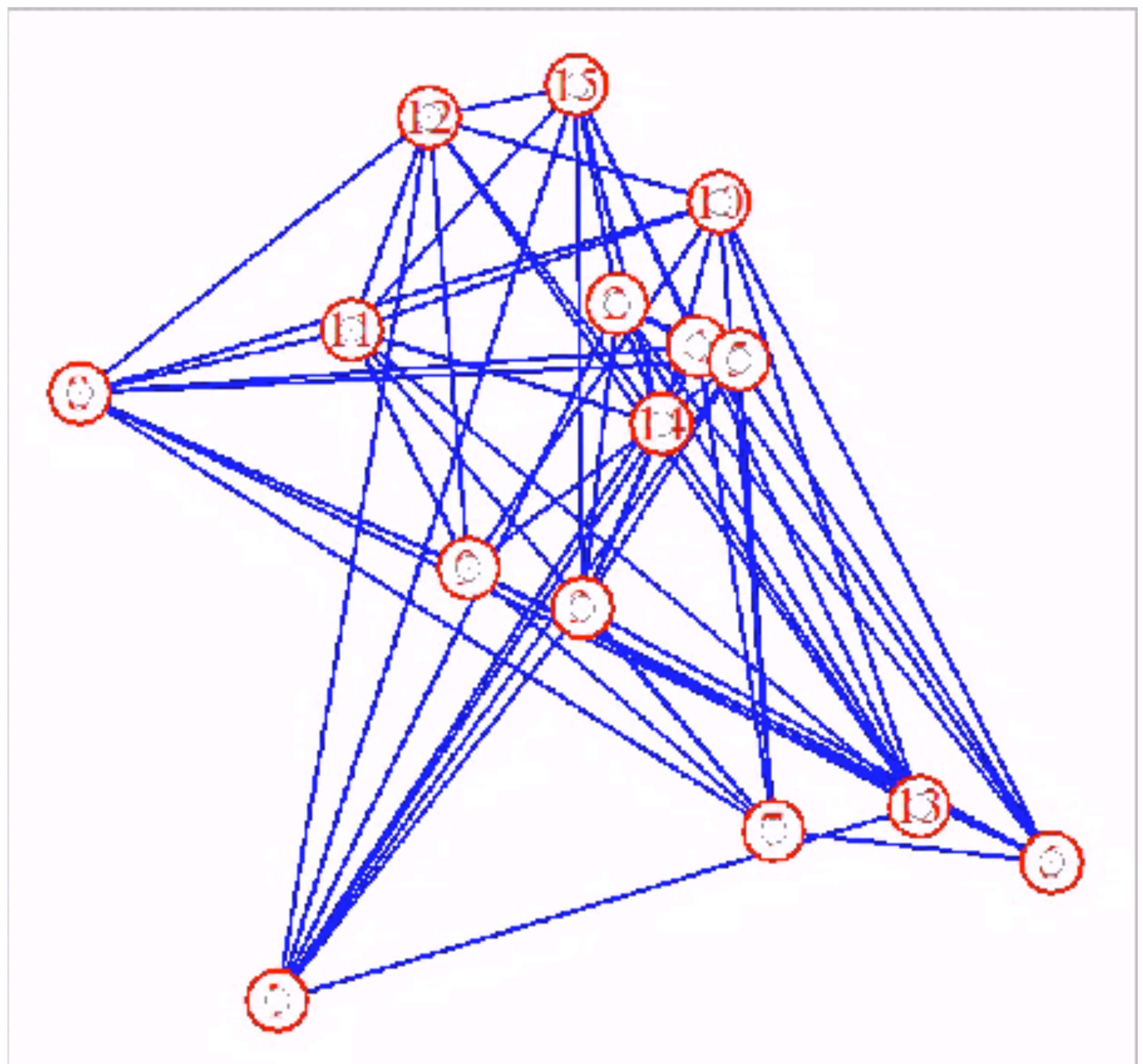


Does the control strategy need to change with different sensing/communication?

Are there common architectural requirements that do not depend on the choice of sensing?

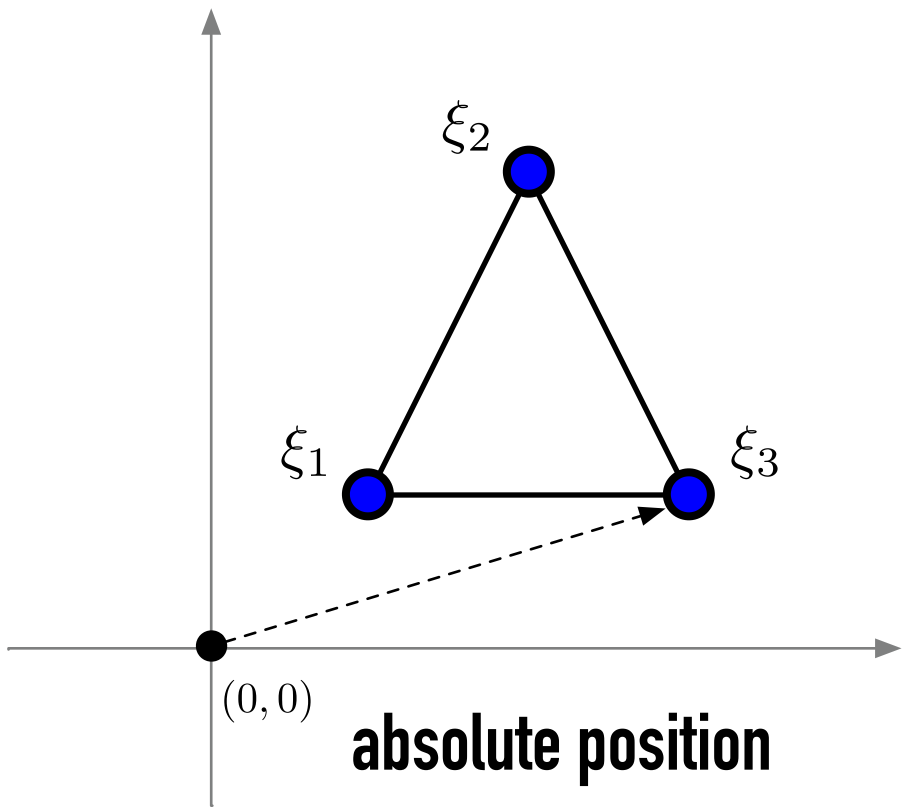
FORMATION CONTROL

Given a team of robots endowed with the ability to sense/ communicate with neighboring robots, design a control for each robot using only *local information* that moves the team into a desired formation shape.

