PASSIVITY THEORY IN COOPERATIVE CONTROL:

A NETWORK OPTIMIZATION PERSPECTIVE

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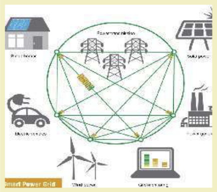
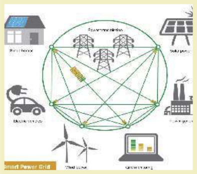
INTRODUCTION

NETWORKED DYNAMIC SYSTEMS

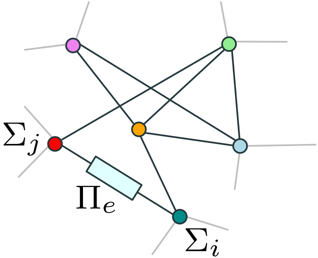


Networks of dynamical systems are one of the enabling technologies of the future.



- how do we analyze these systems
- how do we design these systems

IN THIS TALK...

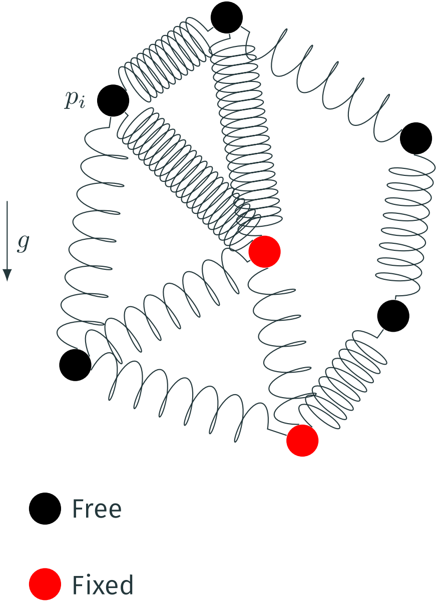


Explore the structure and mechanisms of networked systems to reveal deep connections between properties of dynamical systems and optimization theory.

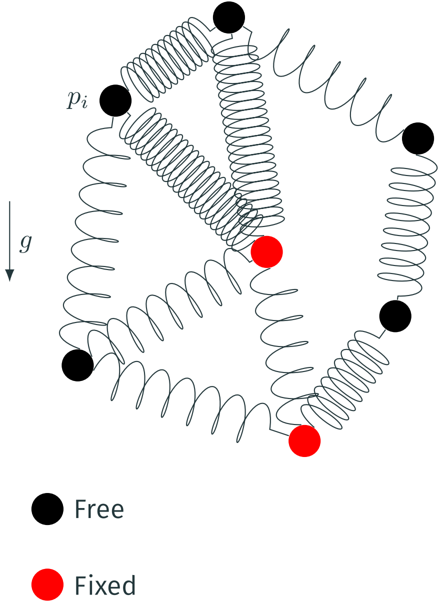
- A general model of diffusively coupled networks
- Characterization of network equilibriums via Network Optimization
- Convergence properties of dynamic networks via passivity theory
- Exploring further connections between passivity and optimization

A PHYSICS WARM-UP

A MASS-SPRING NETWORK

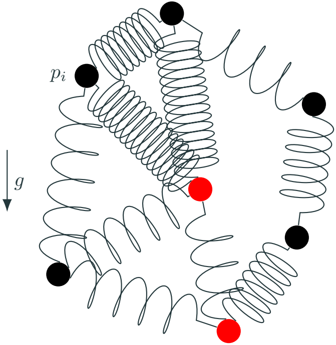


- A fixed network of (linear) springs
- springs connected to masses with position $p_i \in \mathbb{R}^2$ and mass m_i
- r masses have a fixed position (anchors)



- A fixed network of (linear) springs
- springs connected to masses with position $p_i \in \mathbb{R}^2$ and mass m_i
- r masses have a fixed position (anchors)

> Determine the positions of the free masses that minimize the total potential energy of the mass-spring network.



- Potential Energy due to gravity

$$m_i g^T p_i$$

- Elastic Potential Energy of springs

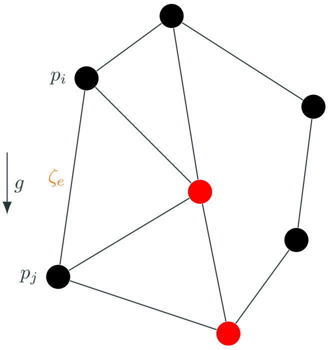
$$\frac{1}{2} k_{ij} (\|p_i - p_j\| - r_{ij})^2$$

an optimization problem (take 1)

$$\min_{p_i} \quad \sum_i m_i g^T p_i + \sum_{i \sim j} \frac{1}{2} k_{ij} (\|p_i - p_j\| - r_{ij})^2$$

$$\texttt{s.t.}\, p_i = \mathbf{p}_i^*, \; i = 1, \ldots, r \;\; \texttt{(fixed nodes)}$$

A MASS-SPRING NETWORK



- Potential Energy due to gravity (nodes)

$$m_i g^T p_i, \; i = 1, \ldots, n$$

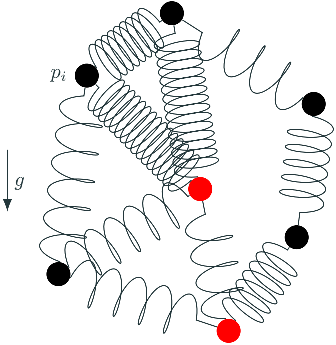
- Elastic Potential Energy of springs (edges)

$$\frac{1}{2} k_e (\| \underbrace{p_i - p_j}_{\zeta_e} \| - r_e)^2, \; e = 1, \ldots, m$$

an optimization problem (take 2)

$$\min_{p_i, \zeta_e} \quad \sum_{i=1}^{r} (m_i g^T p_i + \mathbb{I}_{\mathbf{p_i^*}}(p_i)) + \sum_{i=r+1}^{n} m_i g^T p_i + \sum_{e} \frac{1}{2} k_e (\|\zeta_e\| - r_e)^2$$

$$\texttt{s.t.} \, p_i - p_j = \zeta_e, \; \forall e = (i, j)$$



A Convex Program!

an optimization problem (take 2)

$$\min_{p_i, \zeta_e} \quad \sum_{i}^{r} (m_i g^T p_i + \mathbb{I}_{\mathbf{p_i^*}}(p_i)) + \sum_{i=r+1}^{n} m_i g^T p_i + \sum_{e} \frac{1}{2} k_{ij} (\|\zeta_e\| - r_e)^2$$

$$\texttt{s.t.} p_i - p_j = \zeta_e, \forall e = (i, j)$$

A MASS-SPRING NETWORK - THE DYNAMICS

- dynamic model for the masses

$$\Sigma_i \ : \ \begin{cases} \begin{bmatrix} \dot{p}_i \\ \ddot{p}_i \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_i \\ \dot{p}_i \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u_i + m_i g \\ y_i \ = \begin{cases} \begin{bmatrix} p_i \\ 0 \end{bmatrix}, & i = 1, \ldots, r \ \texttt{(anchors)} \\ \begin{bmatrix} p_i \\ \dot{p}_i \end{bmatrix}, & i = r+1, \ldots, n \end{cases} \end{cases}$$

