RIGIDITY THEORY IN MULTI-AGENT COORDINATION

A FUNDAMENTAL SYSTEM ARCHITECTURE

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Geometric Constraint Systems: Rigidity, Flexibility and Applications
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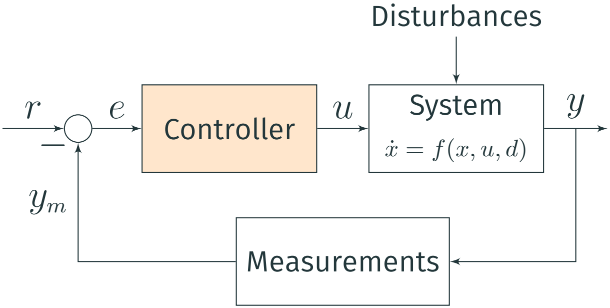
SpaceX Flacon 9 autonomous landing on autonomous drone ship

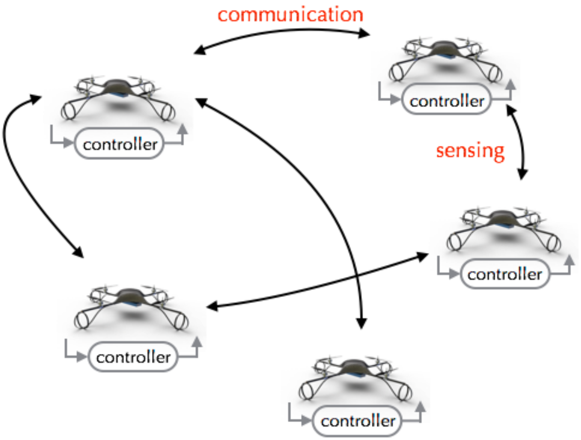
A CLASSIC CONTROL SYSTEM



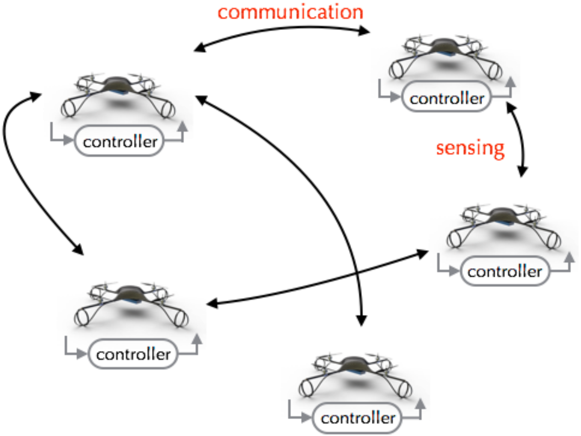
A control systems engineer aims to design a controller that ensures the closed-loop system

- is stable
- satisfies some performance criteria

WHAT ARE MULTI-AGENT SYSTEMS?



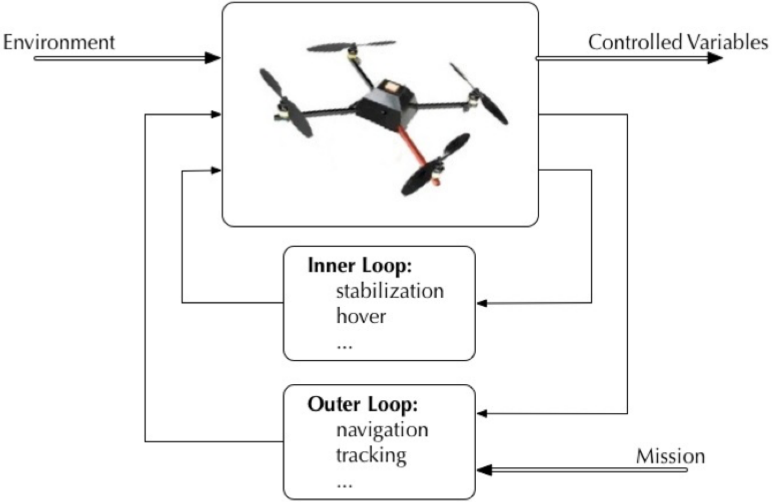
WHAT ARE MULTI-AGENT SYSTEMS?

