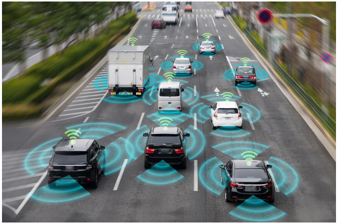
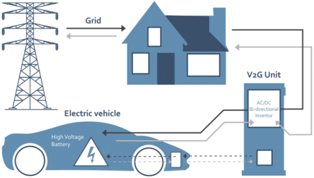
A CHARACTERIZATION OF ALL PASSIVIZING INPUT-OUTPUT TRANSFORMATIONS OF A PASSIVE-SHORT SYSTEM

Miel Sharf (Jether Energy) and **Daniel Zelazo**

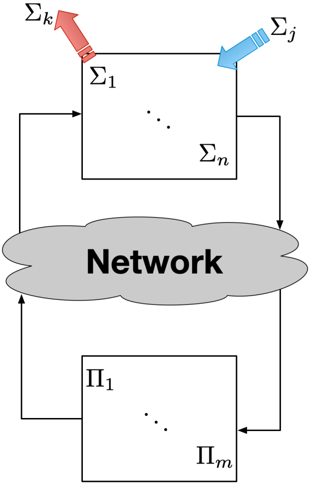
November 5, 2024
New Jersey Institute of Technology

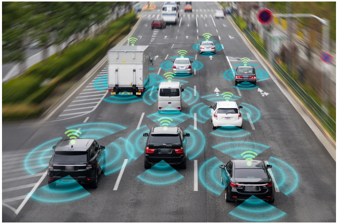


network of self-driving cars

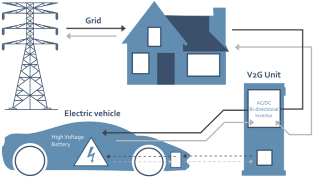


smart-grid with EV integration

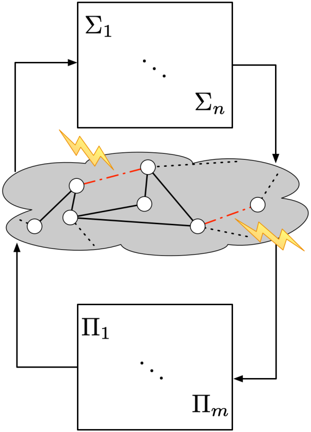


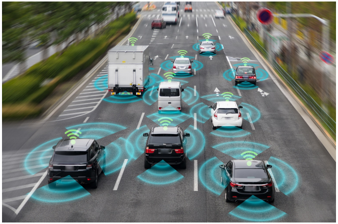


network of self-driving cars

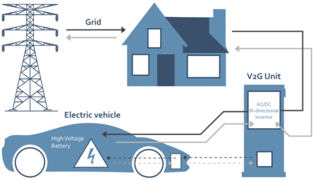


smart-grid with EV integration

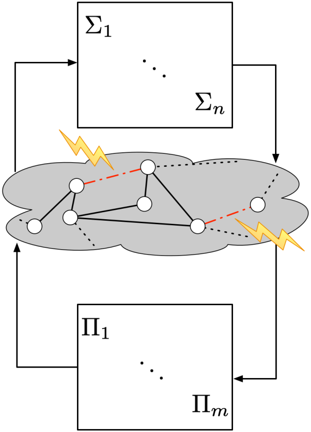




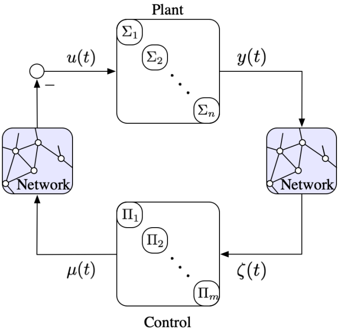
network of self-driving cars



smart-grid with EV integration



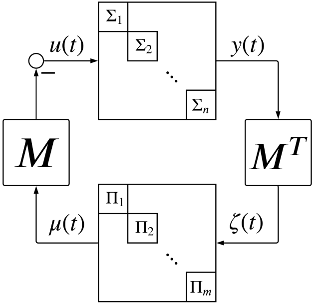
Resillience and robustness of network systems required for safe operations



$(\Sigma, \Pi, \mathcal{G})$

Components of a networked system

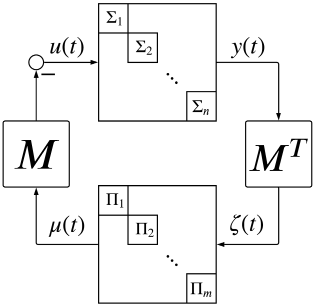
- agents - dynamical systems that should interact with eachother to achieve some goal
- network - communication and sensing infrastructure for sharing of information
- controllers - computational nodes that process information from the network to make decisions for each agent



(Σ, Π, M)

Network Interconnection

- Network is encoded by a matrix $M \in \mathbb{R}^{n \times m}$
- $[M]_{ij} = \begin{cases} \star, & \text{controller } j \text{ access to agent } i \\ 0, & \text{otherwise} \end{cases}$



(Σ, Π, M)

Network Interconnection

- Network is encoded by a matrix $M \in \mathbb{R}^{n \times m}$
- $[M]_{ij} = \begin{cases} \star, & \text{controller } j \text{ access to agent } i \\ 0, & \text{otherwise} \end{cases}$

A Stability Result

The stability of the dynamic network (Σ, Π, M) can be guaranteed for output-strictly passive agent dynamics Σ_i and passive controller dynamics Π_e.