- springs couple masses together

$$\Pi_e \ : \ \begin{cases} u_i & = \sum_{i \sim j} k_{ij} (\|p_i - p_j\| - r_{ij}) \frac{p_j - p_i}{\|p_j - p_i\|} + \\ & \quad b_{ij} (\dot{p}_j - \dot{p}_i) \\ & = \sum_{i \sim j} \kappa_{ij} (y_i - y_j) \end{cases}$$

A MASS-SPRING NETWORK - THE DYNAMICS

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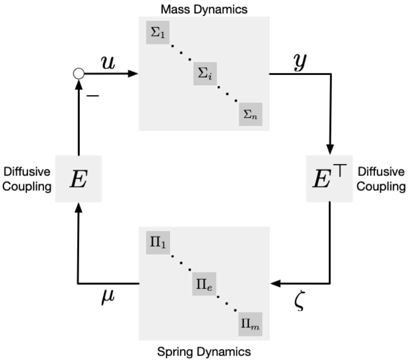
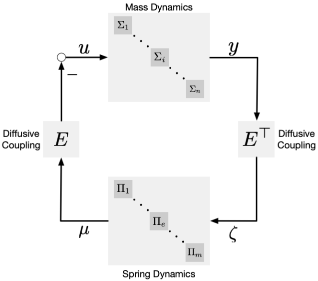


An example of a **diffusively coupled network**!

- System Equilibrium

$$\begin{cases} 0 & = \dot{p}_i \\ 0 & = m_i g + \sum_{i\sim j} k_{ij}(\|p_i - p_j\| - r_{ij}) \frac{p_j - p_i}{\|p_j - p_i\|} \end{cases}$$

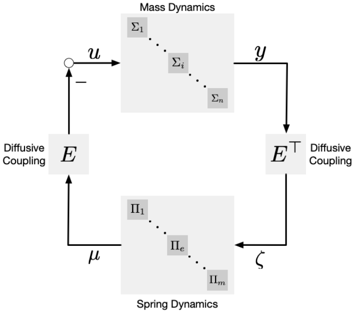


Minimum Total Potential Energy Principle (MTPE)

Equilibrium configurations extremize the total potential energy. Stable equilibriums correspond to minimizers of the total potential energy.

Dynamics

- Diffusively Coupled Network

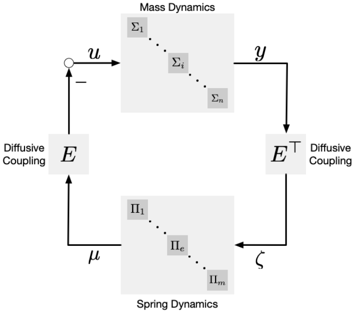


- Dissipasivity Theory

$$V(x) = \frac{1}{2}\sum_{i} \|\dot{p}_i\|^2 + \frac{1}{2}\sum_{i\sim j} k_{ij}\|p_i - p_j\|_2^2$$

Dynamics

- Diffusively Coupled Network



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Optimization

- Convex Optimization

$$\min_{p_i,\zeta_e} \quad J(p,\zeta)$$

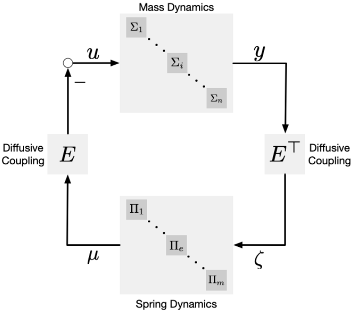
$$\texttt{s.t.}\, p_i - p_j = \zeta_e, \forall\, e = (i,j)$$

- Optimality Conditions

$$0 \in \partial J(p,\zeta)$$

Dynamics

- Diffusively Coupled Network



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Optimization

- Convex Optimization

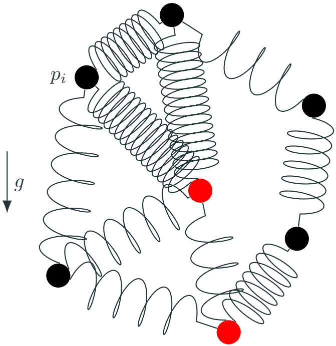
$$\min_{p_i,\zeta_e} \quad J(p,\zeta)$$

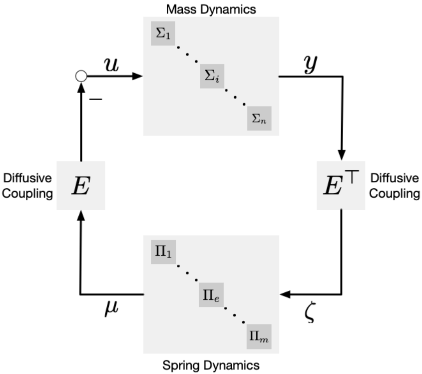
$$\texttt{s.t.}\, p_i - p_j = \zeta_e, \forall\, e = (i,j)$$

- Optimality Conditions

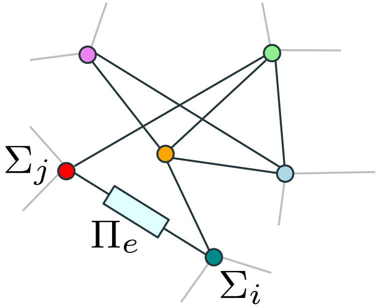
MTPE Principle ensures that the dynamics of the diffusively coupled network solve the optimization problem, and vice versa.

- What class of systems can be "solved" by examining a related optimization problem?
- What class of optimization problems can be be "solved" by a dynamical system?



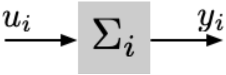


DIFFUSIVELY COUPLED NETWORKS



A network system is comprised of dynamical systems that interact with eachother over an information exchange network (a graph).

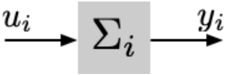
Agent dynamics:



$$\Sigma_i : \begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = h_i(x_i, u_i) \end{cases}$$

A NETWORK MODEL

Agent dynamics:



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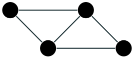
Information Exchange Network:



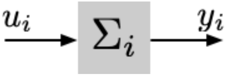
$$E = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

$$\mathcal{G} = (\mathbb{V}, \mathbb{E})$$

$$[E]_{ij} = \begin{cases} \pm 1 & (i,j) \in \mathbb{E} \\ 0 & \texttt{o.w.} \end{cases}$$

$$E^\top \mathbf{1} = 0$$

Agent dynamics:



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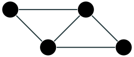