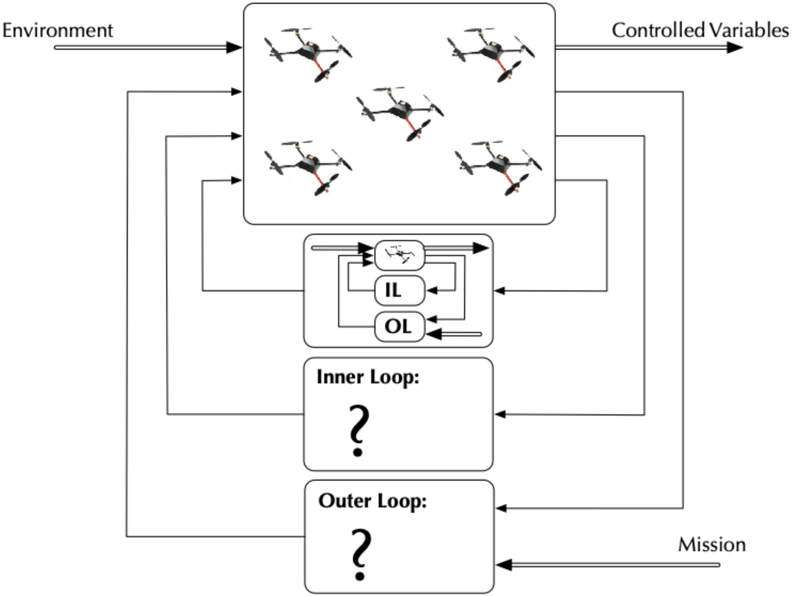


What is the right control architecture?

- of each agent
- of the information exchange layer

CONTROL ARCHITECTURES




Environment
Controlled Variables
IL
OL
Inner Loop:
?
Outer Loop:
?
Mission

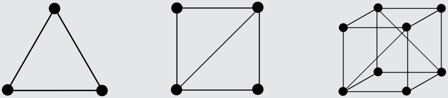
COORDINATION OBJECTIVES

rendezvous formation control localization

- Does the control strategy need to change with different sensing/communication?
- Are there common architectural requirements that do not depend on the choice of sensing?

Formation Control Objective

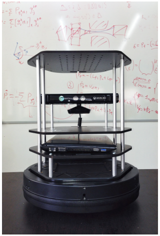
Given a team of robots endowed with the ability to sense/communicate with neighboring robots, design a control for each robot using only local information that moves the team into a desired spatial configuration - the formation



Control Theory provides us with an analytical justification for using simple models!

INTEGRATOR DYNAMICS

$$\dot{x} = u_x$$
$$\dot{y} = u_y$$
$$\dot{z} = u_z$$



UNICYCLE DYNAMICS

$$\dot{x} = v_{\text{lin}} \cos(\psi)$$
$$\dot{y} = v_{\text{lin}} \sin(\psi)$$
$$\dot{\psi} = v_{\text{ang}}$$

AGENT CONFIGURATIONS

- we consider a team of n agents in a d-dimensional Euclidean space

$$p_i(t) \in \mathbb{R}^d$$

- the configuration of the agents at time t is the vector

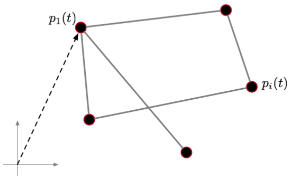
$$p(t) = \begin{bmatrix} p_1(t) \\ \vdots \\ p_n(t) \end{bmatrix} \in \mathbb{R}^{nd}$$

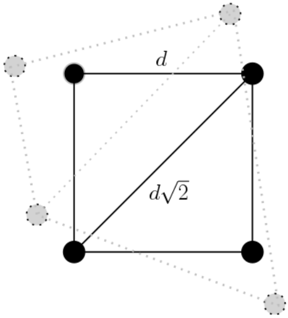
- agents modelled by single integrator dynamics

$$\dot{p}_i(t) = u_i(t), \ i = 1, \ldots, n$$

- agents interact according to a sensing graph

$$\mathcal{G} = (\mathcal{V}, \mathcal{E})$$





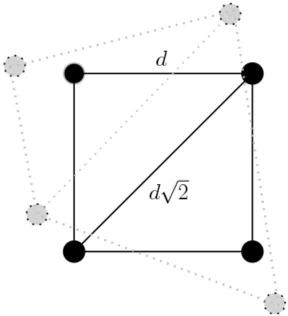
THE "FORMATION" POTENTIAL

$$\Phi(p) = \frac{1}{4}\sum_{i\sim j}\left(\|p_i - p_j\|^2 - d_{ij}^2\right)^2$$

A GRADIENT FLOW

$$\dot{p} = -\nabla_p \Phi(p)$$

A FORMATION POTENTIAL



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A GRADIENT FLOW

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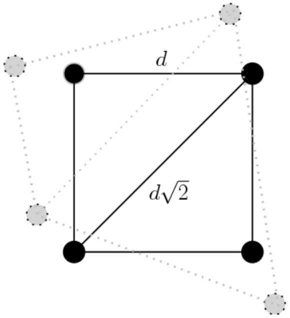
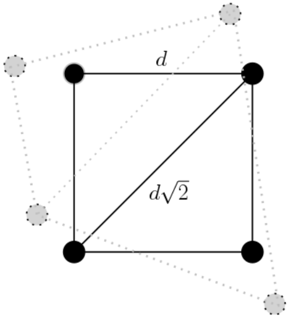
Theorem

The gradient dynamical system asymptotically converges to the critical points of the formation potential.



