Rigidity Theory for Multi-Robot Coordination

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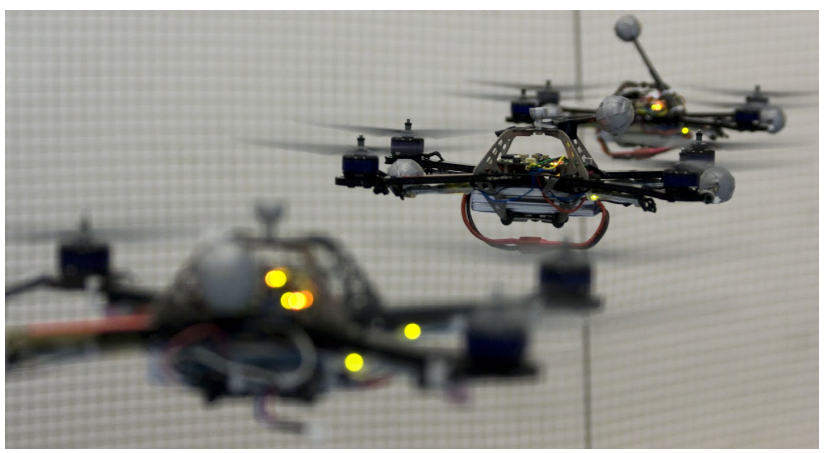
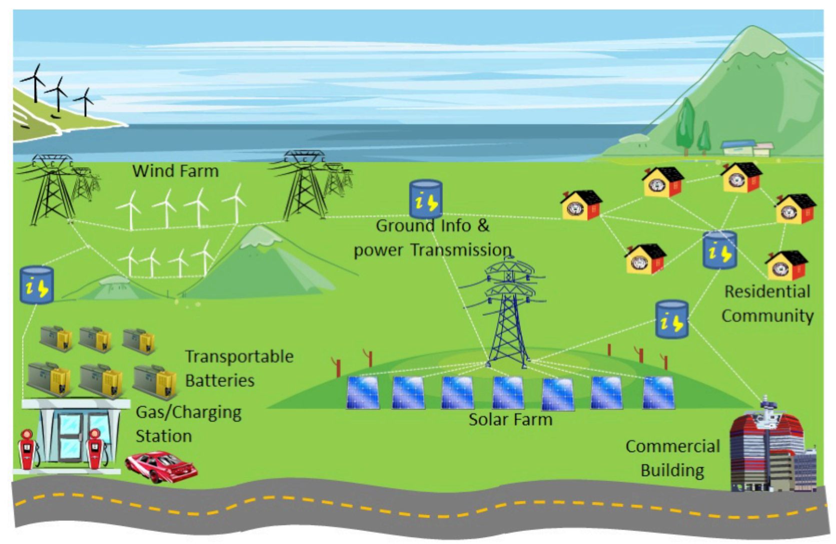
IAAC workshop on "Motion Control Methods in Robotics"

Multi-Agent Systems

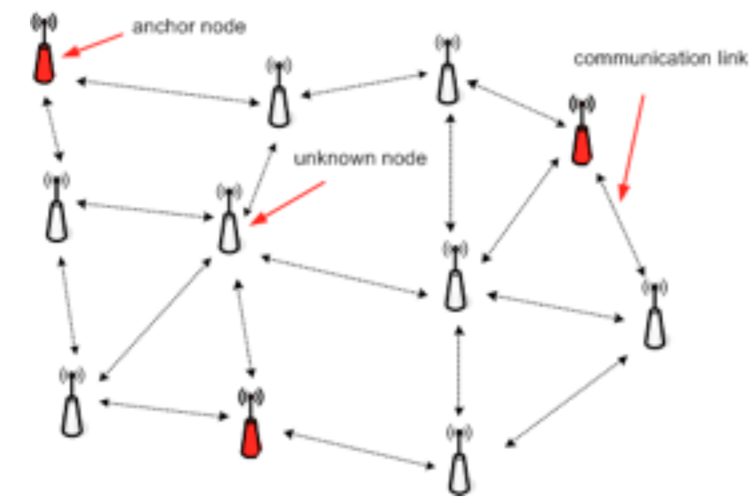
Multi-Agent Systems are systems composed of multiple interacting dynamic units.



formation control
& multi-robot coordination



energy management &
the “smart-grid”



sensor networks

Challenges in Multi-Robot Systems

Solutions to coordination problems in multi-robot systems are *highly* dependent on the sensing and communication mediums available!

selection criteria depends on mission requirements, cost, environment…

<u>Sensing</u>

- GPS
- Relative Position Sensing
- Range Sensing
- Bearing Sensing

<u>Communication</u>

- Internet
- Radio
- Sonar
- MANet

Challenges in Multi-Robot Systems

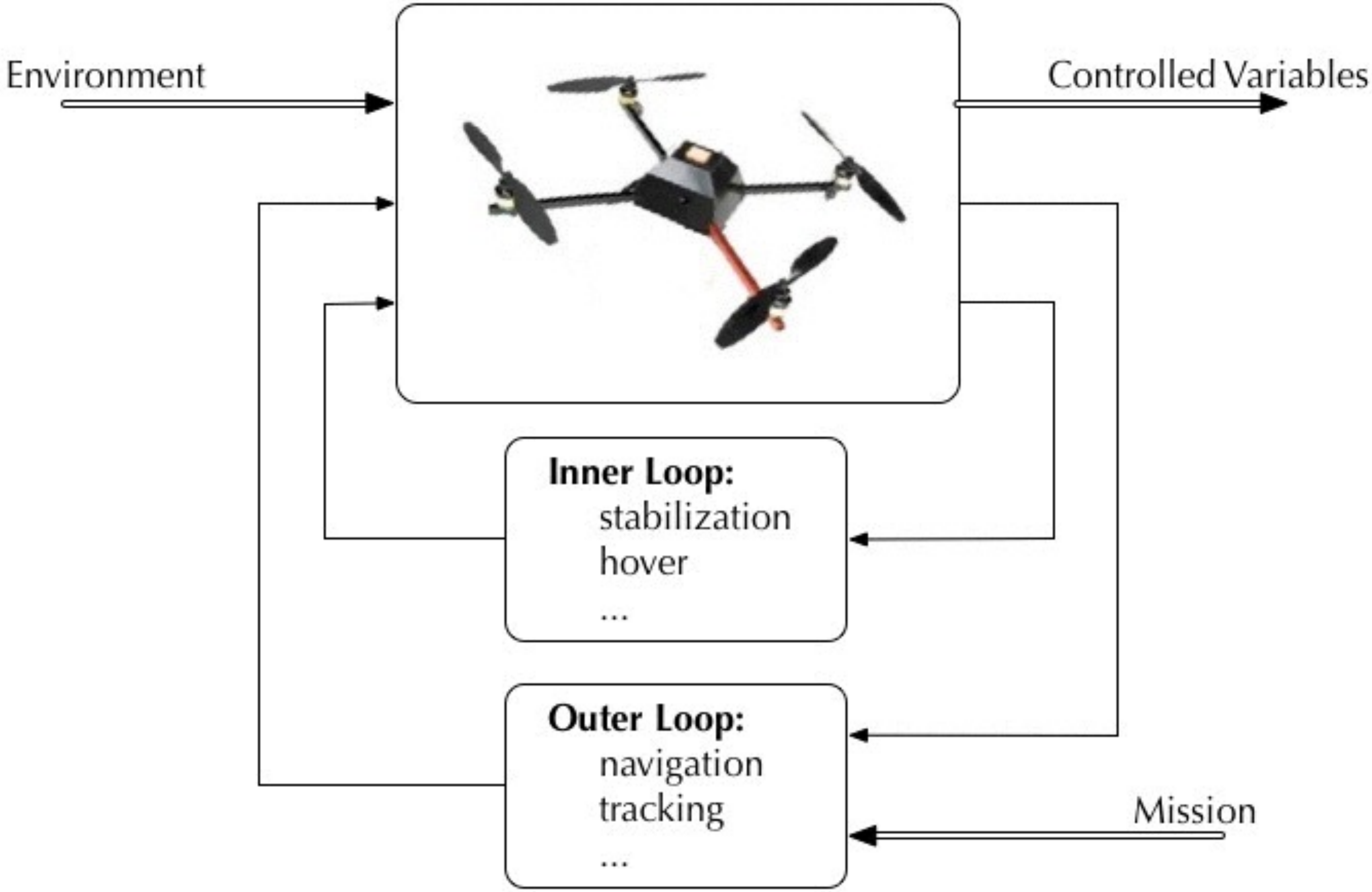
Solutions to coordination problems in multi-robot systems are *highly* dependent on the sensing and communication mediums available!

selection criteria depends on mission requirements, cost, environment…

Are there *architectural features* of a multi-agent system that are independent of any particular mission or hardware capabilities?