[Corollary of B&Z 2014]

PASSIVATION BY THE NETWORK

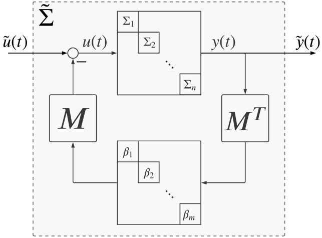
- stability result requires a passivity property to hold

PASSIVATION BY THE NETWORK

- stability result requires a passivity property to hold
- what if this cannot be guaranteed?

PASSIVATION BY THE NETWORK

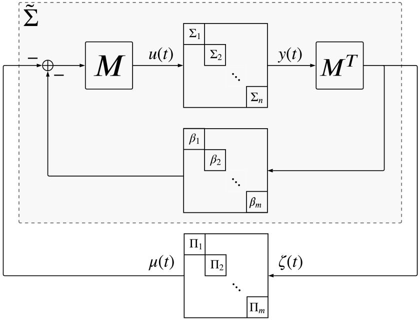
- stability result requires a passivity property to hold
- what if this cannot be guaranteed?



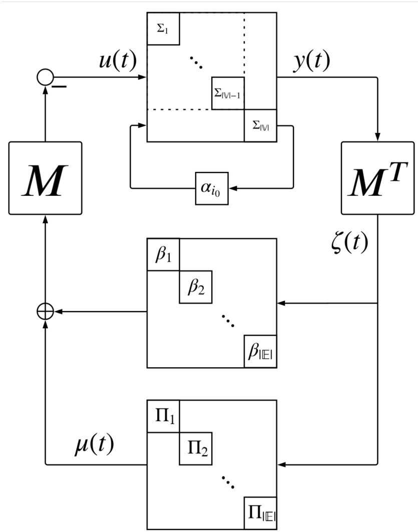
- ρ_i is passivity index of each agent
  - $\rho_i = 0$: passive
  - $\rho_i > 0$: strictly output-passive
  - $\rho_i < 0$: output passive short
- $R = \mathrm{diag}(\rho_1, \ldots, \rho_n)$

Lemma [Belabbas, Chen, Z 2023]

Assume that $\rho_i < 0$ for at least one agent. If $R + M\mathrm{diag}(\beta)M^T$ is positive definite, then $\tilde{\Sigma} : \tilde{u}(t) \mapsto \tilde{y}(t)$, is output-strict passive with respect to any steady-state input-output pair. Furthermore, there exists scalars β_i, $i = 1, \ldots, m$ such that $R + M\mathrm{diag}(\beta)M^T > 0$ if and only if $x^T Rx > 0$ for any $x \in \ker(M^T)$.

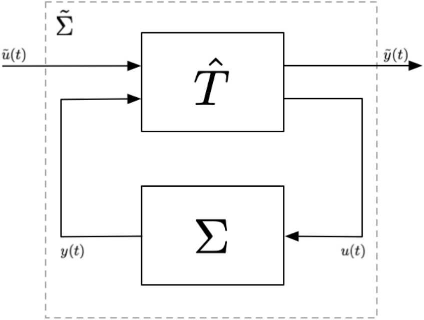


- if $M^T M$ is full-rank, we can always passivy the systems with a constant network gain β
- stability of network is guaranteed for any passive controllers and correct gain β
- gain depends on spectral properties of M

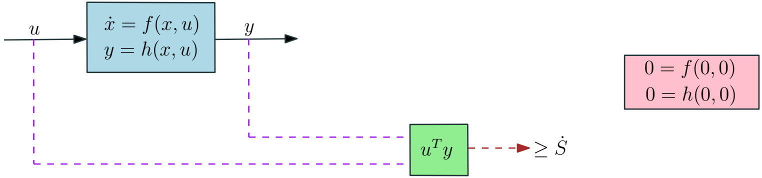


- in fact, a single agent can be used to passivy the entire network!
- design goal is to ensure agent has sufficient excess of passivity to compensate for any shortage of passivity in the network

PASSIVATION GOALS



- how do we passivy as dynamical system?
  - $\rightarrow$ feedback passivation
  - $\rightarrow$ loop-transformations (classic)
- can we passivy a system to achieve arbitrary passivity indices?
- can we characterize all transformations that map a system with given passivty index to a system with prescribed passivity index?



