# Cyclically-Monotone Relations and their use in Passivity-Based Cooperative Control

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# Duality in Cooperative Control Problems

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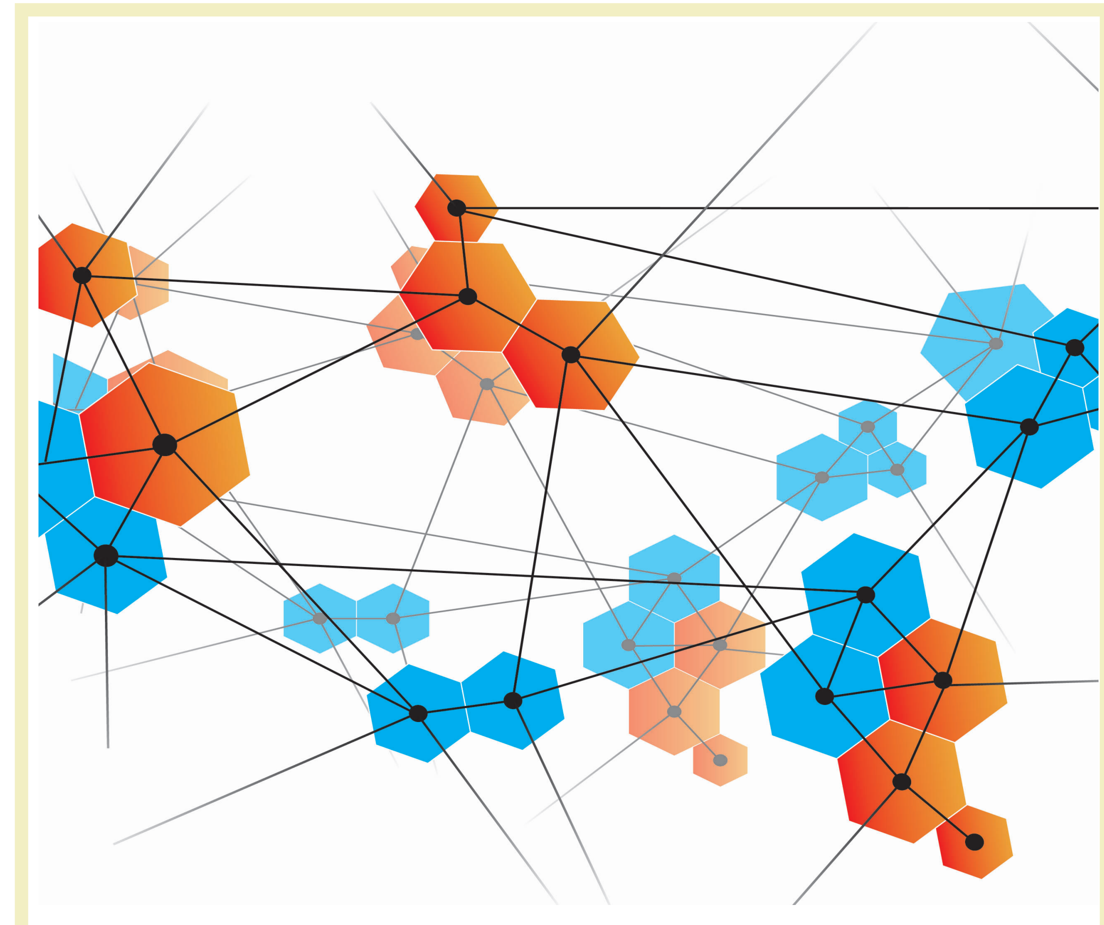
# Networked Dynamic Systems

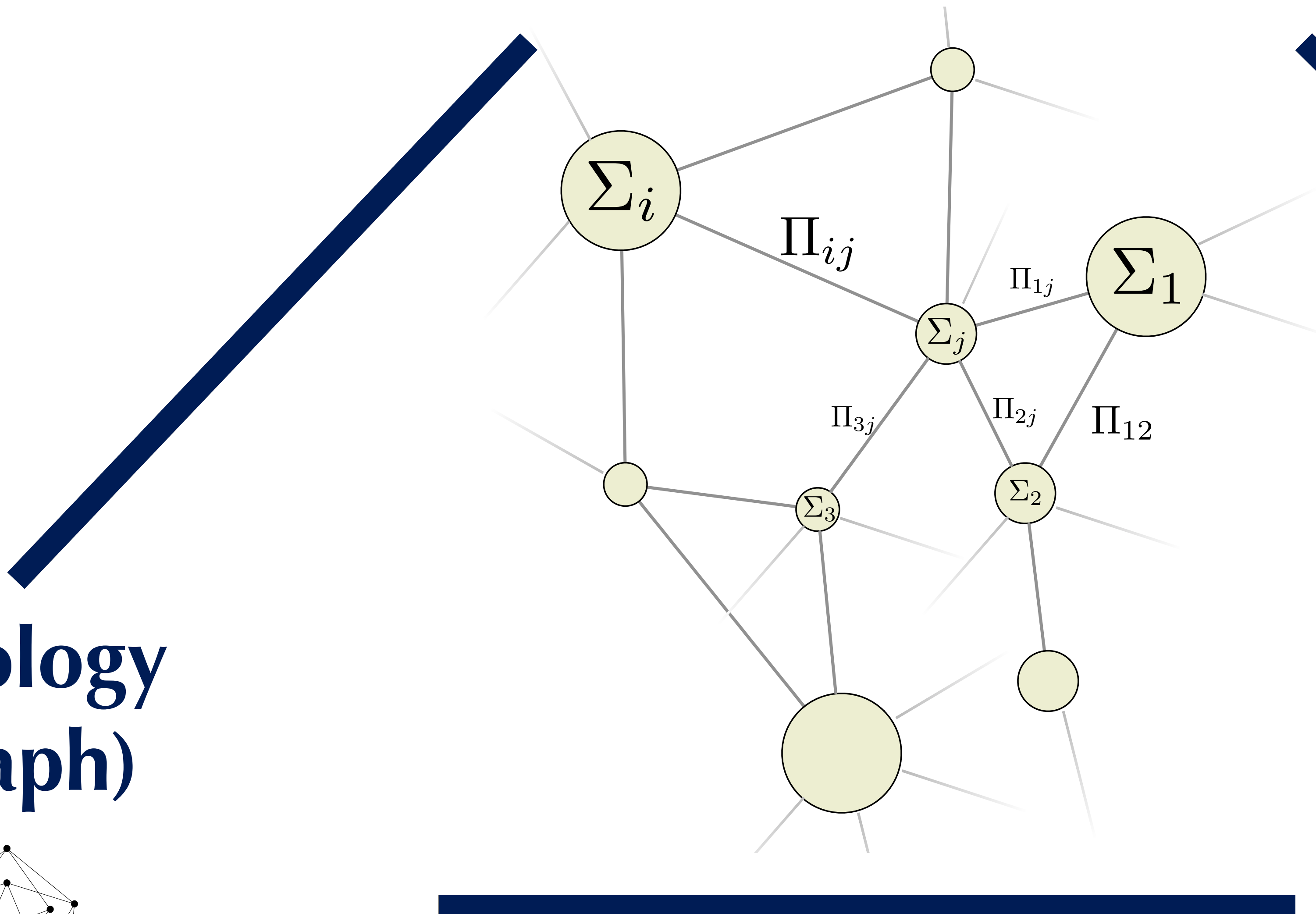



networks of dynamical systems are one of ***the*** enabling technologies of the future

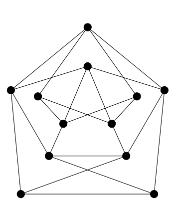
# Networked Dynamic Systems

**dynamics**

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t))$$



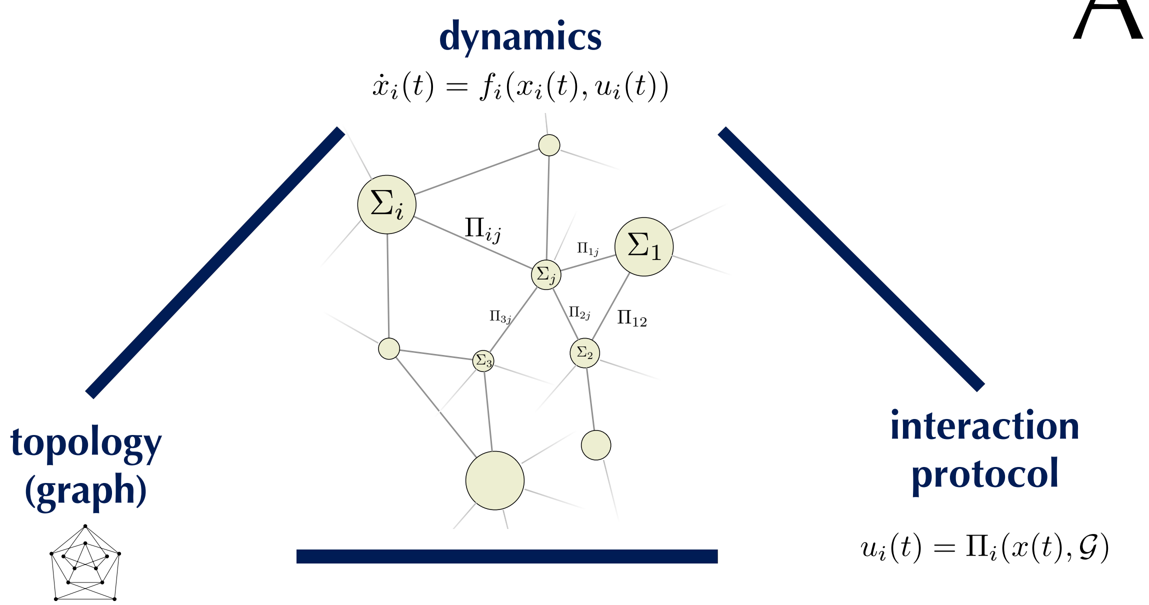

**topology (graph)**



**interaction protocol**

$$u_i(t) = \Pi_i(x(t), \mathcal{G})$$

# Networked Dynamic Systems




## Analysis

- steady-state behavior
- interplay between dynamics and graph
- equilibrium configurations

## Synthesis

- design of distributed protocols
- design of “good” network structures
- robust

**can we reveal *deep* results describing the underlying behavior of these systems?**

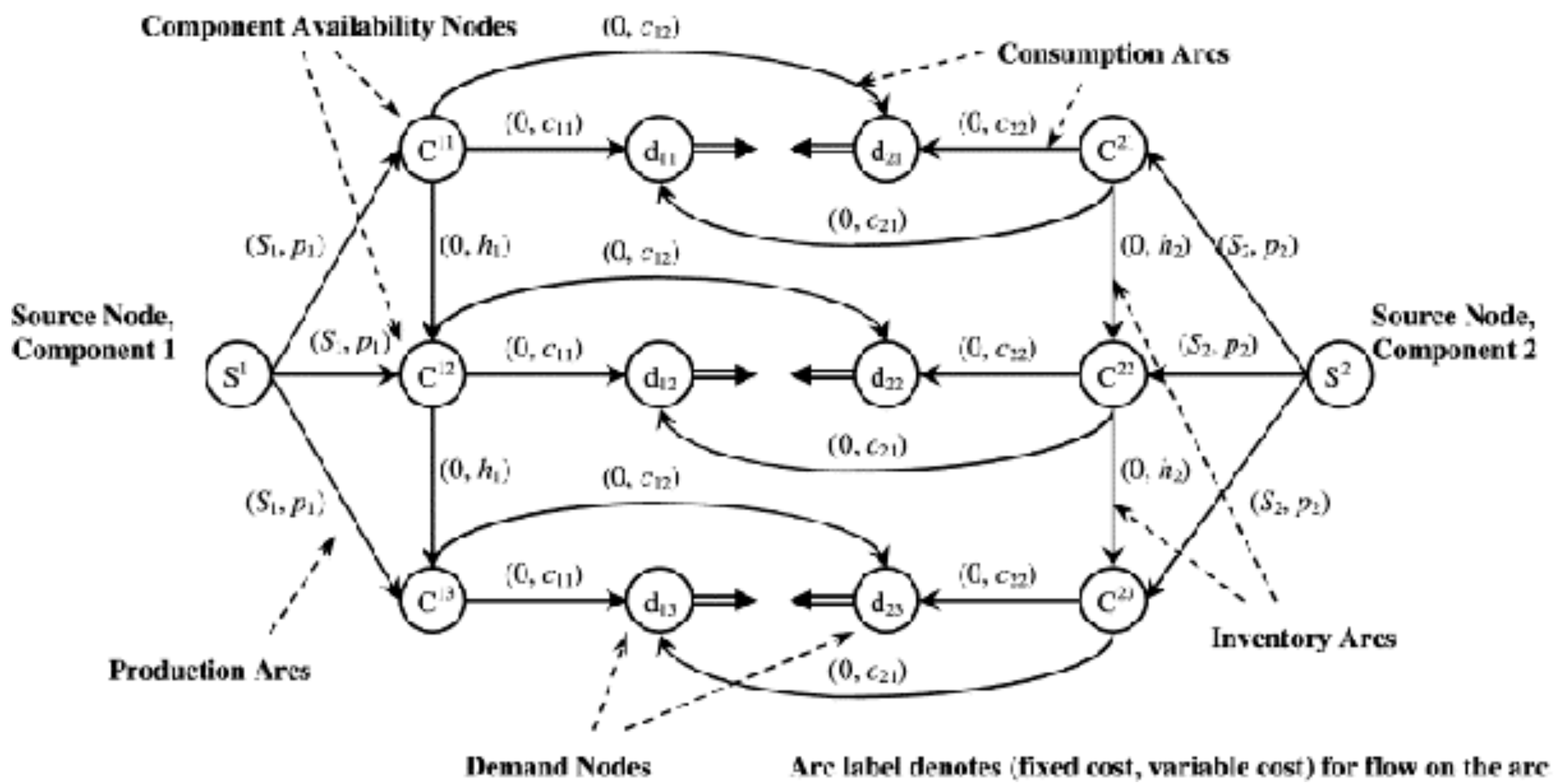
# in this talk...