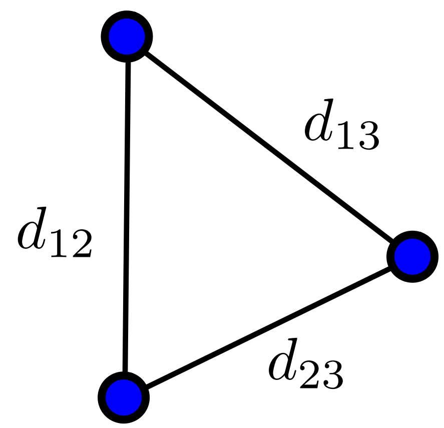


FORMATION DETERMINATION = SENSOR SELECTION

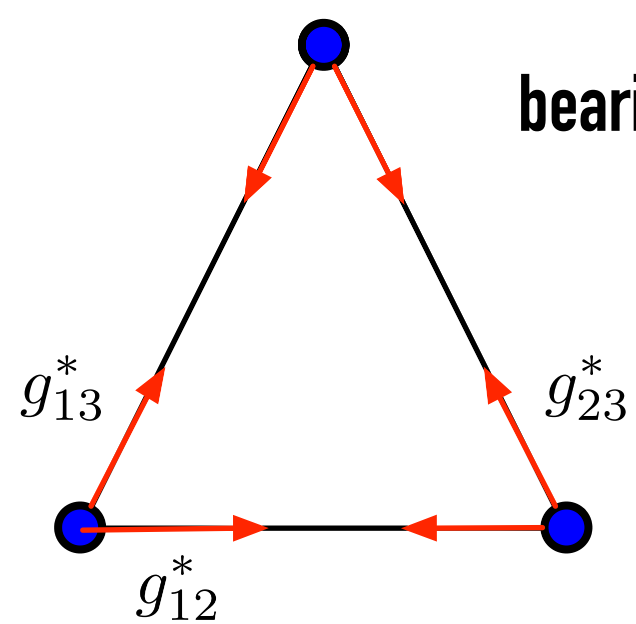
HOW TO DEFINE A SHAPE



absolute position



distances



bearings

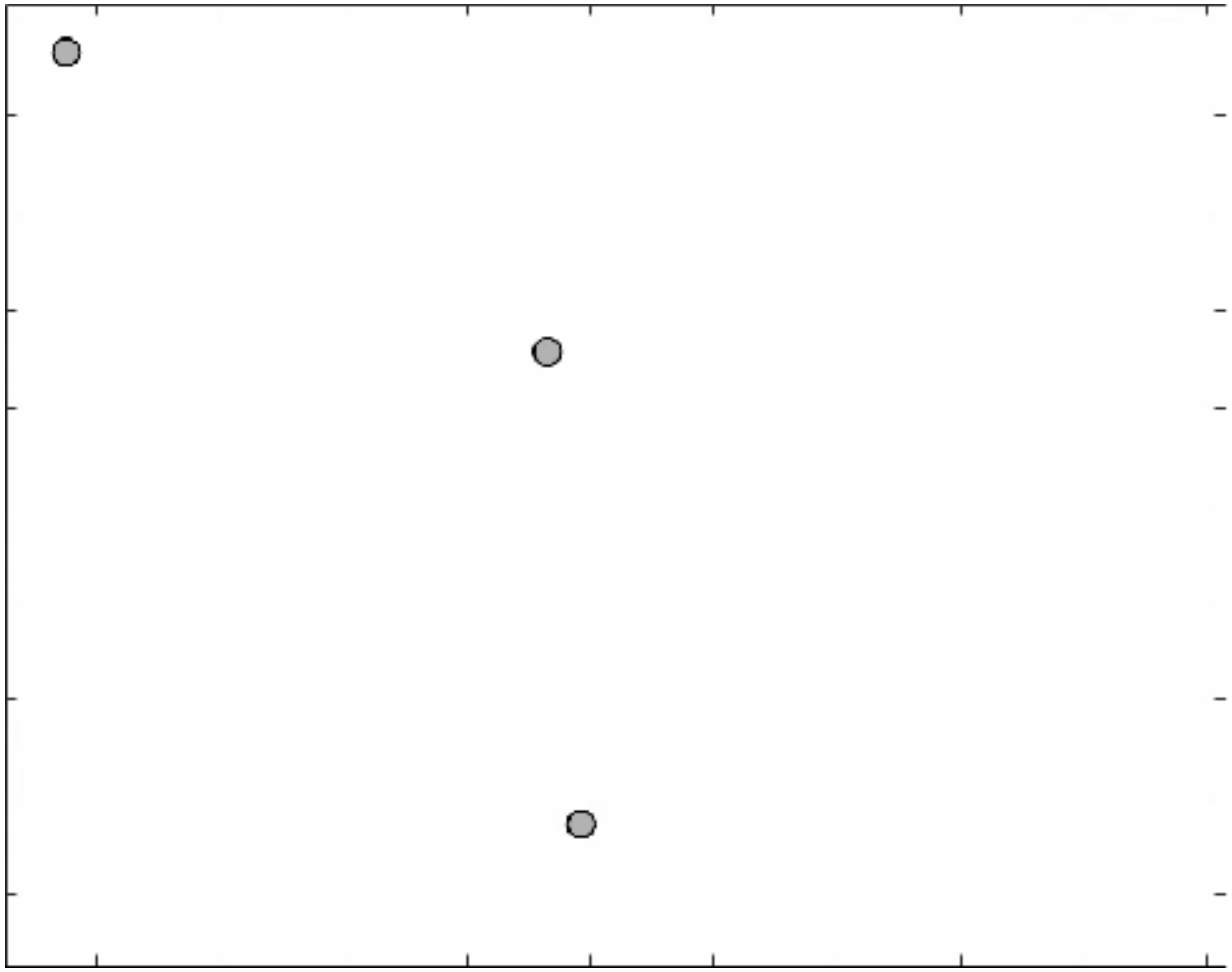
EXAMPLE: FORMATION CONTROL

"robots" – modeled as kinematic point mass

$$\dot{x}_i = u_i$$

Assumptions

- GLOBAL COORDINATE FRAME
- RELATIVE POSITION MEASUREMENTS
- NO SENSING CONSTRAINTS (360°)
- SENSING AND COMMUNICATION



Formation

- SPECIFIED BY (ABSOLUTE) TARGET POSITIONS

$$\xi_i \in \mathbb{R}^2$$

Control

$$u_i = \sum_{i \sim j} ((x_j - \xi_j) - (x_i - \xi_i))$$

THE "CONSENSUS" PROTOCOL

CONSENSUS

Formation

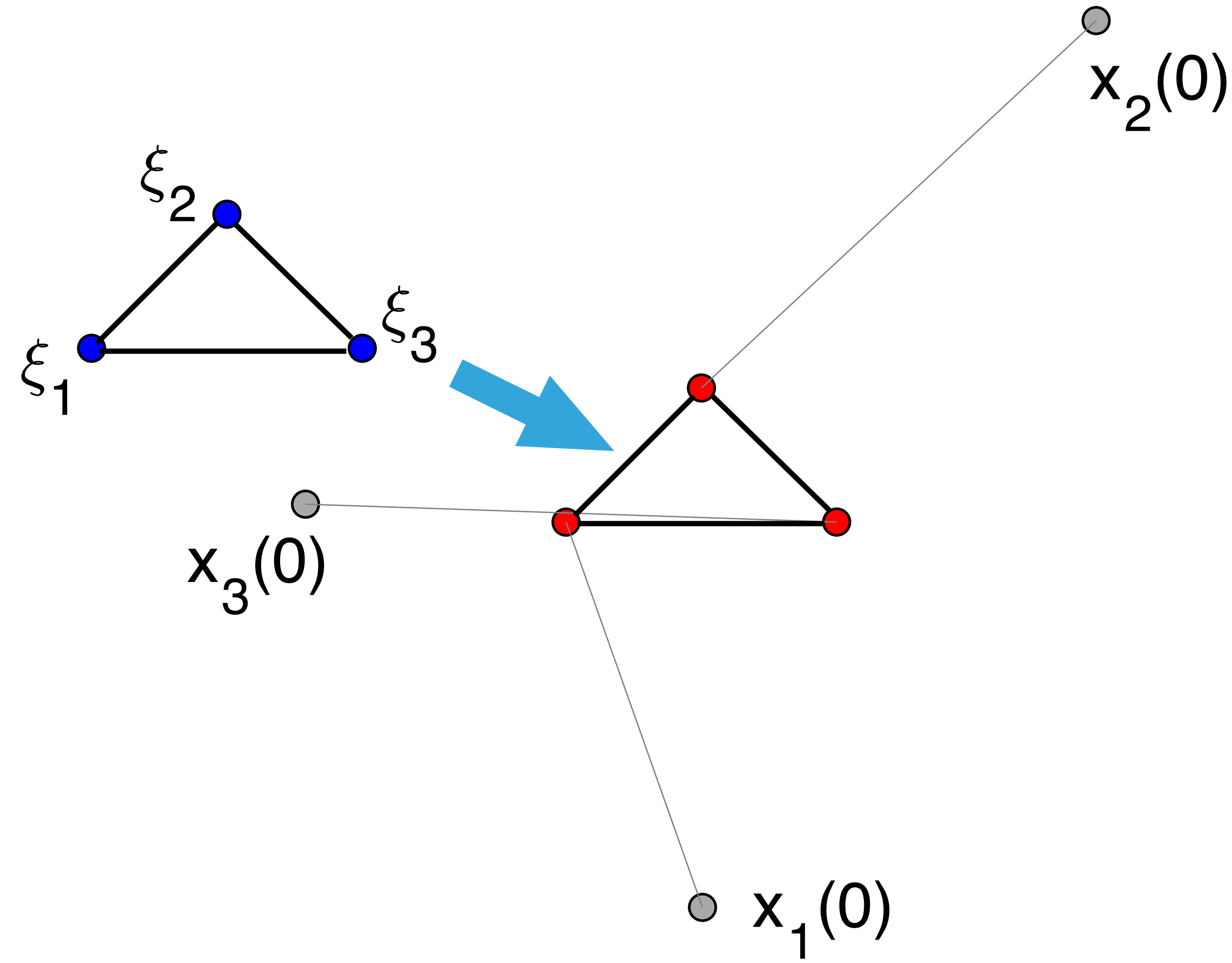
- SPECIFIED BY (ABSOLUTE) TARGET POSITIONS

$$\xi_i \in \mathbb{R}^2$$

Control

$$u_i = \sum_{i \sim j} ((x_j - \xi_j) - (x_i - \xi_i))$$

- FINAL FORMATION WILL BE A TRANSLATION OF THE TARGET FORMATION
- AGENTS MUST COMMUNICATE THEIR TARGET POSITION
- REQUIRES GLOBAL POSITIONING

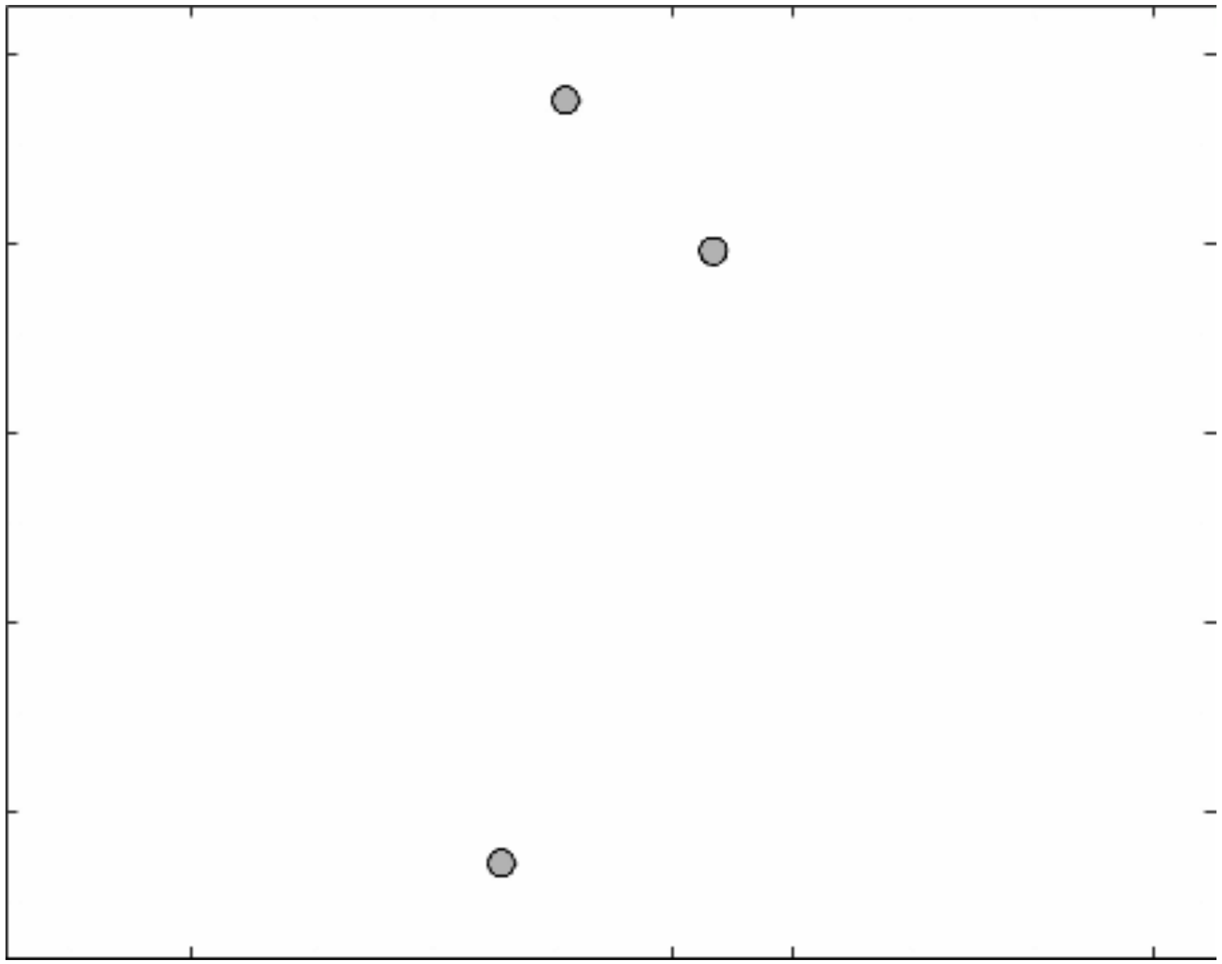


"robots" – modeled as kinematic point mass

$$\dot{x}_i = u_i$$

Assumptions

- GLOBAL COORDINATE FRAME
- RELATIVE POSITION MEASUREMENTS
- DISTANCE MEASUREMENTS
- NO SENSING CONSTRAINTS (360°)
- SENSING



Formation

- SPECIFIED BY DISTANCES BETWEEN PAIRS OF ROBOTS

$$d_{ij} \in \mathbb{R}$$

Control

$$u_i = \sum_{i \sim j} (\|x_i - x_j\|^2 - d_{ij}^2)(x_j - x_i)$$

[Krick2009]