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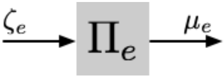
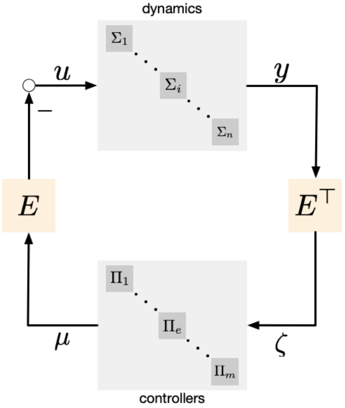
Controller dynamics:



$$\Pi_e : \begin{cases} \dot{\eta}_e = \phi_e(\eta_e, \zeta_e) \\ \mu_e = \psi_e(\eta_e, \zeta_e) \end{cases}$$



$$(\Sigma, \Pi, \mathcal{G})$$

- Consensus Dynamics

$$\dot{x}_i = -\sum_{i\sim j} w_{ij}(x_j - x_i)$$

- Kumamoto Model

$$\dot{\theta}_i = -k\sum_{i\sim j} \sin(\theta_i - \theta_j)$$

- Traffic Dynamics

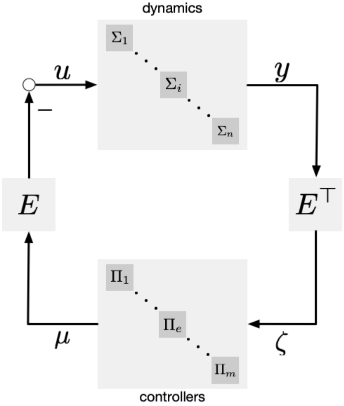
$$\dot{v}_i = \kappa_i\left(V_i^0 - v_i + V_i^1\sum_{i\sim j}\tanh(p_j - p_i)\right)$$

- Neural Network

$$C\dot{V}_i = f(V_i, h_i) + \sum_{i\sim j} g_{ij}(V_j - V_i)$$

$$\dot{h}_i = g(V_i, h_i)$$

STEADY-STATE NETWORK SOLUTIONS



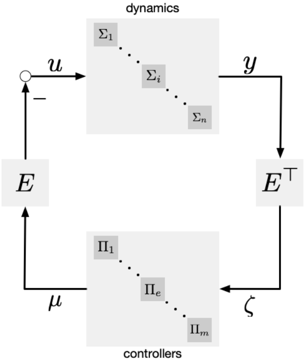
What properties must the systems Σ_i and Π_e possess such that $(\Sigma, \Pi, \mathcal{G})$ admits and converges to a steady-state solution?

$$u(t) \to \mathrm{u}$$
$$y(t) \to \mathrm{y}$$
$$\zeta(t) \to \zeta$$
$$\mu(t) \to \mu$$

- Consensus: $\mathrm{y} = \alpha \mathbf{1}$ $(\zeta = 0)$
- Formation: $\zeta \neq 0$ constant

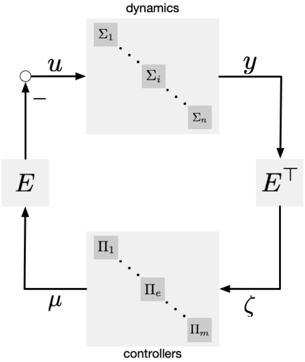
All signals converge to a constant steady-state

NETWORK OPTIMIZATION MEETS PASSIVITY THEORY



Assumption 1

Each agent Σ_i and controller Π_e admit forced steady-state solutions.

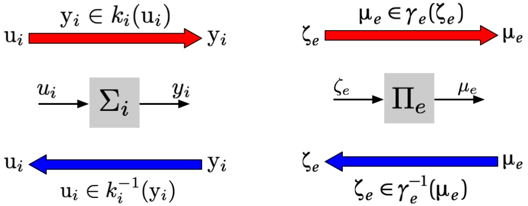


Assumption 1

Each agent Σ_i and controller Π_e admit forced steady-state solutions.

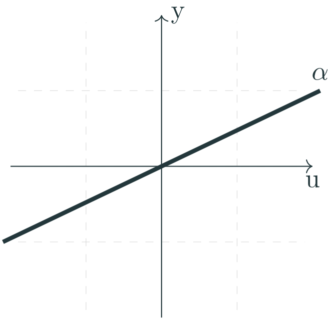
Input-Output Maps

The *steady-state input-output map* $k : \mathcal{U} \to \mathcal{Y}$ associated with Σ is the set consisting of all steady-state input-output pairs (u, y) of the system.



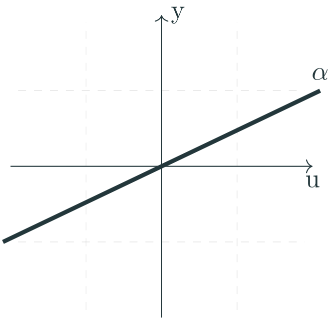
INPUT-OUTPUT RELATIONS

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$\Rightarrow k(\mathrm{u}) = \{\mathrm{y} \,|\, \underbrace{(-CA^{-1}B + D)}_{\alpha}\,\mathrm{u}\}$$



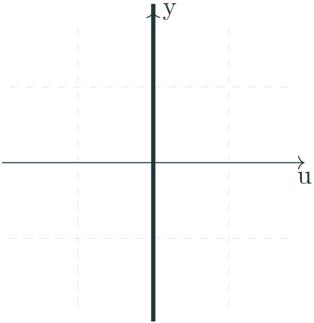
SISO and stable linear system

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$\Rightarrow k(\mathrm{u}) = \{\mathrm{y} \mid \underbrace{(-CA^{-1}B + D)}_{\alpha}\,\mathrm{u}\}$$



SISO and stable linear system

$$\dot{x} = u$$
$$y = x$$
$$\Rightarrow k = \{(0, \mathrm{y}),\ \mathrm{y} \in \mathbb{R}\}$$



simple integrator

NETWORK CONSISTENCY EQUATIONS

The network interconnection imposes constraints on allowable steady-states

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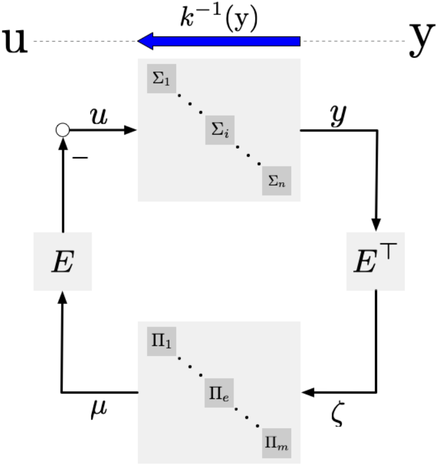
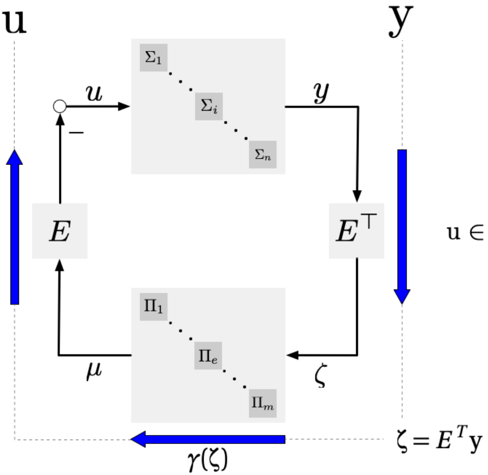
The network interconnection imposes constraints on allowable steady-states