Distributed Control

$$\dot{p}_i = \sum_{i \sim j} (\|p_i - p_j\|^2 - d_{ij}^2)(p_j - p_i)$$



Distributed Control

$$\dot{p}_i = \sum_{i \sim j} (\|p_i - p_j\|^2 - d_{ij}^2)(p_j - p_i)$$

- Does this strategy solve the formation control problem?
- Does it reveal a necessary control architecture for the multi-agent system?

RIGIDITY MEETS FORMATION CONTROL

For a framework $(\mathcal{G}, p)$, we have

Edge Function

$$f_D(p) = \frac{1}{2}\begin{bmatrix} \vdots \\ \|p_i - p_j\|^2 \\ \vdots \end{bmatrix}$$

Rigidity Matrix

$$R_D(p) = \frac{\partial f_D(p)}{\partial p}$$

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Edge Function

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Rigidity Matrix

$$R_D(p) = \frac{\partial f_D(p)}{\partial p}$$

$$\begin{aligned} \dot{p} = \nabla\Phi(p) &= \frac{\partial}{\partial p}\|f_D(p) - \frac{1}{2}d^2\|^2 \\ &= -R_D(p)^T R_D(p)p - R_D(p)^T d^2 \end{aligned}$$

Theorem (Stability and Rigidity)

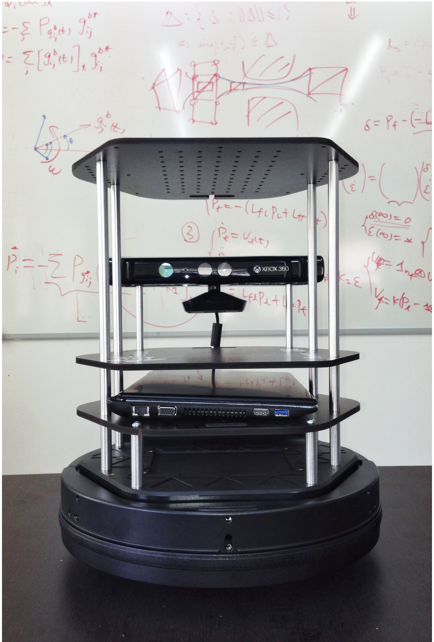
If the target formation is infinitesimally rigid, then the dynamics are (locally) asymptotically stable and satisfy

$$\lim_{t\to\infty} p(t) = p^\star$$

where $\|p_i^\star - p_j^\star\|^2 = d_{ij}^2$ *for all* $\{i,j\} \in \mathcal{E}$.

L. Krick, M. E. Broucke & B. A. Francis, *Stabilisation of infinitesimally rigid formations of multi-robot networks*, International Journal of Control, 82(3):423-439, 2009.

A REAL ROBOT



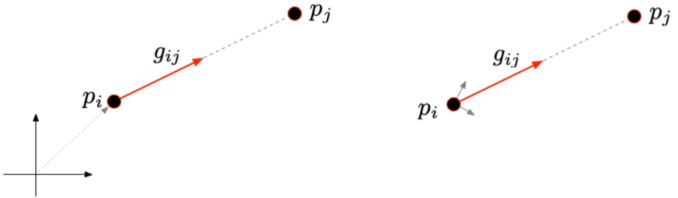
Bearing Sensing

The bearing between two agents is defined as the unit vector

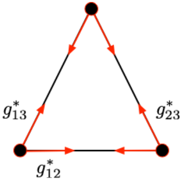
$$g_{ij}(t) = \frac{p_j(t) - p_i(t)}{\|p_j(t) - p_i(t)\|},$$

where $p_i(t)$ is the position of agent i.

- NOTE: g_{ij} can be expressed in a common frame or local frame



target formation specified by desired bearings



Formation Control Objective

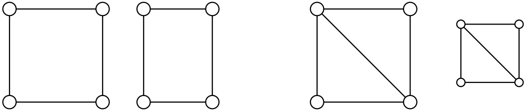
Design u_i for each agent using only bearing measurements such that

$$\lim_{t\to\infty} g_{ij}(t) = g^*_{ij}$$

for all pairs (i, j) in the sensing graph.

Bearing Rigidity

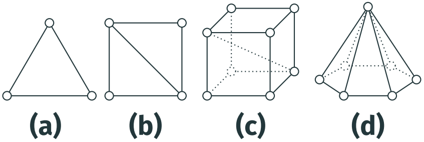
- If we fix the bearing of each edge in a network, can the geometric pattern of the network be uniquely determined?



- Intuitive definition: a network is bearing rigid if its bearings can uniquely determine its geometric pattern.

BEARING-EDGE FUNCTION

◇ How can one determine if a given network is bearing rigid?