THE "DISTANCE CONSTRAINED" FORMATION CONTROL PROBLEM

DISTANCE CONSTRAINED

Formation

- SPECIFIED BY DISTANCES BETWEEN PAIRS OF ROBOTS

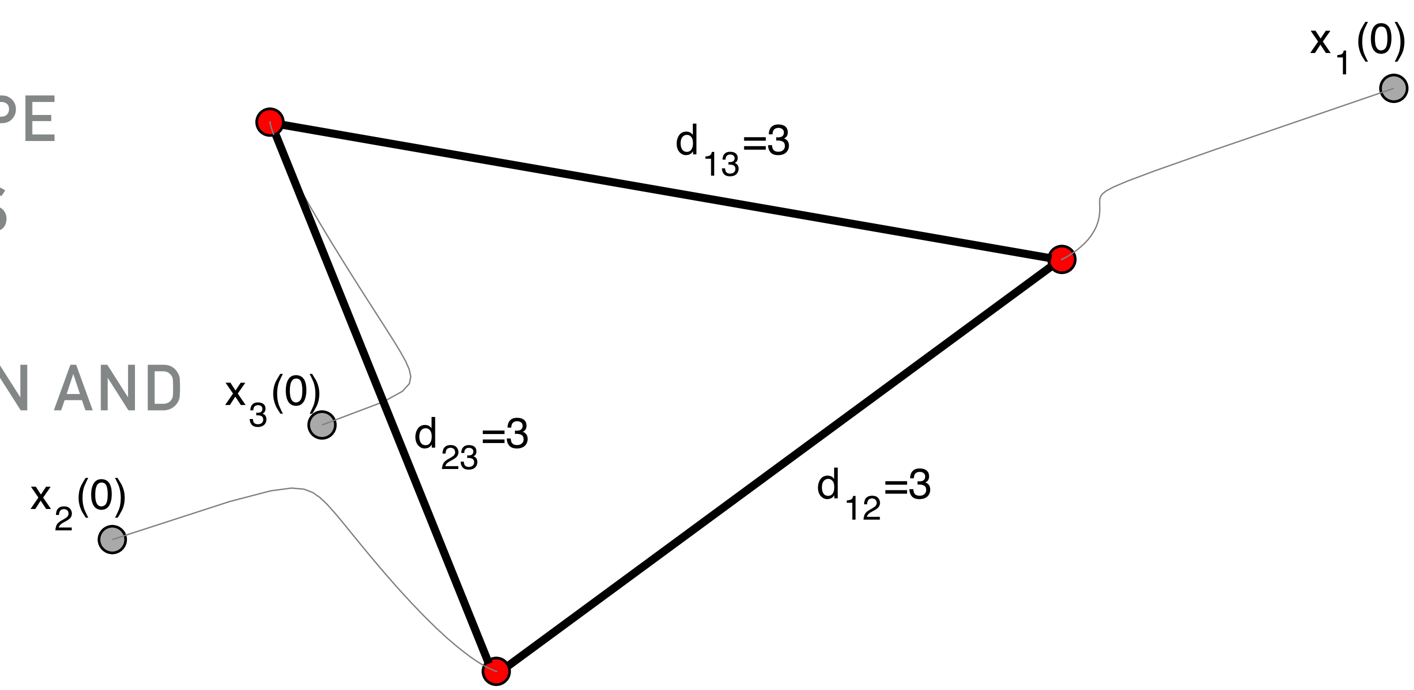
$$d_{ij} \in \mathbb{R}$$

Control

$$u_i = \sum_{i \sim j} (\|x_i - x_j\|^2 - d_{ij}^2)(x_j - x_i)$$

- FINAL FORMATION WILL BE A TRANSLATION OR ROTATION OF SHAPE SATISFYING DISTANCE CONSTRAINTS

- AGENTS REQUIRE RELATIVE POSITION AND DISTANCES

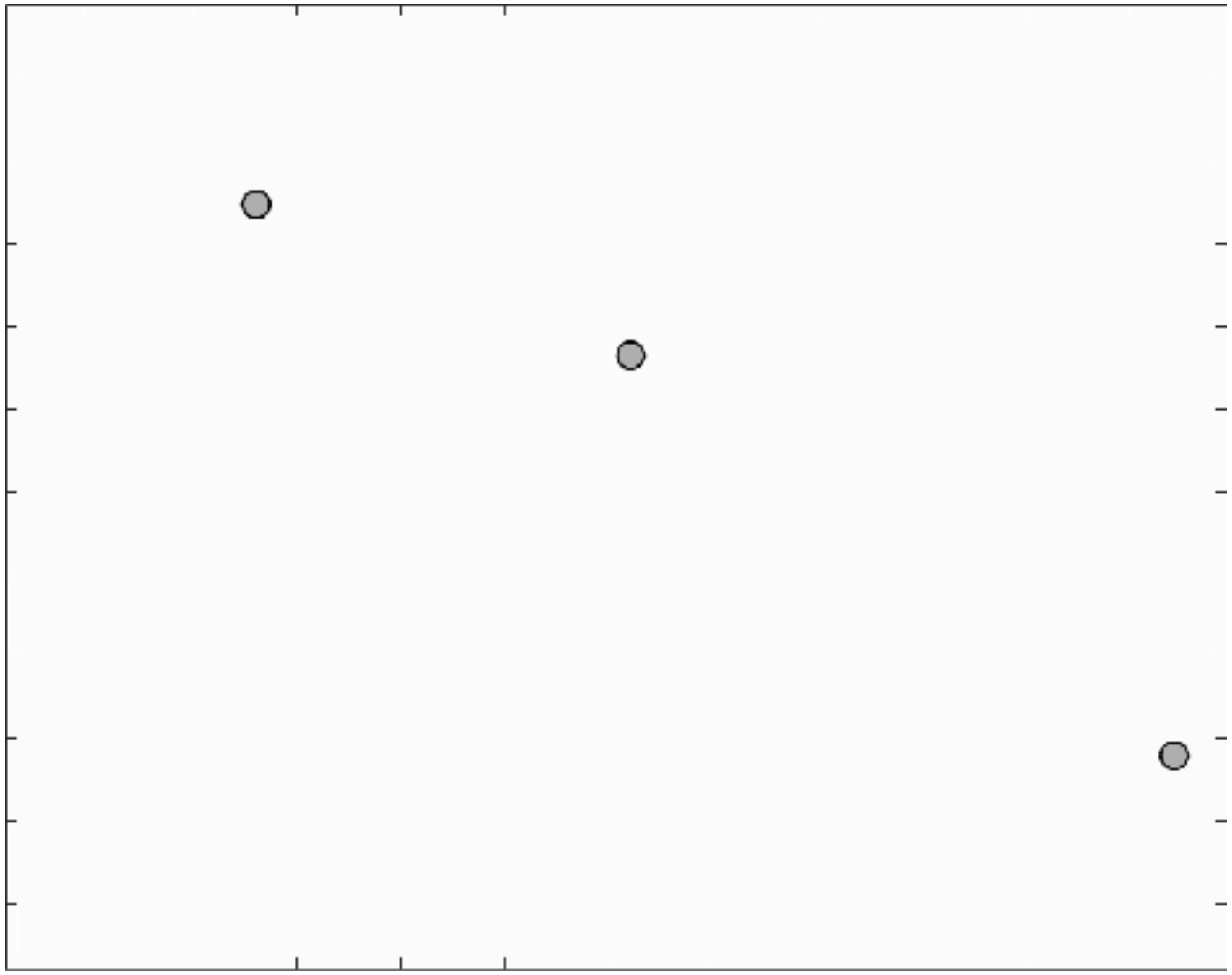


"robots" - modeled as kinematic point mass

$$\dot{x}_i = u_i$$

Assumptions

- GLOBAL COORDINATE FRAME
- BEARING MEASUREMENTS
- NO SENSING CONSTRAINTS (360°)
- SENSING



Formation

- SPECIFIED BY BEARING VECTORS

$$g_{ij}^* \in \mathbb{R}^2, \ \|g_{ij}^*\| = 1$$

Control

$$u_i = -\sum_{i \sim j} (I - g_{ij} g_{ij}^T) g_{ij}^*$$

THE "BEARING ONLY" FORMATION CONTROL PROBLEM

[Zhao,Zelazo2016]

BEARING ONLY

Formation

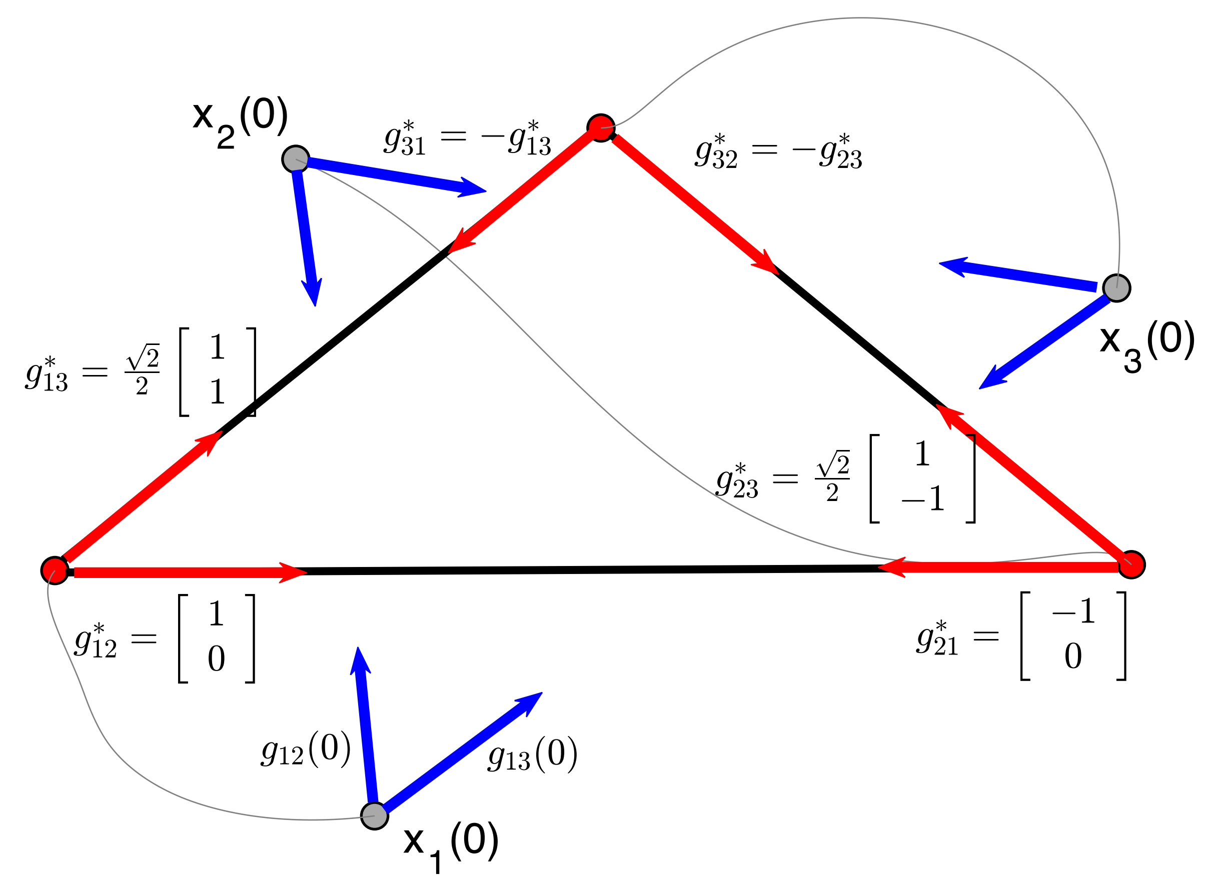
- SPECIFIED BY BEARING VECTORS

$$g_{ij}^* \in \mathbb{R}^2, \ \|g_{ij}^*\| = 1$$

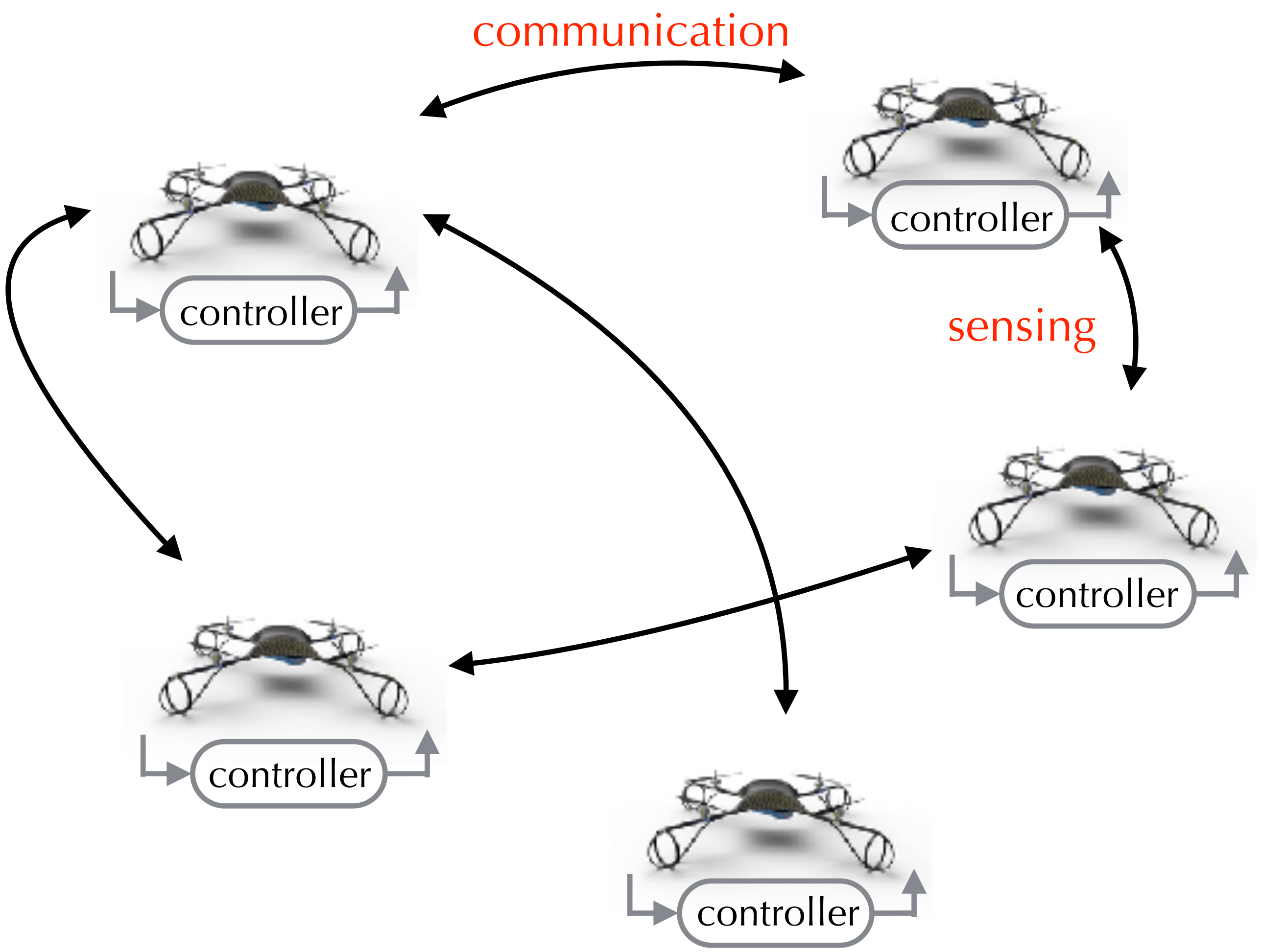
- FINAL FORMATION WILL BE A TRANSLATION OR SCALING OF SHAPE SATISFYING BEARING CONSTRAINTS
- AGENTS REQUIRE BEARING MEASUREMENTS

Control

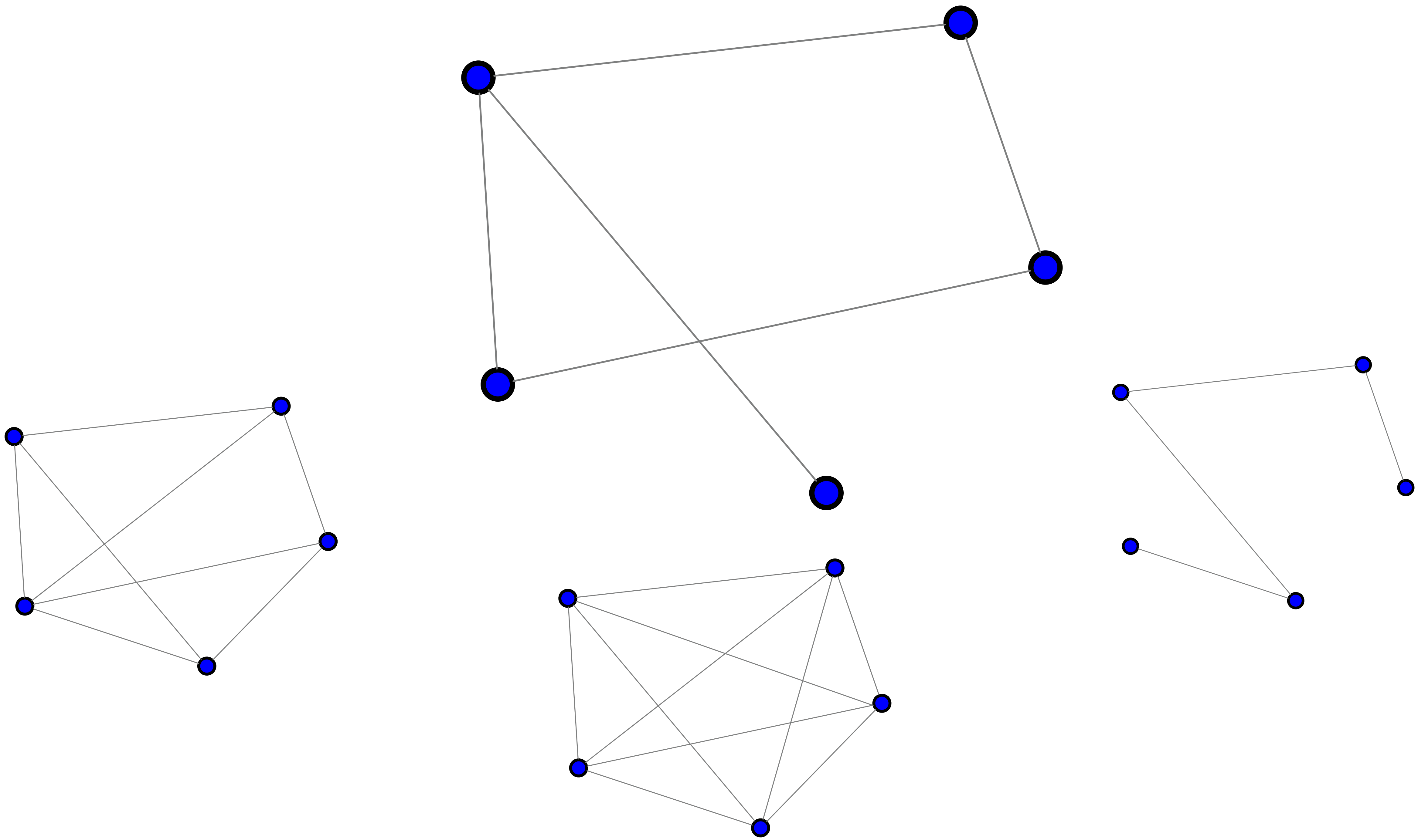
$$u_i = -\sum (I - g_{ij} g_{ij}^T) g_{ij}^*$$