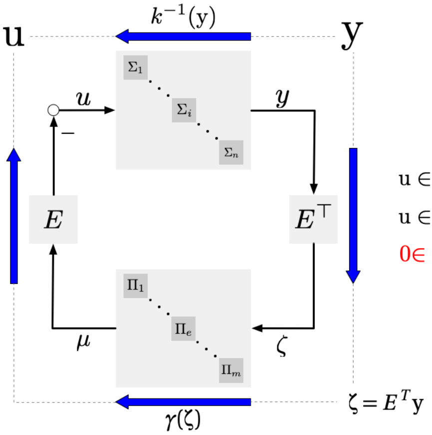
$\mathrm{u} \in k^{-1}(\mathrm{y})$

The network interconnection imposes constraints on allowable steady-states



$$\mathrm{u} \in -E\gamma(E^T\mathrm{y})$$

The network interconnection imposes constraints on allowable steady-states

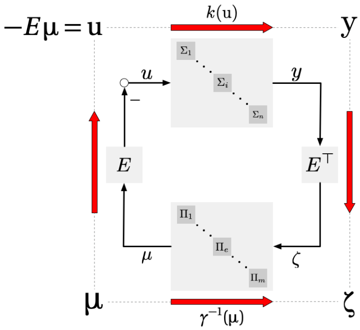


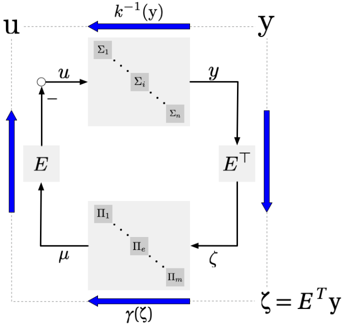
$\mathrm{u} \in k^{-1}(\mathrm{y})$

$\mathrm{u} \in -E\gamma(E^T\mathrm{y})$

$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T\mathrm{y})$

The network interconnection imposes constraints on allowable steady-states





$\zeta \in \gamma^{-1}(\mu)$

$\zeta \in E^T k(-E\mu)$

$0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$

$u \in k^{-1}(y)$

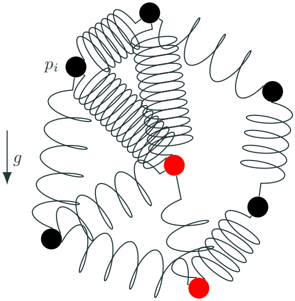
$u \in -E\gamma(E^T y)$

$0 \in k^{-1}(y) + E\gamma(E^T y)$

The network system $(\Sigma, \Pi, \mathcal{G})$ admits a steady-state if and only if there exists a solution to the system of non-linear inclusions

$$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T\mathrm{y})$$
$$0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$$

- When do solutions exist?
- How do we find them?

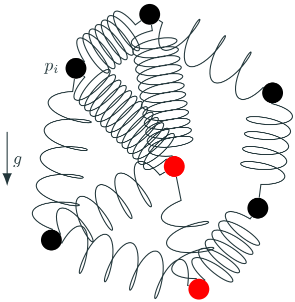


A Convex Program!

Minimum Potential Energy Problem

$$\min_{p_i,\zeta_e} \quad \sum_{i}^{r}(m_i g^T p_i + \mathbb{I}_{\mathbf{p_i^*}}(p_i)) + \sum_{i=r+1}^{n} m_i g^T p_i + \sum_{e} \frac{1}{2} k_{ij}(\|\zeta_e\| - r_e)^2$$

$$\texttt{s.t.}\, p_i - p_j = \zeta_e, \forall e = (i,j)$$

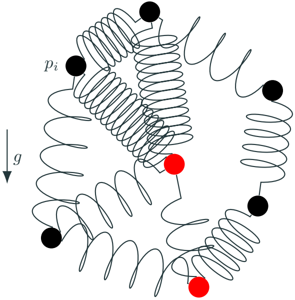


A Convex Program!

Minimum Potential Energy Problem

$$\min_{p_i,\zeta_e} \quad \sum_i J_i(p_i) + \sum_e \Gamma_e(\zeta_e)$$

$$\texttt{s.t.}\, E^T p = \zeta$$



A Convex Program!

Minimum Potential Energy Problem

$$\min_{p} \quad J(p) + \Gamma(E^T p)$$

First-order Optimality Condition:

$$0 \in \partial J(p) + E\partial\Gamma(E^T p)$$

SOLUTION OF NETWORK EQUATIONS

The network system $(\Sigma, \Pi, \mathcal{G})$ admits a steady-state if and only if there exists a solution to the system of non-linear inclusions

$$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T \mathrm{y})$$
$$0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$$

RECALL First-order Optimality Condition:

$$0 \in \partial J(p) + E\partial\Gamma(E^T p)$$

Network equations are the first-order optimality conditions of a corresponding optimization problem!

SOLUTION OF NETWORK EQUATIONS

The network system $(\Sigma, \Pi, \mathcal{G})$ admits a steady-state if and only if there exists a solution to the system of non-linear inclusions

$$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T \mathrm{y})$$
$$0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$$

RECALL First-order Optimality Condition:

$$0 \in \partial J(p) + E\partial\Gamma(E^T p)$$

Network equations are the first-order optimality conditions of a corresponding optimization problem!

What is it?

Definition

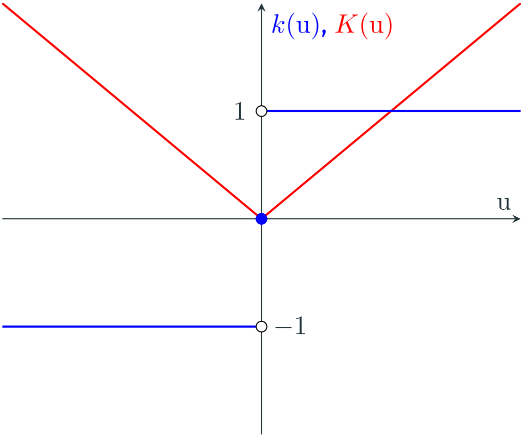
Let k_i be the input-output relation for system Σ_i. Define the function $K_i : \mathbb{R} \to \mathbb{R}$ such that $\partial K_i(\mathrm{u}_i) = k_i(\mathrm{u}_i)$ and $K = \sum_i K_i$. The function K is called the *cost function* associated with the system Σ_i.