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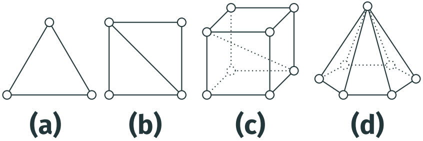


The Bearing-Edge Function

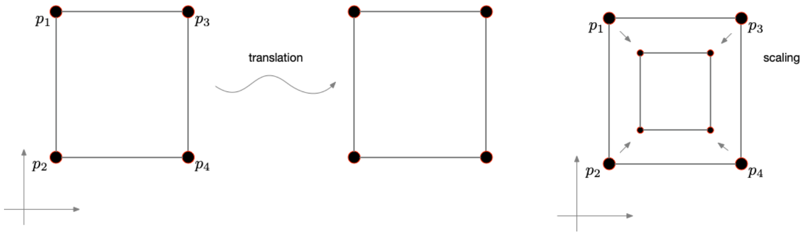
For a network with $|\mathcal{E}| = m$ edges, the bearing-edge function is defined as

$$f_B(p) \triangleq \begin{bmatrix} g_1 \\ \vdots \\ g_m \end{bmatrix} \in \mathbb{R}^{dm}.$$

Bearing Trivial Motions

Trivial motions preserve the bearing between all pairs of agents for any framework

- (rigid body) translations
- scaling



Consider the Taylor-series expansion of the bearing-edge function:

$$f_B(p + \delta_p) = f_B(p) + \frac{\partial f_B(p)}{\partial p}\delta_p + h.o.t.$$

Infinitesimal Motions

An infinitesimal motion, δ_p, of a network satisfies

$$\frac{\partial f_B(p)}{\partial p}\delta_p = 0.$$

- first order ”bearing-preserving” motions
- trivial motions are always infinitesimal motions

The Rigidity Matrix

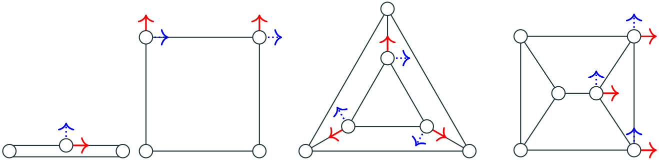
$$R_B(p) \triangleq \frac{\partial f_B(p)}{\partial p}$$

Rank-Test for Bearing Rigidity

A network is infinitesimally bearing rigid if and only if

$$\text{rank}(R_B(p)) = dn - d - 1.$$

Examples:



(a) (b) (c) (d)

The Rigidity Matrix

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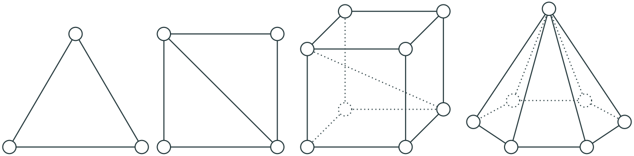
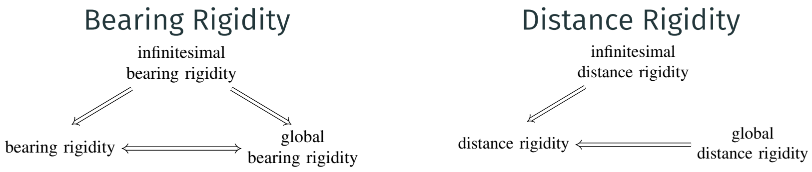
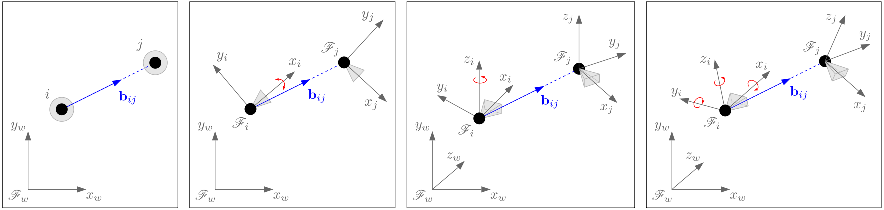


(a) (b) (c) (d)

SOME FEATURES OF BEARING RIGIDITY



- in $\mathbb{R}^2$, infinitesimal distance rigidity and infinitesimal bearing rigidity are equivalent
- infinitesimal bearing rigidity is preserved in lifted spaces
- Laman graphs are generically bearing rigid in arbitrary dimension
- at most $2n - 3$ edges are sufficient to ensure bearing rigidity in arbitrary dimension
- infinitesimal bearing rigid frameworks uniquely define a shape (modulo scale and translation)



(a) $\mathbb{R}^2$ (b) $\mathbb{R}^2 \times \mathbb{S}^1$ (c) $\mathbb{R}^3 \times \mathbb{S}^1$ (d) $SE(3)$