Definition

Let Σ be a SISO system with a constant input-output steady-state pair (u, y). The system is said to be input-output (ρ, ν)-passive wrt (u, y) if there exists a C^1 positive semi-definite storage function $S(x)$ and numbers $\rho, \nu \in \mathbb{R}$, such that $\rho\nu < 1/4$ and

$$\dot{S} = \frac{\partial S}{\partial x} f(x,u) \leq (y - \mathrm{y})(u - \mathrm{u}) - \rho(y - \mathrm{y})^2 - \nu(u - \mathrm{u})^2,$$

for any trajectory u, y.

Definition

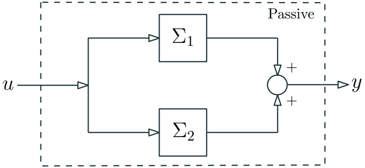
Let Σ be a SISO system with a constant input-output steady-state pair (u, y). The system is said to be input-output (ρ, ν)-passive wrt (u, y) if there exists a C^1 positive semi-definite storage function $S(x)$ and numbers $\rho, \nu \in \mathbb{R}$, such that $\rho\nu < 1/4$ and

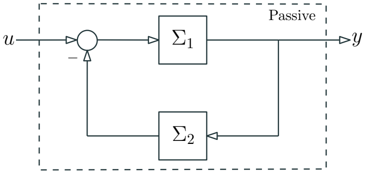
$$\dot{S} = \frac{\partial S}{\partial x} f(x, u) \leq (y - \mathrm{y})(u - \mathrm{u}) - \rho(y - \mathrm{y})^2 - \nu(u - \mathrm{u})^2,$$

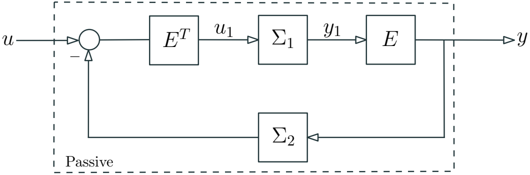
for any trajectory u, y.

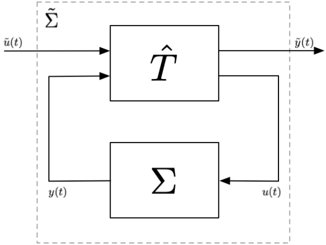
- $\rho = \nu = 0 \Rightarrow$ **passivity**
- $\rho, \nu > 0 \Rightarrow$ **strict input/output passivity**
- $\rho, \nu < 0 \Rightarrow$ **passive short**

- Parallel Interconnection
- Negative Feedback Interconnection
- Symmetric Interconnection

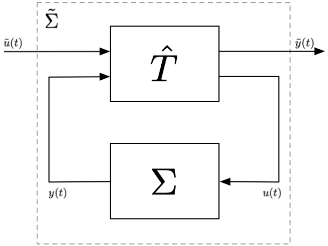








For a passive-short system $\Sigma : u \mapsto y$, we aim to find a map $\hat{T}$ such that the closed-loop system $\tilde{\Sigma} : \tilde{u} \mapsto \tilde{y}$ is passive. This is known as feedback passivation.



For a passive-short system $\Sigma : u \mapsto y$, we aim to find a map $\hat{T}$ such that the closed-loop system $\tilde{\Sigma} : \tilde{u} \mapsto \tilde{y}$ is passive. This is known as feedback passivation.

Problem Statement

Let Σ be a dynamical system with equal input and output dimensions, which is I/O (ρ, ν)-passive, and let $\rho_\star, \nu_\star$ be numbers such that $\rho_\star \nu_\star < 1/4$. Characterize all I/O transformations $\hat{T}$ such that the transformed system $\tilde{\Sigma}$ is I/O $(\rho_\star, \nu_\star)$-passive.

Consider the following system:

$$\dot{x} = -\sqrt[3]{x} + 0.5x + 0.5u$$
$$y = 0.5x - 0.5u$$

the system is passive-short

$$S(x) = \frac{1}{6}x^2$$
$$\dot{S} = yu + \frac{2}{3}y^2 + \frac{1}{3}u^2 - \frac{1}{3}(2y+u)\sqrt[3]{2y+u} \leq yu + \frac{2}{3}y^2 + \frac{1}{3}u^2$$

system has $\rho = -2/3, \nu = -1/3$

AN EXAMPLE

we can consider the following transformation:

$$\begin{cases} u(t) &= \tilde{u}(t) - y(t) \\ \tilde{y}(t) &= u(t) + 2y(t) \end{cases} \Rightarrow \begin{bmatrix} u(t) \\ \tilde{y}(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y(t) \\ \tilde{u}(t) \end{bmatrix}$$