Similarly,

$$\partial K_i^\star(\mathrm{y}_i) = k_i^{-1}(\mathrm{y}_i),\; K^\star = \sum_i K_i^\star$$

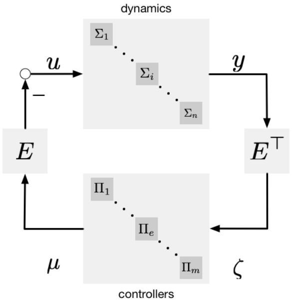
$$\partial \Gamma_e(\zeta_e) = \gamma_e(\zeta_e),\, \Gamma = \sum_e \Gamma_e$$

$$\partial \Gamma_e^\star(\mu_e) = \gamma_e^{-1}(\mu_e)\, \Gamma^\star = \sum_e \Gamma_e^\star$$



$$K(\mathrm{u}) = |\mathrm{u}|$$

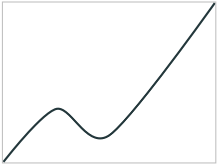
$$y = k(\mathrm{u}) = \mathrm{sgn}(\mathrm{u})$$



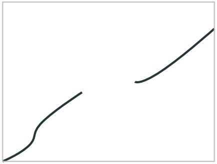
Steady-state values $\mathrm{u}, \mathrm{y}, \zeta$ and μ are the solutions of the following pair of optimization problems[1]:

$\min_{\mathrm{y},\zeta} \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e)$ $s.t. \quad E^T\mathrm{y} = \zeta.$	$\min_{\mathrm{u},\mu} \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e)$ $s.t. \quad \mathrm{u} = -E\mu.$
First-order Optimality Condition $0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T\mathrm{y})$	First-order Optimality Condition $0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$

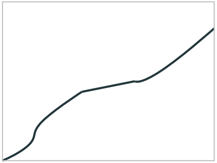
[1] [Bürger, Z, Allgower, 2014]



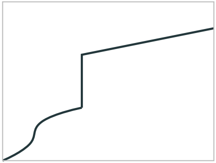
Not Monotone



Monotone but not maximal

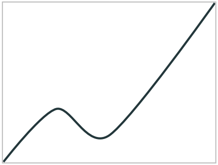


Maximal monotone function

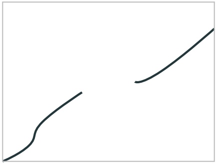


Maximal monotone relation

A relation on $\mathbb{R}$ is monotone
if they are non-decreasing curves in $\mathbb{R}^2$



Not Monotone



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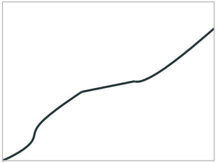
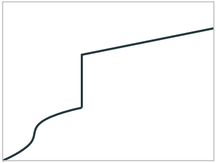


Maximal monotone function

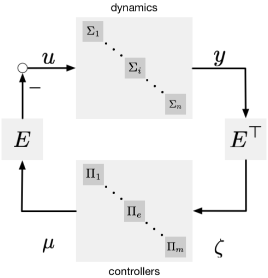


Maximal monotone relation

Theorem

The subdifferentials of convex functions on $\mathbb{R}$ are maximally monotone relations from $\mathbb{R}$ to $\mathbb{R}$.[a]

[a] [R. T. Rockafellar, Convex Analysis. Princeton University Press, 1997]



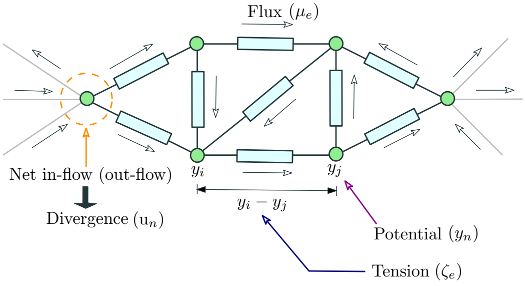
Theorem

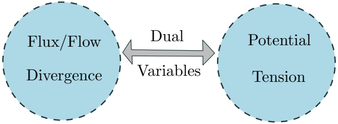
If the input-output maps k_i and γ_e are maximally monotone, then the steady-state values $\mathrm{u}, \mathrm{y}, \zeta$ and μ are the solutions of the following pair of convex dual optimization problems[1]:

Optimal Flow Problem (OFP)	**Optimal Potential Problem (OPP)**
$\min_{\mathrm{y},\zeta} \quad \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e)$	$\min_{\mathrm{u},\mu} \quad \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e)$
$s.t. \quad E^T \mathrm{y} = \zeta.$	$s.t. \quad \mathrm{u} = -E\mu.$

[1] [Bürger, Z, Allgower, 2014]

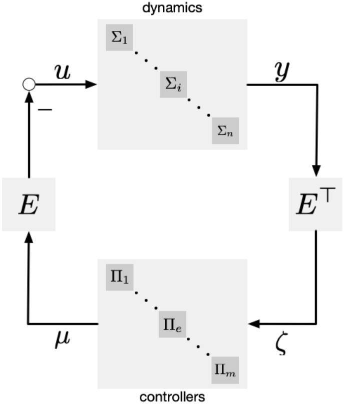
NETWORK OPTIMIZATION





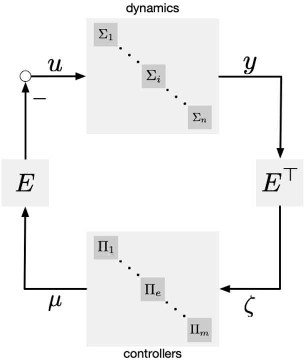
Optimal Flow Problem[1]	**Optimal Potential Problem**[1]
$\min_{u,\mu} \quad \sum_{n=1}^{\lvert\mathcal{V}\rvert} C_n^{\text{div}}(u_n) + \sum_{e=1}^{\lvert\mathcal{E}\rvert} C_e^{\text{flux}}(\mu_e)$	$\min_{y,\zeta} \quad \sum_{n=1}^{\lvert\mathcal{V}\rvert} C_n^{\text{pot}}(y_n) + \sum_{e=1}^{\lvert\mathcal{E}\rvert} C_e^{\text{ten}}(\zeta_e)$
$s.t. \quad u + E\mu = 0.$	$s.t. \quad E^T y = \zeta.$

[1] [R. T. Rockafellar, Network Flows and Monotropic Optmizations. John Wiley and Sons, Inc., 1984]



Diffusively coupled dynamic networks can be associated to static network optimization problems!

Monotone steady-state maps $\Leftrightarrow$ Network Duality



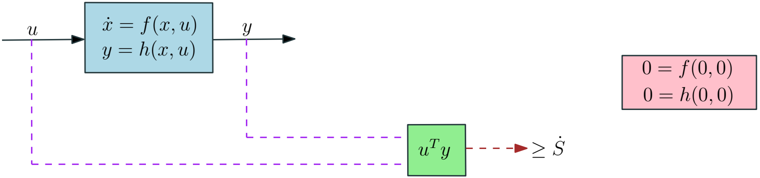
Assumption 1

Each agent Σ_i and controller Π_e admit forced steady-state solutions.

Assumption 2

The input-output maps of each agent, k_i, and controller, γ_e, are maximally monotone.

Under what conditions does the network actually *converge* to these steady states?