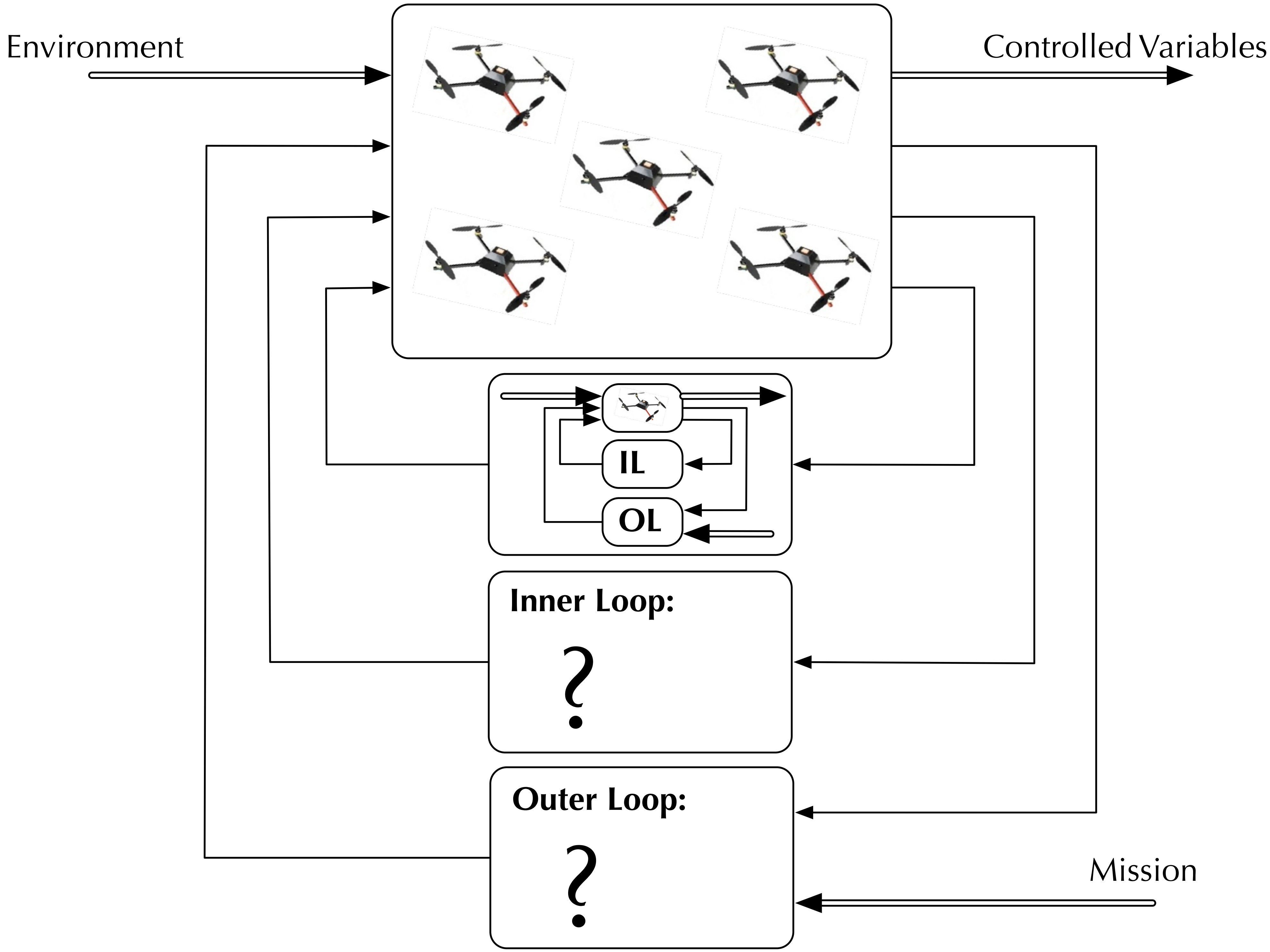
Towards a Multi-Robot Control Architecture

control architecture for a *single* quadrotor



Towards a Multi-Robot Control Architecture

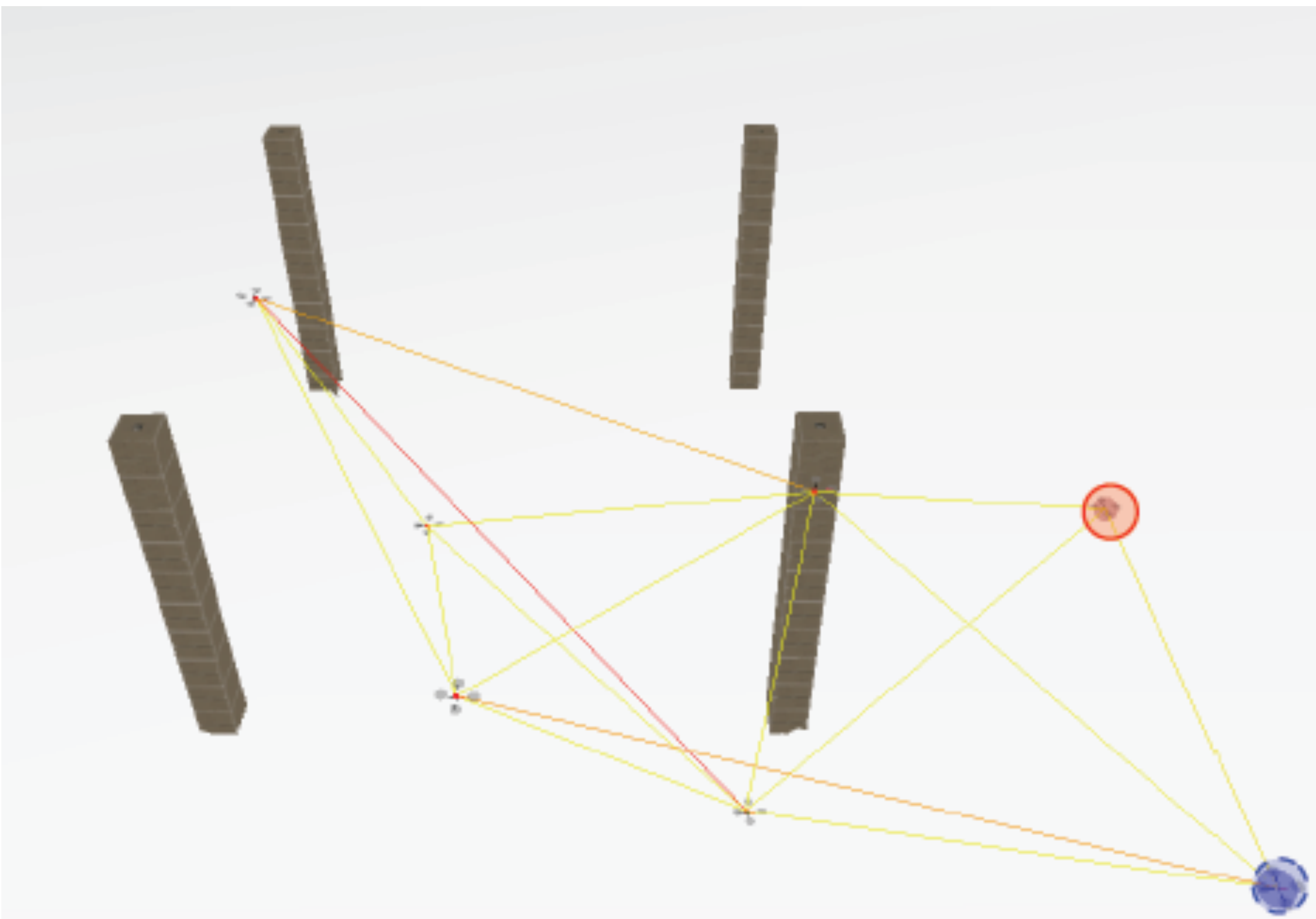
what is the architecture for a *multi-robot* system?



Towards a Multi-Robot Control Architecture

what is the architecture for a *multi-robot* system?

