Temporal Circular Formation Control with Bounded Trajectories in a Uniform Flowfield

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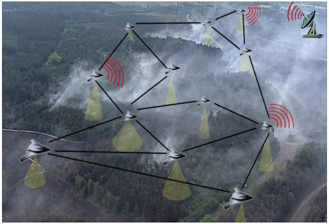
Motivation

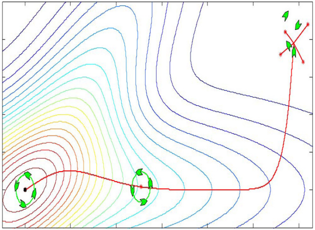
Formation Control of Unmanned Vehicles

Surveillance



Search and Rescue

Tracking and Monitoring



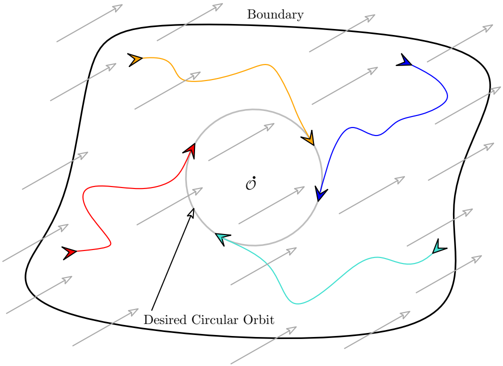
Source Seeking

Ocean Explorations

Oil Spill Tracking

Some Challenges

- Trajectories of the vehicles remains confined to a given workspace while stabilizing to a *desired* formation
- Robustness against disturbance from an external source: for instance, wind in UAVs and ocean currents in AUVs applications



Agent Model
Unicycle Model

Equations of motion (**EOM**)

$$\boxed{\begin{aligned}\dot{r}_k &= v_k e^{i\theta_k}\\ \dot{\theta}_k &= u_k, \quad k = 1, \ldots, N.\end{aligned}}$$

- $r_k \rightarrow$ position
- $\theta_k \rightarrow$ heading angle
- $v_k \rightarrow$ speed
- $u_k \rightarrow$ control input (To be designed)

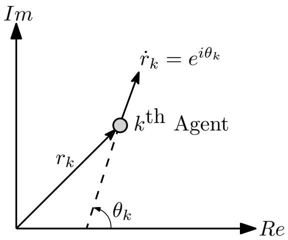
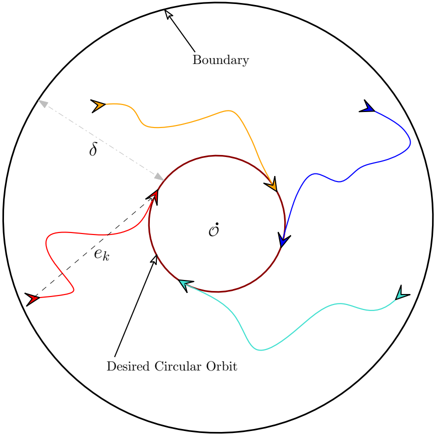
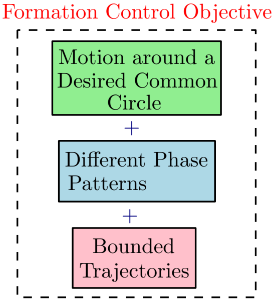


Figure: The k^{th} agent in a plane

Previous Work

No Exterbal Disturbance $\Longrightarrow$ Spatial phase pattens can be achieved around the desired circle[1]

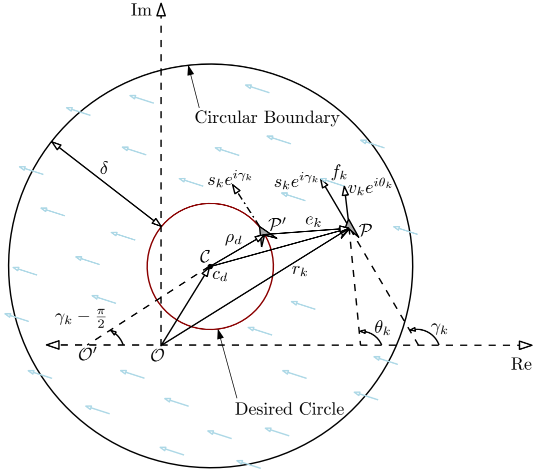




[1] A. Jain *et. al.*, "Trajectory-constrained collective circular motion with different phase arrangements," IEEE Transactions on Automatic Control, Conditionally Accepted, June 2019.