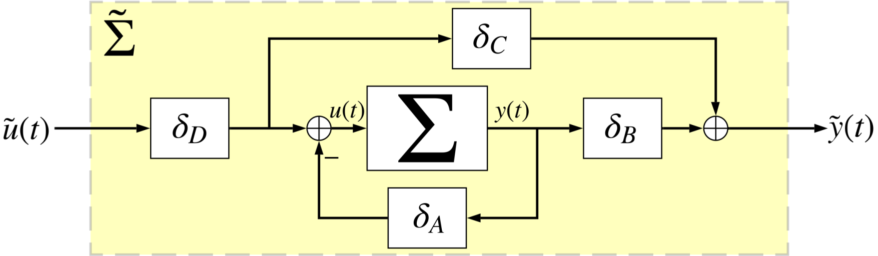
yields the transformed system

$$\dot{x} = -\sqrt[3]{x} + \tilde{u}$$
$$\tilde{y} = x$$

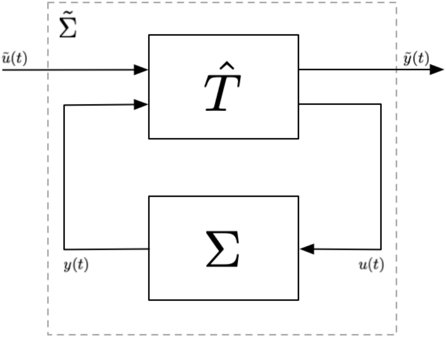
which is passive with storage function $S(x) = \frac{1}{2}x^2$ satisfying

$$\dot{S}(x) = \tilde{y}\tilde{u} - \tilde{y}\sqrt[3]{\tilde{y}} \leq \tilde{y}\tilde{u}$$

The loop transformation, combination of feedback, feedforward, pre-, and post-multiplication is the classic approach to feedback passivation



The loop transformation, combination of feedback, feedforward, pre-, and post-multiplication is the classic approach to feedback passivation



for this work, we prefer to consider the map $T : \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \mapsto \begin{bmatrix} \tilde{u}(t) \\ \tilde{y}(t) \end{bmatrix}$

A geometric approach to finding our map T...

Projective Quadratic Inequalities

A *projective quadratic inequality (PQI)* is an inequality with variables $\xi, \chi \in \mathbb{R}$ of the form

$$0 \leq a\xi^2 + b\xi\chi + c\chi^2 = \mathbf{f}_{(a,b,c)}(\xi, \chi),$$

for some numbers a, b, c, not all zero. The inequality is called *non-trivial* if $b^2 - 4ac > 0$. The associated solution set $\mathcal{C}_{\xi,\chi}$ of the PQI is the set of all points $(\xi, \chi) \in \mathbb{R}^2$ satisfying the inequality.

PQI:

$$0 \leq a\xi^2 + b\xi\chi + c\chi^2 = \mathbf{f}_{(a,b,c)}(\xi, \chi),$$

recall our definition for I/O (ρ, ν)-passivity

$$\dot{S} \leq yu - \rho y^2 - \nu u^2$$

PQI captures passivity

$$\dot{S} \leq \mathbf{f}_{(-\nu,1,-\rho)}(u, y)$$

Solution set

$$\mathcal{C}_{\rho,\nu} = \{(\xi, \chi) \in \mathbb{R} \times \mathbb{R} \,:\, \mathbf{f}_{(-\nu,1,-\rho)}(\xi, \chi) \geq 0\}$$

- we are interested in maps $T : \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \mapsto \begin{bmatrix} \tilde{u}(t) \\ \tilde{y}(t) \end{bmatrix}$
- original system has a PQI solution set $\mathcal{C}_{\rho,\nu}$ for some (ρ, ν)
- transformed system has PQI solution set $\mathcal{C}_{\rho^\star,\nu^\star}$ for some $(\rho^\star, \nu^\star)$

- we are interested in maps $T : \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \mapsto \begin{bmatrix} \tilde{u}(t) \\ \tilde{y}(t) \end{bmatrix}$
- original system has a PQI solution set $\mathcal{C}_{\rho,\nu}$ for some (ρ, ν)
- transformed system has PQI solution set $\mathcal{C}_{\rho^\star,\nu^\star}$ for some $(\rho^\star, \nu^\star)$

An I/O transformation T maps an I/O (ρ, ν)-passive system to an I/O $(\rho_\star, \nu_\star)$-passive system if and only if it maps the PQI $0 \leq \mathbf{f}_{(-\nu,1,-\rho)}(\xi, \chi)$ to the PQI $0 \leq \mathbf{f}_{(-\nu^\star,1,-\rho^\star)}(\xi, \chi)$ (or to a stricter inequality) recall our earlier example...

$$\dot{x} = -\sqrt[3]{x} + 0.5x + 0.5u$$
$$y = 0.5x - 0.5u$$

satisfies

$$\frac{1}{3}\chi^2 + \chi\xi + \frac{2}{3}\xi^2 = \mathbf{f}_{(1/3,1,2/3)}(\xi, \chi) \geq 0$$

we considered the transformation $\begin{bmatrix} \tilde{\chi} \\ \tilde{\xi} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \chi \\ \xi \end{bmatrix}$