- **Network Optimization**
  - optimal flow/optimal potential problems
  - monotone/cyclically monotone relations and convex functions

- **Passivity-based Cooperative Control**
  - equilibrium independent passivity
  - steady-state input/output maps

## Duality Theory for Cooperative Control

# Network Optimization




*Network Flows and Monotropic Optimization*

*"...in fact, the great watershed in optimization isn't between linearity and nonlinearity, but convexity and nonconvexity."*

R. Tyrrell Rockafellar
SIAM Review, 1993

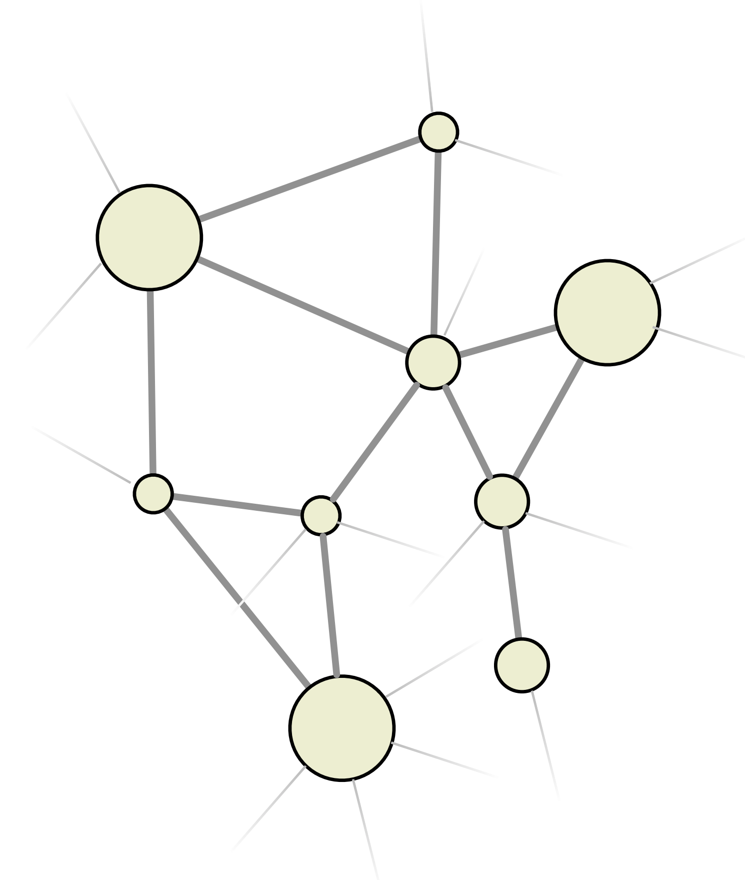
Shortest Path Problem

Max-Flow Problem

Minimum Cost Flow Problem

# Network Definitions

A **network (graph)** is a mathematical structure used to model pairwise relations between objects.



$$\mathcal{G} = (\mathbf{V}, \mathbf{E})$$

## Incidence Matrix

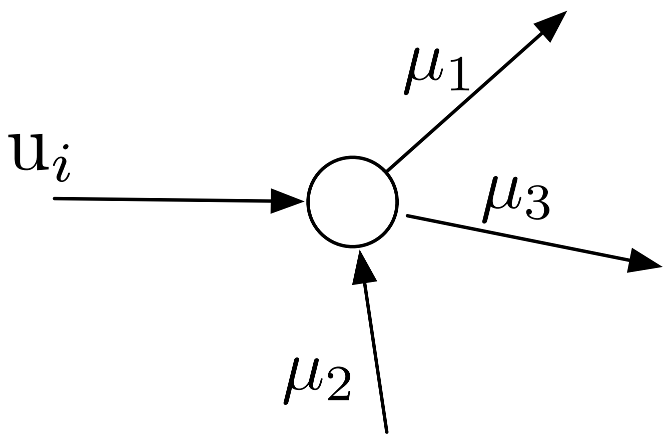
$$E(\mathcal{G}) = \mathbb{R}^{|\mathbf{V}| \times |\mathbf{E}|}$$

$$E(\mathcal{G})^T \mathbf{1} = 0$$

$$[E]_{ik} = \begin{cases} +1 & \text{if } i \text{ is positive end of } k \\ -1 & \text{if } i \text{ is negative end of } k \\ 0 & \text{otherwise} \end{cases}$$

# Network Optimization

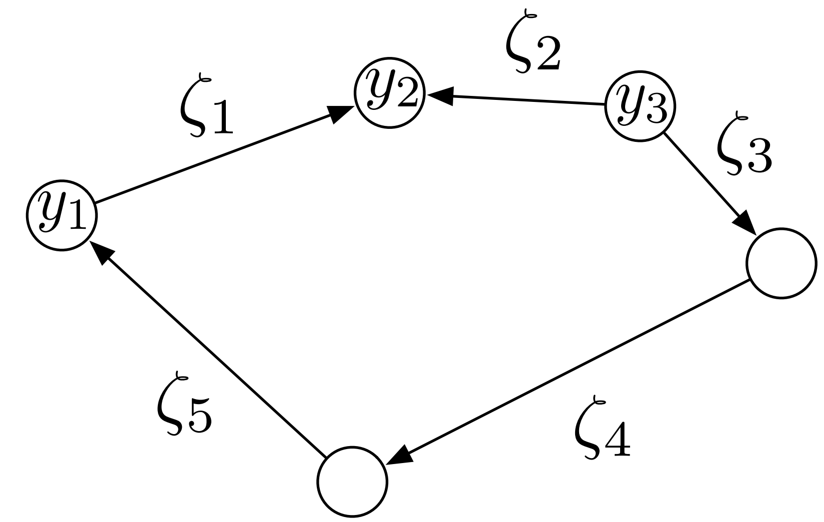
Flow "conservation" laws




$$u + E(\mathcal{G})\mu = 0$$

"flow networks"

"Cycle" laws




$$\zeta = E^T(\mathcal{G})y$$

"potential networks"

*conversion formula*

$$\boldsymbol{\mu}^\top \boldsymbol{\zeta} = -\mathbf{y}^\top \mathbf{u}.$$

# Network Optimization

## *Optimal Flow Problem*

$$\min_{\mathbf{u},\mu} \quad \sum_{i=1}^{|\mathbf{V}|} C_i^{div}(\mathrm{u}_i) + \sum_{k=1}^{|\mathbf{E}|} C_k^{flux}(\mu_k)$$

$$\text{s.t.} \quad \mathbf{u} + E\mu = 0.$$

$\mathrm{u}_i$: ***divergence*** (in/out-flow) at a node

$\mu_k$: ***flow*** on an edge

## *Optimal Potential Problem*

$$\min_{\mathbf{y},\zeta} \quad \sum_{i=1}^{|\mathbf{V}|} C_i^{pot}(\mathrm{y}_i) + \sum_{k=1}^{|\mathbf{E}|} C_k^{ten}(\zeta_k)$$

$$\text{s.t.} \quad \zeta = E^{\top}\mathbf{y}.$$

$\mathrm{y}_i$: ***potential*** at a node

$\zeta_k$: ***tension*** (potential difference) across an edge

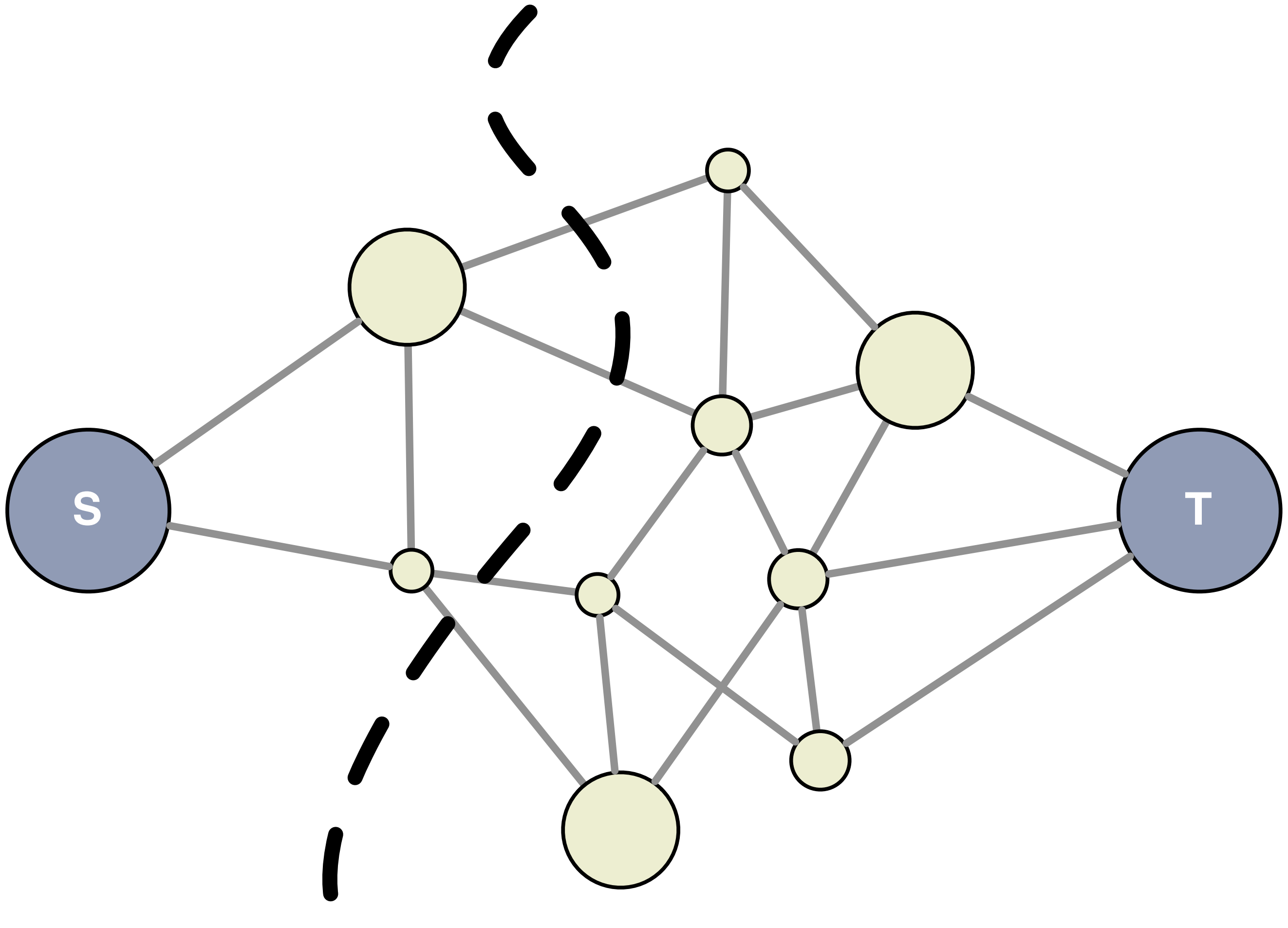
*Dual Optimization Problems*
defined over the "same" network

$$C_i^{pot}(\mathrm{y}_i) := C_i^{div,*} = -\inf_{\tilde{\mathrm{u}}_i} \left\{ C_i^{div}(\tilde{\mathrm{u}}_i) - \mathrm{y}_i \tilde{\mathrm{u}}_i \right\}$$

# Network Optimization

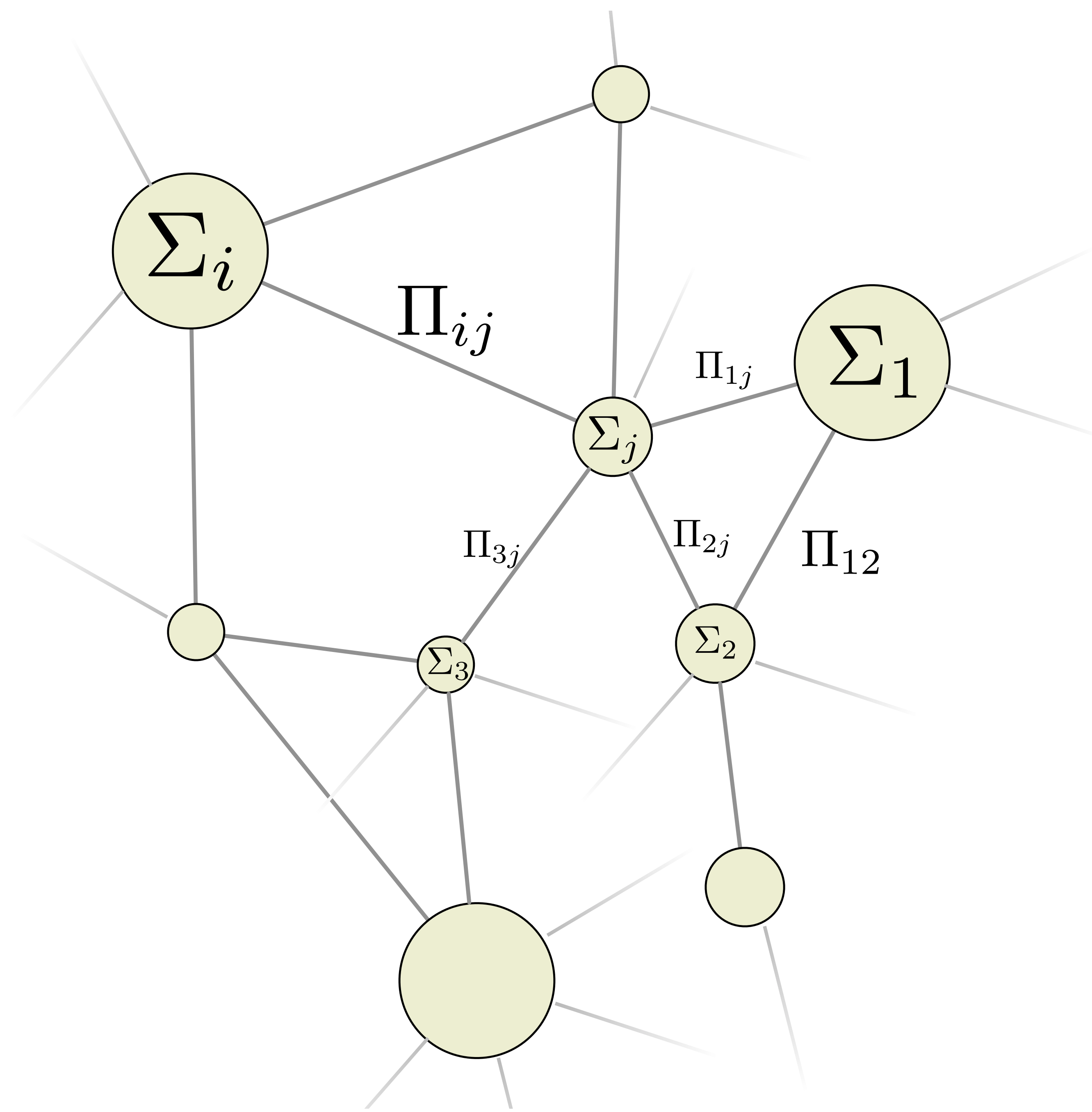
## *Max Flow-Min Cut Theorem*

The maximum value of an S-T flow is equal to the minimum capacity over all s-t cuts.