Objective of this Work

Stabilize collective motions of vehicles in different phases around the desired circular orbit with bounded trajectories in presence of **external disturbance**



Agents Model under External Disturbance

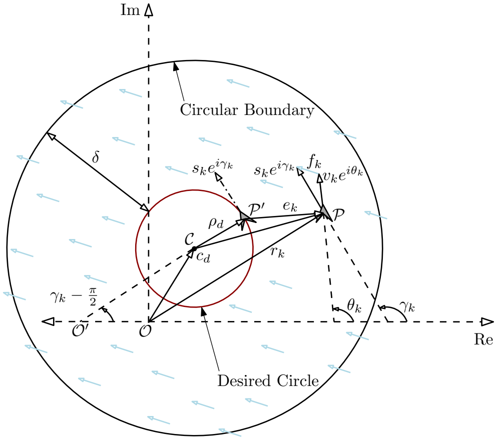
Agent's Model:

$$\dot{r}_k = v_k e^{i\theta_k} + f_k$$
$$\dot{\theta}_k = u_k(\boldsymbol{r}, \boldsymbol{\theta}), \quad k = 1, \dots, N.$$

Modified Model (Inertial Frame):

$$\dot{r}_k = s_k e^{i\gamma_k}$$
$$\dot{\gamma}_k = \zeta_k(\boldsymbol{r}, \boldsymbol{\gamma}), \quad k = 1, \dots, N,$$

- $s_k = |v_k e^{i\theta_k} + f_k| \rightarrow$ magnitude of resultant velocity
- $\gamma_k = \arg(v_k e^{i\theta_k} + f_k) \rightarrow$ resultant heading angle



$$\boxed{u_k = \frac{\zeta_k - \langle f_k', i \rangle}{1 - s_k^{-1}\langle e^{i\gamma_k}, f_k \rangle}}$$

where $f'(r_k) = \partial f(r_k)/\partial r_k$ for all k.

Uniform Flowfield and Control Relation

- We consider uniform Flowfield, given by

$$f = \lambda e^{i\mu},$$

where λ is the strength of the flowfield in the direction μ, and satisfies the assumption that $|\lambda| < \min_k v_k, k = 1, \ldots, N$

- Control interconnection

$$\boxed{u_k = \frac{\zeta_k}{1 - \lambda s_k^{-1} \cos(\mu - \gamma_k)}}$$

- Speed s_k is given by

$$s_k = \sqrt{v_k^2 + \lambda^2 + 2\lambda v_k \cos(\mu - \theta_k)}$$

Lemma

For $|\lambda| < \min_k v_k, k = 1, \ldots, N$, u_k exists and finite.

Phase Patterns Around Desired Circular Orbit

With and Without Flowfield

Main Difference

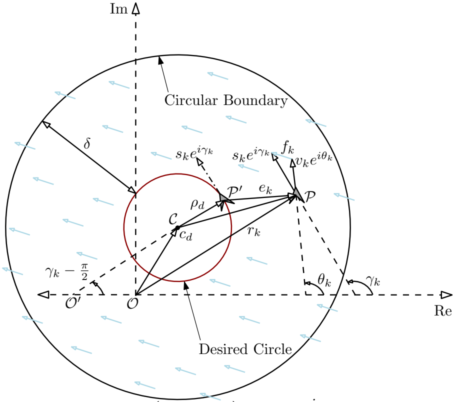
- No flowfield: Spatial patterns can be achieved around the desired circular orbit (i.e, phases repeats spatially) [Jain *et. al.* 2019]
- In flowfield: Only temporal formation can be achieved due to nonidentical speeds of the vehicles (i.e, phases repeats in time)

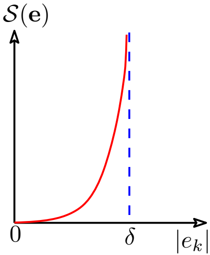
Temporal Phase Pattern

The temporal phase patterns around the common circle are generated by the minimization of the rotationally invariant phase potential function $\mathcal{W}(\boldsymbol{\psi})$, where $\boldsymbol{\psi}$ is the time phase defined as

$$\psi_k = \frac{2\pi\rho_d}{T}\int_0^{\gamma_k} \frac{d\gamma}{s(\gamma)},$$

where $t = \rho_d \int_0^{\gamma_k(t)} \frac{d\gamma}{s(\gamma)}$ is time taken for the revolution by angle γ_k, and $T = \rho_d \int_0^{2\pi} \frac{d\gamma}{s(\gamma)}$ is the time for one complete revolution around the circular orbit.