Connectivity

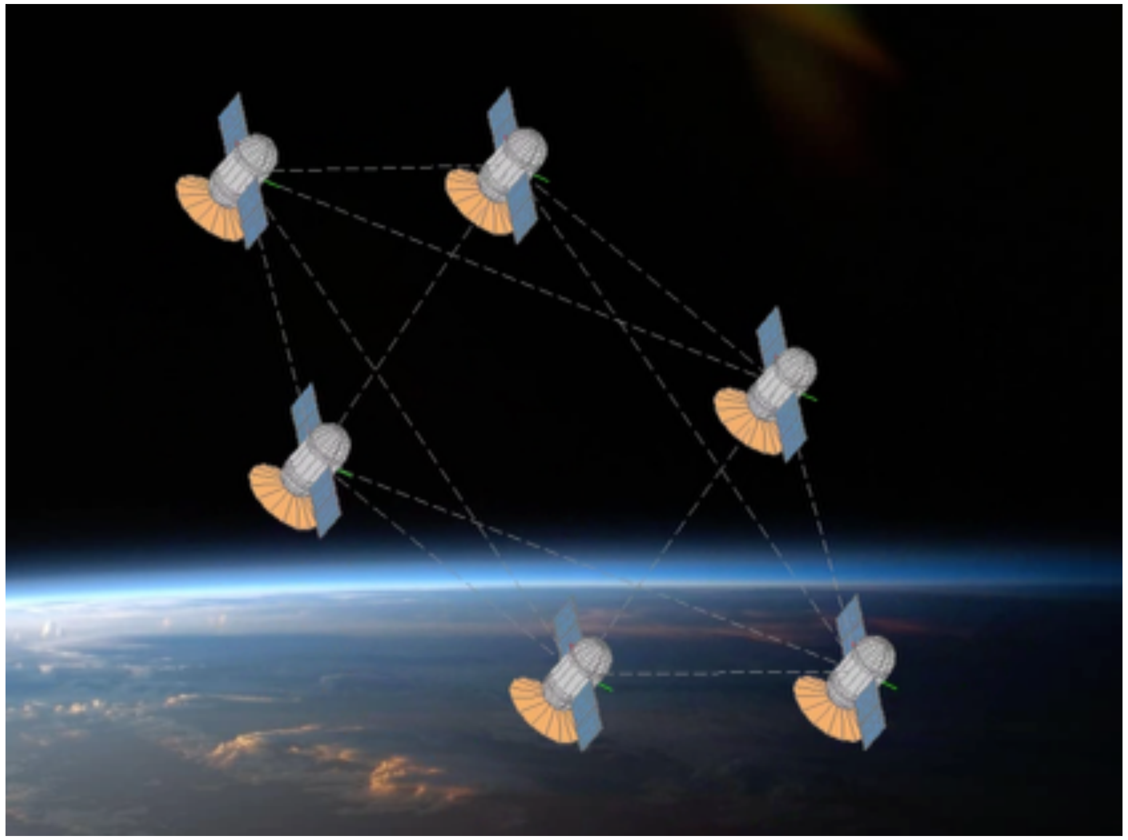


Ji and Egerstedt, 2007
Dimarogonas and Kyriakopoulos, 2008
Yang *et al.*, 2010
Robuffo Giordano *et al.*, 2013

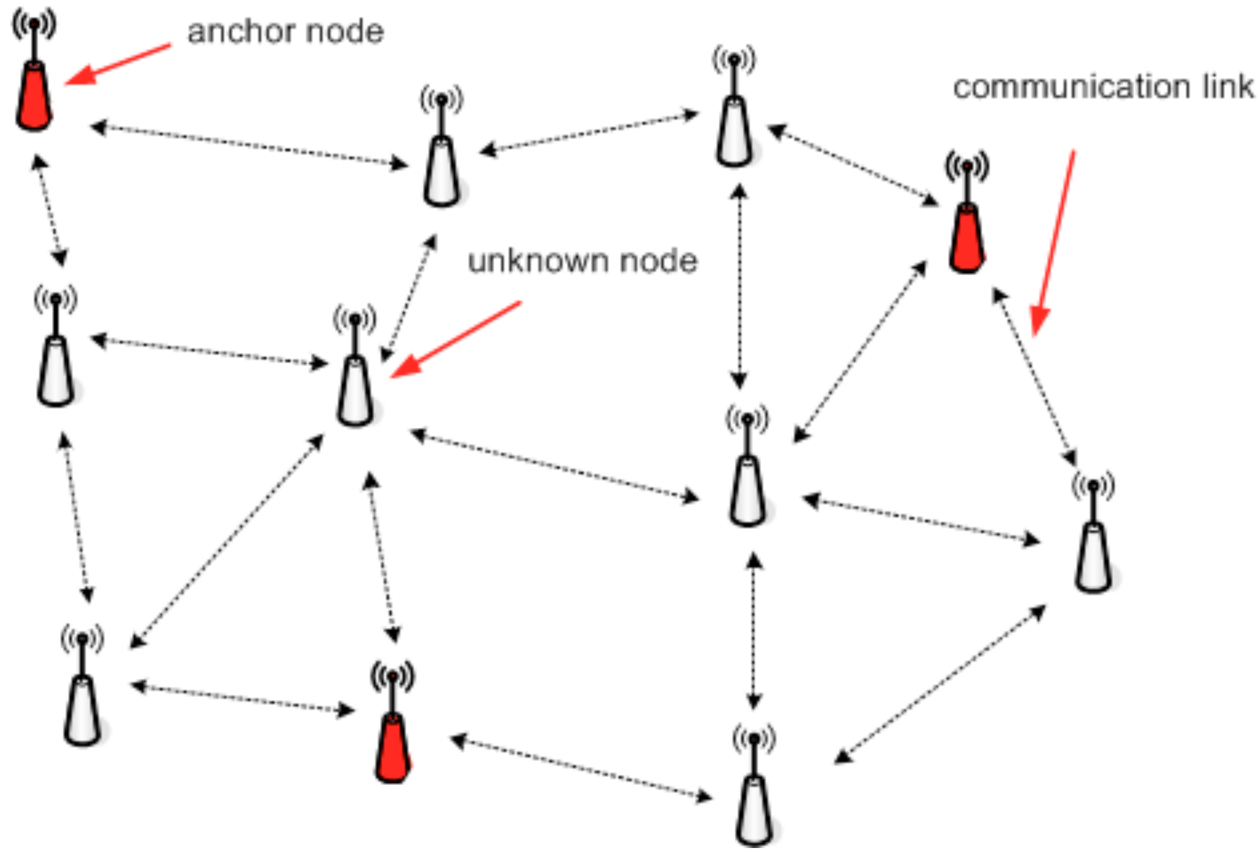
Towards a Multi-Robot Control Architecture

is connectivity enough for higher-level objectives?

formation control



localization

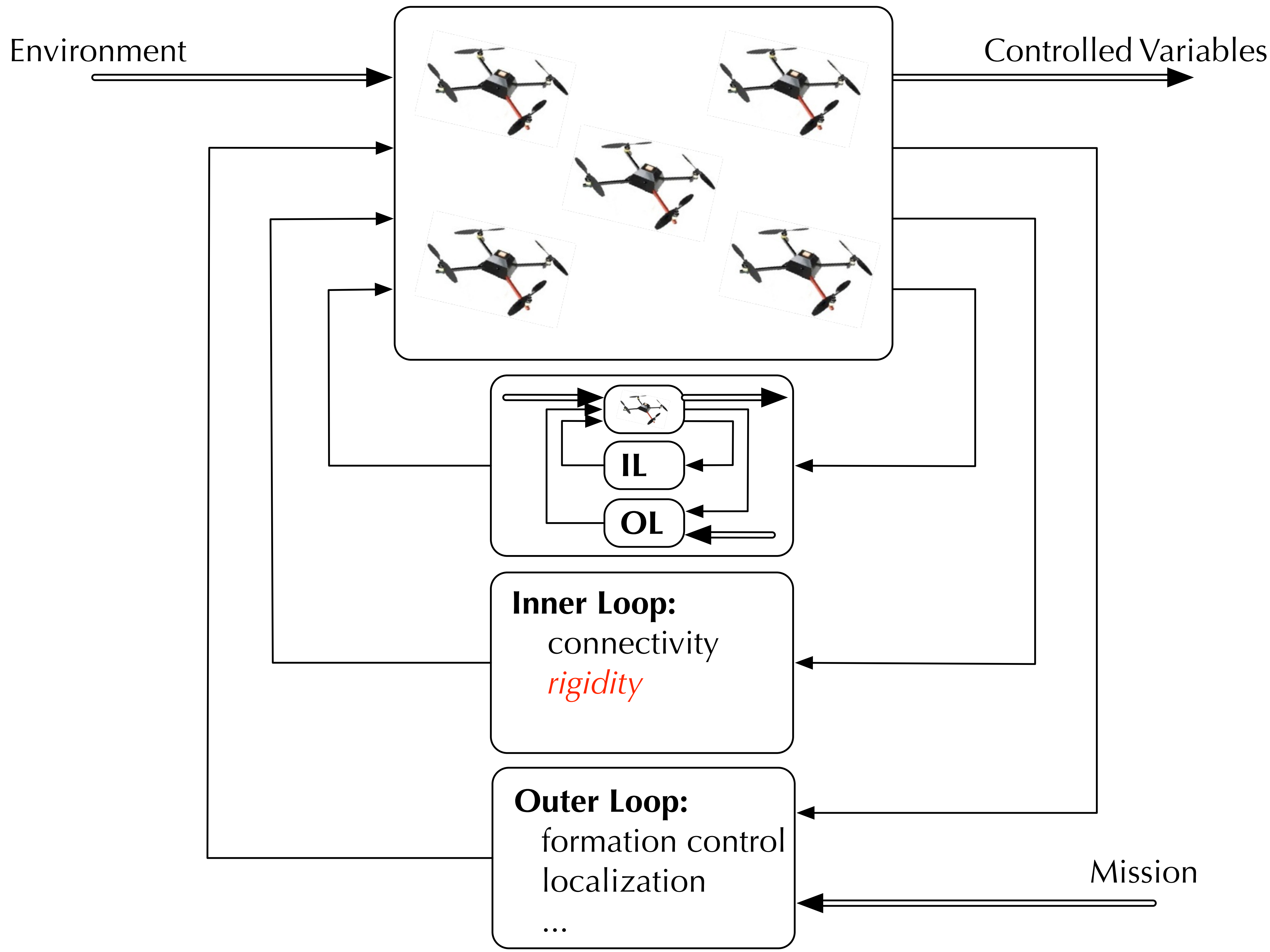


http://www.commsys.isy.liu.se/en/research

Rigidity Theory provides the correct framework to address many multi-agent mission objectives

Towards a Multi-Robot Control Architecture

what is the architecture for a *multi-robot* system?