transformed system satisfies some PQI

$$a\tilde{\chi}^2 + b\tilde{\chi}\tilde{\xi} + c\tilde{\xi}^2 \geq 0$$

we should recover original PQI by inverting the map

$$\begin{aligned} 0 &\leq a\tilde{\chi}^2 + b\tilde{\chi}\tilde{\xi} + c\tilde{\xi}^2 \\ &= a(\chi+\xi)^2 + b(\chi+\xi)(\chi+2\xi) + c(\chi+2\xi)^2 \\ &= (a+b+c)\chi^2 + (2a+3b+4c)\chi\xi + (a+2b+4c)\xi^2 \end{aligned}$$

solving for (a, b, c) using

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ 1 \\ \frac{2}{3} \end{bmatrix}$$

gives $a = c = 0,\ b = 1/3$ implying that

$$0 \leq \frac{1}{3}\tilde{\chi}\tilde{\xi}$$

i.e., the transformed system is passive

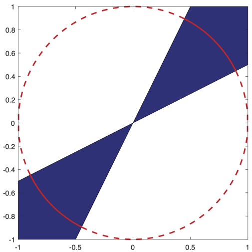
main idea

Let $\mathcal{A}$ be the solution set of the original PQI. The solution set of the new PQI under the transformation T is

$$T(\mathcal{A}) = \{T(\chi, \xi) \,:\, (\chi, \xi) \in \mathcal{A}\}.$$

We can therefore study the effect of linear transformations on PQIs by studying their actions on the solution sets.

The solution set of any non-trivial PQI is a symmetric double-cone. Moreover, any symmetric double-cone is the solution set of some non-trivial PQI.



Theorem⋆ [Sharf, Jain, Z 2021]

Let (ξ_1, χ_1), (ξ_2, χ_2) be non-colinear solutions of $a_1\xi^2 + \xi\chi + c_1\chi^2 = 0$, and $(\tilde{\xi}_1, \tilde{\chi}_1)$,$(\tilde{\xi}_2, \tilde{\chi}_2)$ be non-colinear solutions of $a_2\xi^2 + \xi\chi + c_2\chi^2 = 0$. Define

$$T_1 = \begin{bmatrix} \tilde{\xi}_1 & \tilde{\xi}_2 \\ \tilde{\chi}_1 & \tilde{\chi}_2 \end{bmatrix} \begin{bmatrix} \xi_1 & \xi_2 \\ \chi_1 & \chi_2 \end{bmatrix}^{-1}, T_2 = \begin{bmatrix} \tilde{\xi}_1 & -\tilde{\xi}_2 \\ \tilde{\chi}_1 & -\tilde{\chi}_2 \end{bmatrix} \begin{bmatrix} \xi_1 & \xi_2 \\ \chi_1 & \chi_2 \end{bmatrix}^{-1}.$$

Then one of T_1, T_2 transforms the PQI $a_1\xi^2 + \xi\chi + c_1\chi^2 \geq 0$ to the PQI $\tau a_2\xi^2 + \tau\xi\chi + \tau c_2\chi^2 \geq 0$ for some $\tau > 0$.

EXAMPLE CONTINUED

...back to our original system with PQI

$$\frac{1}{3}\chi^2 + \chi\xi + \frac{2}{3}\xi^2 = \mathbf{f}_{(1/3,1,2/3)}(\xi, \chi) \geq 0$$

can be rewritten as

$$\frac{1}{3}(\chi + \xi)(\chi + 2\xi) = 0$$

so two solutions are $(2, -1), (-1, 1) \in \mathcal{C}_{1/3,2/3}$

...back to our original system with PQI

$$\frac{1}{3}\chi^2 + \chi\xi + \frac{2}{3}\xi^2 = \mathbf{f}_{(1/3,1,2/3)}(\xi, \chi) \geq 0$$

can be rewritten as

$$\frac{1}{3}(\chi + \xi)(\chi + 2\xi) = 0$$

so two solutions are $(2, -1), (-1, 1) \in \mathcal{C}_{1/3,2/3}$

the new PQI satisfies

$$\frac{1}{3}\tilde{\chi}\tilde{\xi} \geq 0$$

with solutions $(1, 0), (0, 1) \in \mathcal{C}_{0,0}$

EXAMPLE CONTINUED

...back to our original system with PQI

$$\frac{1}{3}\chi^2 + \chi\xi + \frac{2}{3}\xi^2 = \mathbf{f}_{(1/3,1,2/3)}(\xi,\chi) \geq 0$$

can be rewritten as

$$\frac{1}{3}(\chi+\xi)(\chi+2\xi) = 0$$

so two solutions are $(2,-1),(-1,1) \in \mathcal{C}_{1/3,2/3}$

the new PQI satisfies

$$\frac{1}{3}\tilde{\chi}\tilde{\xi} \geq 0$$

with solutions $(1,0),(0,1) \in \mathcal{C}_{0,0}$ applying theorem

$$T_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

i.e., the transformation we found earlier!

summary

A map T transforms an I/O (ρ, ν)-passive system to an I/O $(\rho_\star, \nu_\star)$-passive system if and only if it sends $C_{\rho,\nu}$ into $C_{\rho_\star,\nu_\star}$, which we denote by $C_{\rho,\nu} \hookrightarrow C_{\rho_\star,\nu_\star}$

