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Distributed Consensus Kalman Filtering Over Time-Varying Graphs

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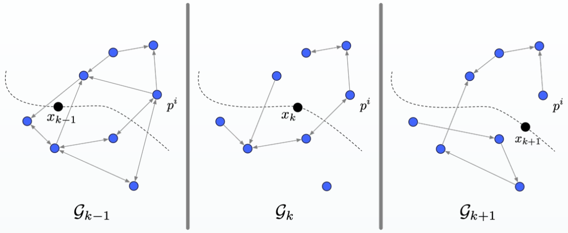
Cooperative Estimation



Observed Process

$$\mathcal{P} \,:\, x_{k+1} = Ax_k + Bw_k$$

Observation Model

$$z_k^i = H^i x_k + v_k^i$$

A network of sensors aim to cooperatively estimate the state of a linear discrete-time stochastic process. The sensors can share information over a *time-varying* network.

Assumptions

- measurement and process noise assumed to be AGWN
- process is observable by each agent
- communication graph is time varying

Classic Approach: Consensus Kalman Filter

sub-optimal approach

[Olfati-Saber 2009]

Estimation

$$K_k^i = P_k^i H^{i^T}\left(R^i + H^i \bar{P}_k^i H^{i^T}\right)^{-1}$$
$$\hat{P}_k^i = F_k^i \bar{P}_k^i F_k^{i^T} + K_k^i R^i K_k^{i^T}$$
$$\hat{x}_k^i = \bar{x}_k^i + K_k^i\left(z_k^i - H^i \bar{x}_k^i\right) + C_k^i \sum_{j\in\mathcal{N}_i}\left(\bar{x}_k^j - \bar{x}_k^i\right)$$

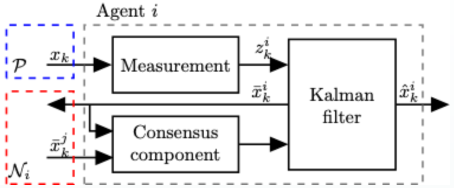
Prediction

$$\bar{x}_{k+1}^i = A\hat{x}_k^i$$
$$\bar{P}_{k+1}^i = A\hat{P}_k^i A^T + BQB^T,$$

Consensus Gain

- Nominally calculated using global graph properties
- Nominally not robust to time varying graphs
- May lead to in degraded performance if consensus gain is chosen to be small.

Consensus Kalman Filtering: The Decentralized Case



Estimation

$$K_k^i = P_k^i H^{i^T}\left(R^i + H^i \bar{P}_k^i H^{i^T}\right)^{-1}$$
$$\hat{P}_k^i = F_k^i \bar{P}_k^i F_k^{i^T} + K_k^i R^i K_k^{i^T}$$
$$\hat{x}_k^i = \bar{x}_k^i + K_k^i\left(z_k^i - H^i \bar{x}_k^i\right) + C_k^i \sum_{j\in\mathcal{N}_i}\left(\bar{x}_k^j - \bar{x}_k^i\right)$$

Prediction

$$\bar{x}_{k+1}^i = A\hat{x}_k^i$$
$$\bar{P}_{k+1}^i = \frac{1}{|\mathcal{N}_{i,k}|+1} A \sum_{j\in\mathcal{N}_{i,k}\cup\{i\}} \hat{P}_k^j A^T + BQB^T$$

Consensus Gain

$$C_k^i = \begin{cases} \frac{1}{|\mathcal{N}_{i,k}|+1}\underbrace{(I - K_k^i H^i)}_{:=F_k^i}, & |\mathcal{N}_{i,k}| > 0 \\ 0, & |\mathcal{N}_{i,k}| = 0 \end{cases}$$

- depends only on neighborhood size

Covariance Update

$$\bar{P}_{k+1}^i = \frac{1}{|\mathcal{N}_{i,k}|+1} A \sum_{j\in\mathcal{N}_{i,k}\cup\{i\}} \hat{P}_k^j A^T + BQB^T$$

- communication of error covariance matrix with neighbors
- local averaging

Theorem

The noiseless error dynamics are asymptotically stable.