Rigidity Theory

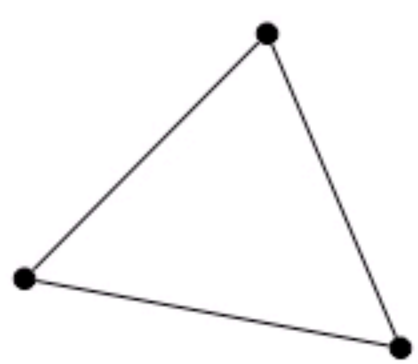
Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

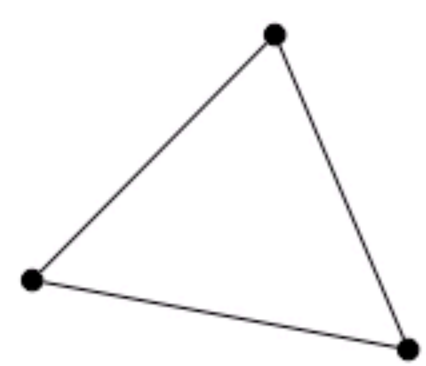
Distance Rigidity

- maintain distance pairs
- rigid body rotations and translations

Bearing (Parallel) Rigidity

- maintain angles (shape)
- rigid body translations and dilations



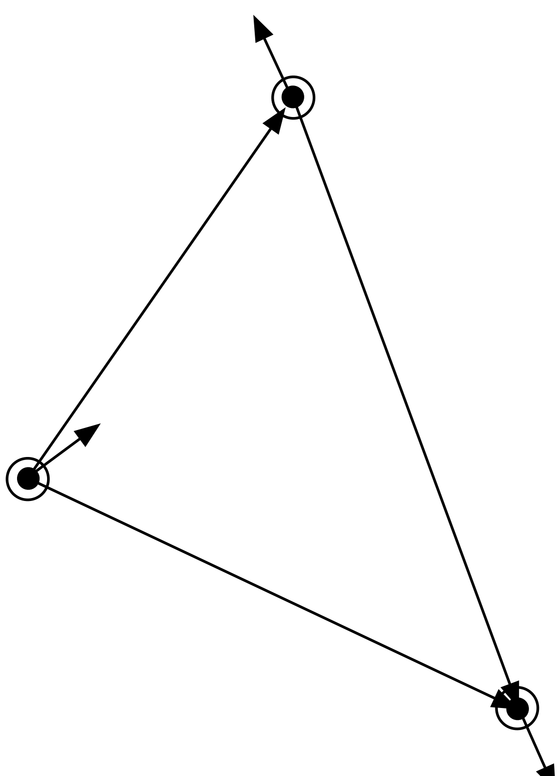


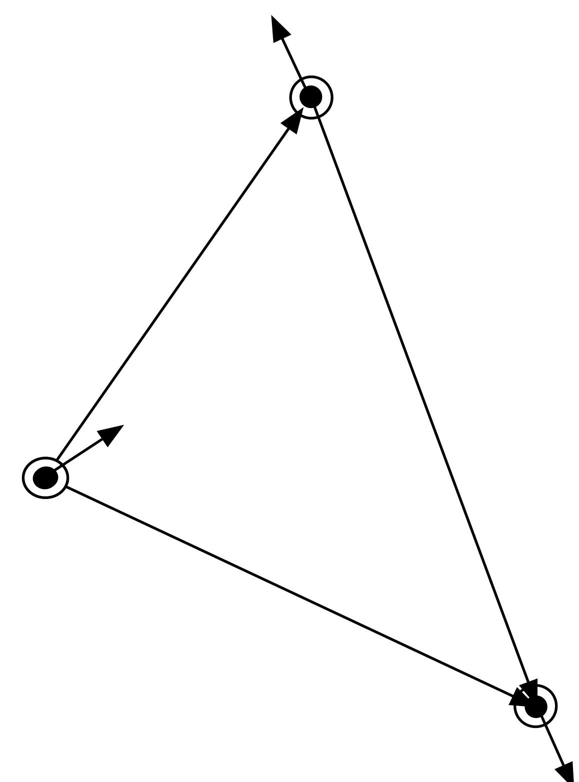
Infinitesimal Motions in SE(2)

Rigidity is a combinatorial theory for characterizing the “stiffness” or “flexibility” of structures formed by rigid bodies connected by flexible linkages or hinges.

SE(2) Rigidity

- maintain bearings in *local* frame
- rigid body rotations and translations + coordinated rotations





Rigidity Theory

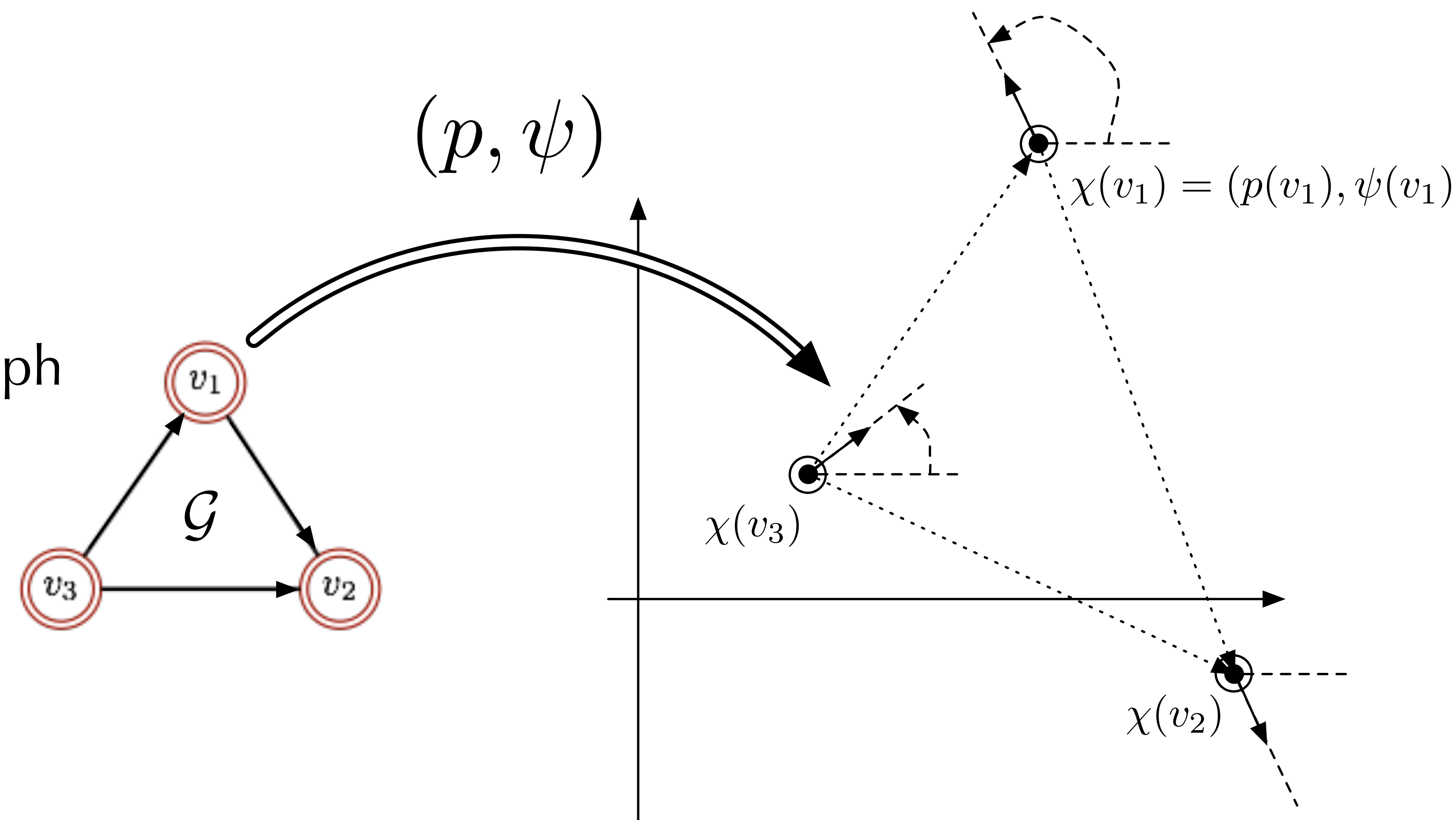
bar-and-joint frameworks in SE(2)