- earlier theorem gives a characterization for these maps - allows to find a map from one double cone to another double cone
- we would like to characterize all possible maps

TOWARDS A CHARACTERIZATION

summary

A map T transforms an I/O (ρ, ν)-passive system to an I/O $(\rho_\star, \nu_\star)$-passive system if and only if it sends $C_{\rho,\nu}$ into $C_{\rho_\star,\nu_\star}$, which we denote by $C_{\rho,\nu} \hookrightarrow C_{\rho_\star,\nu_\star}$

- earlier theorem gives a characterization for these maps - allows to find a map from one double cone to another double cone
- we would like to characterize all possible maps

main idea

show that all maps from an arbitrary double cone into another arbitrary double cone can be built using maps from $\mathcal{C}_{0,0}$ into iteslf

MAPPING $C_{0,0}$ INTO ITSELF

Proposition

Let $\rho, \nu, \rho_\star, \nu_\star$ be any four numbers such that $\rho\nu, \rho_\star\nu_\star < 1/4$, and let $T : C_{\rho,\nu} \hookrightarrow C_{\rho_\star,\nu_\star}$. Let $S_{\rho,\nu} : C_{0,0} \hookrightarrow C_{\rho,\nu}$ and $S_{\rho_\star,\nu_\star} : C_{0,0} \hookrightarrow C_{\rho_\star,\nu_\star}$ built using Theorem $\star$. Then there exists a matrix $Q : C_{0,0} \hookrightarrow C_{0,0}$, such that $T = S_{\rho_\star,\nu_\star} Q S_{\rho,\nu}^{-1}$ holds.

Proposition

Let $\rho, \nu, \rho_\star, \nu_\star$ be any four numbers such that $\rho\nu, \rho_\star\nu_\star < 1/4$, and let $T : C_{\rho,\nu} \hookrightarrow C_{\rho_\star,\nu_\star}$. Let $S_{\rho,\nu} : C_{0,0} \hookrightarrow C_{\rho,\nu}$ and $S_{\rho_\star,\nu_\star} : C_{0,0} \hookrightarrow C_{\rho_\star,\nu_\star}$ built using Theorem $\star$. Then there exists a matrix $Q : C_{0,0} \hookrightarrow C_{0,0}$, such that $T = S_{\rho_\star,\nu_\star} Q S_{\rho,\nu}^{-1}$ holds.

$$C_{0,0} \overset{S_{\rho,\nu}}{\hookrightarrow} C_{\rho,\nu} \overset{T}{\hookrightarrow} C_{\rho_\star,\nu_\star} \overset{S_{\rho_\star,\nu_\star}^{-1}}{\hookrightarrow} C_{0,0}.$$

Proposition

Let $\rho, \nu, \rho_\star, \nu_\star$ be any four numbers such that $\rho\nu, \rho_\star\nu_\star < 1/4$, and let $T : C_{\rho,\nu} \hookrightarrow C_{\rho_\star,\nu_\star}$. Let $S_{\rho,\nu} : C_{0,0} \hookrightarrow C_{\rho,\nu}$ and $S_{\rho_\star,\nu_\star} : C_{0,0} \hookrightarrow C_{\rho_\star,\nu_\star}$ built using Theorem $\star$. Then there exists a matrix $Q : C_{0,0} \hookrightarrow C_{0,0}$, such that $T = S_{\rho_\star,\nu_\star} Q S_{\rho,\nu}^{-1}$ holds.

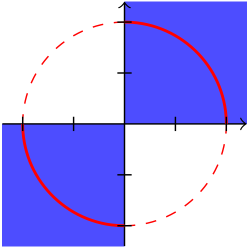
$$C_{0,0} \overset{S_{\rho,\nu}}{\hookrightarrow} C_{\rho,\nu} \overset{T}{\hookrightarrow} C_{\rho_\star,\nu_\star} \overset{S^{-1}_{\rho_\star,\nu_\star}}{\hookrightarrow} C_{0,0}.$$

gives a prescription for finding all matrices mapping $C_{\rho,\nu}$ into $C_{\rho_\star,\nu_\star}$.

- $S_{\mu,\tau}$
- matrices mapping $C_{0,0}$ into itself

Proposition

A matrix $T \in GL_2(\mathbb{R})$ sends $C_{0,0}$ into itself if and only if all of the entries of T have the same sign, i.e., $T_{ij}T_{kl} \geq 0$ for every $i, j, k, l \in \{1, 2\}$.