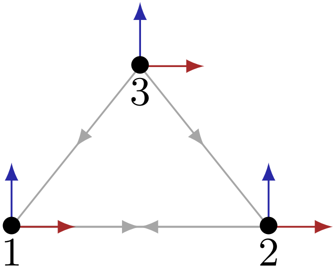
Bearing Rigidity Function

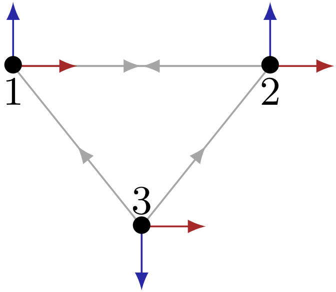
Given a n-agent formation modeled as a framework $(\mathcal{G}, \chi)$ in $\bar{\mathcal{D}}$, the bearing rigidity function is the map

$$\mathbf{b}_\mathcal{G} : \bar{\mathcal{D}} \to \bar{\mathcal{M}}, \quad \chi \mapsto \mathbf{b}_\mathcal{G}(\chi) = [\mathbf{b}_1^T \quad \cdots \quad \mathbf{b}_m^T]^T$$

Trivial Motions

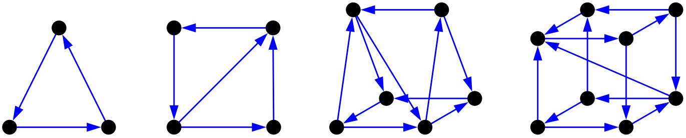
Trivial motions in $SE(2)$ are translations, scaling, and coordinated rotations



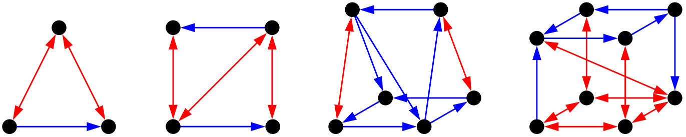


In directed bearing rigidity, local rigidity does not imply global rigidity

EXAMPLES



(a) $n = 3$ (b) $n = 4$ (c) $n = 6$ (d) $n = 8$



(e) $n = 3$ (f) $n = 4$ (g) $n = 6$ (h) $n = 8$

IBF frameworks in $(\mathbb{R}^2 \times \mathbb{S}^1)^n$ ((a),(b)), in $(\mathbb{R}^3 \times \mathbb{S}^1)^n$ with $\mathbf{n} = \mathbf{e}_3$ ((c),(d)). Examples of IBR frameworks in $(\mathbb{R}^2 \times \mathbb{S}^1)^n$ ((e),(f)) and in $(\mathbb{R}^3 \times \mathbb{S}^1)^n$ with $\mathbf{n} = \mathbf{e}_3$ ((g),(h)).

A GENERAL BEARING RIGIDITY MATRIX

For a framework $(\mathcal{G}, \chi)$, the bearing rigidity matrix takes the form

$$\mathbf{B}_{\mathcal{G}}(\chi) = [\mathbf{B}_p \;\; \mathbf{B}_o] \in \mathbb{R}^{3m \times 6n},$$

with

$$\mathbf{B}_p = \mathbf{D}_p \bar{\mathbf{E}}^\top \in \mathbb{R}^{3m \times 3n} \quad \text{and} \quad \mathbf{B}_o = \mathbf{D}_o \bar{\mathbf{E}}_o^\top \in \mathbb{R}^{3m \times 3n} \qquad (1)$$

$\mathcal{D}$	$\mathbf{p}_i$	$\mathbf{R}_i$	$\mathbf{D}_p$	$\mathbf{D}_o$
$SE(3)$	$[p_i^x \;\; p_i^y \;\; p_i^z]^\top$	$\mathbf{R}\left(\alpha_i, \beta_i, \gamma_i, \{\mathbf{e}_h\}_{h=1}^3\right)$	$\text{diag}(d_{ij}\mathbf{R}_i^\top \mathbf{P}(\bar{\mathbf{p}}_{ij}))$	$\text{diag}(\mathbf{R}_i^\top [\bar{\mathbf{p}}_{ij}]_\times \mathbf{I}_3)$
$\mathbb{R}^3 \times \mathbb{S}^1$	$[p_i^x \;\; p_i^y \;\; p_i^z]^\top$	$\mathbf{R}(\alpha_i, \mathbf{n}), \mathbf{n} = \sum_{h=1}^3 n_h \mathbf{e}_h$	$\text{diag}(d_{ij}\mathbf{R}_i^\top \mathbf{P}(\bar{\mathbf{p}}_{ij}))$	$\text{diag}(\mathbf{R}_i^\top [\bar{\mathbf{p}}_{ij}]_\times [\mathbf{0}_{3\times 2} \; \mathbf{n}])$
$\mathbb{R}^2 \times \mathbb{S}^1$	$[p_i^x \;\; p_i^y \;\; 0]^\top$	$\mathbf{R}(\alpha_i, \mathbf{e}_3)$	$\text{diag}(d_{ij}\mathbf{R}_i^\top \mathbf{P}(\bar{\mathbf{p}}_{ij}))$	$\text{diag}(\mathbf{R}_i^\top [\bar{\mathbf{p}}_{ij}]_\times [\mathbf{0}_{3\times 2} \; \mathbf{e}_3])$
$\mathbb{R}^3$	$[p_i^x \;\; p_i^y \;\; p_i^z]^\top$	$\mathbf{R}(\alpha_i, \mathbf{0}_{3\times 1}) = \mathbf{I}_3$	$\text{diag}(d_{ij}\mathbf{I}_3^\top \mathbf{P}(\bar{\mathbf{p}}_{ij}))$	$\text{diag}(\mathbf{I}_3^\top [\bar{\mathbf{p}}_{ij}]_\times \mathbf{0}_{3\times 3})$
$\mathbb{R}^2$	$[p_i^x \;\; p_i^y \;\; 0]^\top$	$\mathbf{R}(\alpha_i, \mathbf{0}_{3\times 1}) = \mathbf{I}_3$	$\text{diag}(d_{ij}\mathbf{I}_3^\top \mathbf{P}(\bar{\mathbf{p}}_{ij}))$	$\text{diag}(\mathbf{I}_3^\top [\bar{\mathbf{p}}_{ij}]_\times \mathbf{0}_{3\times 3})$