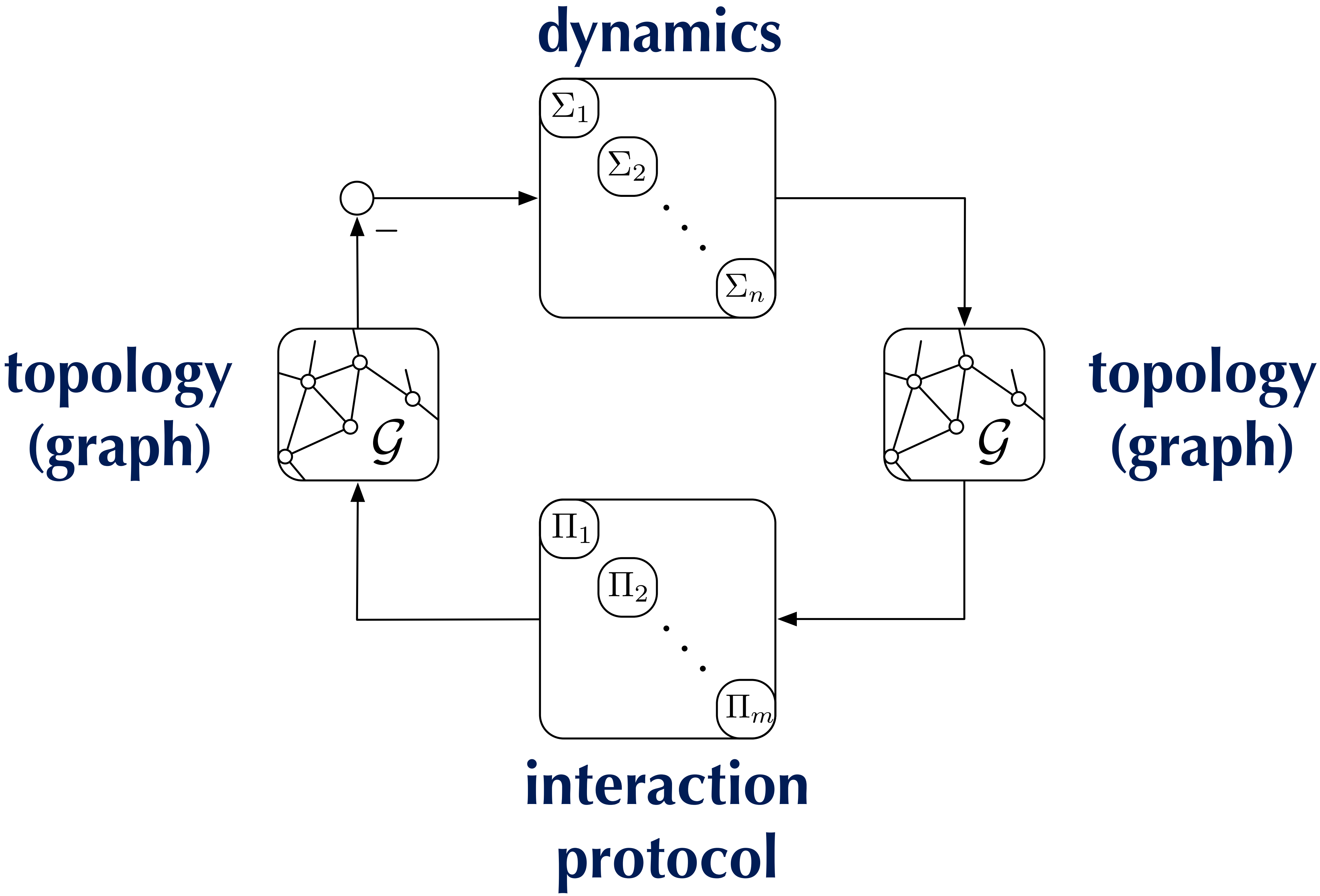



Elegant illustration of *Duality Theory*

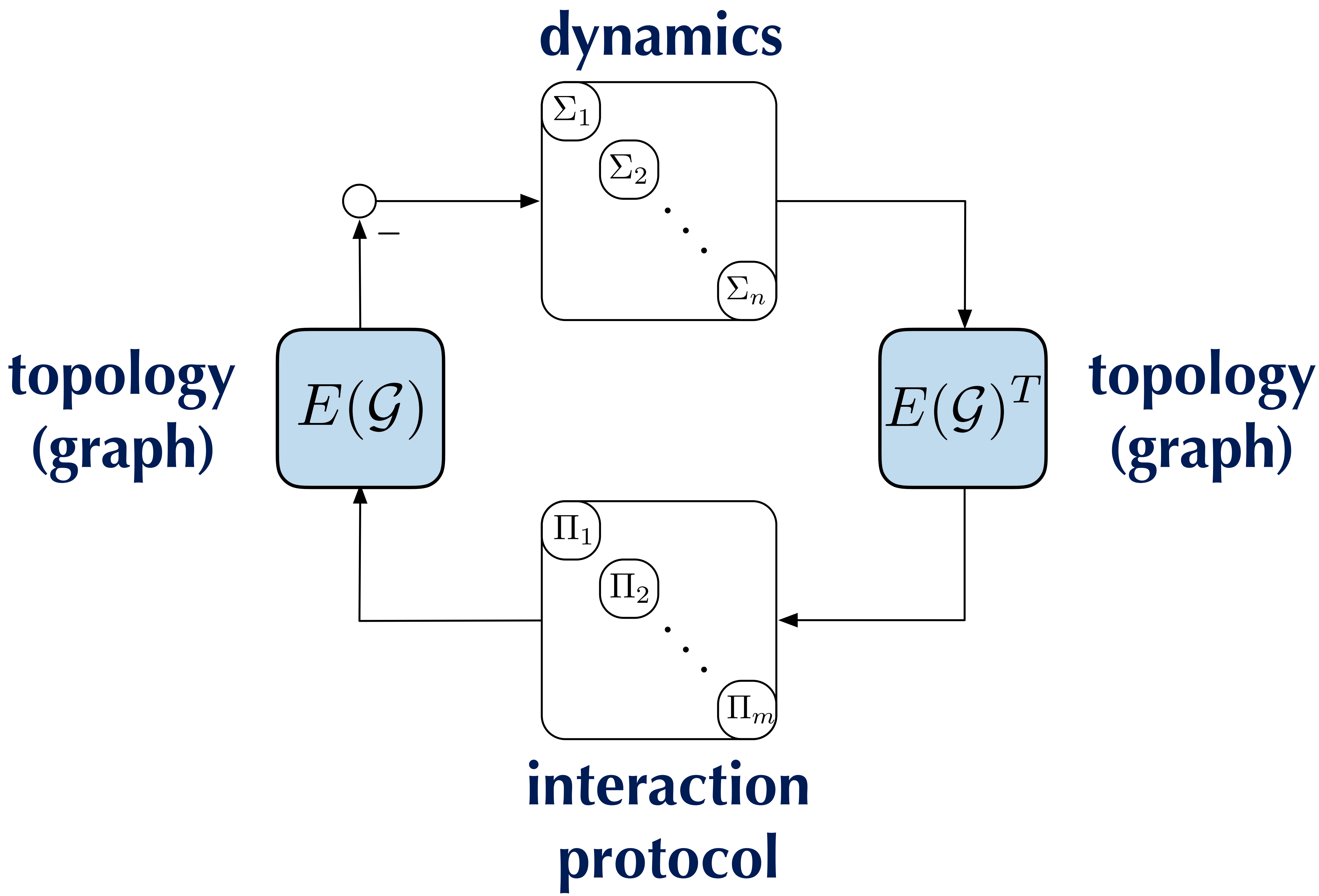
# Networked Dynamic Systems



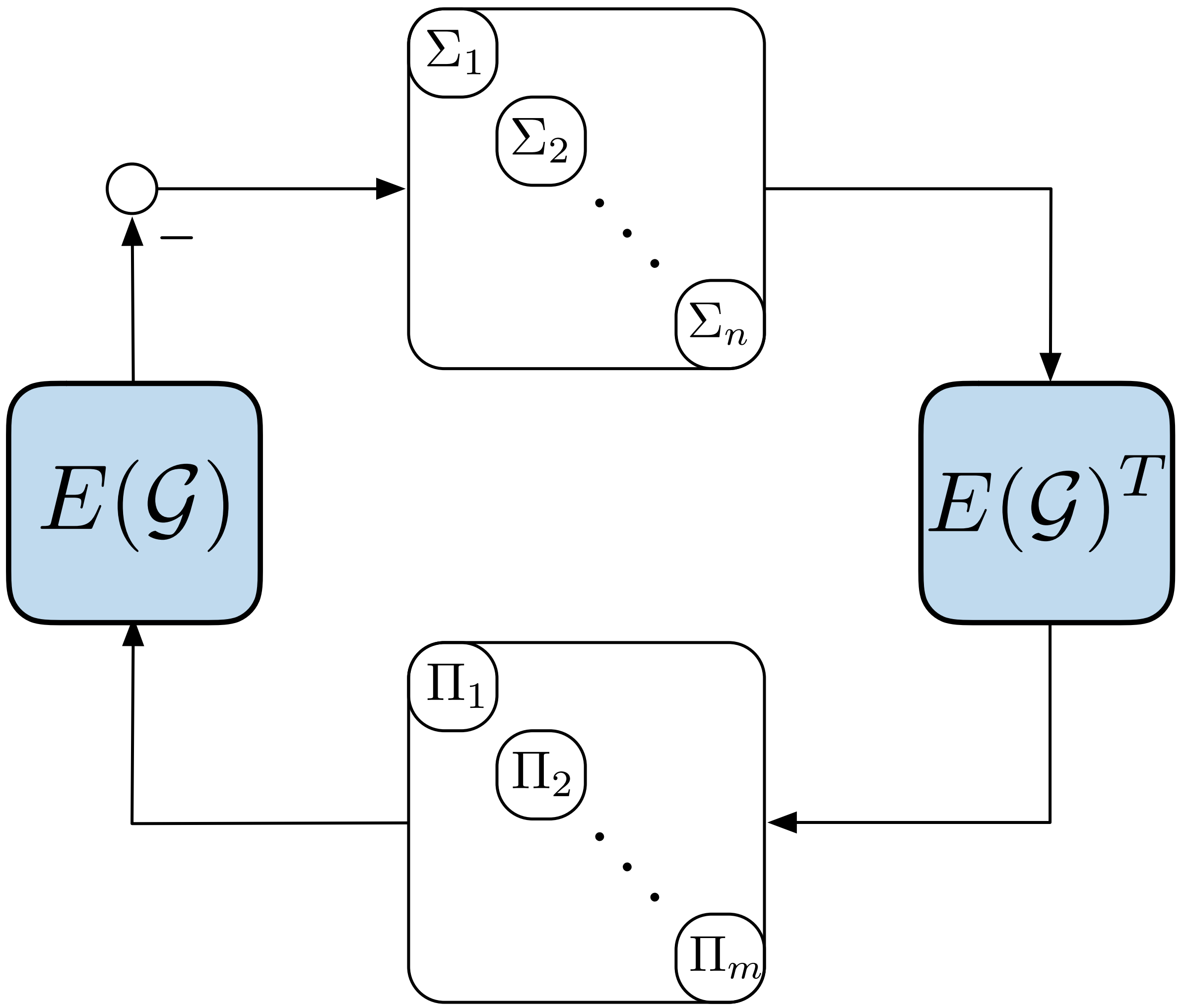
# Networked Dynamic Systems



# Diffusively Coupled Networks



# Diffusively Coupled Networks




## Kumamoto Model

$$\dot{\theta}_i = -k \sum_{i \sim j} \sin(\theta_i - \theta_j)$$

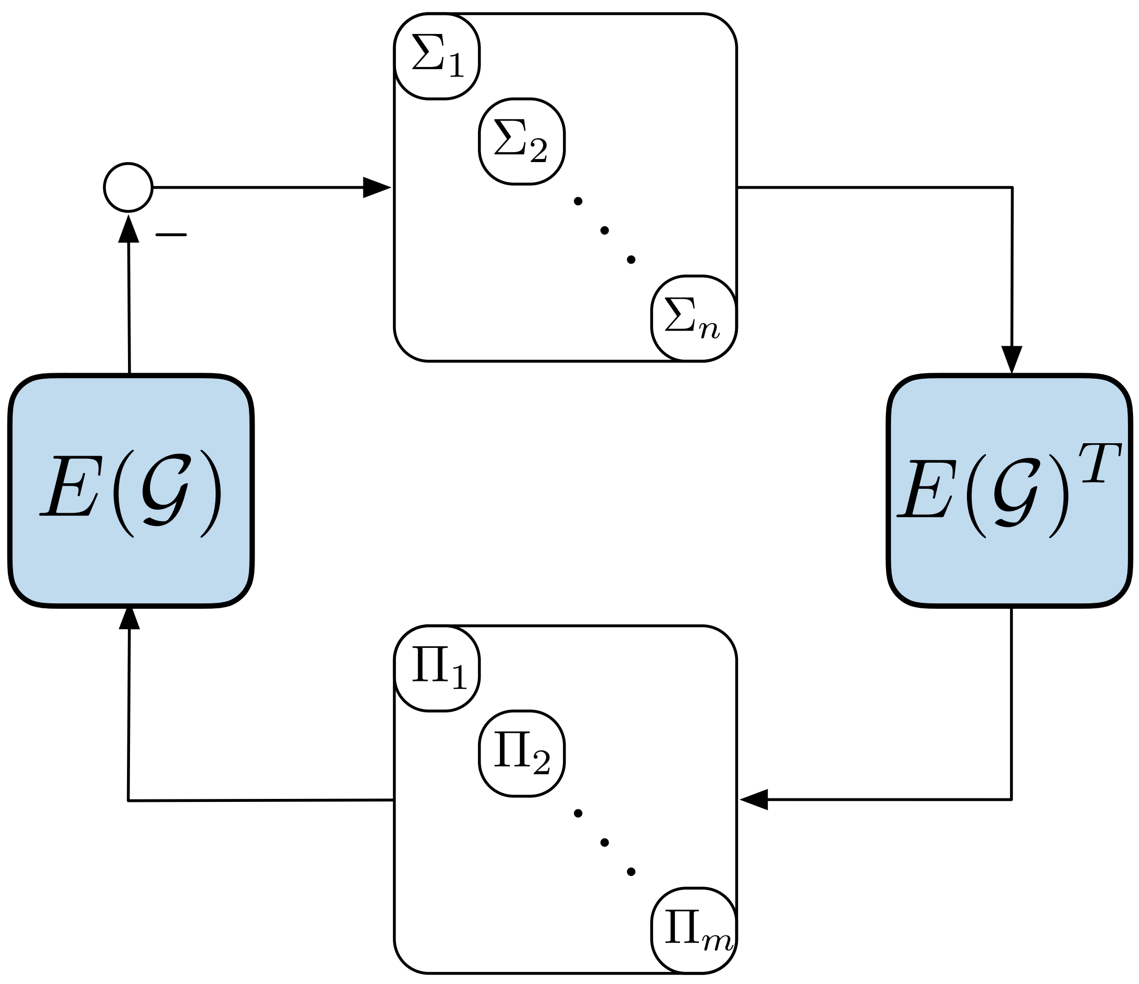
## Traffic Dynamics Model

$$\dot{v}_i = \kappa_i \left( V_i^0 - v_i + V_i^1 \sum_{i \sim j} \tanh(p_j - p_i) \right)$$