- The error is: $e_k = (r_k - c_d) + i\rho_d e^{i\gamma_k}$
- Barrier Lyapunov Function (**BLF**) [δ is a design parameter]

$$\mathcal{S}(\boldsymbol{e}) \triangleq \frac{1}{2}\sum_{k=1}^{N} \ln\left(\frac{\delta^2}{\delta^2 - |e_k|^2}\right)$$

The Controller

Composite Lyapunov Function

$$\mathcal{V}(\boldsymbol{r},\boldsymbol{\gamma}) = \underbrace{\mathcal{S}(\boldsymbol{r},\boldsymbol{\gamma})}_{\text{circular motion}} + \underbrace{(T/2\pi)\mathcal{W}(\boldsymbol{\psi})}_{\text{phase control}}.$$

Theorem

Consider the unicycle vehicle model and assume that the initial states of the vehicles are given such that the condition $|e_k(0)| < \delta$ satisfies for all k. Let the vehicles be governed by the control law

$$\zeta_k = \omega_k \left(1 + K\left[s_k \frac{\langle r_k - c_d, e^{i\gamma_k}\rangle}{\delta^2 - |e_k|^2} - \frac{\partial \mathcal{W}}{\partial \psi_k}\right]\right),$$

where, $\rho_d > 0, \omega_k = s_k/\rho_d$, and K is the controller gain. Then, we have the following:

(i) *If $K > 0$, all the vehicles asymptotically converge to a circular formation in which each vehicle moves around the desired circle of radius ρ_d and center c_d in temporal phase patterns defined by potential $\mathcal{W}(\boldsymbol{\psi})$.*

(ii) *Moreover, the trajectories of the vehicles remain bounded within the circular region $|r_k(t) - c_d| < (\delta + \rho_d)$ centered at c_d for all $t \geq 0$.*

Bounds on Various Signals

Theorem

Consider the closed loop system model, under the proposed control law with $\kappa > 0, K > 0$, and assume that the initial states of the vehicles are given such that the condition $|e_k(0)| < \delta$ is satisfied for all k, where e_k and δ are defined above. Then the following properties hold.

(i) *The absolute values of the error signals e_k, and the trajectories r_k, for all $k = 1, \ldots, N$, are bounded by*

$$|e_k| \leq \delta\sqrt{1 - e^{-\frac{2\mathcal{V}_1(\boldsymbol{r}(0),\boldsymbol{\gamma}(0))}{\kappa}}};$$

$$|r_k - c_d| \leq \rho_d + \delta\sqrt{1 - e^{-\frac{2\mathcal{V}_1(\boldsymbol{r}(0),\boldsymbol{\gamma}(0))}{\kappa}}}.$$

(ii) *The squared summation of the relative temporal phasors belongs to the compact set*

$$\sum_{\{j,k\}\in\mathcal{E}} \left|e^{i\psi_j} - e^{i\psi_k}\right|^2 \in [\chi_{min}, \chi_{max}].$$

where, $\chi_{min} = \max\left\{0, \left(N\sigma_{max} - \frac{4\pi\mathcal{V}_1(\boldsymbol{r}(0),\boldsymbol{\gamma}(0))}{KT}\right)\right\}$ *and* $\chi_{max} = N\sigma_{max}$.

Simulation Parameters

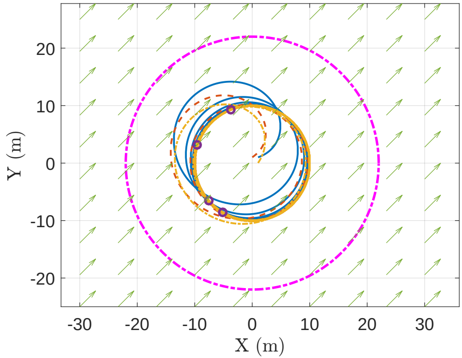
- $N = 4$ agents
- initial positions: $\boldsymbol{x}(0) = [1,\ 0,\ 1,\ -1]^T; \boldsymbol{y}(0) = [1,\ 1,\ 0,\ -1]^T$;
- initial heading angles: $\boldsymbol{\theta}(0) = [0^\circ,\ 30^\circ,\ 60^\circ,\ 90^\circ]^T$;
- initial speeds: $\boldsymbol{v} = [1,\ 1.5,\ 2,\ 2.5]^T$
- flowfield: magnitude $\lambda = 0.75$; direction $\mu = 45^\circ$
- radius and center of the desired circle are $c_d = (0,0)$ and $\rho_d = 10$;
- radius of the constrained boundary $\delta = 12$.
- agents share information according to a graph whose Lapacian is given by