…back to formation control

Consider the potential function of bearing errors:

$$\Phi(t) = \frac{1}{2} \sum \|g_{ij}(t) - g_{ij}^*\|^2$$

A Gradient-descent control

$$\dot{p} = -\nabla_p \Phi(t)$$

$$\dot{p}_i(t) = -\sum_{j \in \mathcal{N}_i} \frac{1}{\|e_{ij}(t)\|} P_{g_{ij}(t)} g_{ij}^*$$

- $e_{ij}(t) = p_j(t) - p_i(t)$
- implementation requires distance and bearing measurements!
- $P_{g_{ij}(t)}$ is an orthogonal projection matrix

Proposed Control Law

$$\dot{p}_i(t) = -\sum_{j \in \mathcal{N}_i} P_{g_{ij}(t)} g_{ij}^*$$

$$\dot{p}(t) = R_B^T(p)\mathrm{diag}\{\|e_{ij}\|\}g^*$$

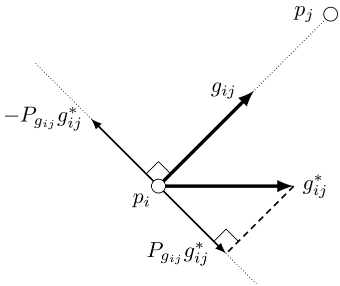


Figure 1: Geometric interpretation

EXAMPLES

EXAMPLES

Centroid and Scale Invariance

- Centroid of the formation

$$\bar{p} \triangleq \frac{1}{n}\sum_{i=1}^{n} p_i$$

- Scale of the formation

$$s \triangleq \sqrt{\frac{1}{n}\sum_{i=1}^{n} \|p_i - \bar{p}\|^2}.$$

BEARING-ONLY FORMATION CONTROL – STABILITY ANALYSIS

Centroid and Scale Invariance

- Centroid of the formation

$$\bar{p} \triangleq \frac{1}{n}\sum_{i=1}^{n} p_i$$

- Scale of the formation

$$s \triangleq \sqrt{\frac{1}{n}\sum_{i=1}^{n} \|p_i - \bar{p}\|^2}.$$

Almost global convergence

- Two isolated equilibriums: one stable, one unstable

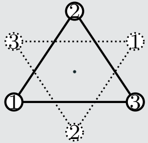


Figure 2: Solid line is target formation.

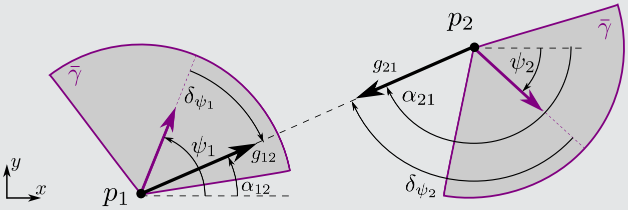
Reference: S. Zhao and D. Zelazo, "Bearing rigidity and almost global bearing-only formation stabilization," IEEE Transactions on Automatic Control, vol. 61, no. 5, pp. 1255-1268, 2016.

EXTENSIONS - ATTITUDE SYNCHRONIZATION

EXTENSIONS - FORMATION MANEUVERING

Sensing Model

Visual sensors are bounded by a limited field-of-view



- Sensing graph can become **directed**
- Neighbors are not static
- α_{ij} is the angle of the bearing g_{ij}
- δ_{ψ_i} is the facing direction error

LIMITED FIELD-OF-VIEW SENSING

$$\dot{p}_i(t) = -\sum_{j \in \mathcal{N}_i} P_{g_{ij}(t)} g_{ij}^*$$

Facing direction is *not* controlled

Problem

Design the control inputs u_i and ω_i such that the desired bearing is reached using only bearing measurements and a given limited field-of-view of the visual sensor.