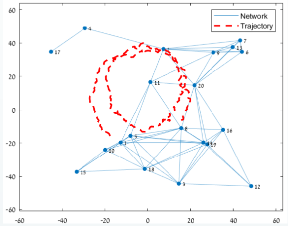
Numerical Example

Process model

$$x_{k+1} = \underbrace{\begin{bmatrix} 0.9996 & -0.0283 \\ 0.0283 & 0.9996 \end{bmatrix}}_{A} x_k + \underbrace{0.375 \cdot I_2}_{B} w_k.$$

Observation model

$$H^i = \begin{cases} \begin{bmatrix} 1 & 0 \end{bmatrix}, & i \in \{1,3,\ldots,19\} \\ \begin{bmatrix} 0 & 1 \end{bmatrix}, & i \in \{2,4,\ldots,20\} \end{cases}$$

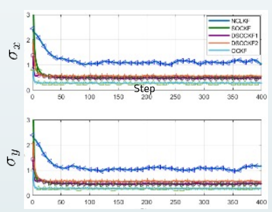


- static network with 20 nodes

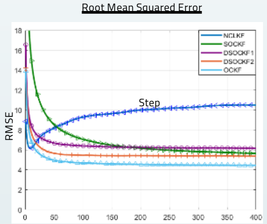
Results

$$\sigma = \frac{1}{MC}\sum_{j=1}^{MC}\sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\hat{x}^{i,j} - \frac{1}{N}\sum_{i=1}^{N}\hat{x}^{i,j}\right)^2} \quad \text{RMSE} = \frac{1}{MC}\sum_{j=1}^{MC}\sqrt{\sum_{i=1}^{N}\left(\mathbb{E}[(\eta^{i,j})^T\eta^{i,j}]\right)}$$

State estimation STD



Root Mean Squared Error

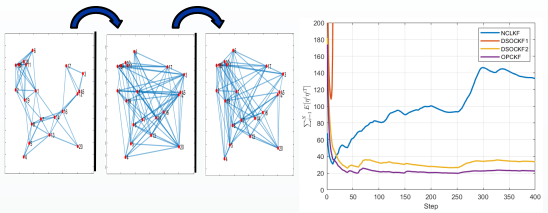


Comparison between several Solutions:

- **NCLKF**: the non-cooperative local KF filter with null consen- sus gain;
- **SOCKF**: the sub-optimal consensus KF with a centralized consensus factor as presented in [1];
- **DSOCKF1**: the decentralized sub-optimal consensus KF with consensus gain taken from [2];
- **DSOCKF2**: our decentralized sub-optimal consensus KF and proposed consensus gain;
- **OCKF**: the optimal consensus Kalman filter as derived in [3].

Results

Performance under graph switching topology.



The decentralized consensus Kalman Filter Is robust to graph topology switches.

Conclusion

- proposed a simple modification to consensus Kalman filter that requires no global network information
- built in robustness to time-varying graphs
- fully distributed design

Future work:

- constructing similar decentralized consensus-based techniques for the EKF and UKF.
- Modifying algorithm to be robust for the general case where not all agents observe the target.

References

[1] Priel, A. and Zelazo, D. (2021). An improved distributed consensus kalman filter design approach. In 2021 60th IEEE conference on Decision and Control, 502–507. IEEE.

[2] Sandell, N.F. and Olfati-Saber, R. (2008). Distributed data association for multi-target tracking in sensor networks. In 2008 47th IEEE Conference on Decision and Control, 1085 – 1090. IEEE.

[3] Deshmukh, R., Kwon, C., and Hwang, I. (2017). Optimal discrete-time Kalman consensus filter. In 2017 American Control Conference (ACC), 5801–5806. IEEE.

[4] Olfati-Saber, R. (2009). Kalman-consensus filter: Optimality, stability, and performance. In Proceedings of the 48h IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference, 7036–7042. IEEE.