## Neural Network

$$\begin{aligned} C\dot{V}_i &= f(V_i, h_i) + \textstyle\sum_{i \sim j} g_{ij}(V_j - V_i) \\ \dot{h}_i &= g(V_i, h_i) \end{aligned}$$

# Duality and Cooperative Control




*Optimal Flow Problem*

$$\min_{\mathbf{u},\mu} \quad \sum_{i=1}^{|\mathbf{V}|} C_i^{div}(\mathrm{u}_i) + \sum_{k=1}^{|\mathbf{E}|} C_k^{flux}(\mu_k)$$

$$\text{s.t.} \quad \mathbf{u} + E\mu = 0.$$

*Optimal Potential Problem*

$$\min_{\mathbf{y},\zeta} \quad \sum_{i=1}^{|\mathbf{V}|} C_i^{pot}(\mathrm{y}_i) + \sum_{k=1}^{|\mathbf{E}|} C_k^{ten}(\zeta_k)$$

$$\text{s.t.} \quad \zeta = E^\top \mathbf{y}.$$

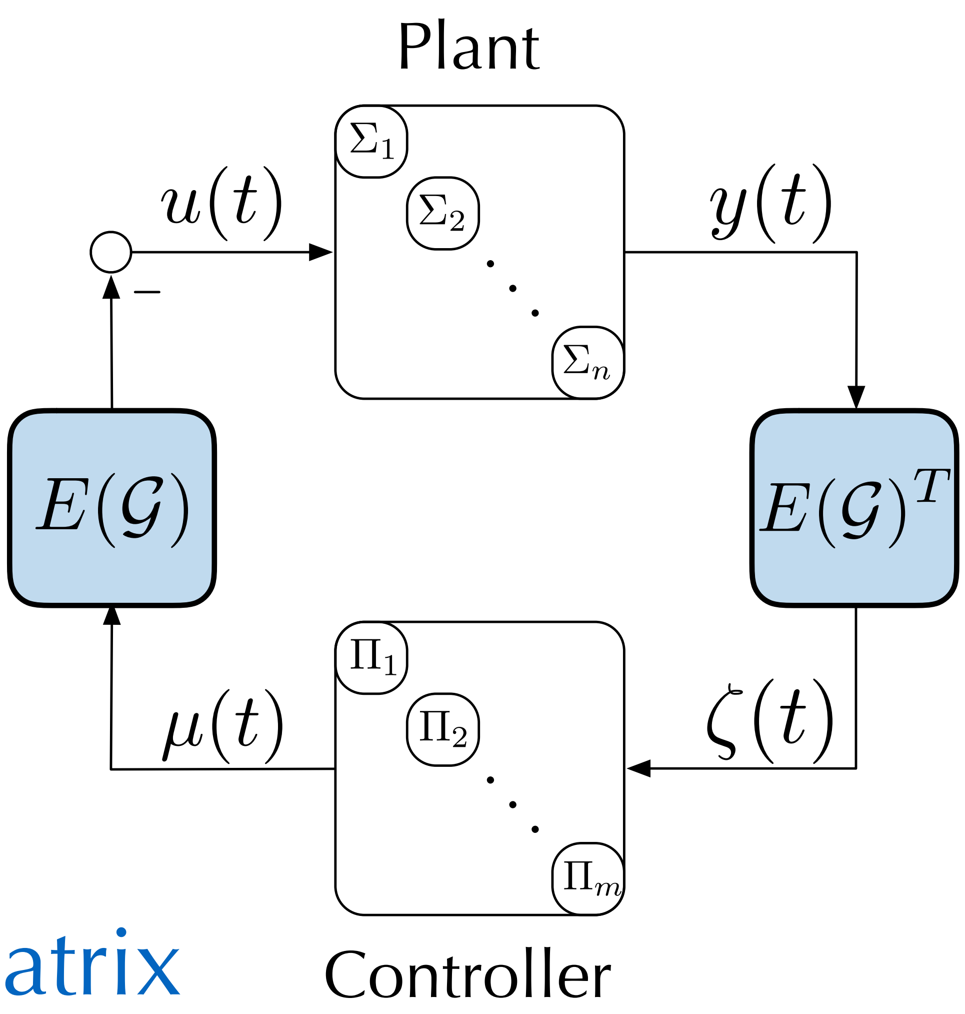
# The Output Agreement Problem

## Plant: Dynamics on nodes

$$\Sigma_i : \quad \begin{aligned} \dot{x}_i(t) &= f_i(x_i(t), u_i(t), \mathrm{w}_i) \\ y_i(t) &= h_i(x_i(t), u_i(t), \mathrm{w}_i) \end{aligned}$$

## Controllers: Dynamics on edges

$$\Pi_k : \quad \begin{aligned} \dot{\eta}_k(t) &= \zeta_k(t) \\ \mu_k(t) &= \psi_k(\eta_k(t), \zeta_k(t)) \end{aligned}$$




## Interconnection via graph incidence matrix

$$\begin{cases} \zeta(t) &= E(\mathcal{G})^T y(t) \\ u(t) &= E(\mathcal{G})\mu(t) \end{cases}$$

**Control Objective**

$$\lim_{t\to\infty} \zeta(t) = 0$$

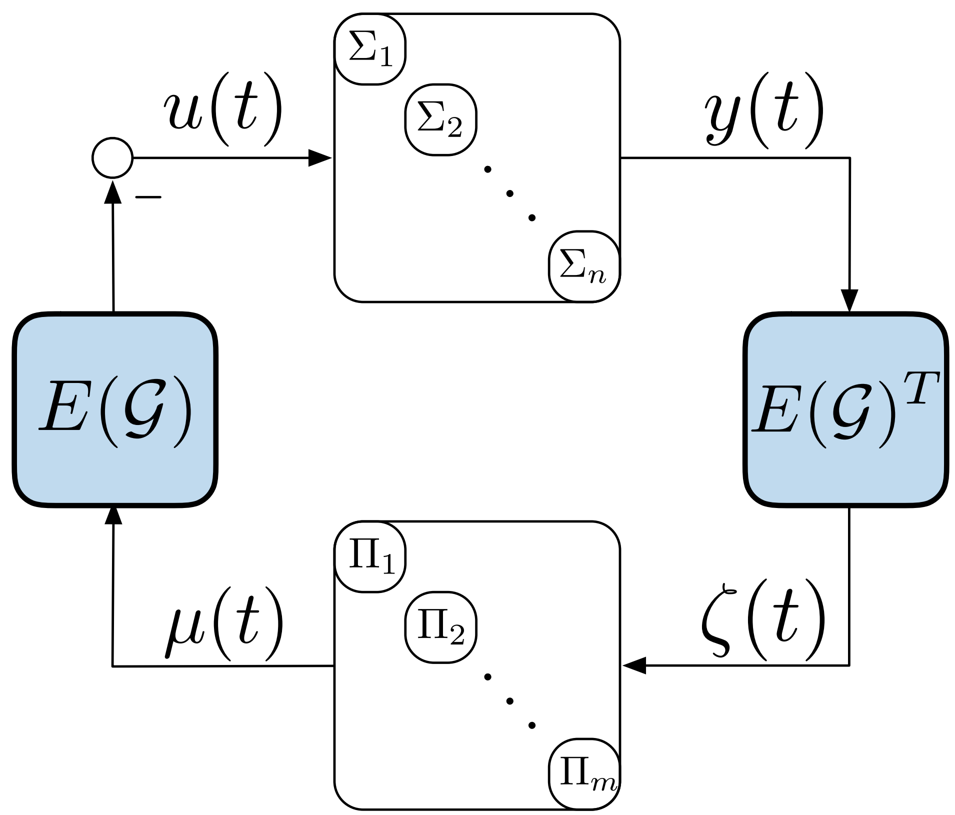
# Necessary Conditions for Output Agreement

## Lemma

If the networked system has a steady-state solution, $\mathbf{u}$, $\mathbf{y}$, then the solution must satisfy

$$\mathbf{u} \in \mathcal{R}(E(\mathcal{G})),\ \mathbf{y} \in \mathcal{N}(E^T(\mathcal{G})) = \text{span}\{\mathbf{1}\}$$

- controller must be able to generate the steady-state input $\mathbf{u}$




- output agreement means output of each agent is identical, i.e., $\mathbf{y} = \beta\mathbf{1}$

# Passivity for Cooperative Control

a "classic" result…

- assume there exists constant signals $\mathbf{u}, \mathbf{y}, \boldsymbol{\mu}, \boldsymbol{\zeta}$ s.t. $\mathbf{u} = -E\boldsymbol{\mu}, , \boldsymbol{\zeta} = E^T\mathbf{y}$
- each dynamic system is output strictly passive with respect to $\mathrm{u}_i, \mathrm{y}_i$

$$\frac{d}{dt}S_i(x_i(t)) \leq (y_i(t) - \mathrm{y}_i)(u_i(t) - \mathrm{u}_i) - \rho_i\|y_i(t) - \mathrm{y}_i\|^2$$

- each controller is passive with respect to $\upzeta_k, \upmu_k$

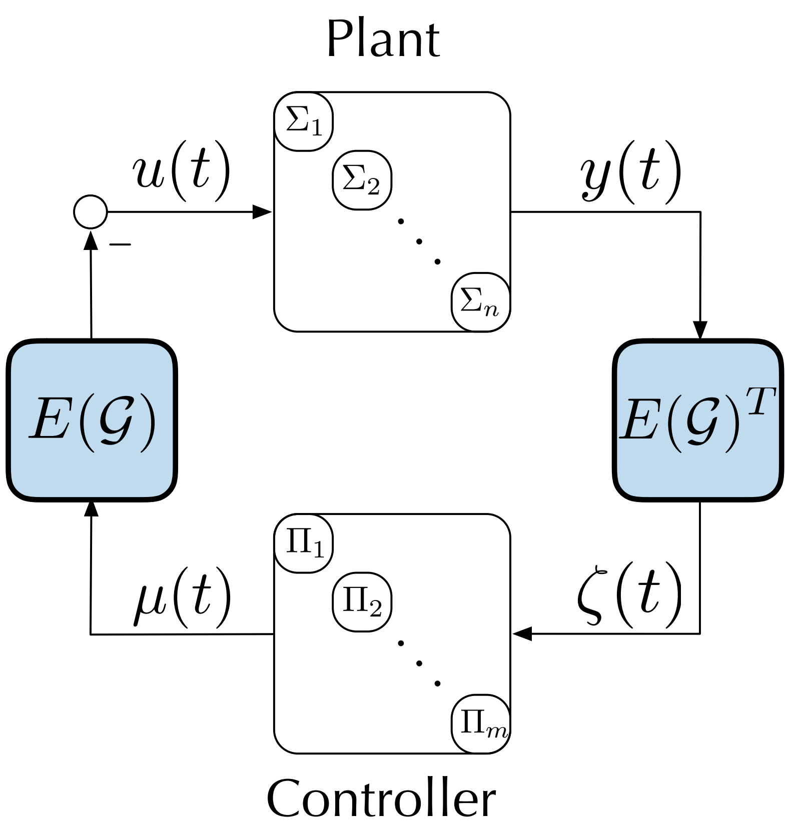
$$\frac{d}{dt}W_k(\eta_k(t)) \leq (\mu_k(t) - \upmu_k)(\zeta_k(t) - \upzeta_k)$$

## Theorem [Arcak 2007]

Suppose the above assumptions are satisfied. Then the network output converges to the constant value **y**, i.e,

$$\lim_{t\to\infty} y(t) = \mathbf{y}$$

# Passivity Shortcomings




a critical assumption is the existence of constant signals

$$\mathbf{u} = -E\mu,\ \zeta = E^T\mathbf{y}$$

→ equilibrium depends on all properties "globally"

→ can not be verified "locally"

# Equilibrium Independent Passivity

## Definition [Hines et. al. Automatica 2011]

A control system

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x,u) \end{aligned}$$

is *equilibrium independent passive* (EIP) if

i) $\exists$ a set $\mathcal{U}$ and *function* $k_x(u)$ s.t. $f(k_x(\mathrm{u}),\mathrm{u}) = 0 \; \forall \, \mathrm{u} \in \mathcal{U}$

ii) the system is passive with respect to the equilibrium input-output pair $\mathrm{u}, \; \mathrm{y} = h(k_x(\mathrm{u}),\mathrm{u})$

# Equilibrium Independent Passivity

**Lemma** [Hines et. al. Automatica 2011]

If $\Sigma$ is EIP, then $k_y(\mathrm{u})$ is monotonically increasing.

Equilibrium input-output maps are *monotone functions!*

# Equilibrium Independent Passivity

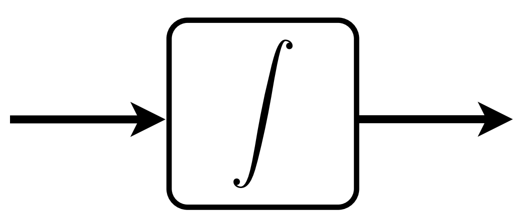
**Lemma** [Hines et. al. Automatica 2011]

If $\Sigma$ is EIP, then $k_y(\mathrm{u})$ is monotonically increasing.