Proposition †

Let μ, τ be any two numbers such that $\mu\tau < 1/4$. Recall that $S_{\mu,\tau}$ is a map $C_{0,0} \hookrightarrow C_{\mu,\tau}$, as constructed in Theorem $\star$. Define $R = \sqrt{1 - 4\tau\mu}$.

i) If $\tau < 0$, we can choose $S_{\mu,\tau} = \frac{1}{2\tau}\begin{bmatrix} -1-R & 1-R \\ -2\tau & 2\tau \end{bmatrix}$.

ii) If $\tau > 0$, we can choose $S_{\mu,\tau} = \frac{1}{2\tau}\begin{bmatrix} 1+R & 1-R \\ 2\tau & 2\tau \end{bmatrix}$.

iii) If $\tau = 0$, we can choose $S_{\mu,\tau} = \begin{bmatrix} 1 & \mu \\ 0 & 1 \end{bmatrix}$.

direct construction

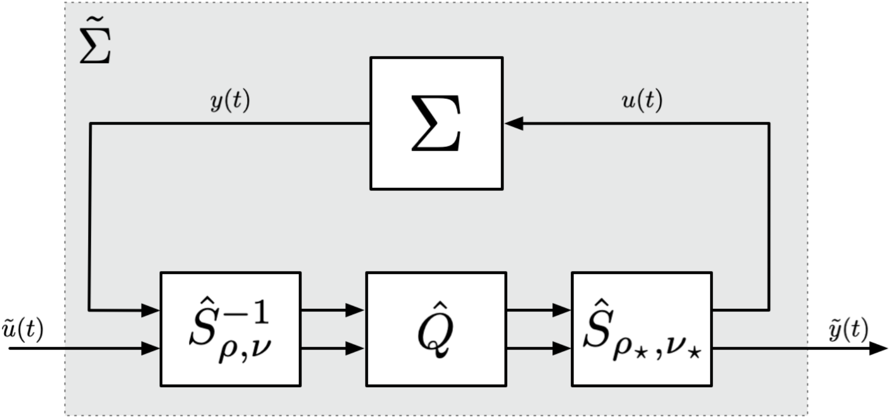
Theorem

Let Σ be a SISO I/O (ρ, ν)-passive system, and let $T \in GL_2(\mathbb{R})$ be an invertible matrix I/O transformation. The transformed system $\tilde{\Sigma}$ is I/O $(\rho_\star, \nu_\star)$-passive if and only if there exists a matrix $M \in GL_2(\mathbb{R})$ such that

i) $M_{ij} \geq 0$ for all $i, j \in \{1, 2\}$;
ii) some $\theta \in \{\pm 1\}$ such that $T = \theta S_{\rho_\star,\nu_\star} M S_{\rho,\nu}^{-1}$, where $S_{\rho,\nu}, S_{\rho_\star,\nu_\star}$ are given in Proposition †.

In other words, the transformed system $\tilde{\Sigma}$ is I/O $(\rho_\star, \nu_\star)$-passive if and only if all of the entries of the matrix $S_{\rho_\star,\nu_\star}^{-1} T S_{\rho,\nu}$ have the same sign.

MAIN RESULT



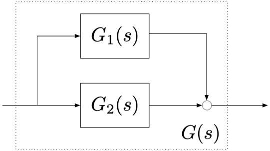
$\tilde{\Sigma}$
$y(t)$
Σ
$u(t)$
$\tilde{u}(t)$
$\hat{S}^{-1}_{\rho,\nu}$
$\hat{Q}$
$\hat{S}_{\rho_\star,\nu_\star}$
$\tilde{y}(t)$

- Motivation: feedback systems with different faulty modes

$$\{\Sigma_i\}_{i \in \mathcal{I}}$$

  - each Σ_i represents a system in different operating mode
  - assume each Σ_i is I/O (ρ_i, ν_i)-passive while the desired passivity index is $(\rho_i^\star, \nu_i^\star)$
  - Transformed systems $\{\tilde{\Sigma}_i\}_{i \in \mathcal{I}}$ are I/O $(\rho_i^\star, \nu_i^\star)$-passive for all i, if and only if there exists matrices M_i with all non-negative entries, and numbers $\theta_i \in \{\pm 1\}$ such that

$$T = \theta_i S_{\rho_i^\star, \nu_i^\star} M_i S_{\rho_i, \nu_i}^{-1}$$



- $G_1(s) = \frac{s-1}{s+1}$, $G_2(s) = \frac{-s^3+6s+5}{s^3+4s^2+5s+2}$
- Parallel interconnection: $G(s) = \frac{2s+3}{s^2+3s+2} = \frac{1}{s+2} + \frac{1}{s+1}$ system is $(2/3, 0)$passive
- assume $G_2(s)$ is faulty and switches to $G_1(s)$ in fault mode
- with fault, $\bar{G}(s) = 2G_1(s)$ and it is $(0, -1.25)$-passive

Find map T that maps fault $G(s)$ to a $(2, 0)$-passive system and $\bar{G}(s)$ to a $(0, 0)$-passive system

- Let $T_1 = S_{2,0}^{-1} T S_{\frac{2}{3},0}$ and $T_2 = S_{0,0}^{-1} T S_{0,-1.25}$
- We want entries of T_1 and T_2 to have same sign. Let

$$T = \begin{bmatrix} 1 & 0.4 \\ 0.4 & 0.2 \end{bmatrix}$$

- leads to

$$T_1 = \frac{1}{15} \begin{bmatrix} 3 & 2 \\ 6 & 7 \end{bmatrix} \text{ and } T_2 = \frac{1}{25} \begin{bmatrix} 10 & 10 \\ 3 & 5 \end{bmatrix}$$