No Sensing: $w_1(0) = w_2(0) = 0$

$|\delta_{\psi_1}(0)| > \bar{\gamma}/2$ and $|\delta_{\psi_2}(0)| > \bar{\gamma}/2$

Complete Sensing: $w_1(0) = w_2(0) = 1$

$|\delta_{\psi_1}(0)| < \bar{\gamma}/2$ and $|\delta_{\psi_2}(0)| < \bar{\gamma}/2$

Partial Sensing: $w_1(0) = 1, w_2(t) = 0, t \geq 0$

$|\delta_{\psi_1}(0)| < \bar{\gamma}/2$ and $|\delta_{\psi_{2(t)}}| > \bar{\gamma}/2$ for all $t \geq 0$

TWO AGENT CASE - SIMULATIONS

Partial Sensing: $w_1(0) = 1, w_2(0) = 0$ **and** $w_2(t) = 1$ **for** $t > T$

$|\delta_{\psi_1}(0)| < \bar{\gamma}/2$ and $|\delta_{\psi_2}(t)| < \bar{\gamma}/2$ for some $t > T$

Analytical Results for $n = 2$

If the following Assumptions hold:

1. Initially one agent can sense the other
2. The visual sensor satisfies $\bar{\gamma}/2 > 1/d_{12}(0)$

Then, the desired formation g^*_{12} will be reached from almost all initial conditions (except for $g_{12}(0) = -g^*_{12}$).

Analytical Results for $n = 2$

If the following Assumptions hold:

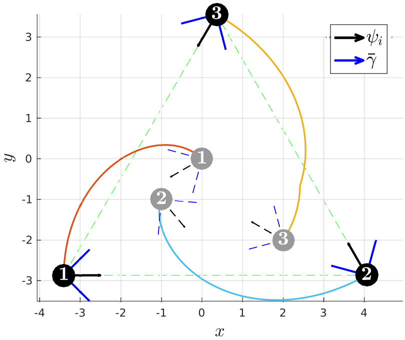
1. Initially one agent can sense the other
2. The visual sensor satisfies $\bar{\gamma}/2 > 1/d_{12}(0)$

Then, the desired formation g_{12}^* will be reached from almost all initial conditions (except for $g_{12}(0) = -g_{12}^*$).

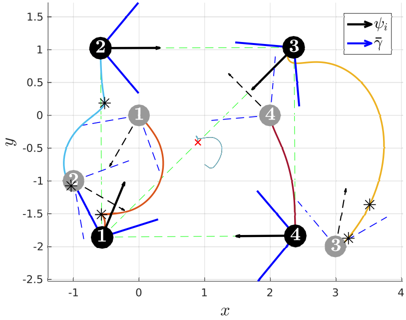
- Holds for two agents only
- Includes directed interactions

What changes?

- Desired facing direction is not intuitive
- Rigidity conditions are required



Faces the closest neighbor.

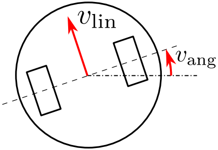


Faces in the middle of the agents that are inside the FOV.

SIMULATION FOR $n > 2$

Facing is controlled by ω_i

TurtleBotII Robots - Unicycle Model



$$\dot{x}_i = v_{i_{\text{lin}}} \cos(\psi_i)$$
$$\dot{y}_i = v_{i_{\text{lin}}} \sin(\psi_i)$$
$$\dot{\psi}_i = v_{i_{\text{ang}}}$$

Vision sensing with Microsoft Kinect Sensor

Figure 3: Kinect used as a bearing-only sensor.

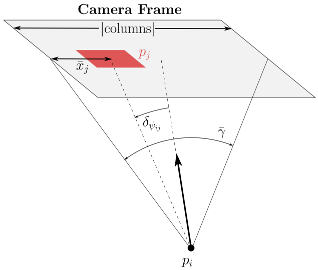


Figure 4: Camera frame that is taken from a visual sensor on agent i, the red square indicates the color of neighbor j within the camera frame.

Bearing-Only Controller for Unicycle Dynamics

$$v_{i_{\text{lin}}} = \left[\cos(\theta_i) \quad \sin(\theta_i)\right]^T u_i$$
$$v_{i_{\text{ang}}} = \left[-\sin(\theta_i) \quad \cos(\theta_i)\right]^T u_i$$

Inspired by S. Zhao et. al, *A general approach to coordination control of mobile agents with motion constraints*, IEEE Transactions on Automatic Control, 63(5):1509-1516.