Equilibrium input-output maps are *monotone functions!*

but…

the integrator is *passive* w.r.t. $\mathcal{U} = \{0\}$ and any output $\mathrm{y} \in \mathbb{R}$

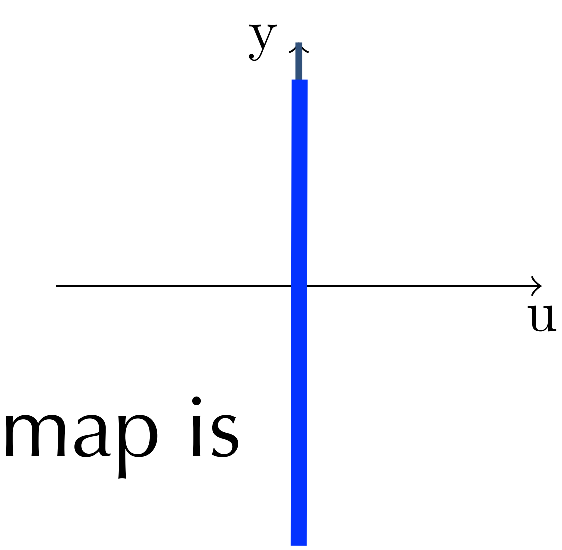


storage function

$$S(x(t)) = \frac{1}{2}\left(x(t) - \mathrm{y}\right)^2$$

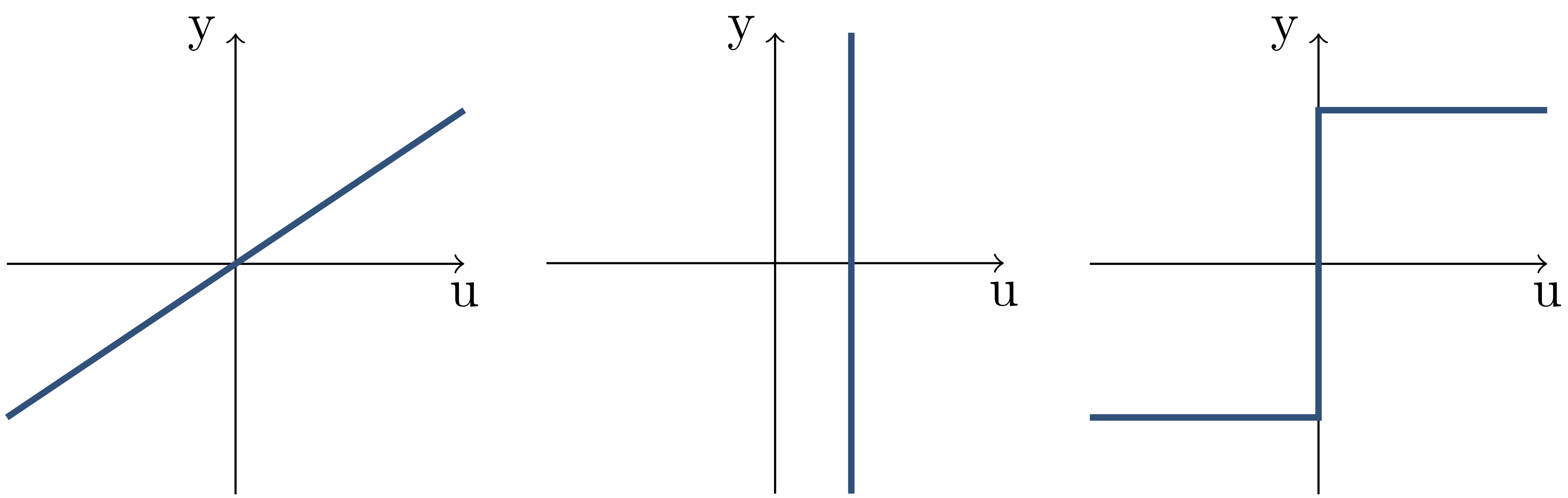
$$\dot{x}(t) = u(t)$$
$$y(t) = x(t)$$

Equilibrium input-output map is *not a function*!



# Monotone Relations

Maximal Monotone Relations - complete non-decreasing curves in $\mathbb{R}^2$




a relation is *maximal monotone* if it cannot be embedded into a larger monotone relation

$$k_y \text{ is maximal monotone} \Leftrightarrow \begin{array}{ll} (i) & \text{for arbitrary } (\mathrm{u},\mathrm{y}) \in k_y \text{ and } (\mathrm{u}',\mathrm{y}') \in k_y \\ & \text{either } (\mathrm{u},\mathrm{y}) \leq (\mathrm{u}',\mathrm{y}') \text{ or } (\mathrm{u},\mathrm{y}) \geq (\mathrm{u}',\mathrm{y}') \\ (ii) & \text{for arbitrary } (\mathrm{u},\mathrm{y}) \notin k_y \; \exists (\mathrm{u}',\mathrm{y}') \in k_y \\ & \text{s.t. neither } (\mathrm{u},\mathrm{y}) \leq (\mathrm{u}',\mathrm{y}') \text{ nor } (\mathrm{u},\mathrm{y}) \geq (\mathrm{u}',\mathrm{y}') \end{array}$$

# Maximal EIP

## Definition

The dynamical SISO system

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), \mathrm{w}) \\ y(t) &= h(x(t), u(t), \mathrm{w}) \end{aligned}$$

is *maximal equilibrium independent passive* if there exists a maximal monotone relation $k_y \subset \mathbb{R}^2$ such that for all $(\mathrm{u}, \mathrm{y}) \in k_y$ there exists a positive semi-definite storage function $S(x(t))$ satisfying

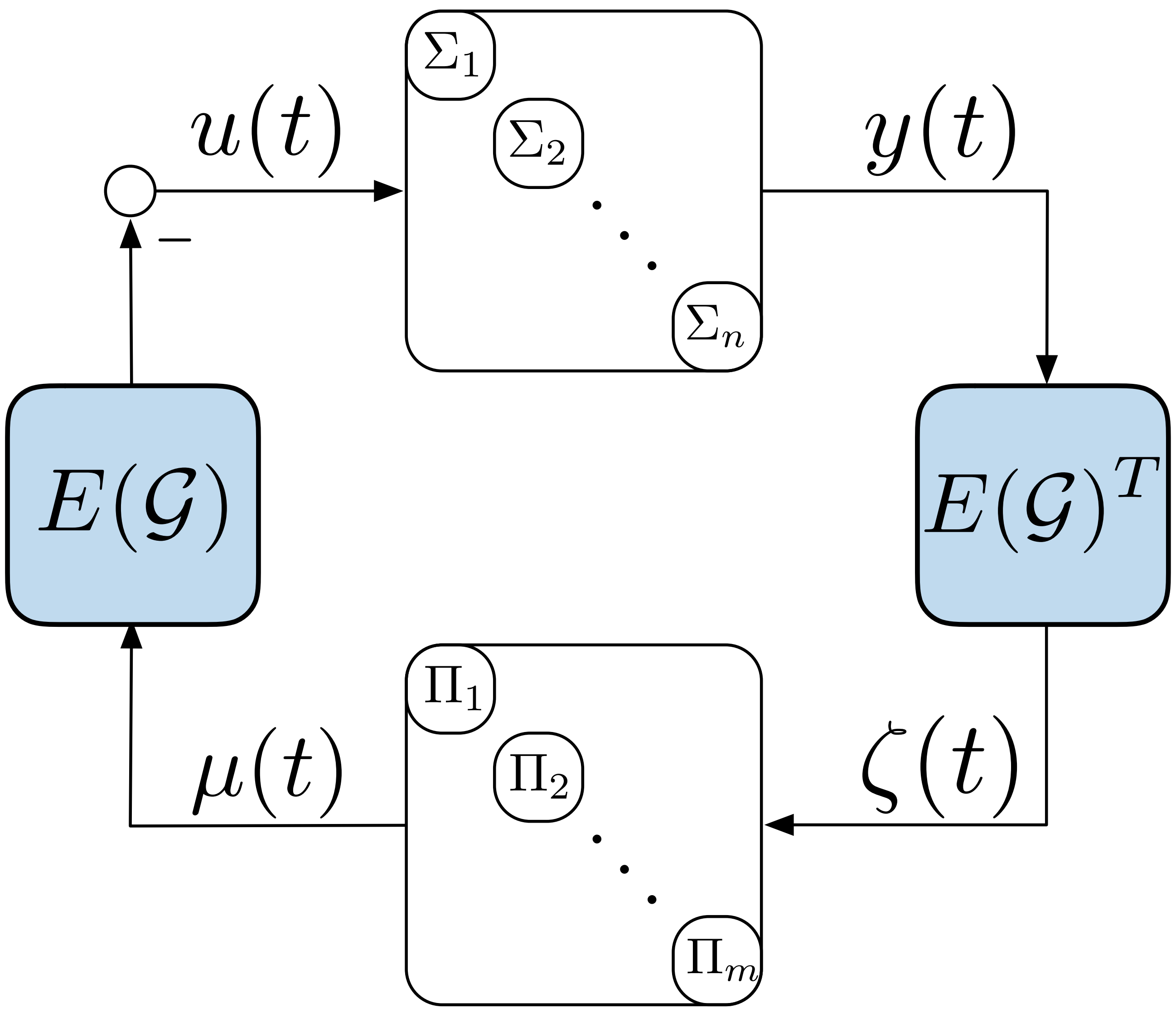
$$\frac{d}{dt} S(x(t)) \leq (y(t) - \mathrm{y})(u(t) - \mathrm{u}).$$

Furthermore, it is *output-strictly maximal equilibrium independent passive* if additionally there is a constant $\rho > 0$ such that

$$\frac{d}{dt} S(x(t)) \leq (y(t) - \mathrm{y})(u(t) - \mathrm{u}) - \rho \| y(t) - \mathrm{y} \|^2.$$

# Maximal EIP

output strictly maximal EIP




maximal EIP

# Necessary Conditions (revisited)

**Lemma**

If the networked system has a steady-state solution, $\mathbf{u}$, $\mathbf{y}$, then the solution must satisfy

$$\mathbf{u} \in \mathcal{R}(E(\mathcal{G})), \ \mathbf{y} \in \mathcal{N}(E^T(\mathcal{G})) = \text{span}\,\{\mathbf{1}\}$$

and

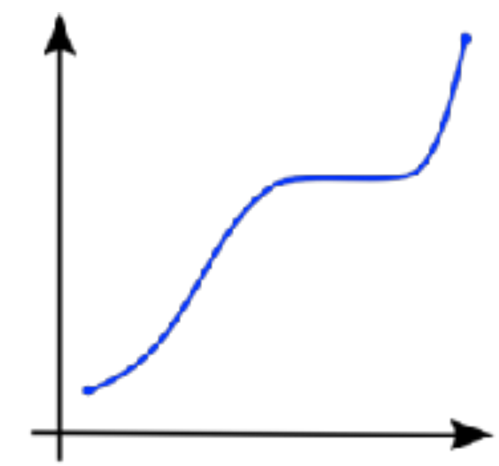
$$\mathbf{y} \in k_y(\mathbf{u})$$

A "network feasibility problem"

# Monotone Relations and Convex Functions

**Theorem** [Rockafellar, *Convex Analysis*]

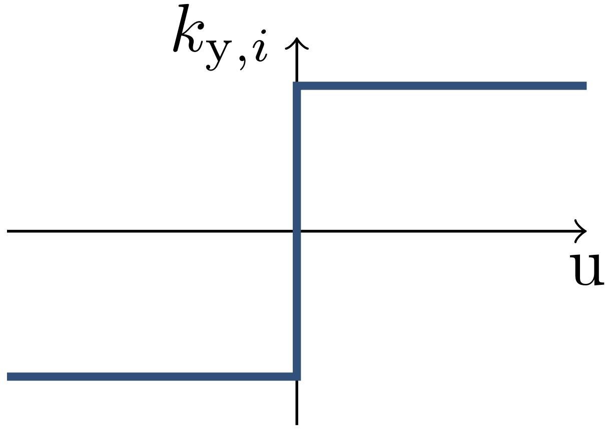
The sub-differential for the closed proper convex functions on $\mathbb{R}$ are the maximal monotone relations from $\mathbb{R}$ to $\mathbb{R}$ .

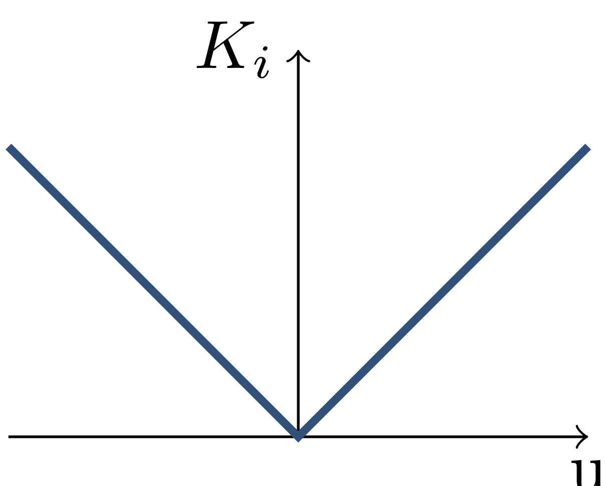


$$k_y(\mathrm{u}) = h(k_x(\mathrm{u}), \mathrm{u}))$$

*integral function* of equilibrium i/o map

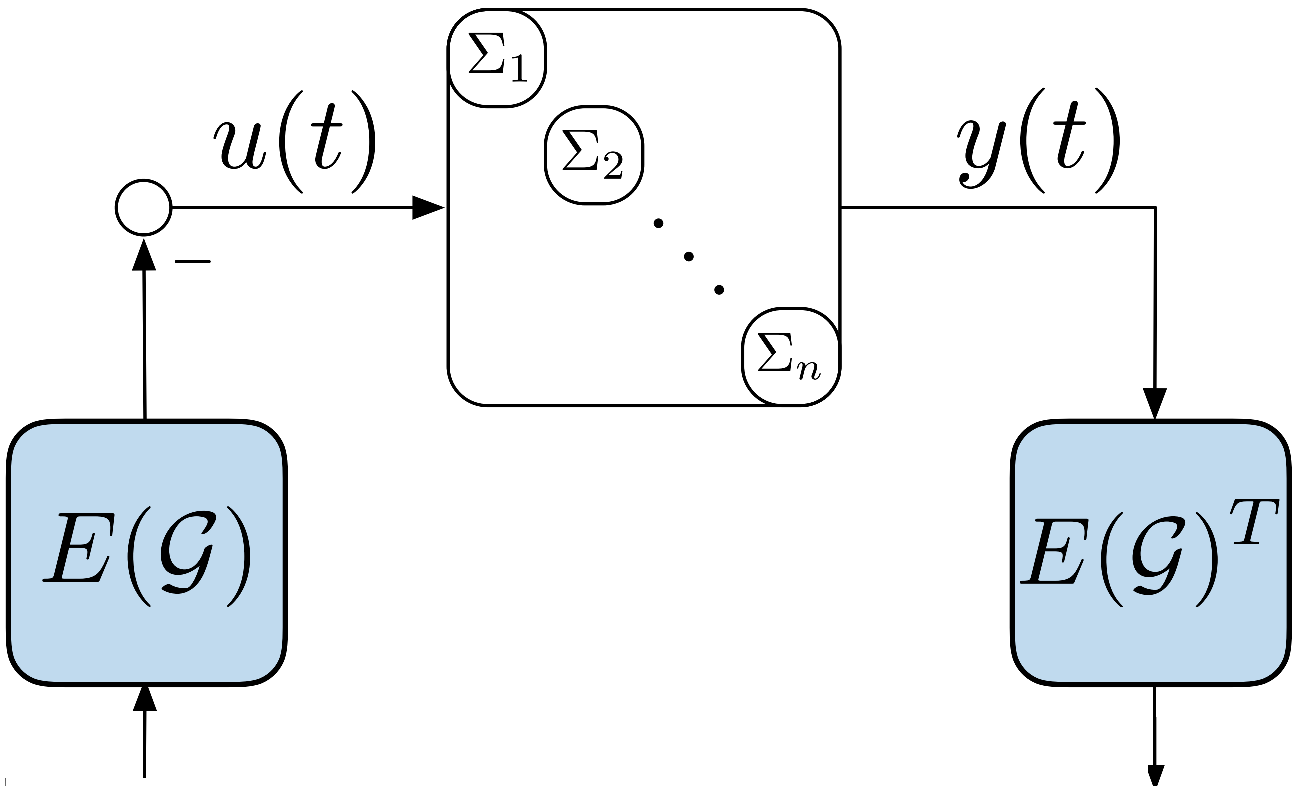
$$\partial K_i(\mathrm{u}_i) = k_{\mathrm{y},i}(\mathrm{u}_i)$$