$$(\mathcal{G}, p, \psi)$$

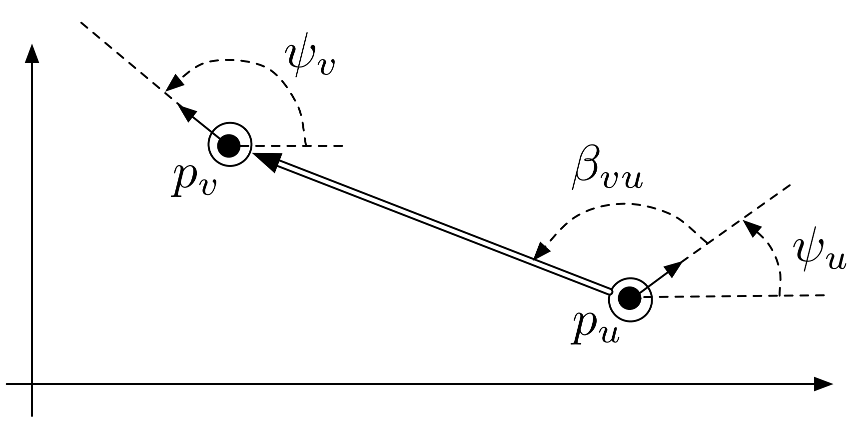
$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ a *directed* graph

$p : \mathcal{V} \to \mathbb{R}^2$

$\psi : \mathcal{V} \to \mathcal{S}^1$



a directed edge indicates availability of relative bearing measurement



stacked vector of entire framework

$$\chi_p = p(\mathcal{V}) \in \mathbb{R}^{2|\mathcal{V}|}$$

$$\chi_\psi = \psi(\mathcal{V}) \in \mathcal{S}^{1^{|\mathcal{V}|}}$$

Rigidity Theory

A framework is **infinitesimally rigid** if all the infinitesimal motions are *trivial* (i.e., translations, rotations, scalings, coordinated rotations).

Distance Rigidity

Rigidity Matrix

$$R(p)\xi = 0$$

Bearing Rigidity

Bearing Rigidity Matrix

$$R_{\|}(p)\xi = 0$$

SE(2) Rigidity

SE(2) Rigidity Matrix

$$\underbrace{\left[\begin{array}{cc} D_{\mathcal{G}}^{-1}(\chi_p)R_{\|}(\chi_p) & \overline{E}(\mathcal{G}) \end{array}\right]}_{\mathcal{B}_{\mathcal{G}}(\chi(\mathcal{V}))}\zeta = 0$$

Theorem

A framework is infinitesimally (distance, parallel) rigid if and only if the rank of the rigidity matrix is $2|\mathcal{V}| - 3$

A framework is SE(2) infinitesimally rigid if and only if the rank of the rigidity matrix is $3|\mathcal{V}| - 4$

Distance and Bearing Rigidity

Theorem

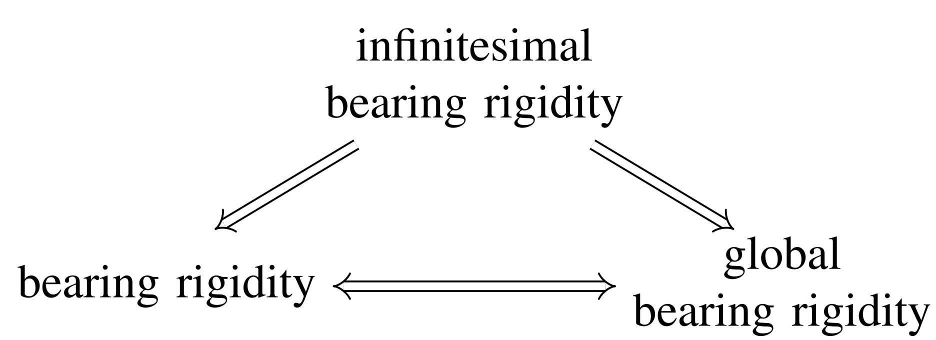
In the plane, a framework is infinitesimally rigid if and only if it is infinitesimally bearing rigid

- does *not* hold for higher dimensions

Theorem

Infinitesimal bearing rigidity implies global bearing rigidity.

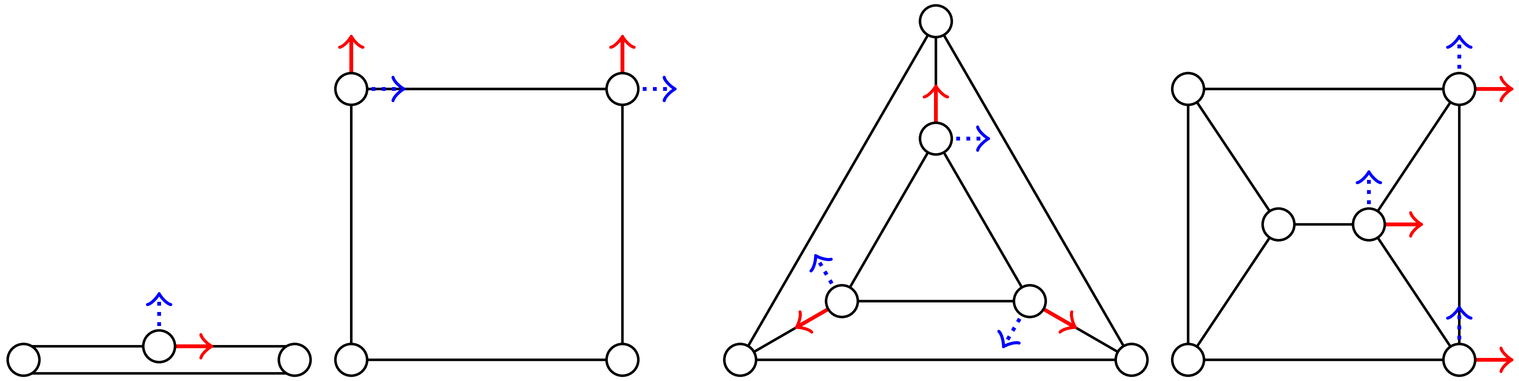
- such a relationship does *not* hold in distance rigidity



[Zhao and Zelazo, TAC2015]

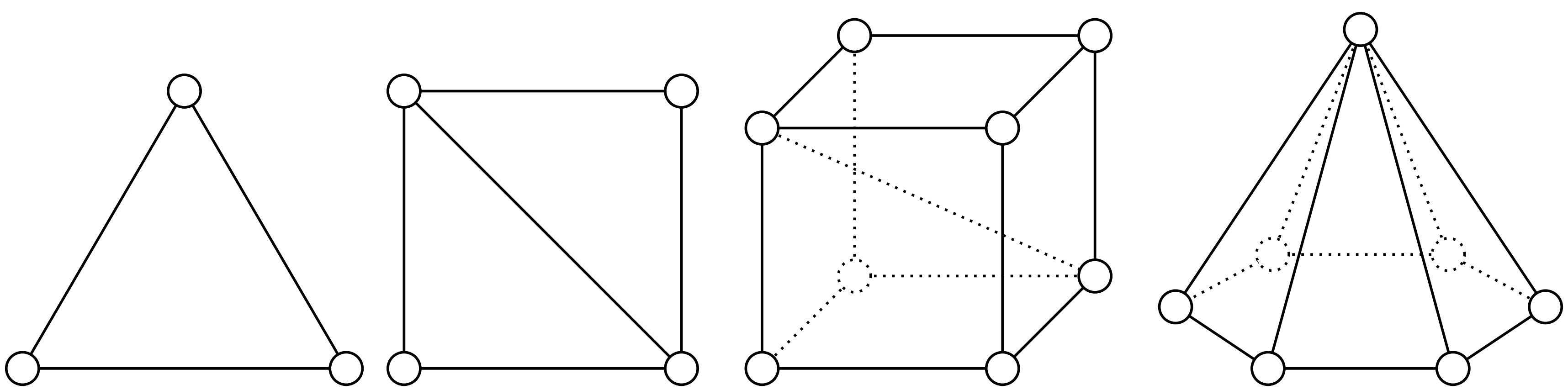
Distance and Bearing Rigidity

non-infinitesimally bearing rigid



(a) (b) (c) (d)

infinitesimally bearing rigid



(a) (b) (c) (d)