Definition [Khalil 2002]

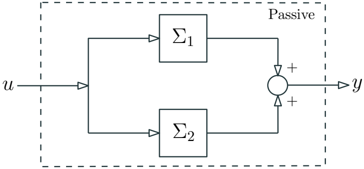
A system is passive if there exists a C^1 storage function $S(x)$ such that

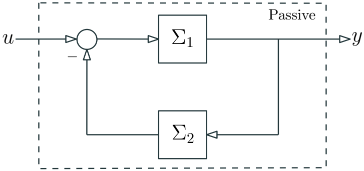
$$u^T y \geq \dot{S} = \frac{\partial S}{\partial x} f(x, u), \quad \forall (x, u) \in \mathbb{R}^n \times \mathbb{R}^p$$

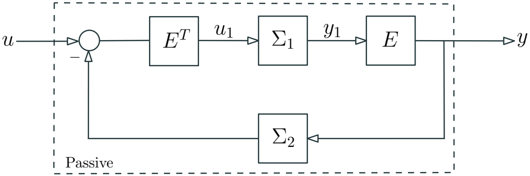
Moreover, it is said to be

- Input-strictly passive if $\dot{S} \leq u^T y - u^T \phi(u)$ and $u^T \phi(u) > 0, \forall u \neq 0$
- Output-strictly passive if $\dot{S} \leq u^T y - y^T \rho(y)$ and $y^T \rho(y) > 0, \forall y \neq 0$

- Parallel Interconnection
- Negative Feedback Interconnection
- Symmetric Interconnection







Theorem

Consider the network system $(\Sigma, \Pi, \mathcal{G})$ comprised of SISO agents and controllers. Suppose that there are vectors $\mathrm{u_i}, \mathrm{y_i}, \zeta_e$ and μ_e such that

i) the systems Σ_i are output strictly-passive with respect to $\mathrm{u_i}$ and $\mathrm{y_i}$;

ii) the systems Π_e are passive with respect to ζ_e and μ_e;

iii) the vectors $\mathrm{u}, \mathrm{y}, \zeta$ and μ satisfy $\mathrm{u} = -\mathcal{E}\mu$ and $\zeta = \mathcal{E}^T\mathrm{y}$.

Then the output vector $y(t)$ converges to y as $t \to \infty$.[1]

[1] [Arcak, 2007], [Bürger, Z, Allgower, 2014]

Theorem

Consider the network system $(\Sigma, \Pi, \mathcal{G})$ comprised of SISO agents and controllers. Suppose that there are vectors $\mathrm{u_i}, \mathrm{y_i}, \zeta_e$ and μ_e such that

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iii) the vectors $\mathrm{u}, \mathrm{y}, \zeta$ and μ satisfy $\mathrm{u} = -\mathcal{E}\mu$ and $\zeta = \mathcal{E}^T\mathrm{y}$.

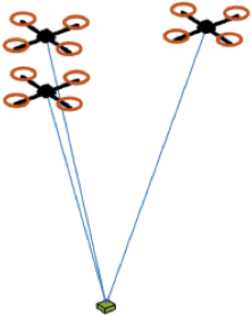
Then the output vector $y(t)$ converges to y as $t \to \infty$.[1]

- requires passivity w.r.t. to specific equilibrium configuration

[1] [Arcak, 2007], [Bürger, Z, Allgower, 2014]

Large-scale Networked Systems

- Not feasible to calculate the equilibrium point for the overall network
- Operate the network at multiple desired equilibrium points (formation of UAVs carrying a suspension load)



[Meissen *et al.*, 2017]

Passivity w.r.t. forced equilibra (u, y)

$$\frac{d}{dt} S(x(t)) \leq (u - \mathrm{u})^T (y - \mathrm{y})$$

Incremental Passivity: A close concept however restricted as passivation inequality must be satisfy along any two arbitrary trajectories

EQUILIBRIUM-INDEPENDENT PASSIVITY (EIP)

EIP[1]

A dynamical system Σ is *equilibrium independent passive* on $\mathcal{U}$ if for every $\mathrm{u} \in \mathcal{U}$ there exists a once-differentiable and positive semi-definite storage function $S(x) : \mathcal{X} \to \mathbb{R}^+$ such that $S(x)|_{\mathrm{x}} = 0$ and

$$\dot{S} \leq (y - \mathrm{y})^T (u - \mathrm{u}) \implies k \text{ monotonically increasing function}$$

for all $u \in \mathcal{U}$ and $y \in \mathcal{Y}$.

[1] [G.H. Hines et al., 2011]

EIP[1]

A dynamical system Σ is *equilibrium independent passive* on $\mathcal{U}$ if for every $\mathrm{u} \in \mathcal{U}$ there exists a once-differentiable and positive semi-definite storage function $S(x) : \mathcal{X} \to \mathbb{R}^+$ such that $S(x)|_{\mathrm{x}} = 0$ and

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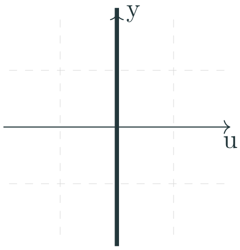


$$\dot{x}(t) = u(t), y(t) = x(t)$$

- Passive with respect to $\mathcal{U} = \{0\}$ and any output value $\mathrm{y} \in \mathbb{R}$ with storage function $S(x) = \frac{1}{2}(x - \mathrm{y})^2$.
- The equilibrium input-output map $k = \{(0, \mathrm{y}) : \mathrm{y} \in \mathbb{R}\}$ is not a single valued function and hence the integrator is **NOT** *EIP*.

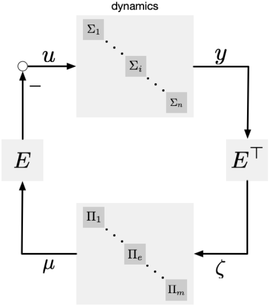
[1] [G.H. Hines et al., 2011]

MEIP[1]

A dynamical SISO system Σ is *maximal equilibrium independent* passive if the following conditions hold:

- The system Σ is passive with respect to any steady-state $(\mathrm{u}, \mathrm{y}) \in k$.
- The relation k is maximally monotone.

[1] [M. Bürger et al., 2014]



Assumption 1

Each agent Σ_i and controller Π_e admit forced steady-state solutions.

Assumption 2

The agent dynamics Σ_i are output-strictly MEIP and the controllers are MEIP.

Theorem

Assume Assumptions 1 and 2 hold. Then the signals $u(t), y(t), \zeta(t), \mu(t)$ converge to the solutions of the following pair of convex dual optimization problems[1]:

Optimal Flow Problem (OFP)	**Optimal Potential Problem (OPP)**
$\min_{\mathrm{y},\zeta} \quad \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e)$	$\min_{\mathrm{u},\mu} \quad \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e)$
$s.t. \quad E^T \mathrm{y} = \zeta.$	$s.t. \quad \mathrm{u} = -E\mu.$

[1] [Bürger, Z, Allgower, 2014]

NEW PERSPECTIVES

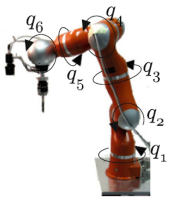
MONOTONICITY AND ITS ROLE IN SYSTEMS THEORY

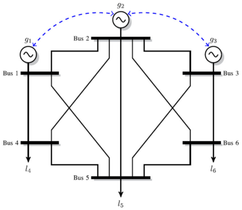
What else can we say about MEIP systems?

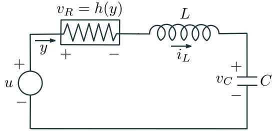
PASSIVITY-SHORT SYSTEMS

In practice, systems are usually passivity-short (or non-passive)!