INFORMATION EXCHANGE NETWORK AND FORMATION DETERMINATION



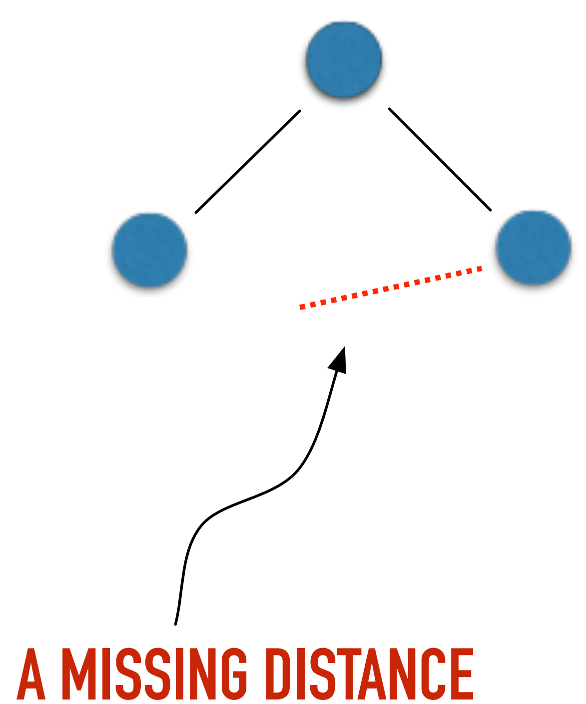
INFORMATION EXCHANGE NETWORK AND FORMATION DETERMINATION



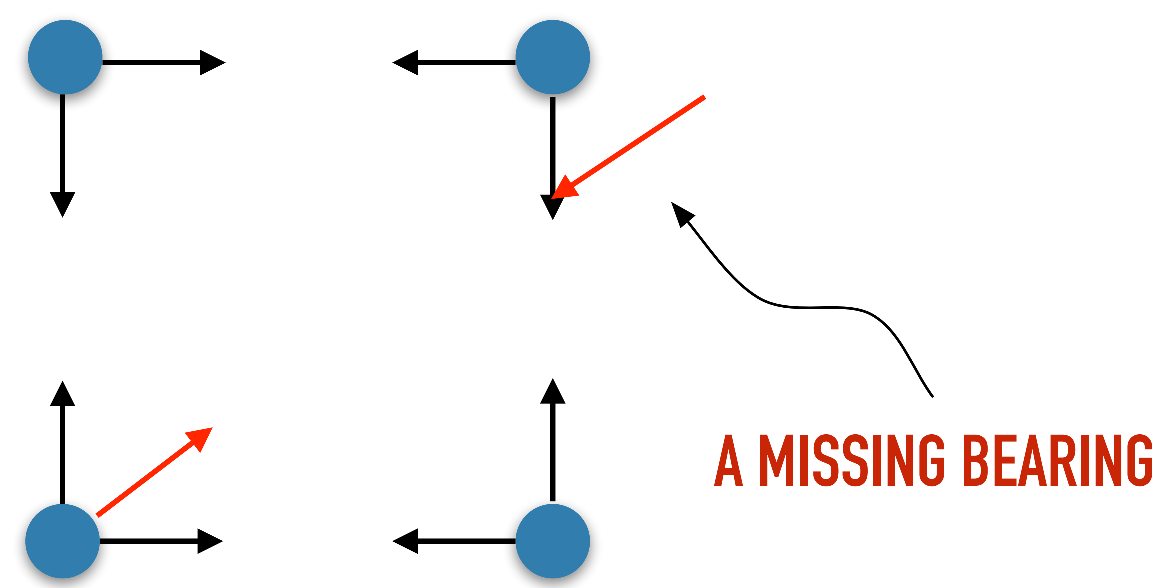
SENSORS, GRAPHS, AND SHAPES

Given a desired formation shape, a sensing modality and its corresponding formation controller, will all information exchange networks (graphs) solve the formation control problem?

The triangle revisited
(distance constrained)



the square
(bearing only)



SENSORS, GRAPHS, AND SHAPES

For a given sensing modality, what kind of information exchange networks can (uniquely) determine a formation shape?

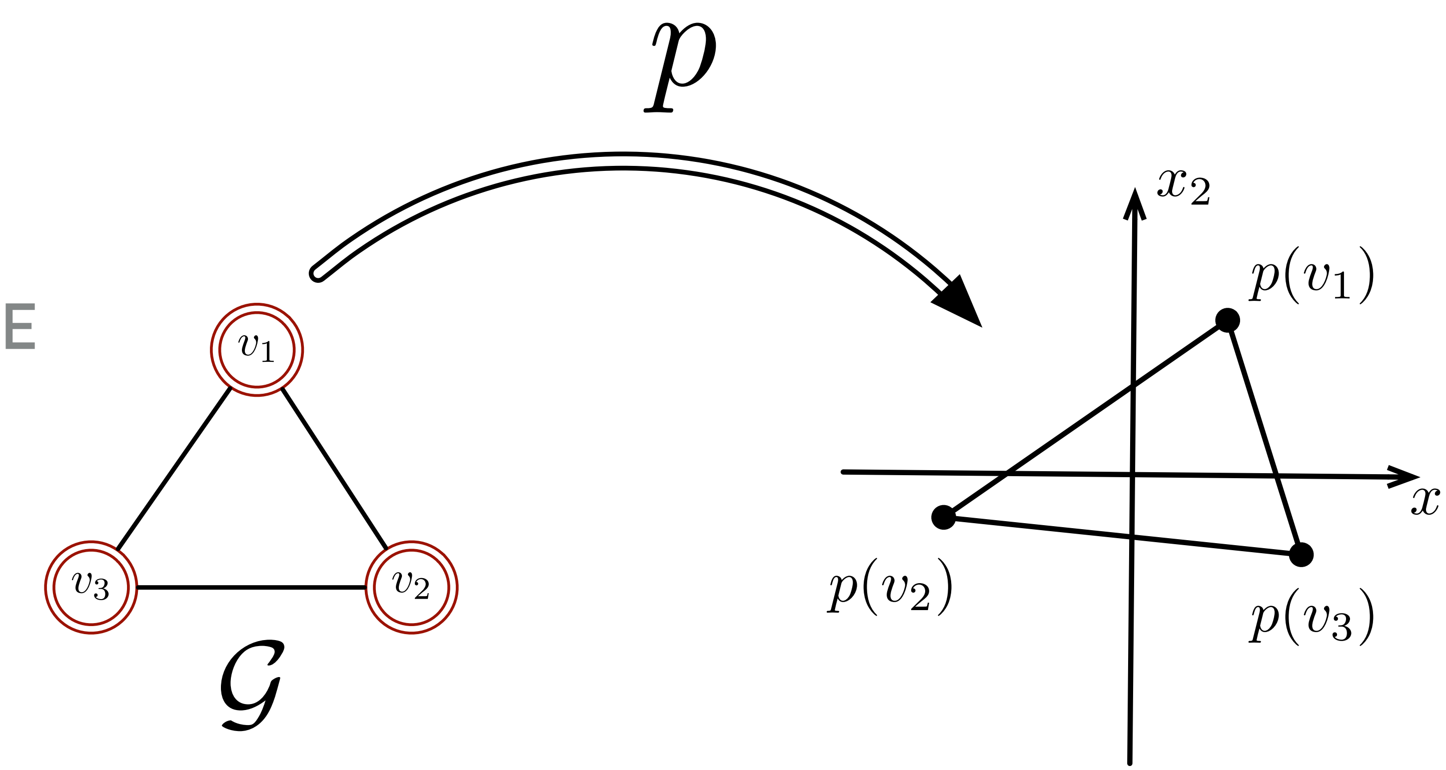
RIGIDITY THEORY

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

(DISTANCE) RIGIDITY THEORY

A *framework*

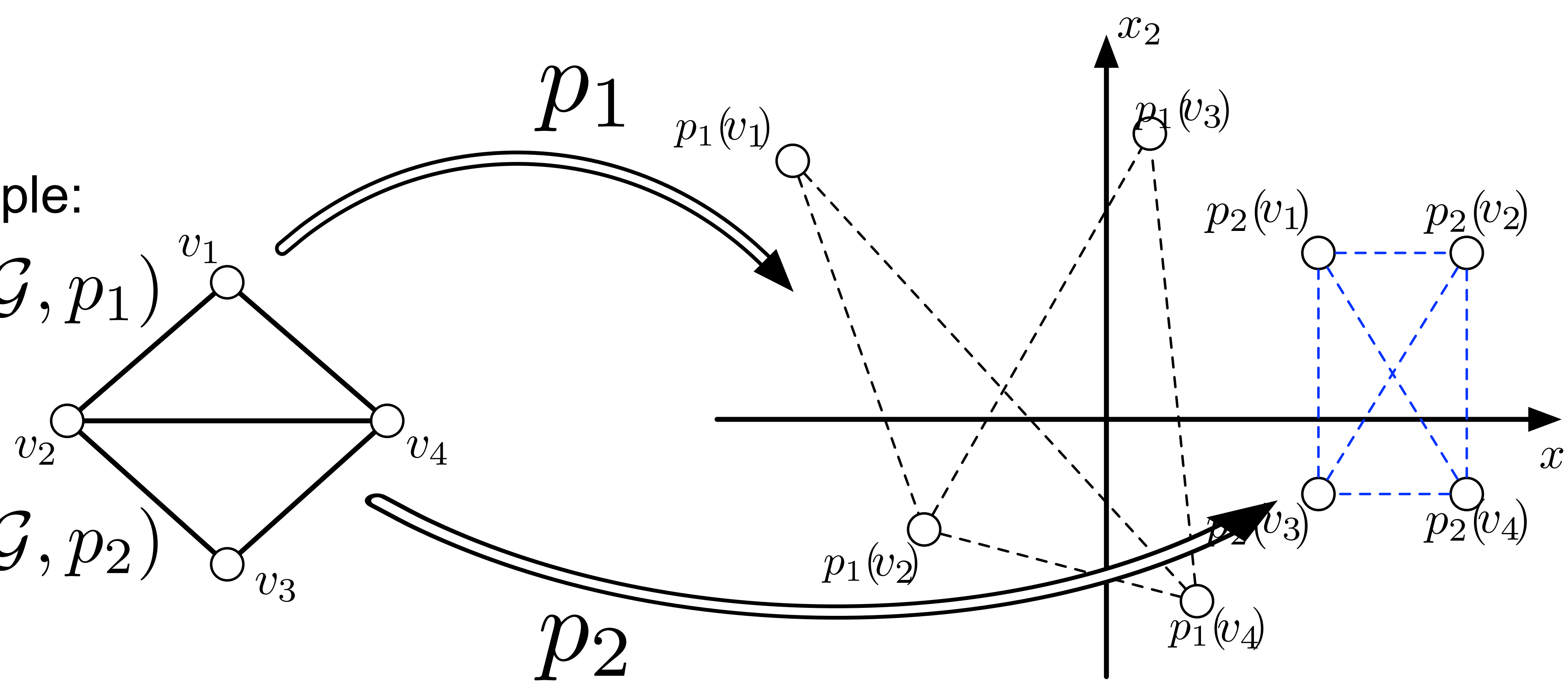
- A GRAPH
- A MAPPING TO A METRIC SPACE



example:

$$\mathcal{F}_1 = (\mathcal{G}, p_1)$$

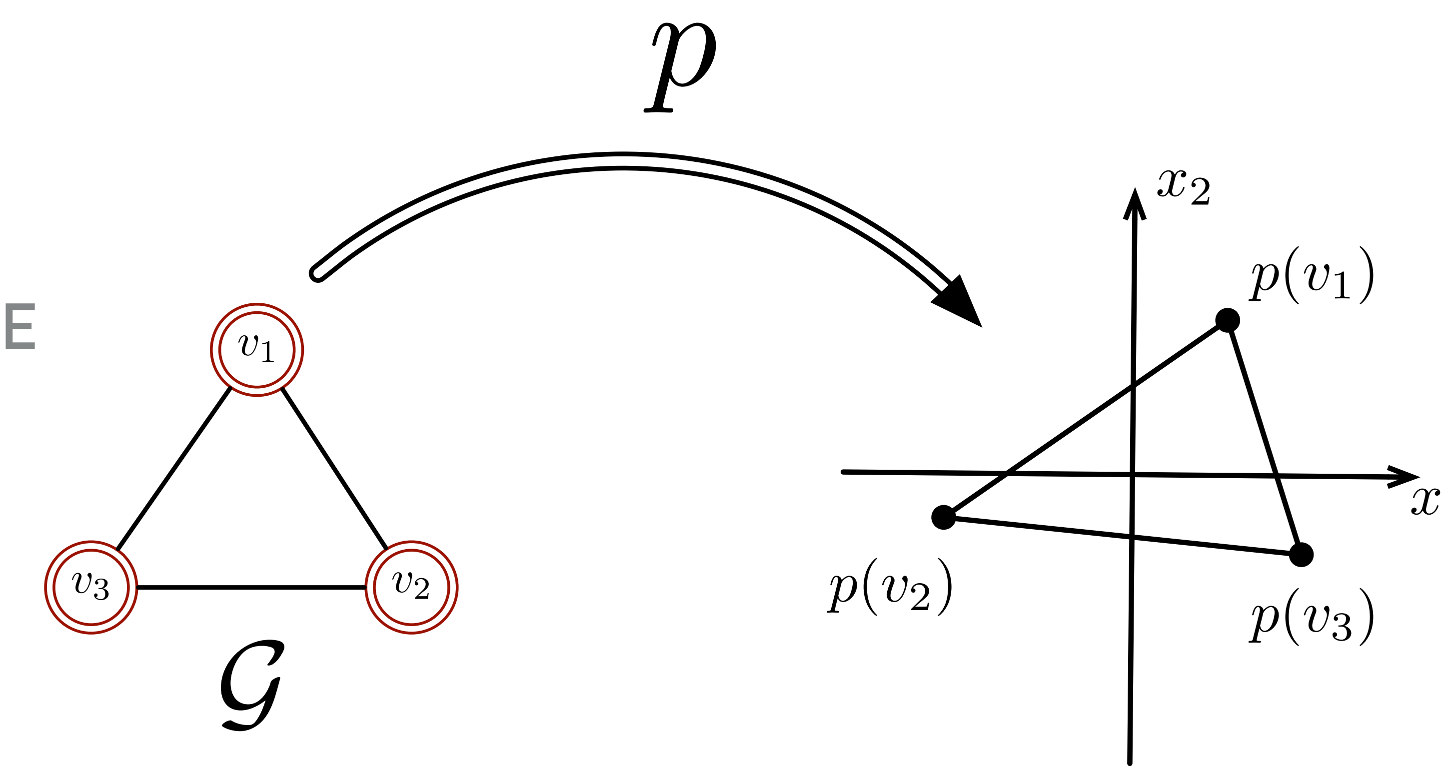
$$\mathcal{F}_2 = (\mathcal{G}, p_2)$$



(DISTANCE) RIGIDITY THEORY

A *framework*

- A GRAPH
- A MAPPING TO A METRIC SPACE



Two frameworks are *equivalent* if $\|p_0(v_i) - p_0(v_j)\| = \|p_1(v_i) - p_1(v_j)\|$

$(\mathcal{G}, p_0)$ $(\mathcal{G}, p_1)$

$\forall \{v_i, v_j\} \in \mathcal{E}$ all edges

Two frameworks are *congruent* if $\|p_0(v_i) - p_0(v_j)\| = \|p_1(v_i) - p_1(v_j)\|$

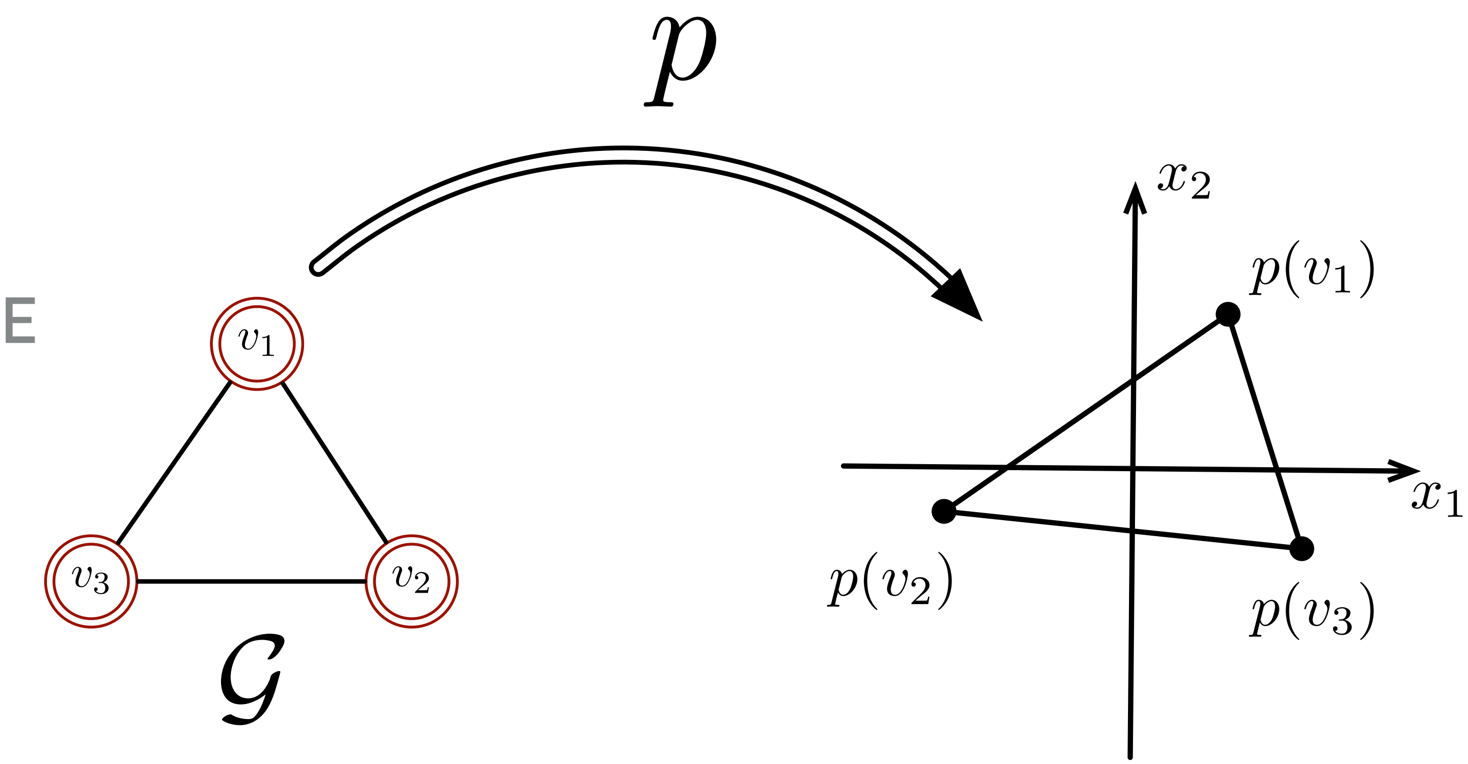
$(\mathcal{G}, p_0)$ $(\mathcal{G}, p_1)$

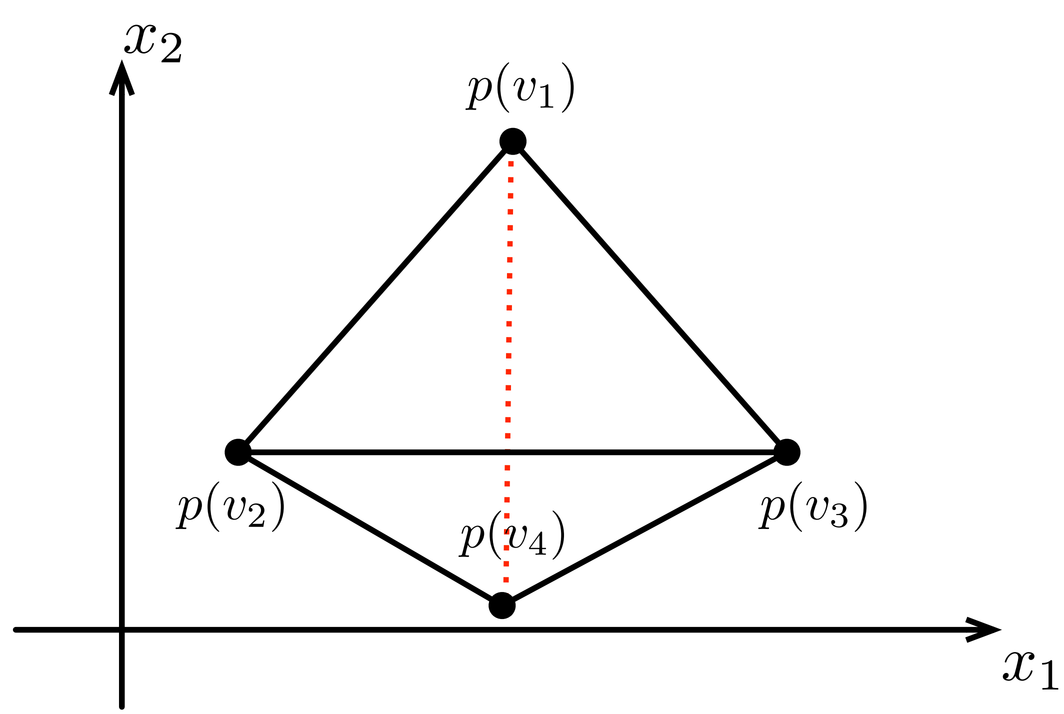
$\forall v_i, v_j \in \mathcal{V}$ all pairs of nodes

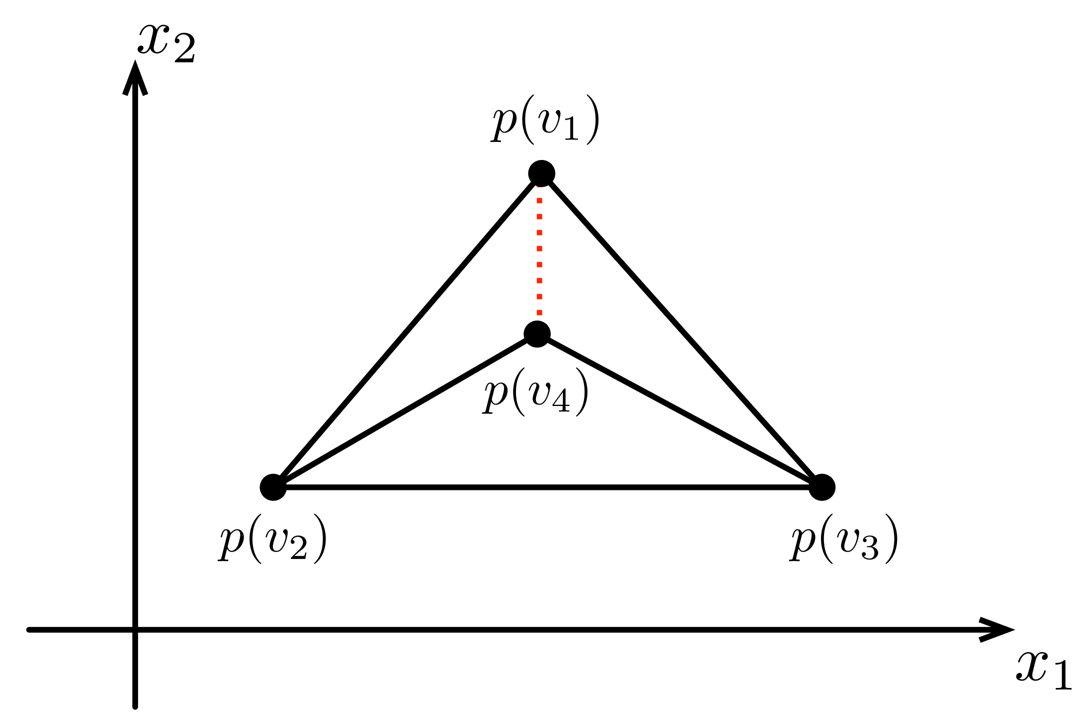
(DISTANCE) RIGIDITY THEORY

A *framework*

- A GRAPH
- A MAPPING TO A METRIC SPACE

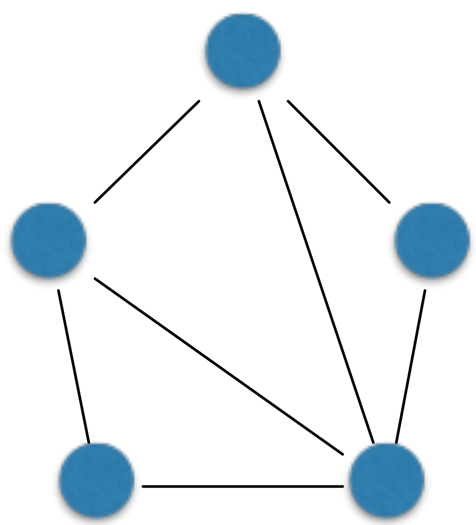






(DISTANCE) RIGIDITY THEORY

A framework is ***globally rigid*** if every framework that is equivalent to it is also congruent.

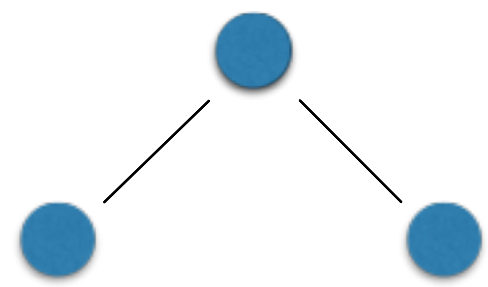


A *rigid* graph can only *rotate* and *translate* to ensure all distances between all nodes are preserved (i.e., preserve the shape)!

A framework is ***infinitesimally rigid*** if every infinitesimal motion is *trivial*

(DISTANCE) RIGIDITY THEORY

A framework is ***globally rigid*** if every framework that is equivalent to it is also congruent.