[Zhao and Zelazo, TAC2015]

Formation Control

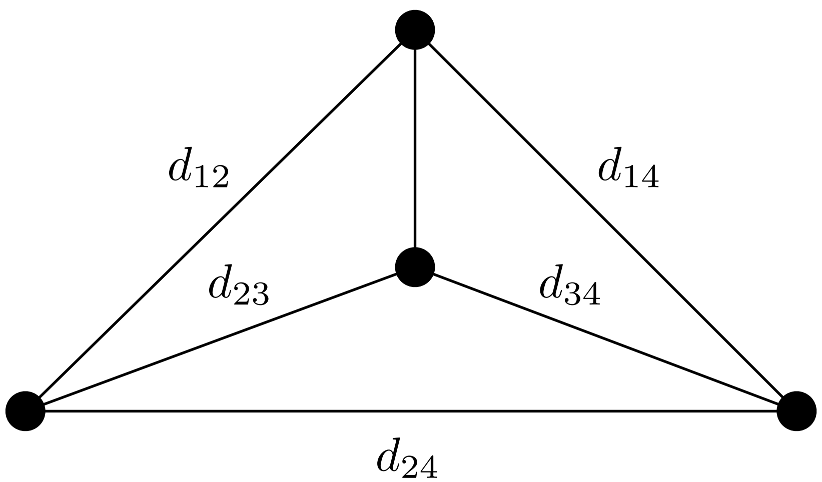
The **formation control** problem is to design a (distributed) control law that drives the agents to a desired spatial configuration determined by interagent distances or bearings.

Gradient Dynamical Systems

$$\dot{p} = -\nabla F(p)$$

distance-based formation control

$$F(p) = \frac{1}{4} \sum_{ij \in \mathcal{E}} \left(\|p_i - p_j\|^2 - d_{ij}^2 \right)^2$$



Formation Control

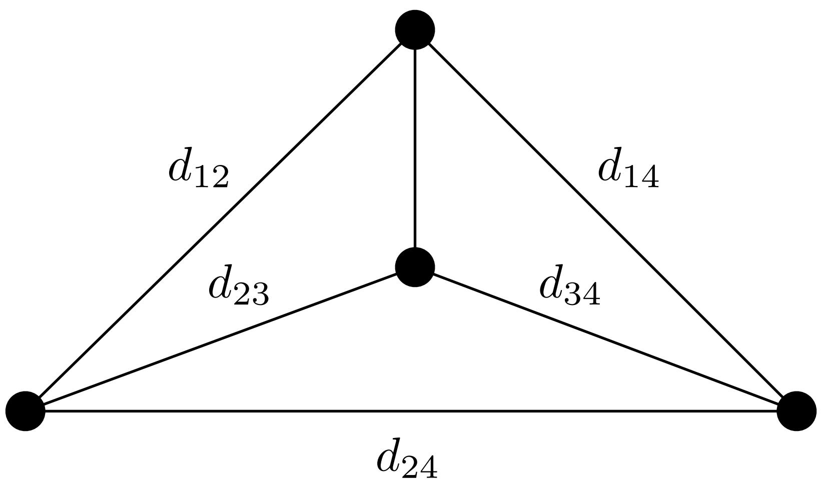
The **formation control** problem is to design a (distributed) control law that drives the agents to a desired spatial configuration determined by interagent distances or bearings.

Gradient Dynamical Systems

$$\dot{p} = -\nabla F(p)$$

distance-based formation control

$$\dot{p} = -R(p)^T(R(p) - d^2)$$



Formation Control

The **formation control** problem is to design a (distributed) control law that drives the agents to a desired spatial configuration determined by interagent distances or bearings.

Distance Rigidity

distance formation control

$$\dot{p}_i = \sum_{j\sim i} \left(\|p_i - p_j\|^2 - d_{ij}^2\right)(p_j - p_i)$$

- control requires distances and relative positions
- distance-only control requires estimation of relative positions

[Krick2007, Anderson2008, Dimarogonas2008, Dörfler2010]

Bearing Rigidity

bearing formation control

$$\dot{p}_i = -\sum_{j\sim i} \frac{1}{\|p_i - p_j\|}\left(I_2 - \frac{(p_j - p_i)(p_j - p_i)^T}{\|p_i - p_j\|^2}\right) g_{ij}^*$$

- control requires bearings and distances

[Zhao and Zelazo, TAC2015]

A Bearing-Only Formation Controller

bearing formation control

$$\dot{p}_i = -\sum_{j\sim i} \frac{1}{\|p_i - p_j\|}\left(I_2 - \frac{(p_j - p_i)(p_j - p_i)^T}{\|p_i - p_j\|^2}\right) g_{ij}^*$$

- requires distance measurements
- orthogonal projection operator

a bearing-only approach

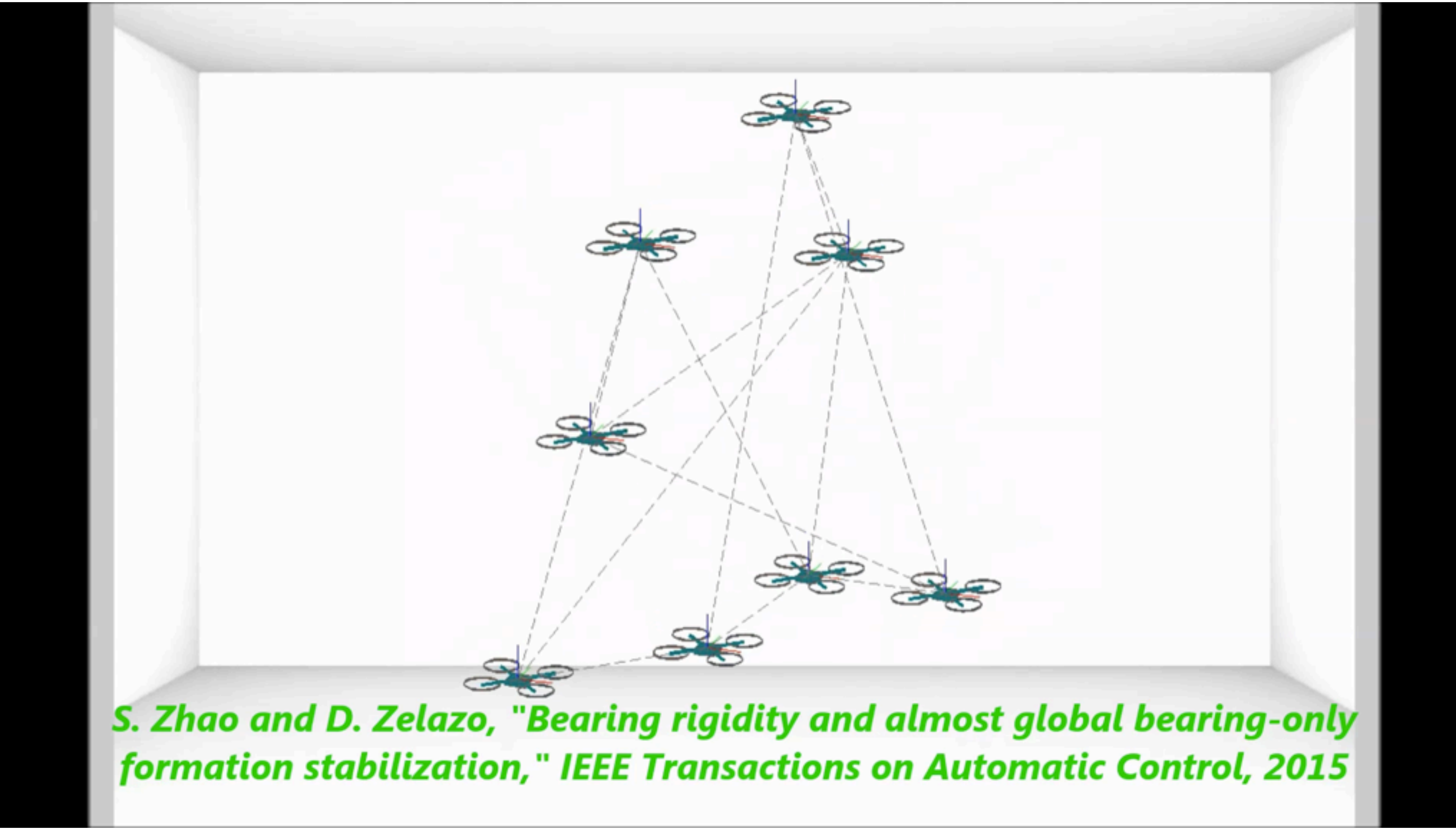
$$\dot{p}_i(t) = -\sum_{j\sim i} P_{g_{ij}(t)} g_{ij}^*$$

stability analysis depends on the **rigidity** of the formation!