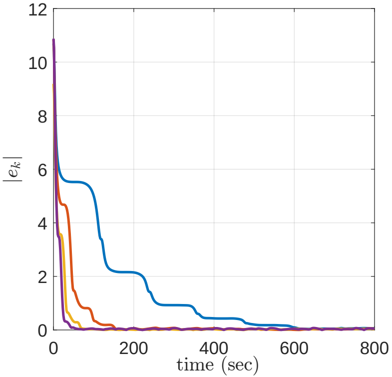
$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}.$$

Note: one can easily verify that the conditions $|e_k(0)| < \delta$ is satisfied for all the vehicles!

Simulation Results

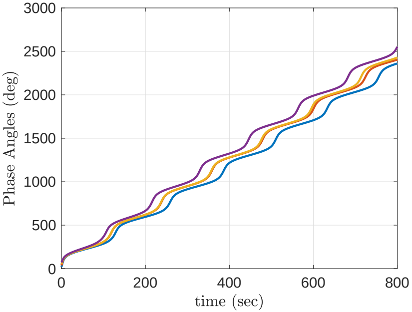
Trajectories and Motion Errors

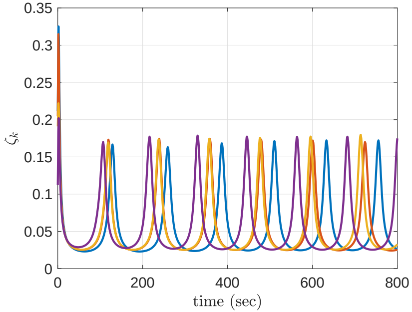




Simulation Results

Time Phases and Controls





Bounded Controls

Practical Considerations

Addresses a practical scenario where physical restrictions on an autonomous vehicle often constrain the force applied on it!

Saturation function

$$\dot{\gamma}_k = \mathsf{sat}(\zeta_k; \zeta_{\mathsf{max}}) \triangleq \begin{cases} \zeta_k, & \text{if } |\zeta_k| \leq \zeta_{\mathsf{max}} \\ \zeta_{\mathsf{max}}\ \mathsf{sign}(\zeta_k), & \text{if } |\zeta_k| \geq \zeta_{\mathsf{max}} \end{cases}$$

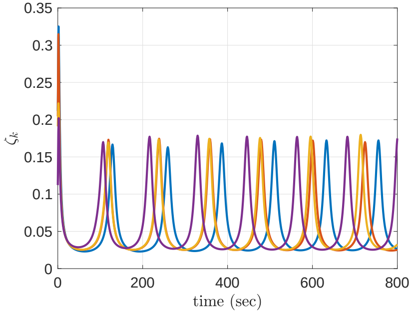
Theorem

Assume that the initial states of the agents are given such that the condition $|e_k(0)| < \delta$ is satisfied for all k. Let the agents be governed by saturated control law. If $K > 0$, all the agents asymptotically converge on a common circle of radius $\rho_d = |\Omega_d^{-1}|$ and center c_d with phase angles in the temporal phase pattern defined by potential $\mathcal{W}(\boldsymbol{\psi})$. Moreover, the trajectories of the agents remain bounded within a circular region $|r_k(t) - c_d| < (\delta + |\Omega_d^{-1}|), \forall t \geq 0$, in the trajectory space.

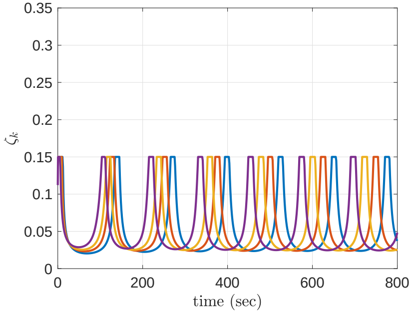
Simulation Results

Normal and Saturated Control

Maximum applied inertial force: $\zeta_{max} = 0.15$



(a) Normal Control



(b) Saturated Control

Conclusions and Future Work

Conclusions

- Asymptotically stabilized trajectory-constrained collective circular formations in *temporal* phase arrangements in a uniform flowfield
- The concept of Barrier Lyapunov Function is exploited in controller design
- Derived bounds on various signals and addressed a practical scenario of bounded force using control saturation

Future Work

- Consideration of flowfield with varying spatial and temporal properties
- Trajectory−constrained stabilization within an arbitrarily-shaped boundary
- Issue of collision avoidance and time-delay in communication network

Acknowledgements

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THANK YOU!