proper, closed convex function

# Duality in Cooperative Control




* Suppose systems are output strictly maximal EIP
* Equilibrium Input-Output Map

$$k_y \subset \mathbb{R} \times \mathbb{R}$$

$$\mathrm{y} \in k_y(\mathrm{u})$$

$$\mathbf{u} \in \mathcal{R}(E(\mathcal{G})),\ \mathbf{y} \in \mathcal{N}(E^T(\mathcal{G})) = \mathrm{span}\{\mathbf{1}\}$$

$$\mathbf{u}^T\mathbf{y} = 0$$

integral function of Equilibrium I/O Maps are *Convex!*

$$\partial K_i(\mathrm{u}_i) = k_{\mathrm{y},i}(\mathrm{u}_i)$$

# Duality in Cooperative Control

*Optimal Flow Problem*

*(OFP1)*

$$\min_{\mathbf{u},\mu} \sum_{i=1}^{|\mathbf{V}|} K_i(\mathrm{u}_i) \quad (= \mathbf{K}(\mathrm{u}))$$
$$\text{s.t.} \quad \mathbf{u} + E\mu = 0.$$

*Optimal Potential Problem*

*(OPP1)*

$$\min_{\mathrm{y}_i} \sum_{i=1}^{|\mathbf{V}|} K_i^\star(\mathrm{y}_i) \quad (= \mathbf{K}^*(\mathrm{y}))$$
$$\text{s.t.} \quad E^\top \mathbf{y} = 0.$$

$$K_i^*(\mathrm{y}_i) = \sup_{\mathrm{u}_i}\{\mathrm{y}_i \mathrm{u}_i - K_i(\mathrm{u}_i)\}$$

# Duality in Cooperative Control

*Optimal Flow Problem*

*(OFP1)*

$$\min_{\mathbf{u},\mu} \sum_{i=1}^{|\mathbf{V}|} K_i(\mathrm{u}_i)$$
$$\text{s.t.} \quad \mathbf{u} + E\mu = 0.$$

*Optimal Potential Problem*

*(OPP1)*

$$\min_{\mathrm{y}_i} \sum_{i=1}^{|\mathbf{V}|} K_i^\star(\mathrm{y}_i)$$
$$\text{s.t.} \quad E^\top \mathbf{y} = 0.$$

**Theorem**

Assume all the node dynamics are maximal EIP. If the networked system has a steady-state solution $\mathbf{u}, \mathbf{y}$, then

1) $\mathbf{u}$ is an optimal solution of OFP1,
2) $\mathbf{y}$ is an optimal solution to OPP1,
3) $\sum_{i=1}^{|\mathcal{V}|} K_i(\mathrm{u}_i) + \sum_{i=1}^{|\mathcal{V}|} K_i^\star(\mathrm{y}_i) = 0$

# Duality in Cooperative Control

*Optimal Flow Problem*

*(OFP1)*

$$\min_{\mathbf{u},\mu} \sum_{i=1}^{|\mathbf{V}|} K_i(\mathrm{u}_i)$$
$$\text{s.t.} \quad \mathbf{u} + E\mu = 0.$$

*Optimal Potential Problem*

*(OPP1)*

$$\min_{\mathrm{y}_i} \sum_{i=1}^{|\mathbf{V}|} K_i^\star(\mathrm{y}_i)$$
$$\text{s.t.} \quad E^\top \mathbf{y} = 0.$$

$\partial K_i(\mathrm{u}_i) = k_{\mathrm{y},i}(\mathrm{u}_i)$ maximal (strongly) monotone relations

$\Rightarrow$ OFP1 is feasible and strictly convex

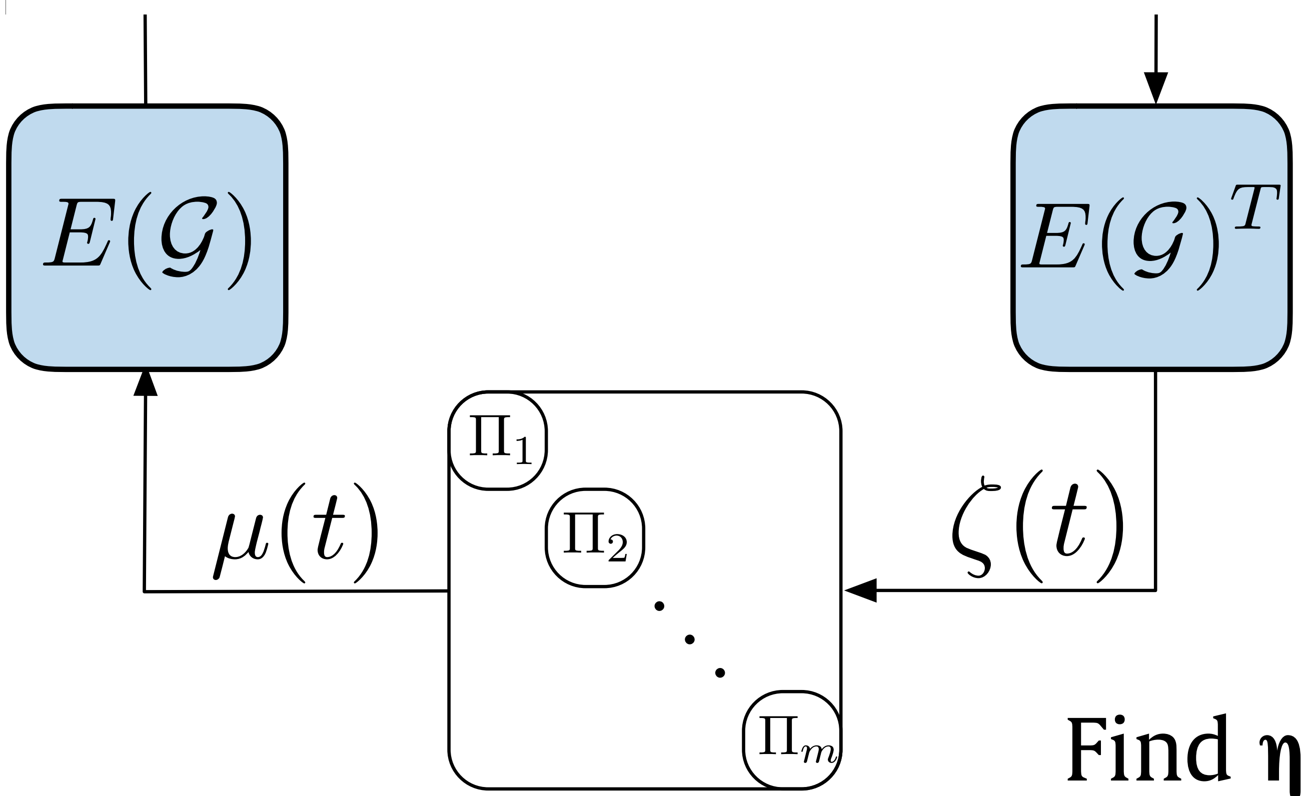
$\Rightarrow$ by strong duality, only one solution to OPP1 exists

$\Rightarrow$ exactly one output agreement solution exists

# Duality in Cooperative Control

How do the controls generate the correct inputs?

$$\Pi_k : \quad \begin{aligned} \dot{\eta}_k(t) &= \zeta_k(t) \\ \mu_k(t) &= \psi_k(\eta_k(t), \zeta_k(t)) \end{aligned}$$



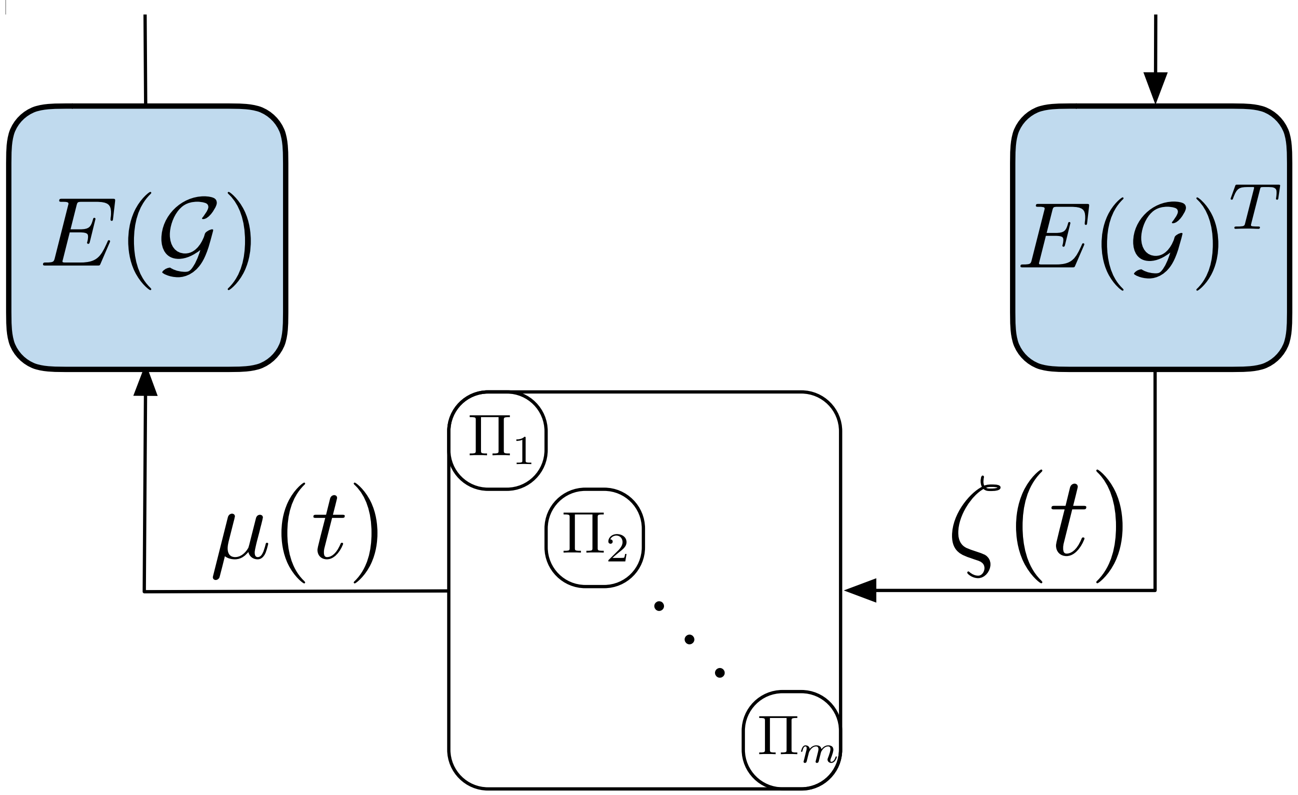

Find $\boldsymbol{\eta} \in \mathcal{R}(E^\top)$

s.t. $\quad \mathbf{u} = -E\boldsymbol{\psi}(\boldsymbol{\eta})$.

# Duality in Cooperative Control

How do the controls generate the correct inputs?

$$\Pi_k : \quad \begin{aligned} \dot{\eta}_k(t) &= \zeta_k(t) \\ \mu_k(t) &= \psi_k(\eta_k(t), \zeta_k(t)) \end{aligned}$$ assume strongly monotone




Let $P_k(\eta)$ be such that $\nabla P_k = \psi_k$.