- almost-global stability exponential stability
- centroid and scale invariance
- works for arbitrary dimension
- collision avoidance

[Zhao and Zelazo, TAC2015]

Formation Control: Bearing-Constrained Formations



Formation Control

The **formation control** problem is to design a (distributed) control law that drives the agents to a desired spatial configuration determined by interagent distances or bearings.

SE(2) Bearing Rigidity

$$\begin{bmatrix} \dot{p}_i \\ \dot{\psi}_i \end{bmatrix} = \begin{bmatrix} -\sum_{(i,j)\in\mathcal{E}} \frac{P_{r_{ij}}}{\|p_i - p_j\|} r_{ij}^d + \sum_{(j,i)\in\mathcal{E}} T(\psi_j - \psi_i) \frac{P_{r_{ji}}}{\|p_i - p_j\|} r_{ji}^d \\ -\sum_{(i,j)\in\mathcal{E}} (r_{ij}^{\perp})^T r_{ij}^d \end{bmatrix}$$

- requires communication
- requires relative orientation

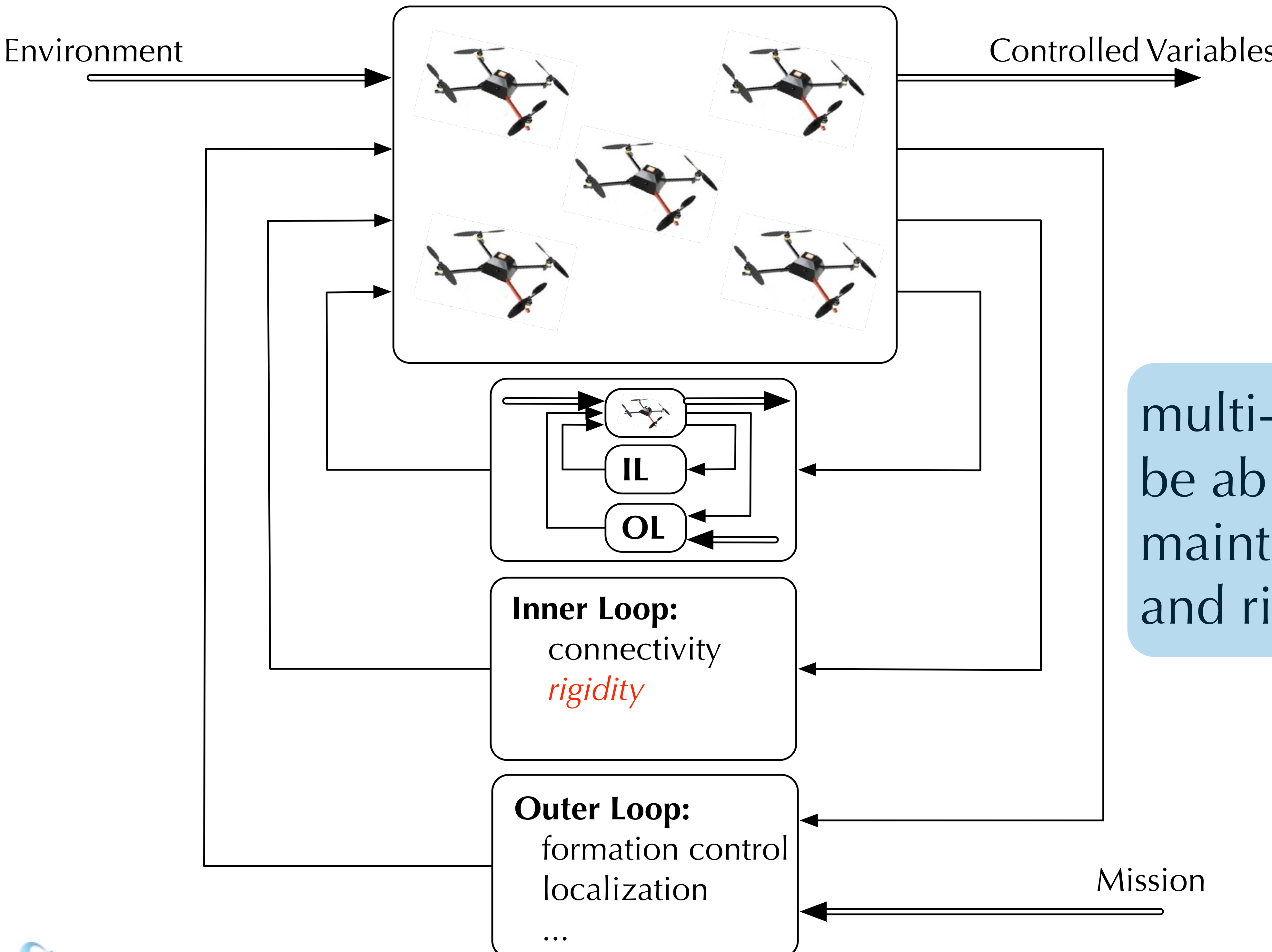
a scale-free SE(2) bearing approach

$$\begin{bmatrix} \dot{p} \\ \dot{\psi} \end{bmatrix} = \hat{\mathcal{B}}_{\mathcal{G}}(\chi)^T \mathbf{b}_{\mathcal{G}}^d$$

[Zelazo, Franchi, Robuffo-Giordano, CDC2015
Schiano, Franchi, Zelazo, Robuffo-Giordano, ICRA2016]

Towards a Multi-Robot Control Architecture

what is the architecture for a *multi-robot* system?



multi-robot systems must be able to *dynamically* maintain the connectivity and rigidity of the team

Rigidity Maintenance

Theorem

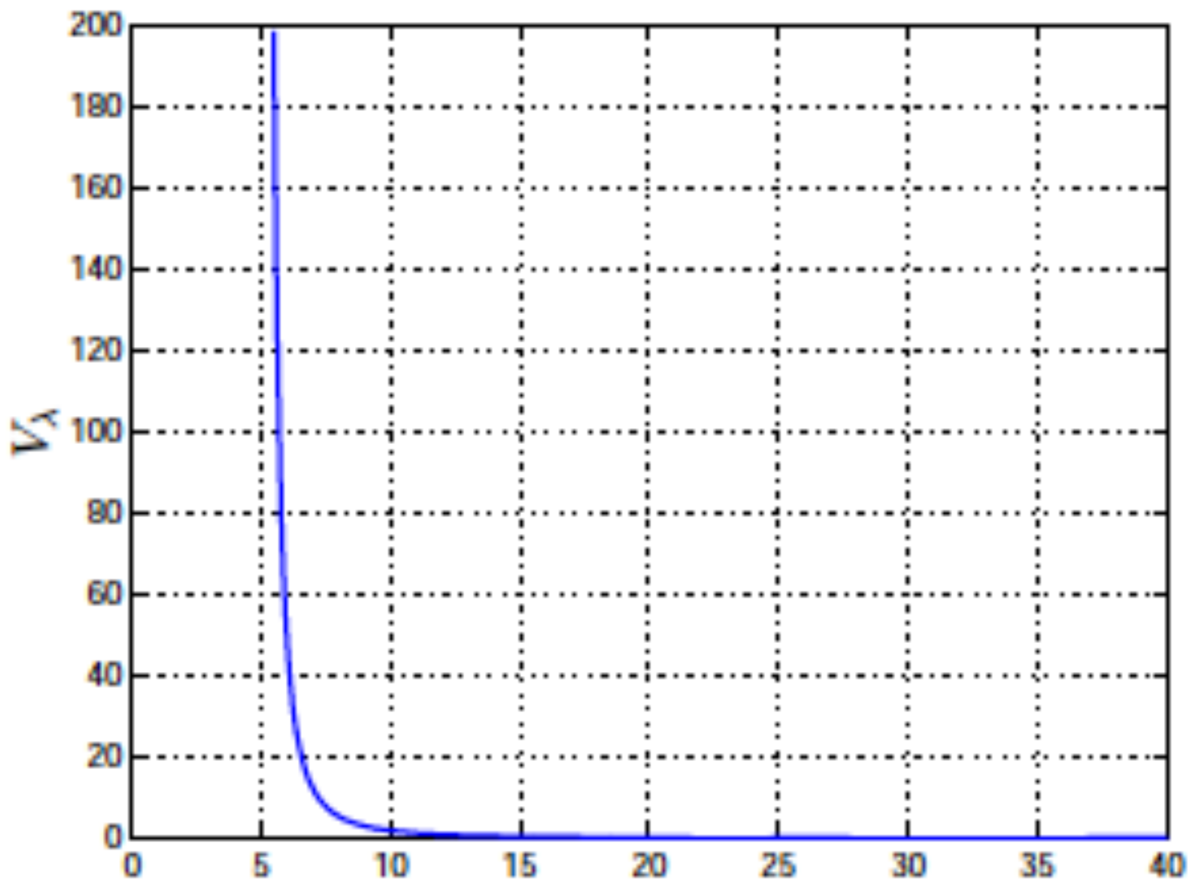
A framework is infinitesimally (distance, parallel) rigid if and only if the *rigidity eigenvalue* is strictly positive.