Bearing Formation Control with Unicycle

$$\dot{x}_i = -\begin{bmatrix}\cos(\theta_i) & \sin(\theta_i)\end{bmatrix} \sum_{j \in \mathcal{N}_i} P_{g_{ij}} g_{ij}^* \cos(\theta_i)$$

$$\dot{y}_i = -\begin{bmatrix}\cos(\theta_i) & \sin(\theta_i)\end{bmatrix} \sum_{j \in \mathcal{N}_i} P_{g_{ij}} g_{ij}^* \sin(\theta_i)$$

$$\dot{\theta}_i = -\begin{bmatrix}-\sin(\theta_i) & \cos(\theta_i)\end{bmatrix} \sum_{j \in \mathcal{N}_i} P_{g_{ij}} g_{ij}^*.$$

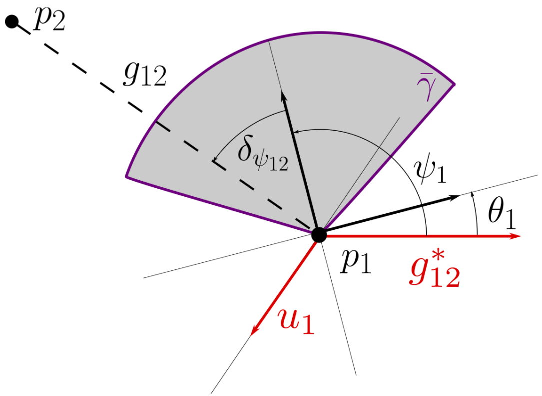


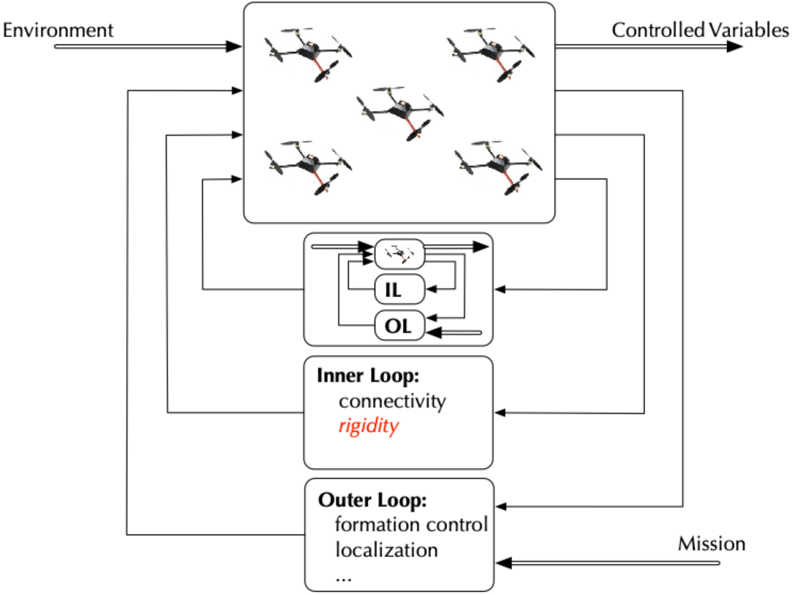
Figure 5: The camera does not align with the moving direction of the unicycle but is turned around $+\pi/2$.

Unique considerations required for unicycle dynamics!

UNICYCLE DYNAMICS BEARING-ONLY CONTROL

UNICYCLE DYNAMICS BEARING-ONLY CONTROL

THE RIGHT ARCHITECTURE



Topics covered by this talk:

- Distance rigidity and formation control
- General Bearing rigidity theory
- Bearing-only formation control law
- Field-of-View constrained systems
- Multi-robot implementation

Where next?

- directed rigidity theory
- general non-linear sensors
- more sophisticated models and robots

ACKNOWLEDGEMENTS

WIAS

lagadic

LAAS-CNRS

GIST

Gwangju Institute of Science and Technology

Dr. Shiyu Zhao

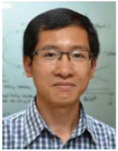
Dr. Paolo Robuffo Giordano

Dr. Antonio Franchi

Prof. Hyo-Sung Ahn

Prof. Angelo Cenedese

Dr. Fabrizio Schiano



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RELATED PUBLICATION

- S. Zhao and D. Zelazo, “Bearing rigidity and almost global bearing-only formation stabilization,”, *IEEE Transactions on Automatic Control*, vol. 61, no. 5, pp. 1255-1268, 2016.
- D. Frank, D. Zelazo, and F. Allgower, “Bearing-Only Formation Control with Limited Visual Sensing: Two Agent Case,” *NeCSys 2018*.
- S. Zhao and D. Zelazo, “Bearing Rigidity Theory and its Applications for Control and Estimation of Network Systems: Life beyond distance rigidity”, *IEEE Control Systems Magazine*, vol. 38, no. 2, pp. 66-83, 2019.