- Can be verified that T_1 sends $G(s) \mapsto \tilde{G}(s) = \frac{0.4s^2+1.6s+1.4}{s^2+3.8s+3.2}$ with passivity index $(2.2857, 0)$ and T_2 send $\bar{G}(s) \mapsto \bar{\tilde{G}}(s) = \frac{0.6s+.2}{1.4s+.6}$ to $(2.333, 0)$-passive system results can be generalized to MIMO systems

Theorem

Let Σ be an I/O (ρ, ν)-passive system with input and output dimension equal to d, and let $T \in GL_{2d}(\mathbb{R})$ be an invertible matrix inducing an I/O transformation. The transformed system $\tilde{\Sigma}$ is I/O $(\rho_\star, \nu_\star)$-passive if and only if there exists a matrix $M \in GL_{2d}(\mathbb{R})$ and some positive $\lambda > 0$ such that:

$$T = (S_{\rho_\star,\nu_\star} \otimes \mathrm{Id}_d) M (S_{\rho,\nu}^{-1} \otimes \mathrm{Id}_d), \quad M^\top J M - \lambda J \geq 0,$$

where $J = \begin{bmatrix} 0 & 0.5\mathrm{Id}_d \\ 0.5\mathrm{Id}_d & 0 \end{bmatrix}$, i.e., $\tilde{\Sigma}$ is I/O $(\rho_\star, \nu_\star)$-passive if and only if there exists $\lambda > 0$ such that $X = (S_{\rho_\star,\nu_\star}^{-1} \otimes \mathrm{Id}_d) T (S_{\rho,\nu} \otimes \mathrm{Id}_d)$ satisfies $X^\top J X - \lambda J \geq 0$.

OUTLOOKS

- framework can allow us to consider optimal passivizing transformations

$$\begin{aligned} \min_{T} \quad & \Phi(T) \\ \text{s.t.} \quad & T \text{ maps I/O } (\rho, \nu) \text{ systems to I/O } (\rho_\star, \nu_\star)\text{-systems.} \end{aligned}$$

OUTLOOKS

- framework can allow us to consider optimal passivizing transformations

$$\begin{aligned} \min_{T,\lambda,M} \quad & \Phi(T) \\ \text{s.t.} \quad & M = (S_{\rho^\star,\nu^\star} \otimes \mathrm{Id}_d)^{-1} T (S_{\rho,\nu} \otimes \mathrm{Id}_d) \\ & M^\top J M - \lambda J \geq 0 \\ & \lambda \geq 0, \end{aligned}$$

- framework can allow us to consider optimal passivizing transformations

$$\begin{aligned} \min_{T,\lambda,M} \quad & \Phi(T) \\ \text{s.t.} \quad & M = (S_{\rho^\star,\nu^\star} \otimes \mathrm{Id}_d)^{-1} T (S_{\rho,\nu} \otimes \mathrm{Id}_d) \\ & M^\top J M - \lambda J \geq 0 \\ & \lambda \geq 0, \end{aligned}$$

- extend to different passivity variations (incremental, equilibrium independent, etc.)

OUTLOOKS

- framework can allow us to consider optimal passivizing transformations

$$\begin{aligned}\min_{T,\lambda,M} \quad & \Phi(T) \\ \text{s.t.} \quad & M = (S_{\rho^\star,\nu^\star} \otimes \mathrm{Id}_d)^{-1} T (S_{\rho,\nu} \otimes \mathrm{Id}_d) \\ & M^\top J M - \lambda J \geq 0 \\ & \lambda \geq 0,\end{aligned}$$

- extend to different passivity variations (incremental, equilibrium independent, etc.)
- applications to plug-and-play networks

THANK-YOU

Dr. Miel Sharf (Jether Energy Research)

Prof. Anoop Jain (IIT-Jodhpur)

- M. Sharf, A. Jain and D. Zelazo, “A Geometric Method for Passivation and Cooperative Control of Equilibrium-Independent Passivity-Short Systems”, *IEEE Transactions on Automatic Control*, 66(12):5877-5892, 2021.
- M. Sharf and D. Zelazo, “A Characterization of All Linear Passivizing Input-Output Transformations of a Passive-Short System: The SISO Case,” *IEEE Control Systems Letters*, 8:532:537, 2024.

CoNeCt
Cooperative Networks and Controls Lab

https://connect-lab-technion.github.io/

Open Faculty Positions

We invite applications for tenure-track positions at all ranks

- We seek individuals with outstanding potential for research and teaching, who will enrich the professional diversity of the academic community
- Research areas of interest include:
  - Aerodynamics, fluid mechanics, and aeroacoustics
  - Astrodynamics and space propulsion
  - Combustion and propulsion
  - Guidance, navigation, control, and autonomous systems
  - Robotics and AI NEW

Visit our website: aerospace.technion.ac.il

DEPARTMENT OF AEROSPACE ENGINEERING | TECHNION Israel Institute of Technology