$$\mathcal{R} = R(p)^T R(p) \quad \mathcal{N}(\mathcal{R}) = \{\text{trivial infinitesimal motions}\}$$

Rigidity Maintenance

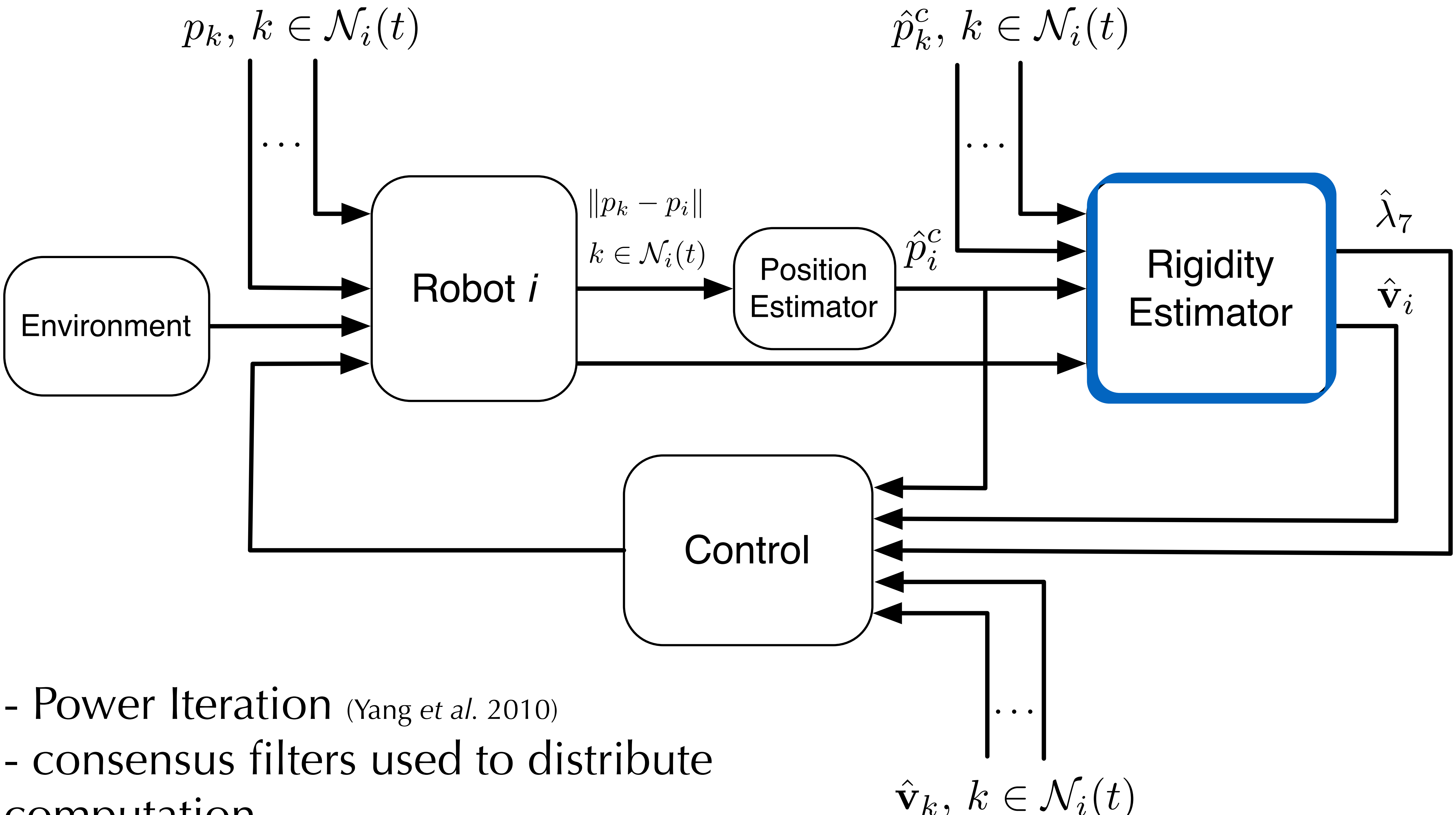
Design a control law to minimize a scalar potential function related to the rigidity eigenvalue

$$\xi_i = -\frac{\partial V_\lambda}{\partial \lambda_4}\left(\frac{\partial \lambda_4}{\partial p_i}\right)$$



rigidity eigenvalue

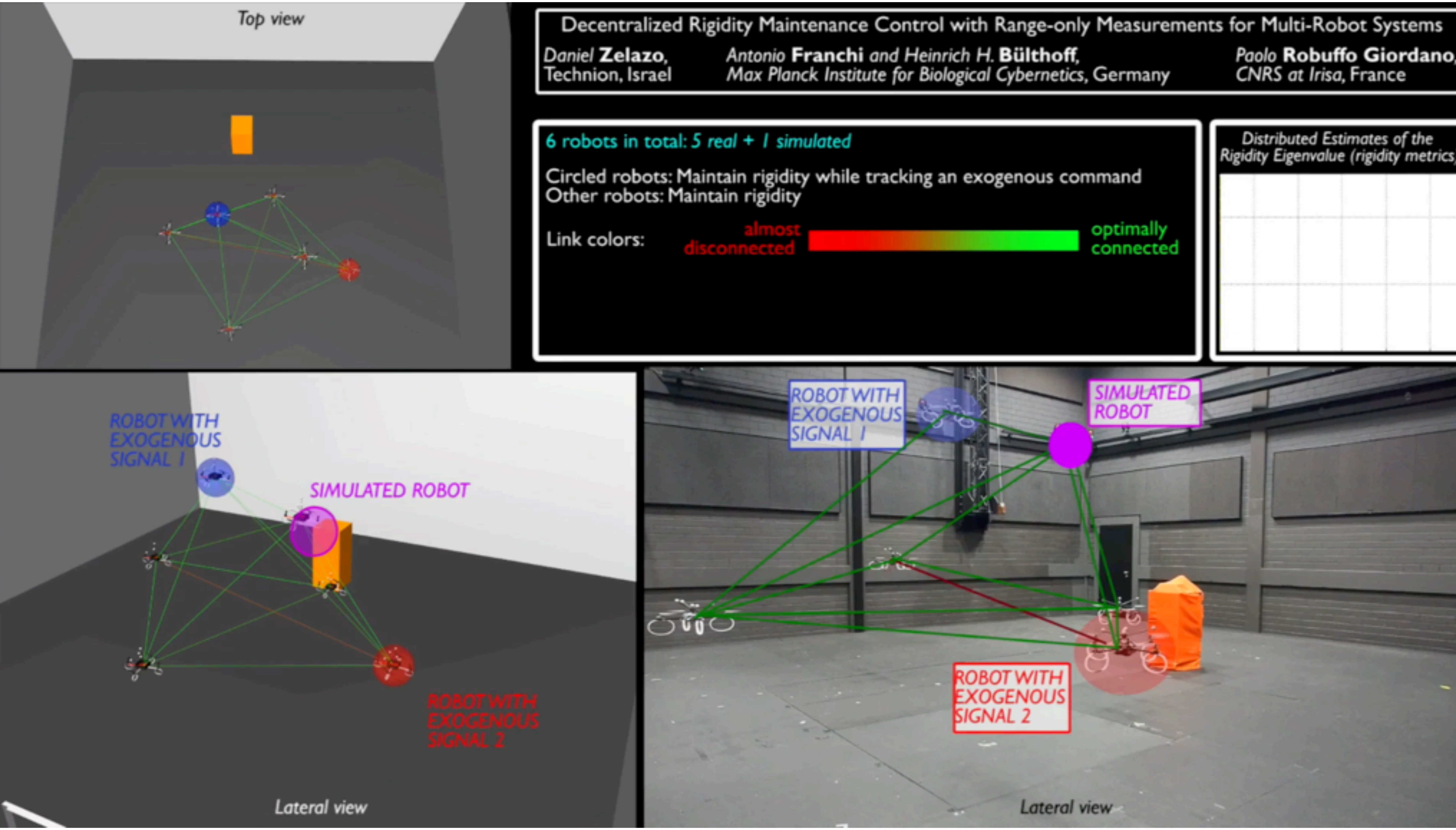
Rigidity Maintenance



- Power Iteration (Yang *et al.* 2010)
- consensus filters used to distribute computation

[Zelazo, Franchi, Robuffo-Giordano, IJRR2015]

Rigidity Maintenance



Conclusions and Outlook

- coordination methods for multi-agent systems depend on sensing and communication mediums
- *rigidity theory* is a powerful framework for handling high-level multi-agent objectives under different sensing and communication constraints
- *rigidity maintenance* is an important “inner-loop” for multi-robot systems

Acknowledgements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Questions?