- Generator (always generates energy) [R. Harvey , 2016]
- Oscillating systems with small or nonexistent damping [R. Harvey, 2017]
- Dynamics of robot system from torque to position [D. Babu, 2018]
- Power-system network (turbine-governor dynamics) [S. Trip, 2018]
- Electrical circuits with nonlinear components
- More general as include non-minimum phase systems and systems with relative degree greater than 1 [Z. Qu, 2014]







$h(\cdot) \in [\sigma, \infty]$ with $\sigma < 0$

EQUILIBRIUM-INDEPENDENT PASSIVITY-SHORT SYSTEMS

Definition

A SISO system $\Sigma : u \mapsto y$ with steady-state input-output relation k is said to be equilibrium independent output ρ-passive (EI-OP(ρ)) if there exists a storage function $S(x)$, and a number $\rho \in \mathbb{R}$, such that the following inequality holds for any trajectory and any equilibrium pair $(\mathrm{u}, \mathrm{y}) \in \mathrm{k}$:

$$\dot{S} \leq (y - \mathrm{y})(u - \mathrm{u}) - \rho(y - \mathrm{y})^2. \quad (1)$$

- If $\rho > 0$, then Σ is output strictly passive.
- If $\rho = 0$, then Σ is passive.
- If $\rho < 0$, then Σ is output passive short.

Similar definitions for input (EI-IP(ν)) and input-output (EI-IOP(ρ, ν)) passive systmes.

Passive short systems can destroy
the developed network optimization framework!

System Type	**Relations**	**Integral Function**
MEIP	k, k^{-1} max. monotone	$K(\mathrm{u}), K^\star(\mathrm{y})$ are convex
Input PS	k is not monotone	$K(\mathrm{u})$ is non-convex
Output PS	k^{-1} is not monotone	$K^\star(\mathrm{y})$ is non-convex
Input-output PS	k, k^{-1} are not monotone	May not exist

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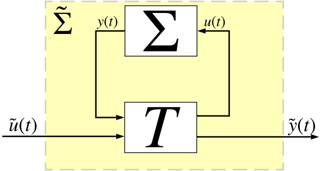
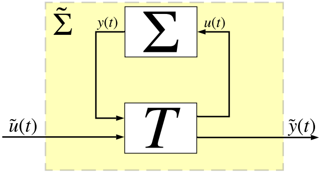
$s.t.\quad E^T\mathrm{y} = \zeta.$	$s.t.\quad \mathrm{u} = -E\mu.$

FEEDBACK PASSIVATION



For a passive-short system $\Sigma : u \mapsto y$, we aim to find a map T such that the closed-loop system $\tilde{\Sigma} : \tilde{u} \mapsto \tilde{y}$ is passive. This is known as feedback passivation.



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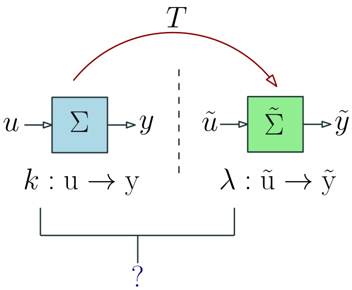
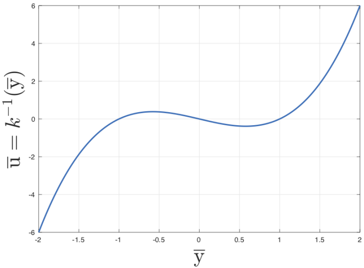


an example

$$\dot{x} = -x + \sqrt[3]{x} + u$$
$$y = \sqrt[3]{x}$$
$$\overline{\mathrm{u}} = k^{-1}(\overline{\mathrm{y}}) = \overline{\mathrm{y}}^3 - \overline{\mathrm{y}}$$

not a monotone input-output relation!

equilibrium input-output map

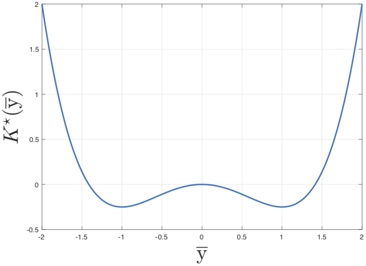


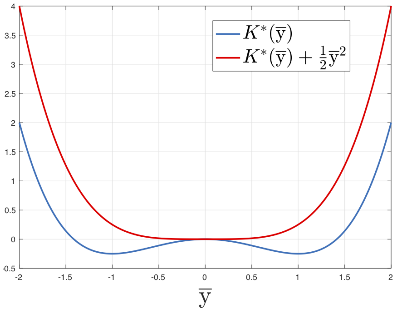
System is output passivity-short

$$S(x) = \frac{3}{4}x^{4/3} - \overline{\mathrm{y}}x + \frac{1}{4}\overline{\mathrm{y}}$$
$$\dot{S} \leq (y - \bar{y})(u - \bar{u}) + (y - \bar{y})^2$$

(passivity index $\rho = -1$)

integral function





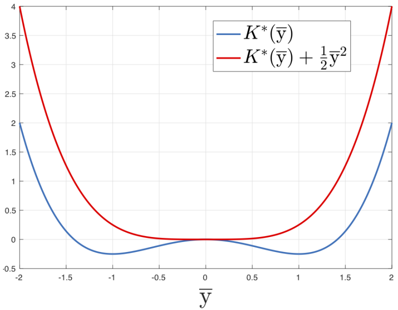
what is the system interpretation of a "convexified" integral function?

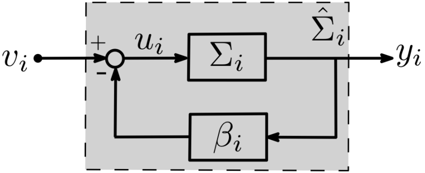
$$K^\star(\overline{y}) = \frac{1}{4}\overline{y}^4 - \frac{1}{2}\overline{y}^2$$

$$\tilde{K}^\star(\overline{y}) = K^\star(\overline{y}) + \frac{1}{2}\overline{y}^2$$

(Tikhonov regularization term)

AN EXAMPLE





what is the system interpretation of a "convexified" integral function?

$$K^\star(\overline{\mathrm{y}}) = \frac{1}{4}\overline{\mathrm{y}}^4 - \frac{1}{2}\overline{\mathrm{y}}^2$$

$$\tilde{K}^\star(\overline{\mathrm{y}}) = K^\star(\overline{\mathrm{y}}) + \frac{1}{2}\overline{\mathrm{y}}^2$$

(Tikhonov regularization term)

regularization is realized by output feedback!