# Duality in Cooperative Control

*Optimal Potential Problem*

*(OPP2)*

$$\min_{\eta,\mathbf{v}} \quad \sum_{k=1}^{|\mathbf{E}|} P_k(\eta_k) + \sum_{i=1}^{|\mathbf{V}|} \mathrm{u}_i \mathrm{v}_i$$
$$\text{s.t.} \quad \eta = E^\top \mathbf{v}.$$

*Optimal Flow Problem*

*(OFP2)*

$$\min_{\mu} \sum_{k=1}^{|\mathbf{E}|} P_k^\star(\mu_k)$$
$$\text{s.t. } \mathbf{u} + E\mu = 0,$$

## Theorem

Assume all the node dynamics are maximal EIP. and the controller dynamics are such that all output maps $\psi_k$ are strongly monotone and define $P_k$ such that $\nabla P_k(\eta_k) = \psi_k(\eta_k)$. Then the networked system has an output agreement steady-state solution. Furthermore, let $\eta$ be the state-state of the controller in output agreement, then

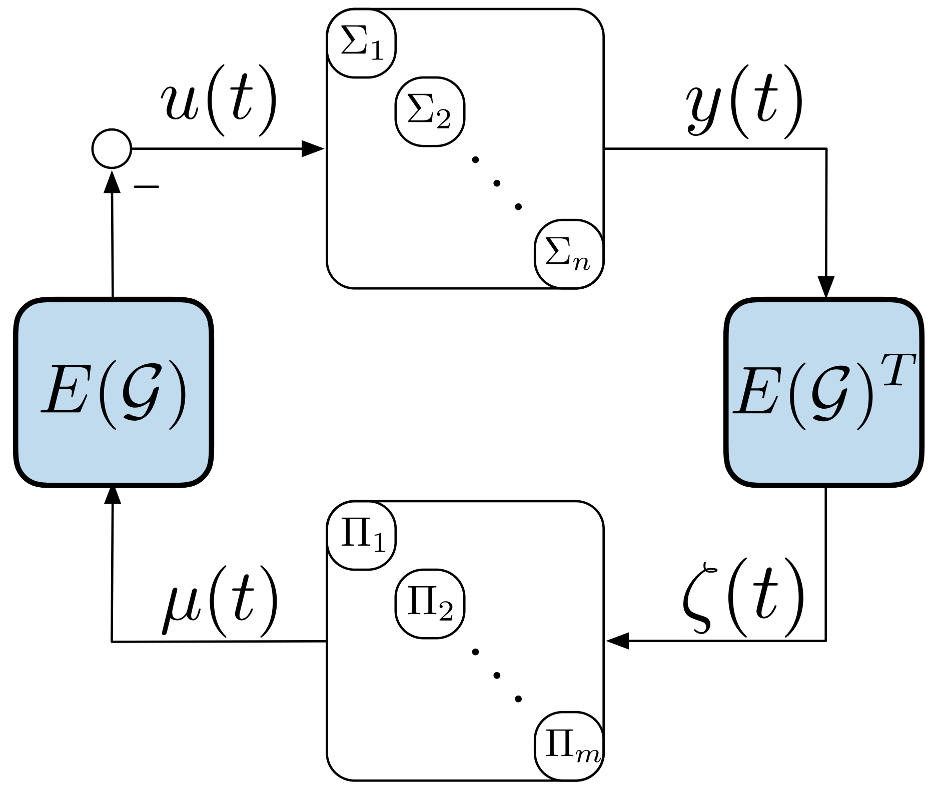
1) $\eta$ is an optimal solution of OPP2,

2) $\mu = \psi(\eta)$ is an optimal solution to OFP2,

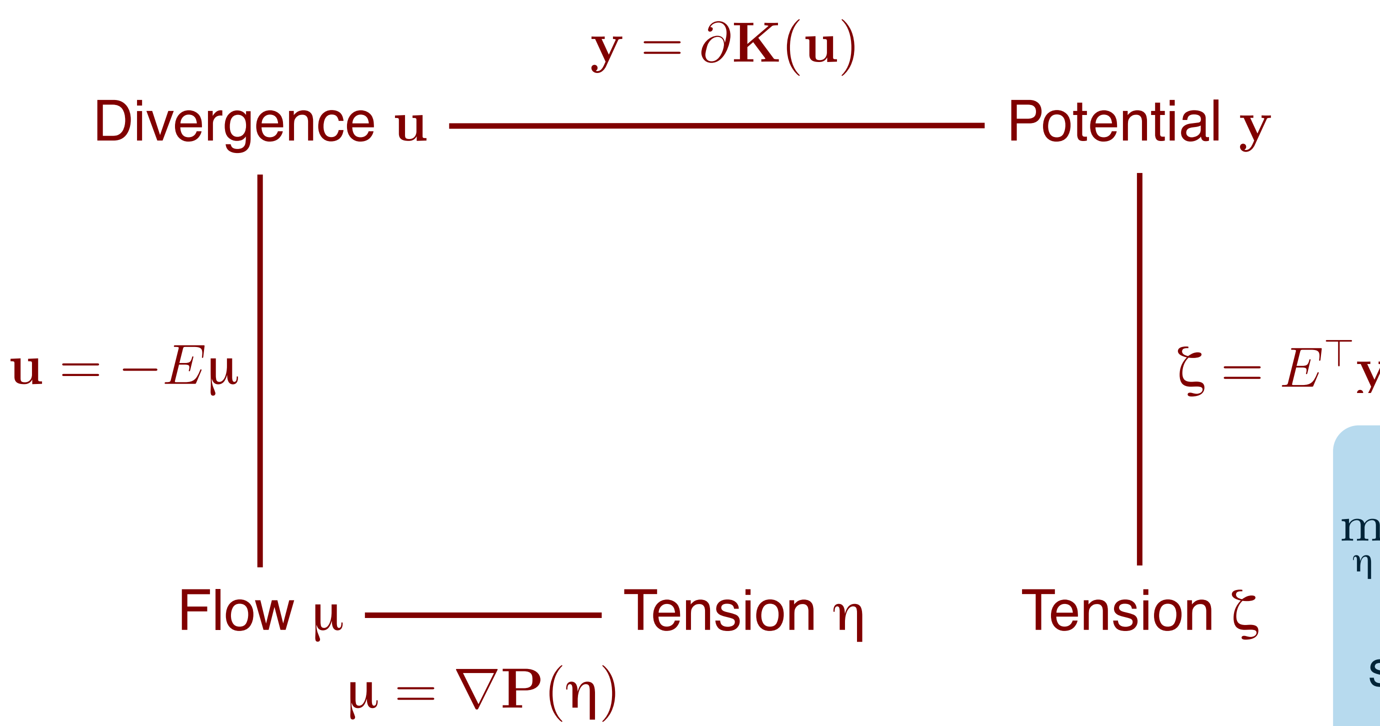
3) $\sum_{k=1}^{|\mathcal{E}|} P_k^\star(\mu_k) + \sum_{k=1}^{|\mathcal{E}|} P_k(\eta_k) = \mu^T \eta$

# The Closed Loop

$$\min_{\mathbf{u},\mu} \sum_{i=1}^{|\mathbf{V}|} K_i(\mathrm{u}_i) \quad \text{s.t.} \quad \mathbf{u} + E\mu = 0.$$




$$\min_{\mathrm{y}_i} \sum_{i=1}^{|\mathbf{V}|} K_i^\star(\mathrm{y}_i), \quad \text{s.t.} \quad E^\top \mathbf{y} = 0.$$




$$\min_{\mu} \sum_{k=1}^{|\mathbf{E}|} P_k^\star(\mu_k) \quad \text{s.t.} \ \mathbf{u} + E\mu = 0,$$

$$\min_{\eta,\mathbf{v}} \sum_{k=1}^{|\mathbf{E}|} P_k(\eta_k) - \sum_{i=1}^{|\mathbf{V}|} \mathrm{u}_i \mathrm{v}_i, \quad \text{s.t.} \quad \eta = E^\top \mathbf{v}.$$

# From SISO to MIMO - Cyclic Monotonicity

## Definition

Consider $R \subset \mathbb{R}^n \times \mathbb{R}^n$. The relation $R$ is *Cyclicly Monotone* if for any $N \geq 1$ and any pairs $(u_i, y_i) \in R$, $i = 1, \ldots, N$,

$$\sum_{i=1}^{N} y_i^T (u_i - u_{i-1}) \geq 0.$$

## Theorem [Rockafellar, 1966]

A relation $R \subset \mathbb{R}^n \times \mathbb{R}^n$ is cyclicly monotone if and only if it is contained in the subgradient of a convex function $\psi : \mathbb{R}^n \to \mathbb{R}$.

# An Example - Vehicle Platooning

## Microscopic Traffic Model

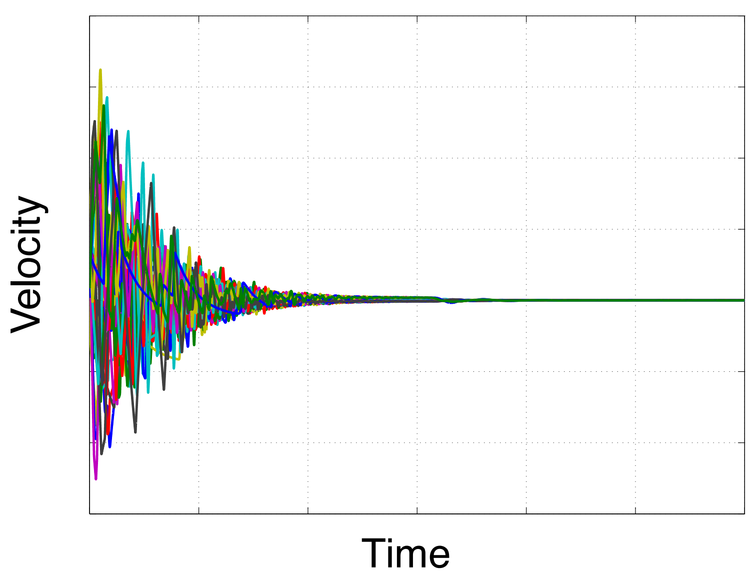
[Helbing *et al.*, 2001]

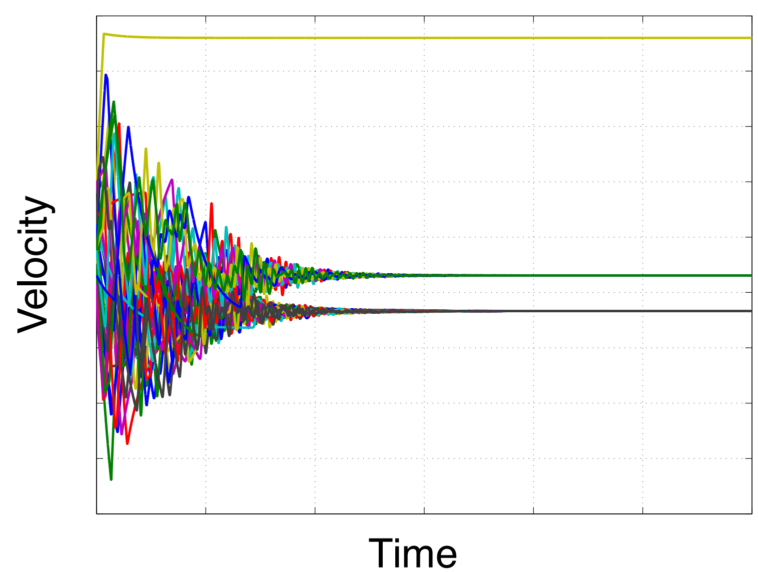
$$\dot{p}_i = v_i$$

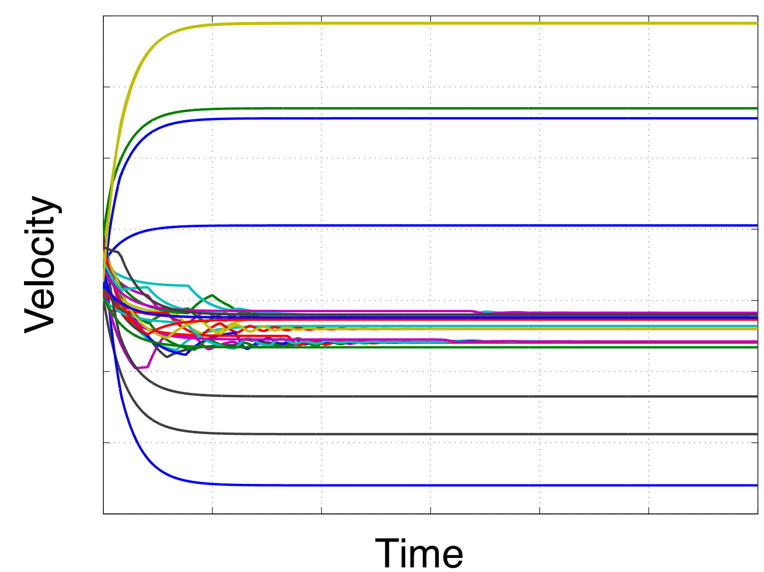
$$\dot{v}_i = \kappa_i [V_i^0 + V_i(\nabla \mathbf{p}) - v_i]$$

## Velocity adjustment (control)

$$V_i(\nabla \mathbf{p}) = V_i^1 \sum_{i \in \mathcal{N}(i)} \tanh(p_j - p_i)$$



# An Example - Vehicle Platooning


## Microscopic Traffic Model

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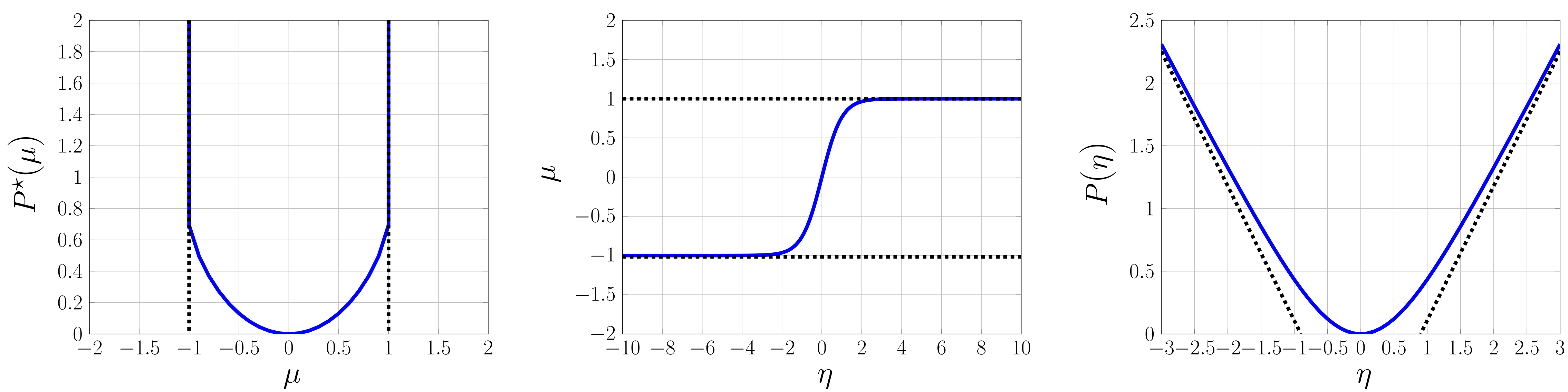
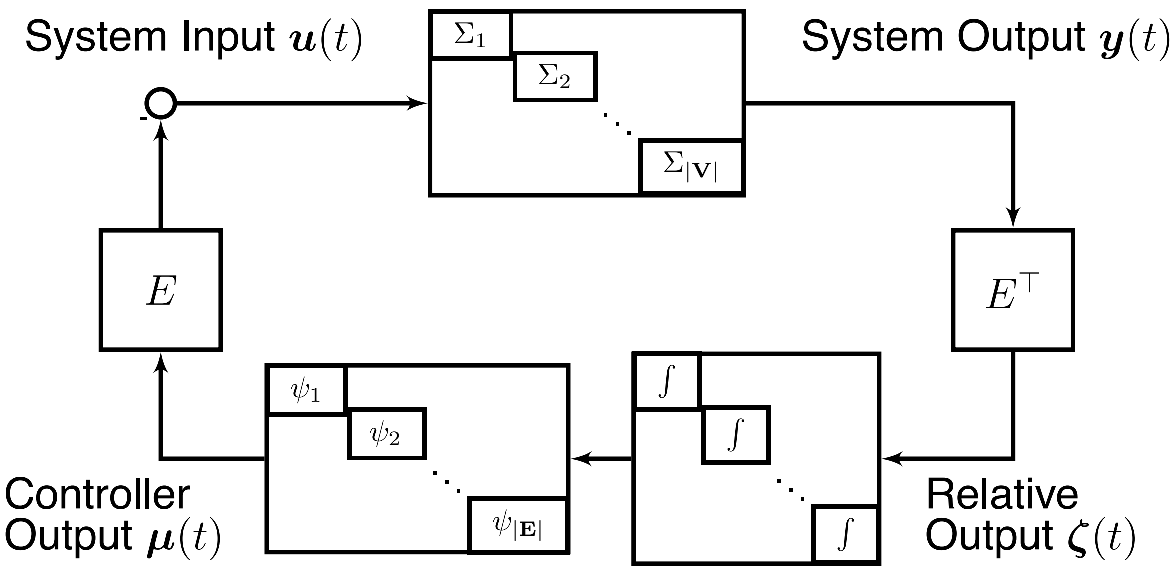
output coupling function is monotone, but not strongly monotone!