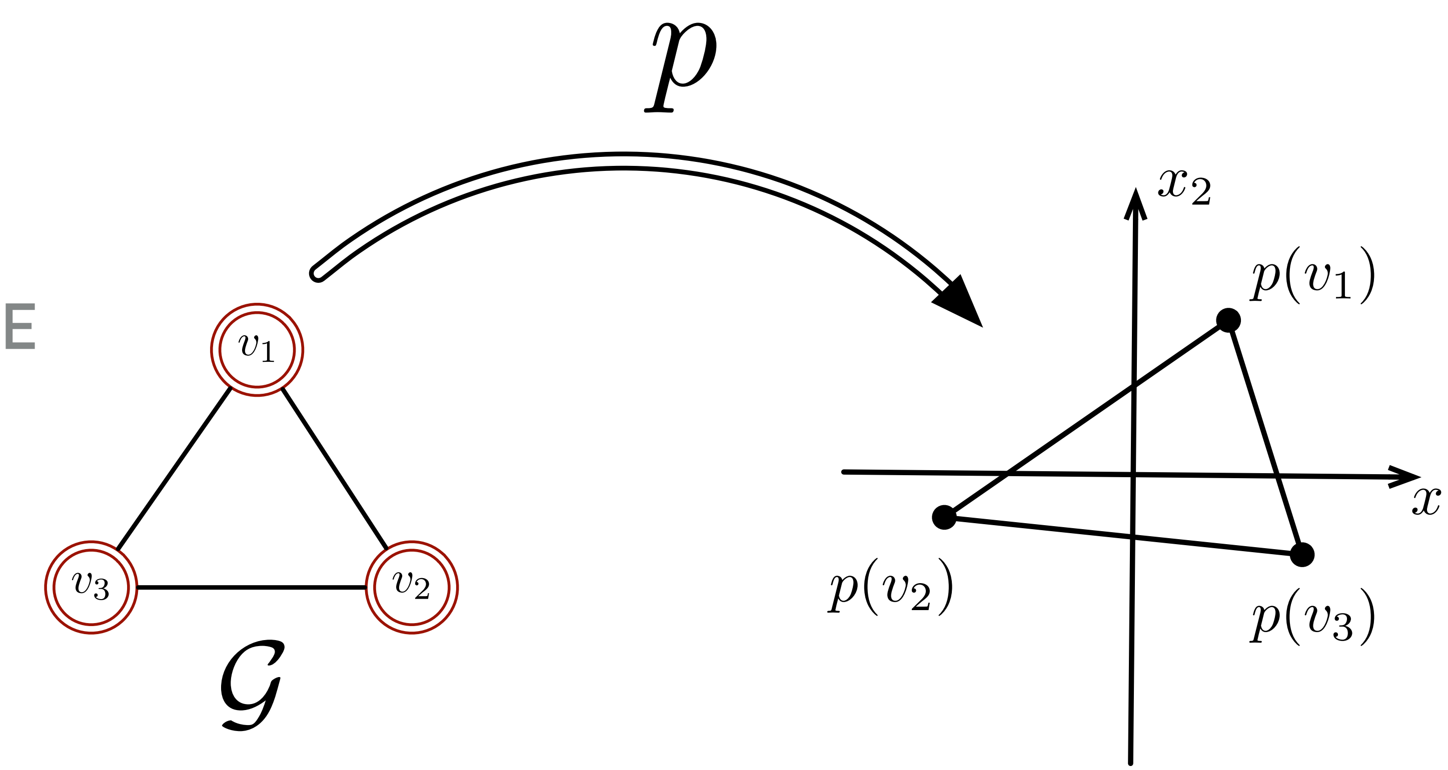
A *rigid* graph can only *rotate* and *translate* to ensure all distances between all nodes are preserved (i.e., preserve the shape)!

A framework is ***infinitesimally rigid*** if every infinitesimal motion is *trivial*

BEARING RIGIDITY THEORY

A *framework*

- A GRAPH
- A MAPPING TO A METRIC SPACE



Two frameworks are *equivalent* if

$$(\mathcal{G}, p_0) \quad (\mathcal{G}, p_1)$$

$$\frac{p_0(v_j) - p_0(v_i)}{\|p_0(v_j) - p_0(v_i)\|} = \frac{p_1(v_j) - p_1(v_i)}{\|p_1(v_j) - p_1(v_i)\|}$$

$$\forall \{v_i, v_j\} \in \mathcal{E}$$

Two frameworks are *congruent* if

$$(\mathcal{G}, p_0) \quad (\mathcal{G}, p_1)$$

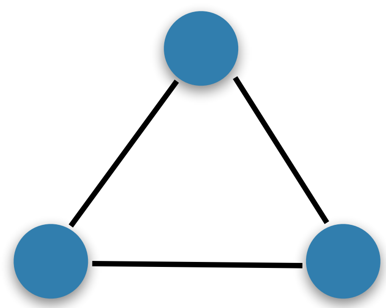
$$\frac{p_0(v_j) - p_0(v_i)}{\|p_0(v_j) - p_0(v_i)\|} = \frac{p_1(v_j) - p_1(v_i)}{\|p_1(v_j) - p_1(v_i)\|}$$

$$\forall v_i, v_j \in \mathcal{V}$$

[Zhao,Zelazo2016]

BEARING RIGIDITY THEORY

A framework is ***globally rigid*** if every framework that is equivalent to it is also congruent.



A bearing *rigid* graph can only *scale* and *translate* to ensure all bearings between all nodes are preserved (i.e., preserve the shape)!

A framework is ***infinitesimally rigid*** if every infinitesimal motion is *trivial*

[Zhao,Zelazo2016]

INFINITESIMAL RIGIDITY

A framework is ***infinitesimally rigid*** if every infinitesimal motion is *trivial*

Distance Function

$$F_D(p) = \frac{1}{2}\begin{bmatrix} \vdots \\ \|p(v_i) - p(v_j)\|^2 \\ \vdots \end{bmatrix}$$

Bearing Function

$$F_B(p) = \begin{bmatrix} \vdots \\ \frac{p(v_j)-p(v_i)}{\|p(v_i)-p(v_j)\|} \\ \vdots \end{bmatrix}$$

Distance Rigidity Matrix

$$R_D(p) = \frac{\partial F_D(p)}{\partial p}$$

Bearing Rigidity Matrix

$$R_B(p) = \frac{\partial F_B(p)}{\partial p}$$

Rigidity matrix is the linear term in the Taylor series expansion of the Distance/Bearing functions

$$F(p + \delta_p) = F(p) + \frac{\partial F(p)}{\partial p}\delta_p + h.o.t.$$

INFINITESIMAL RIGIDITY

A framework is ***infinitesimally rigid*** if every infinitesimal motion is *trivial*

Distance Function

$$F_D(p) = \frac{1}{2}\begin{bmatrix} \vdots \\ \|p(v_i) - p(v_j)\|^2 \\ \vdots \end{bmatrix}$$

Bearing Function

$$F_B(p) = \begin{bmatrix} \vdots \\ \frac{p(v_j)-p(v_i)}{\|p(v_i)-p(v_j)\|} \\ \vdots \end{bmatrix}$$

Distance Rigidity Matrix

$$R_D(p) = \frac{\partial F_D(p)}{\partial p}$$

Bearing Rigidity Matrix

$$R_B(p) = \frac{\partial F_B(p)}{\partial p}$$

infinitesimal motions are precisely the motions that satisfy

$$R(p)\delta_p = \frac{\partial F(p)}{\partial p}\delta_p = 0$$

INFINITESIMAL RIGIDITY

Distance Function

$$F_D(p) = \frac{1}{2}\begin{bmatrix} \vdots \\ \|p(v_i) - p(v_j)\|^2 \\ \vdots \end{bmatrix}$$

Bearing Function

$$F_B(p) = \begin{bmatrix} \vdots \\ \frac{p(v_j)-p(v_i)}{\|p(v_i)-p(v_j)\|} \\ \vdots \end{bmatrix}$$

Distance Rigidity Matrix

$$R_D(p) = \frac{\partial F_D(p)}{\partial p}$$

Bearing Rigidity Matrix

$$R_B(p) = \frac{\partial F_B(p)}{\partial p}$$

THEOREM

A framework is infinitesimally (distance, bearing) rigid if and only if the rank of the rigidity matrix is 2n-3.

3 trivial motions in the plane

SENSORS, GRAPHS, AND SHAPES

For a given sensing modality, what kind of information exchange networks can (uniquely) determine a formation shape?