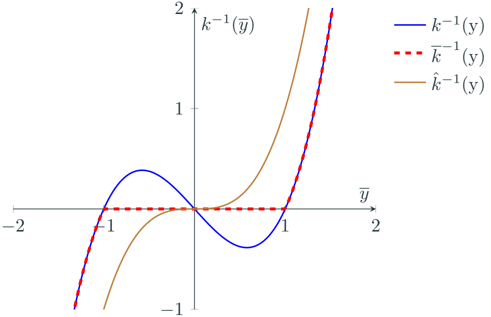
$$u = v - y$$

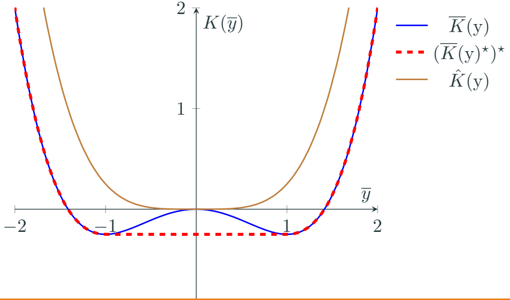
$$\Rightarrow \dot{x} = -x + v$$

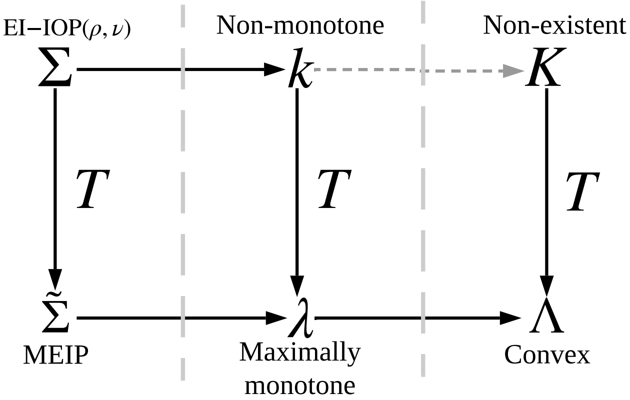
$$\Rightarrow \overline{\mathrm{v}} = \tilde{k}^{-1}(\overline{\mathrm{y}}) = \overline{\mathrm{y}}^3$$

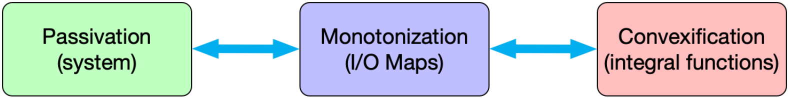
(maximally monotone!)

NEW METRICS FOR PASSIVATION DESIGN

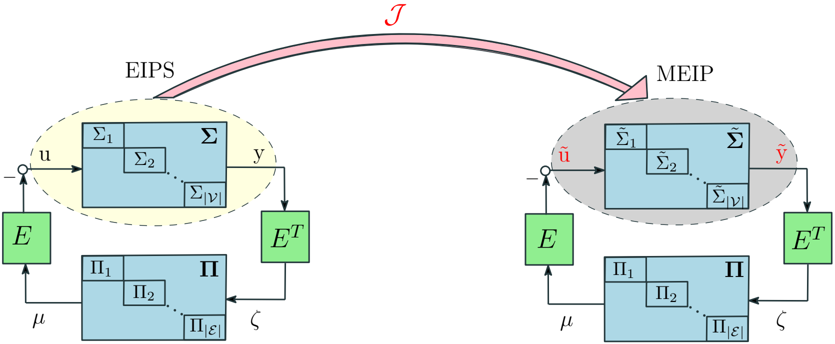





EI−IOP(ρ,ν)
Non-monotone
Non-existent
Σ
k
K
T
T
T
$\tilde{\Sigma}$
λ
Λ
MEIP
Maximally monotone
Convex

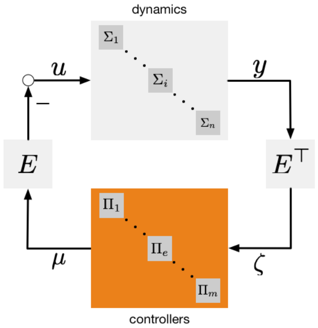

Passivation (system)
Monotonization (I/O Maps)
Convexification (integral functions)

PASSIVATION OF DIFFUSIVELY-COUPLED NETWORKS OF EIPS SYSTEMS



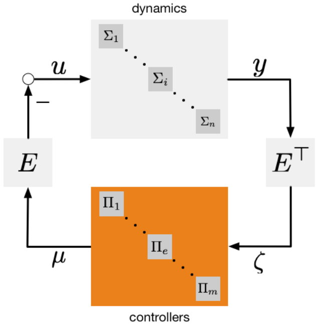
- Without loss of generality assume that the systems at nodes are EIPS (applicable if some of the systems are EIPS)
- Loop Transformation results in a pair of regularized network optimization problems

Controller Synthesis



Idea: shape the integral functions of controllers to achieve desired solution to network optimization problems.

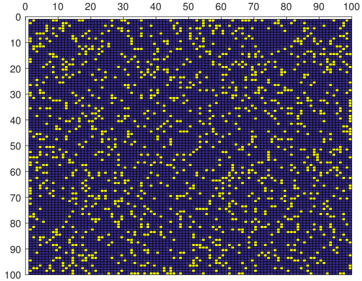
Controller Synthesis



Idea: shape the integral functions of controllers to achieve desired solution to network optimization problems.

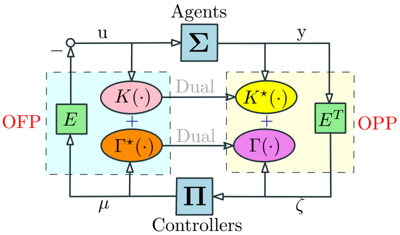
Applications

- Network Reconstruction



Idea: leverage uniqueness of network optimization minima to different exogeneous inputs.

AND MORE...



- an analysis result - convergence of network system and solutions of a pair of network optimization problems [Automatica 14, TAC 19]
- a synthesis result - it is possible to design the controllers and graph to achieve a desired steady by shaping the network optimization problems [L-CSS 17, TAC 19, MED '19]
- passivity-short systems - optimization framework relates regularization to output-feedback passivation of the agents [L-CSS 18, TAC 20 (submitted), Automatica '20 (submitted), L-CSS '19]
- network detection, fault detection, signed nonlinear networks, data-driven control [CDC 18, TCNS 19, TCNS '19 (submitted), TAC '20 (submitted)]

CONCLUDING REMARKS

There is a strong duality theory in cooperative control.

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Cooperative Networks and Controls Lab

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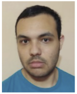

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Frank Allgöwer

QUESTIONS?

SELECTED PUBLICATIONS

- M. Bürger, D. Zelazo and F. Allgower, "Duality and network theory in passivity-based cooperative control,", *Automatica*, 50(8): 2051-2061, 2014.
- M. Sharf and D. Zelazo, "Analysis and Synthesis of MIMO Multi-Agent Systems Using Network Optimization," *IEEE Transactions on Automatic Control*, 64(11):1558-2523, 2019.
- M. Sharf and D. Zelazo, "A Network Optimization Approach to Cooperative Control Synthesis," *IEEE Control Systems Letters*, 1(1):86-91, 2017.
- A. Jain, M. Sharf and D. Zelazo, "Regularization and Feedback Passivation in Cooperative Control of Passivity-Short Systems : A Network Optimization Perspective", *IEEE Control Systems Letters*, (2):4:731-736, 2018.
- M. Sharf, A. Jain and D. Zelazo, "A Geometric Method for Passivation and Cooperative Control of Equilibrium-Independent Passivity-Short Systems", arXiv preprint 2019.
- M. Sharf and D. Zelazo, "Passivity-Based Network Identification Algorithm with Minimal Time Complexity," arXiv preprint, 2019.
- M. Sharf and D. Zelazo, "A Characterization of All Passivizing Input-Output Transformations of a Passive-Short System," arXiv preprint, 2020.

Thank-you!
(wish I were here!)