INFINITESIMALLY RIGID

"robots" - modeled as kinematic point mass

$$\dot{x}_i = u_i$$

Distance Control

$$u_i = \sum_{i \sim j} (\|x_i - x_j\|^2 - d_{ij}^2)(x_j - x_i)$$

$$\dot{x} = -R_D(p)^T R_D(p) - R_D(p)^T d^2$$

locally exponentially stable
undesirable equilibriums

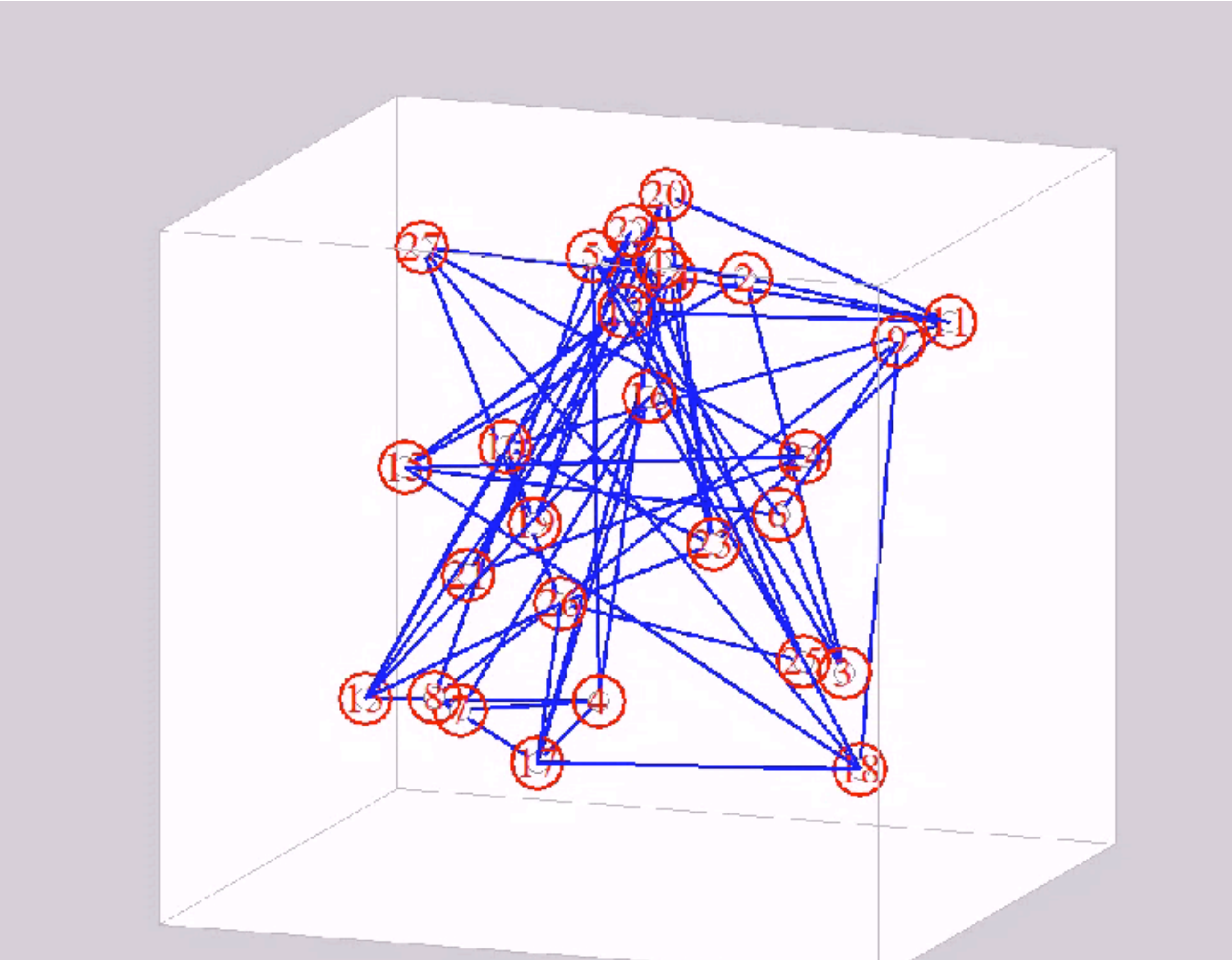
Bearing Control

$$u_i = -\sum_{i \sim j} (I - g_{ij} g_{ij}^T) g_{ij}^*$$

$$\dot{x} = -R_B(p)^T g^*$$

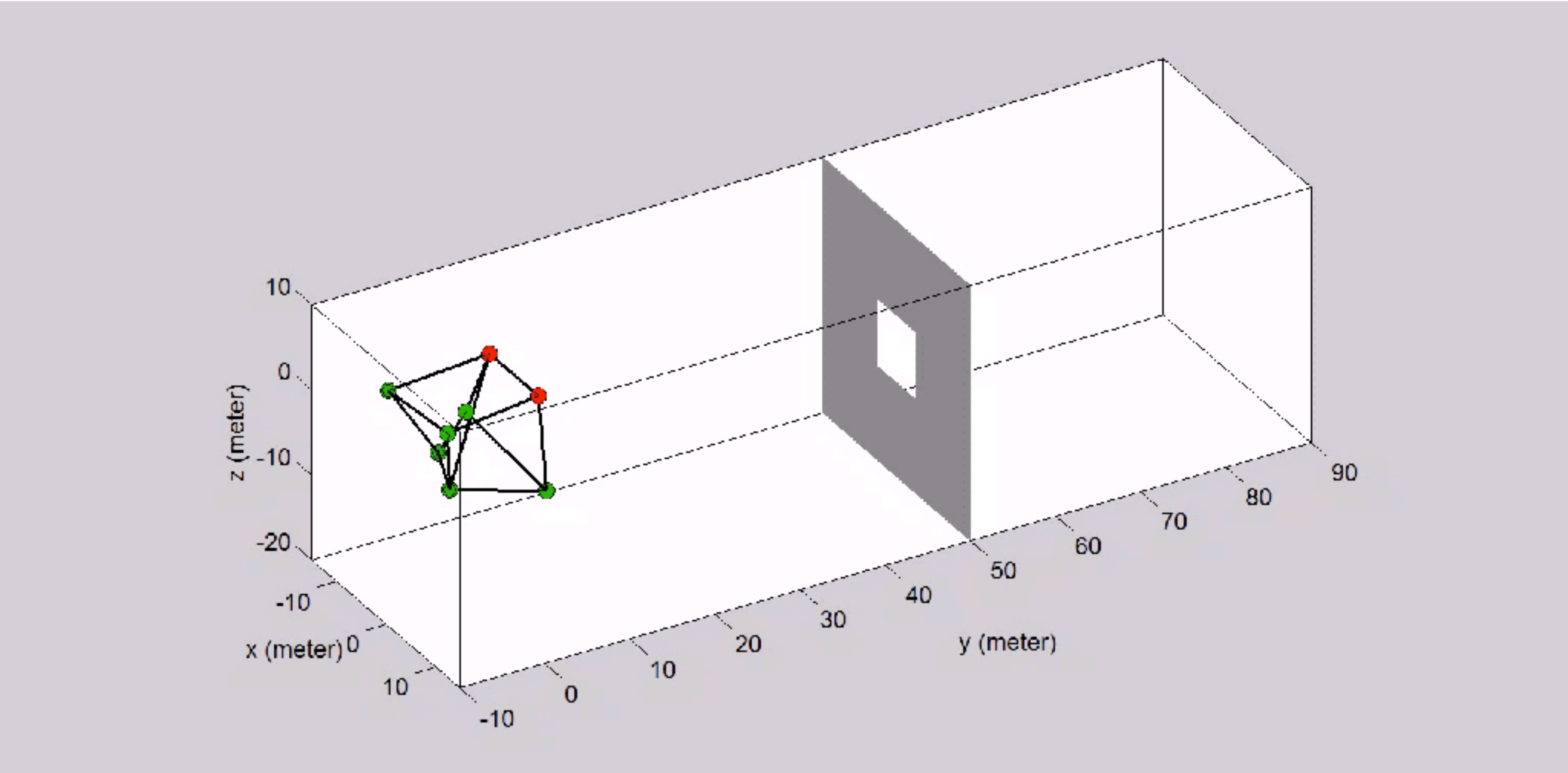
almost global stability
1 undesirable equilibriums

BEARING RIGIDITY THEORY



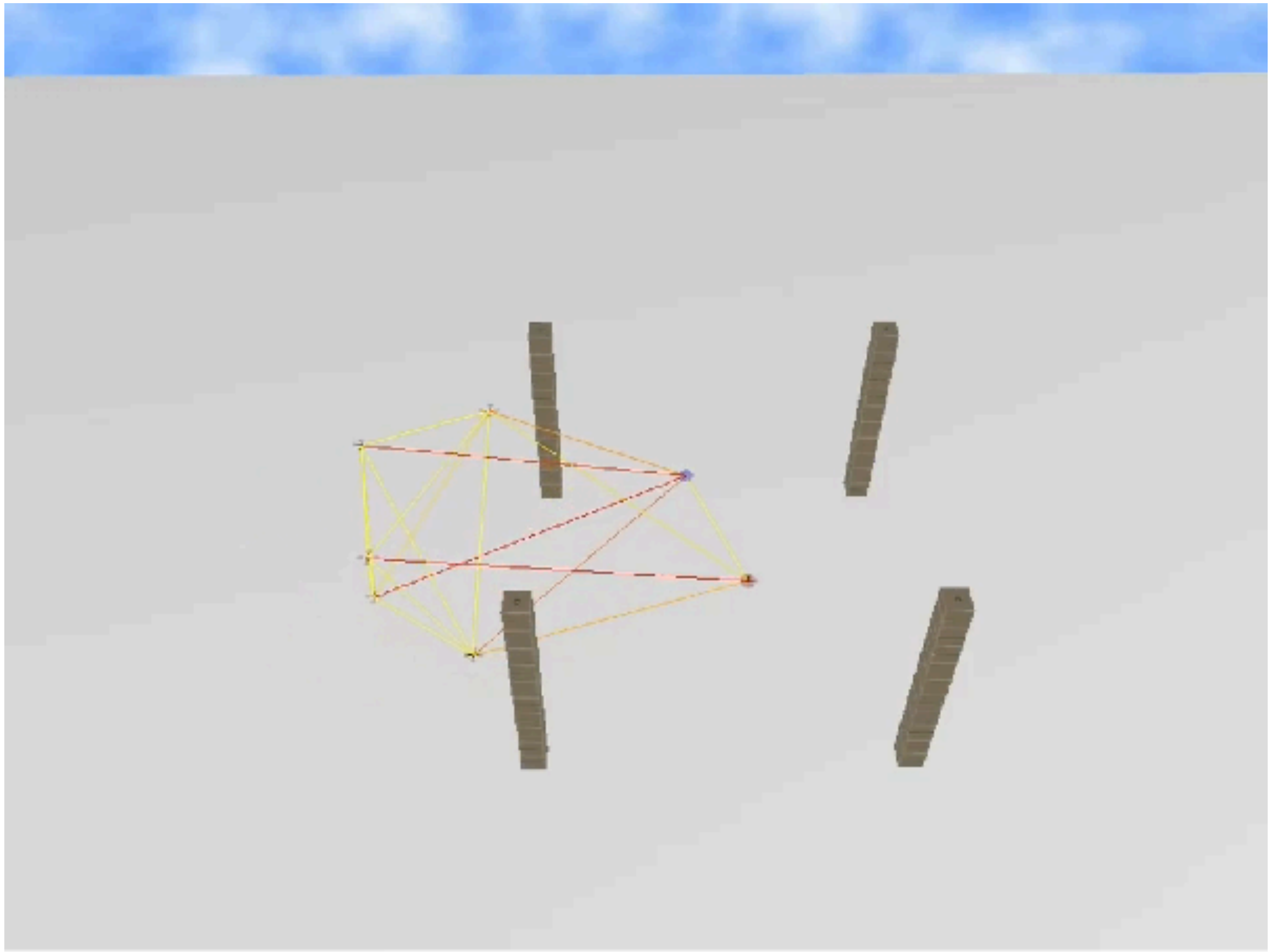
$$u_i = -\sum_{i \sim j} (I - g_{ij} g_{ij}^T) g_{ij}^*$$

BEARING RIGIDITY THEORY



[Zhao,Zelazo2017]

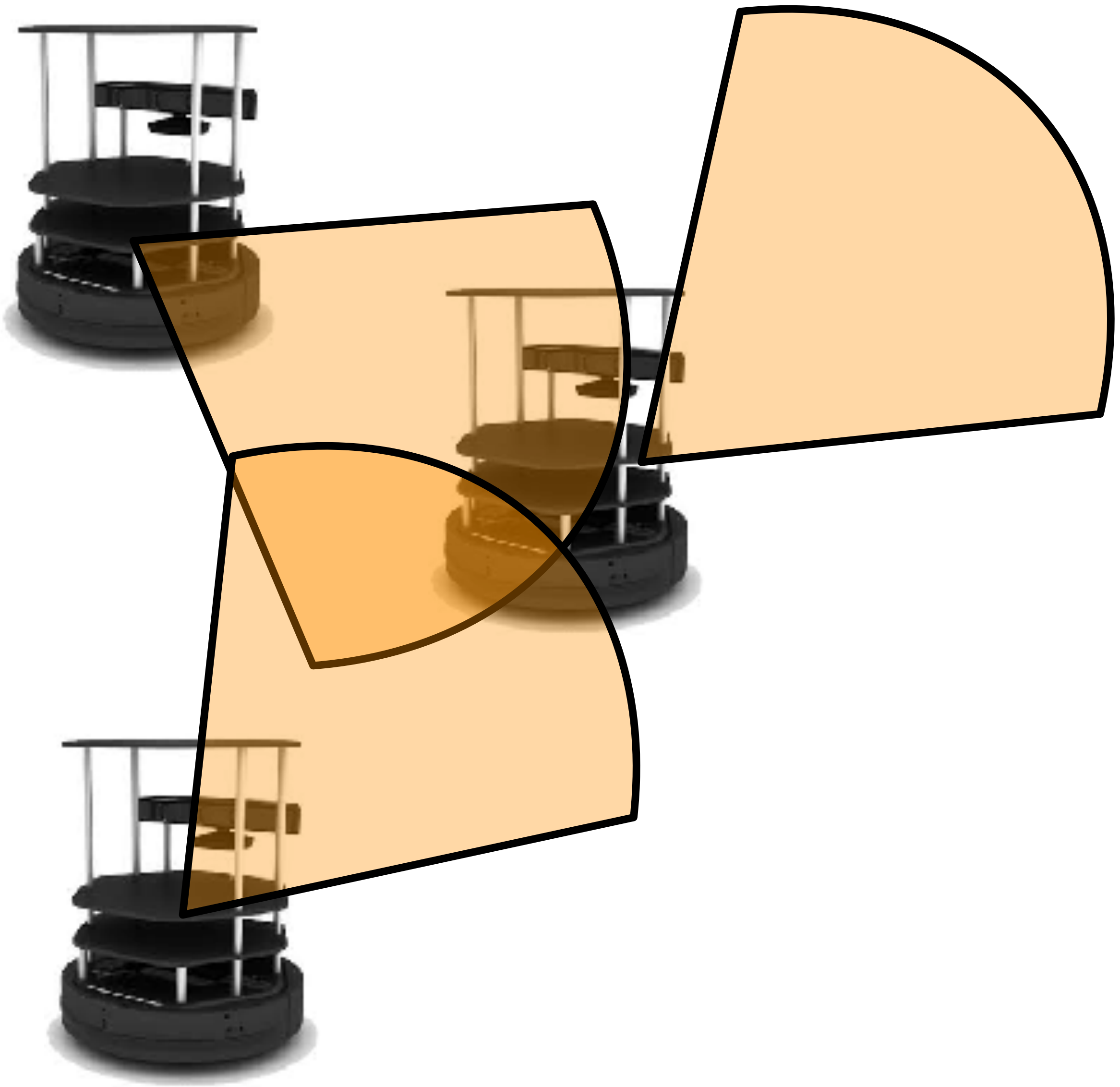
WHAT IS THE ARCHITECTURE OF A MULTI-ROBOT SYSTEM?



CONNECTIVITY

RIGIDITY

FORMATION CONTROL WITHOUT A COMMON FRAME



- sensing is typically *physically attached to the body frame* of the robot
- sensing is inherently directed
- knowledge of common inertial frame is *not* a realistic assumption

rigidity theory extensions for *directed sensing graphs* and *local (body-frame) measurements*

SE(2) RIGIDITY THEORY

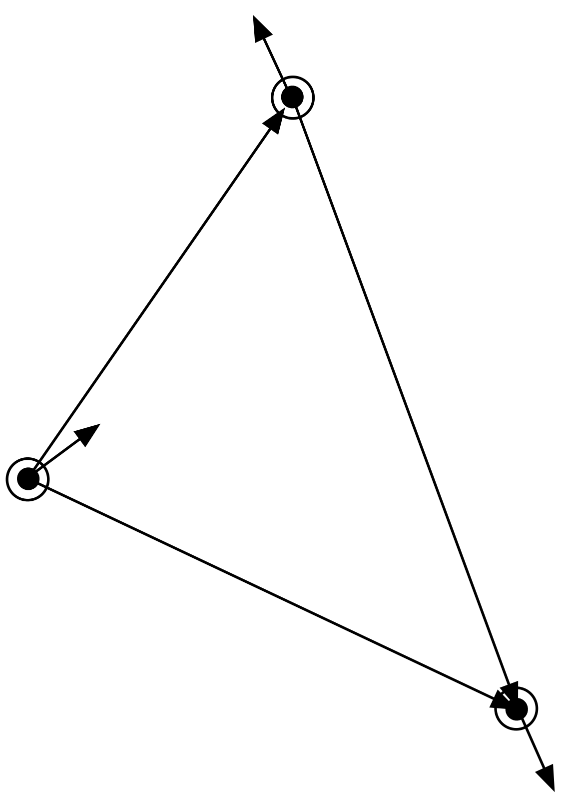
[Zelazo, Giordano, Franchi2015]

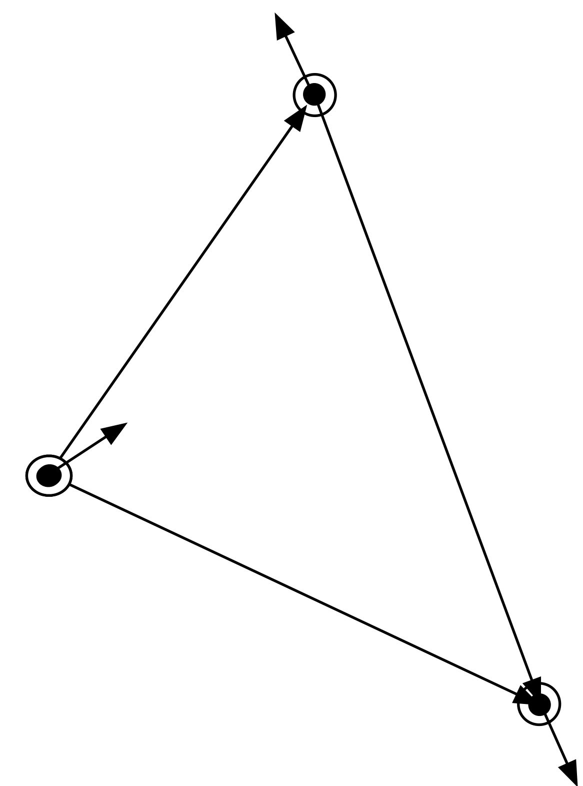
INFINITESIMAL RIGIDITY

A framework is ***infinitesimally rigid*** if every infinitesimal motion is *trivial*

- maintain bearings in *local* frame

- rigid body rotations and translations + coordinated rotations



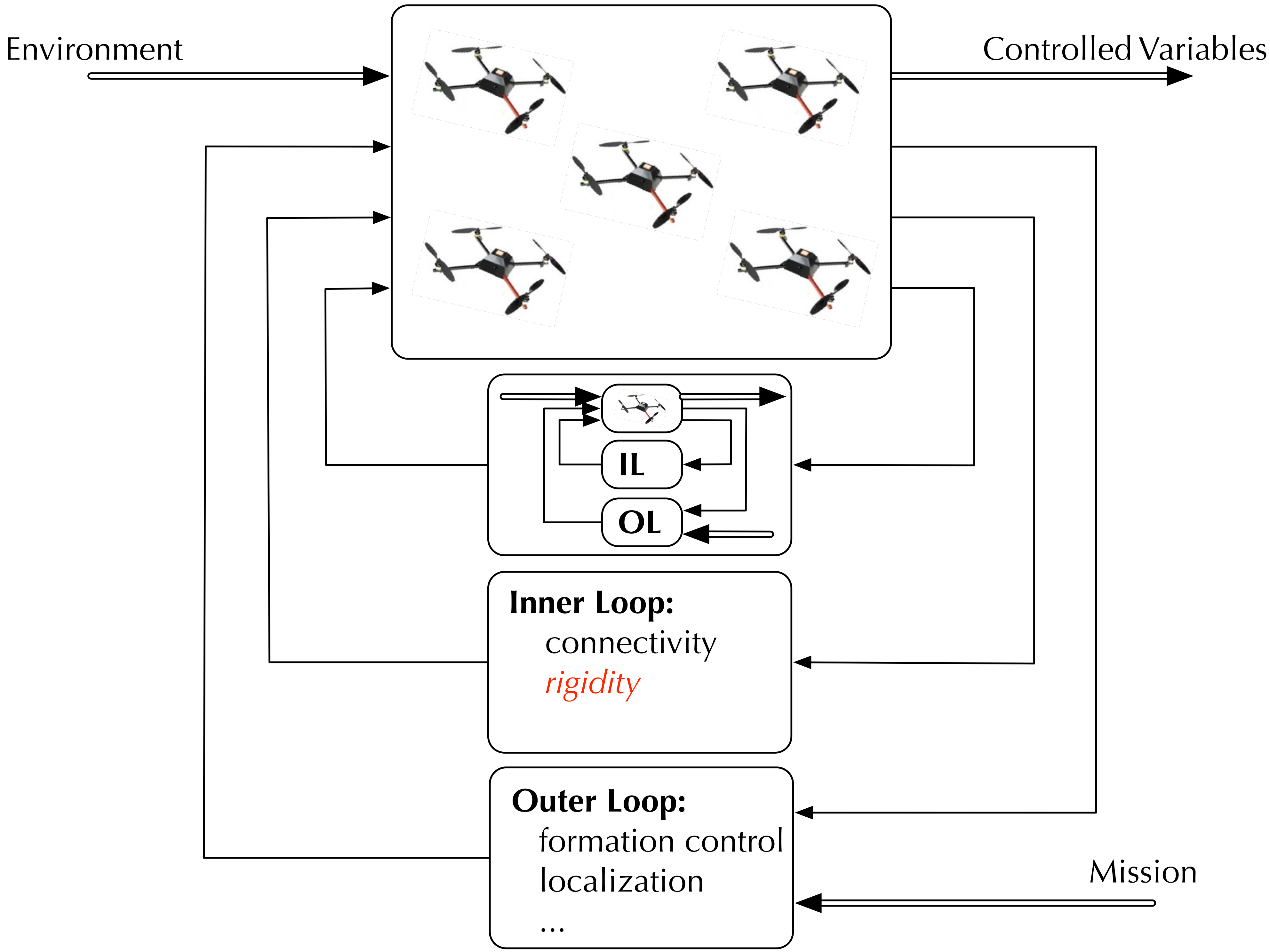


[Zelazo, Giordano, Franchi2015]

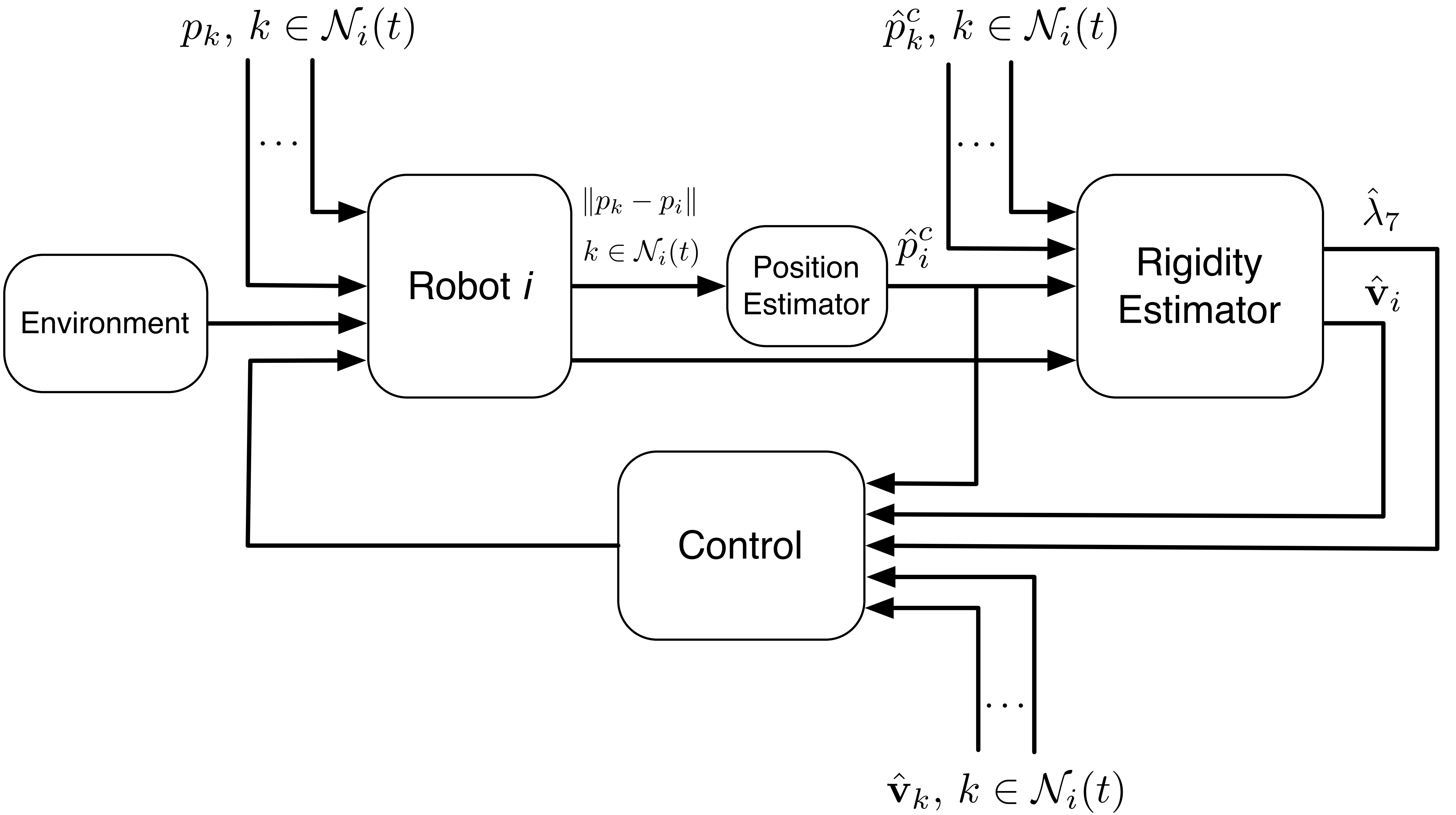
SE(2) FORMATION CONTROL

[Schiano, Franchi, Zelazo, Giordano2016]

RIGIDITY AS AN ARCHITECTURAL REQUIREMENT

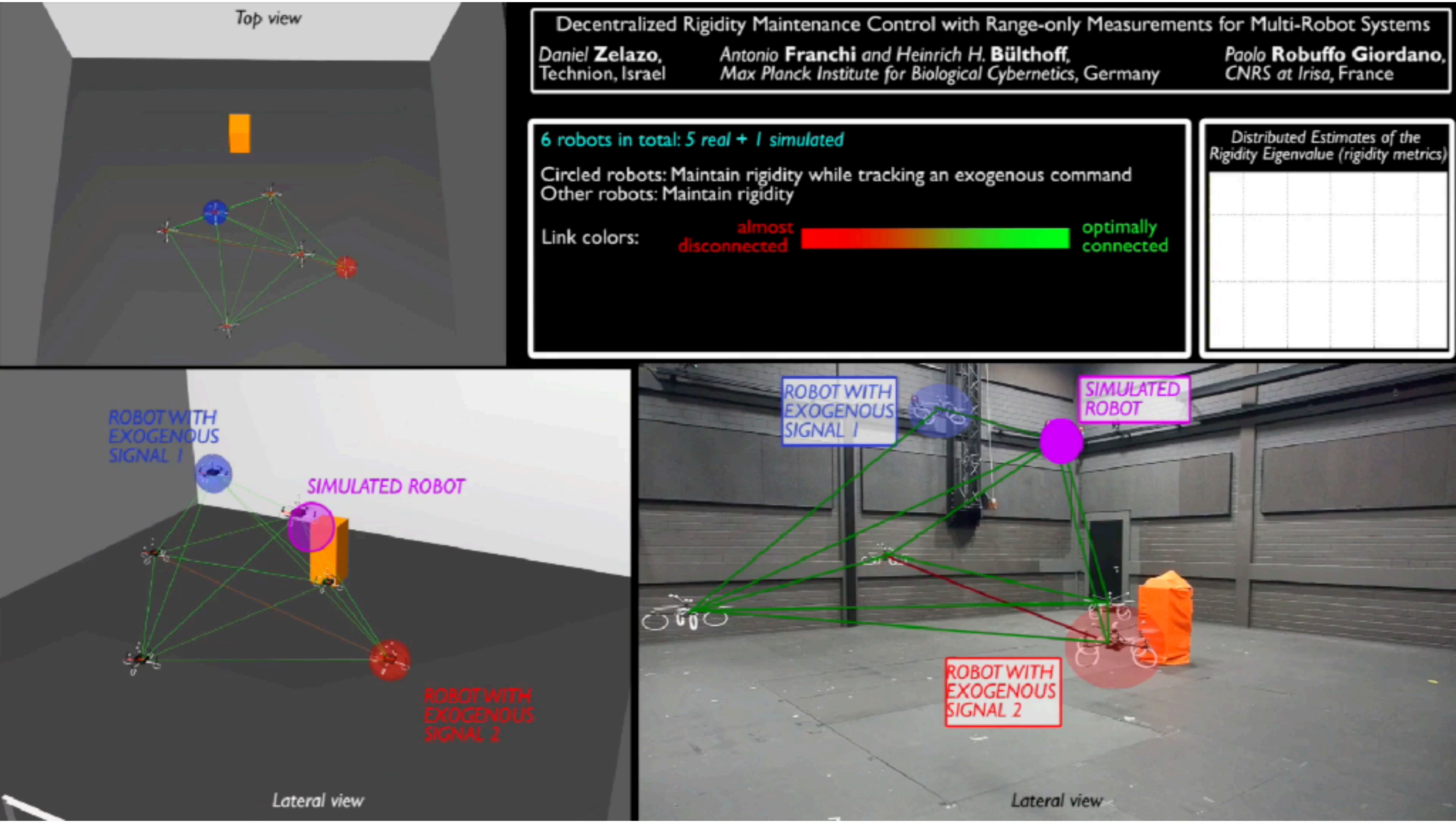


RIGIDITY MAINTENANCE



[Zelazo, Giordano, Franchi2015]

RIGIDITY MAINTENANCE



[Zelazo, Giordano, Franchi2015]

OUTLOOKS

Do we need to develop rigidity theory extensions for every kind of sensor?

G. Stacey and R. Mahony, "*The Role of Symmetry in Rigidity Analysis: A Tool for Network Localisation and Formation Control*," in *IEEE Transactions on Automatic Control*, vol. PP, no. 99, pp. 1-1.

Extensions for directed sensing network

control and estimation algorithms

THEORY ⟷ APPLICATION

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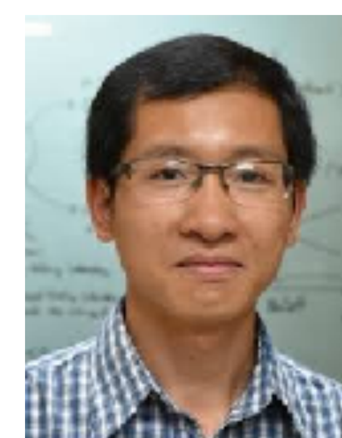

Minh Hoang Trinh