# An Example - Vehicle Platooning

Network Optimization approach can still be used!




*Optimal Flow Problem*

*(OFP1)*

$$\min_{\mathrm{u}_i,\mu_k} \sum_{i=1}^{|\mathbf{V}|} K_i(\mathrm{u}_i)$$

$$\text{s.t. } \mathbf{u} = E\mu$$

$$\|\mu\|_\infty \leq 1$$

*Optimal Potential Problem*

*(OPP1)*

$$\min_{\mathrm{y}_i,\zeta_k} \sum_{i=1}^{|\mathbf{V}|} K_i^\star(\mathrm{y}_i) + \sum_{k=1}^{|\mathbf{E}|} |\zeta_k|$$

$$\text{s.t. } \zeta = E^\top \mathbf{y}$$

# An Example - Vehicle Platooning

*Optimal Flow Problem*
*(OFP1)*

$$\min_{\mathrm{u}_i,\mu_k} \sum_{i=1}^{|\mathbf{V}|} K_i(\mathrm{u}_i)$$

$$\text{s.t. } \mathbf{u} = E\mu$$

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*Optimal Potential Problem*
*(OPP1)*

$$\min_{\mathrm{y}_i,\zeta_k} \sum_{i=1}^{|\mathbf{V}|} K_i^\star(\mathrm{y}_i) + \sum_{k=1}^{|\mathbf{E}|} |\zeta_k|$$

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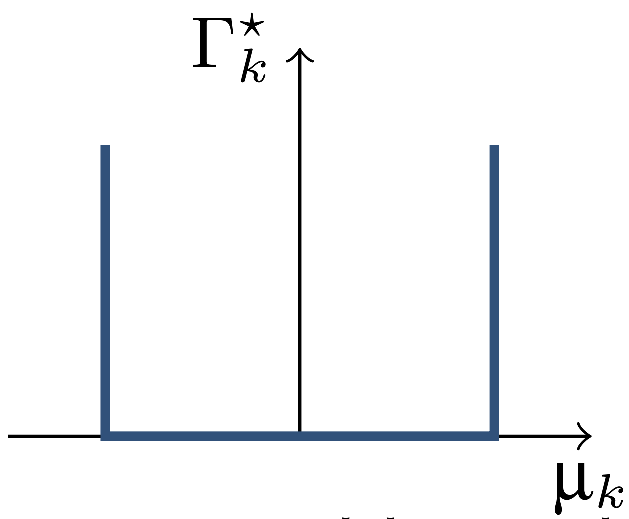
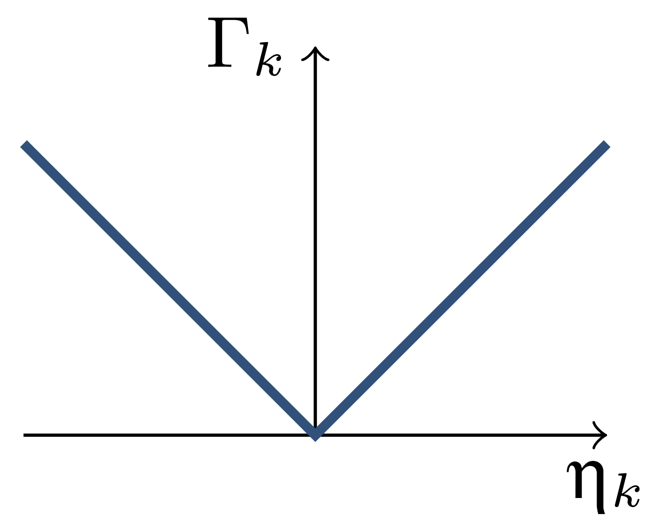
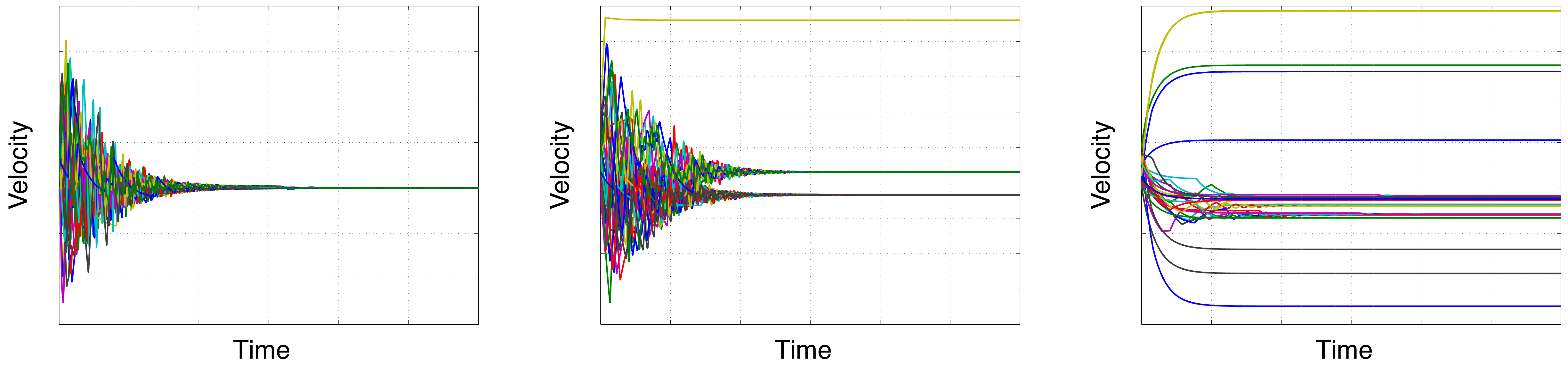



additional objective functions corresponding to "flow capacity" constraints

# An Example - Vehicle Platooning




clustering phenomena can be explained by studying the solutions of the static network optimization problems

$$\min_{\mathrm{u}_i, \mu_k} \sum_{i=1}^{|\mathbf{V}|} K_i(\mathrm{u}_i)$$

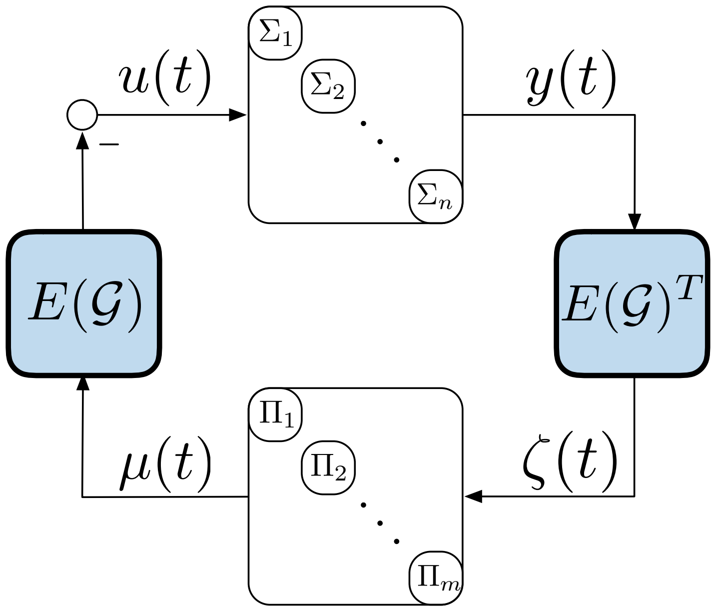
$$\text{s.t. } \mathbf{u} = E\mu$$

$$\|\mu\|_\infty \leq 1$$

$$\min_{\mathrm{y}_i, \zeta_k} \sum_{i=1}^{|\mathbf{V}|} K_i^\star(\mathrm{y}_i) + \sum_{k=1}^{|\mathbf{E}|} |\zeta_k|$$

$$\text{s.t. } \zeta = E^\top \mathbf{y}$$

# Towards a Synthesis Procedure




Design controllers to achieved a *desired* output agreement state.

*Generalized Optimal Potential Problem*

*(OPP1)*

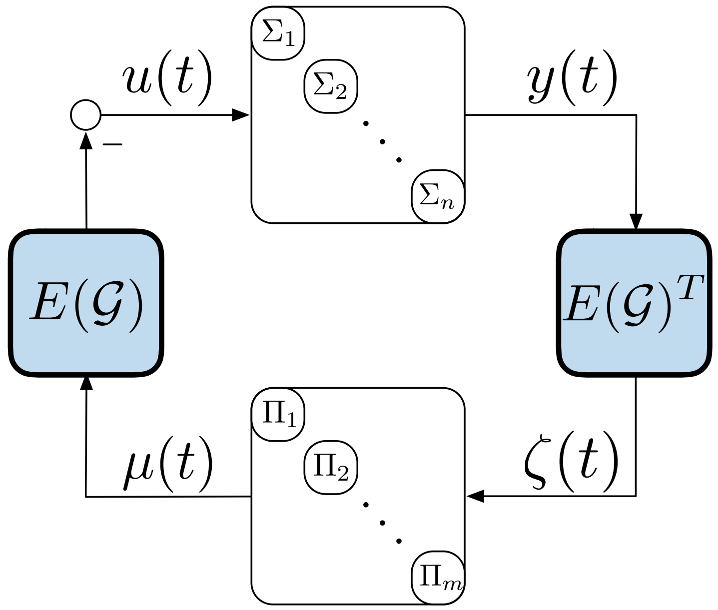
$$\min_{\mathbf{y},\boldsymbol{\zeta}} \sum_{i=1}^{|\mathbf{V}|} K_i^\star(\mathrm{y}_i) + \sum_{k=1}^{|\mathbf{E}|} \Gamma_k(\zeta_k)$$

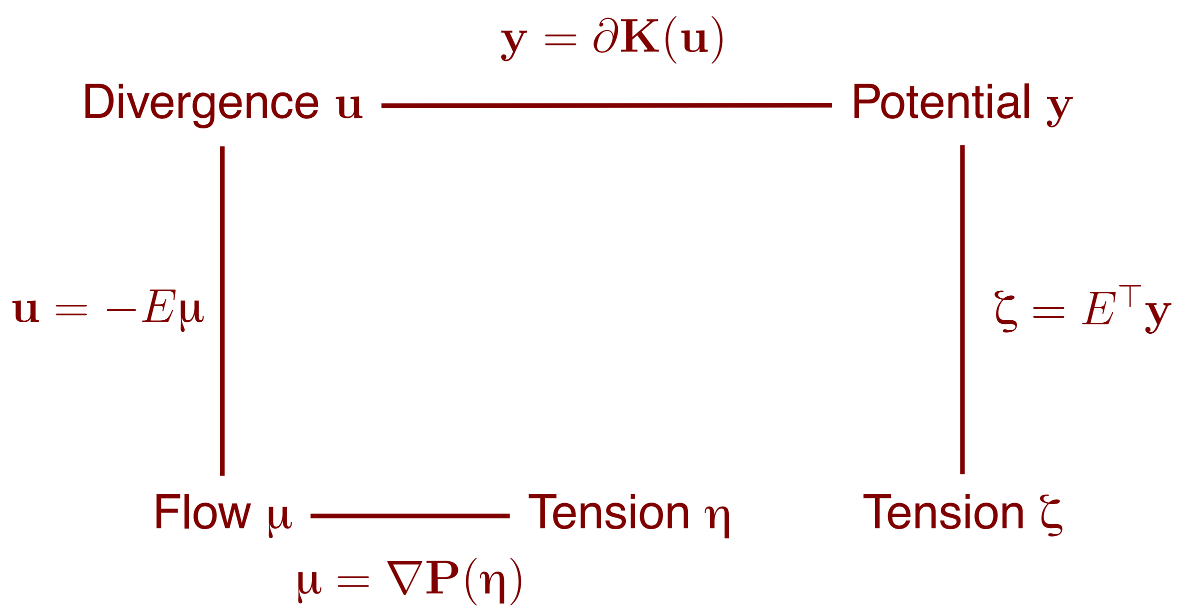
$$\text{s.t. } \boldsymbol{\zeta} = E^\top \mathbf{y}.$$

1. Find a convex function $\Gamma$ such that the desired output agreement state, $y^*$ satisfies $\zeta^* = E^T y^*$ and minimizes the GOPP.
2. Find Maximal EIP systems whose steady-state input-output maps are the subdifferentials of the functions $\Gamma_i$.

# Summary

## Passivity based cooperative control

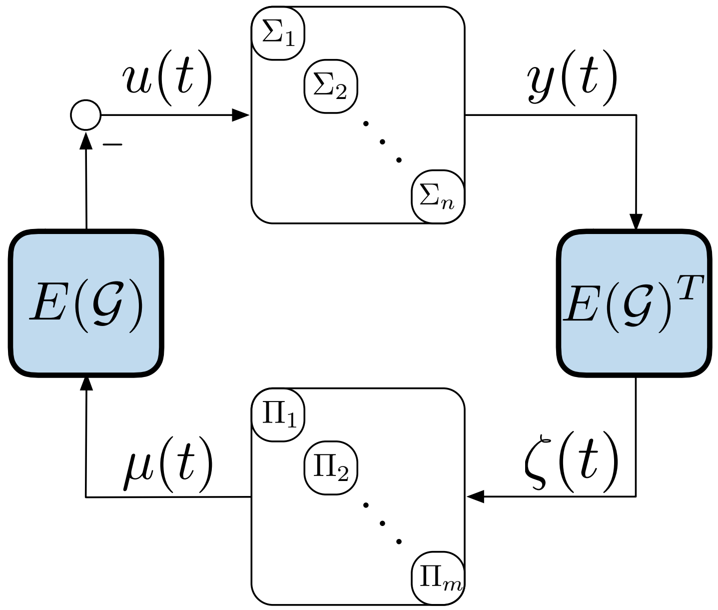


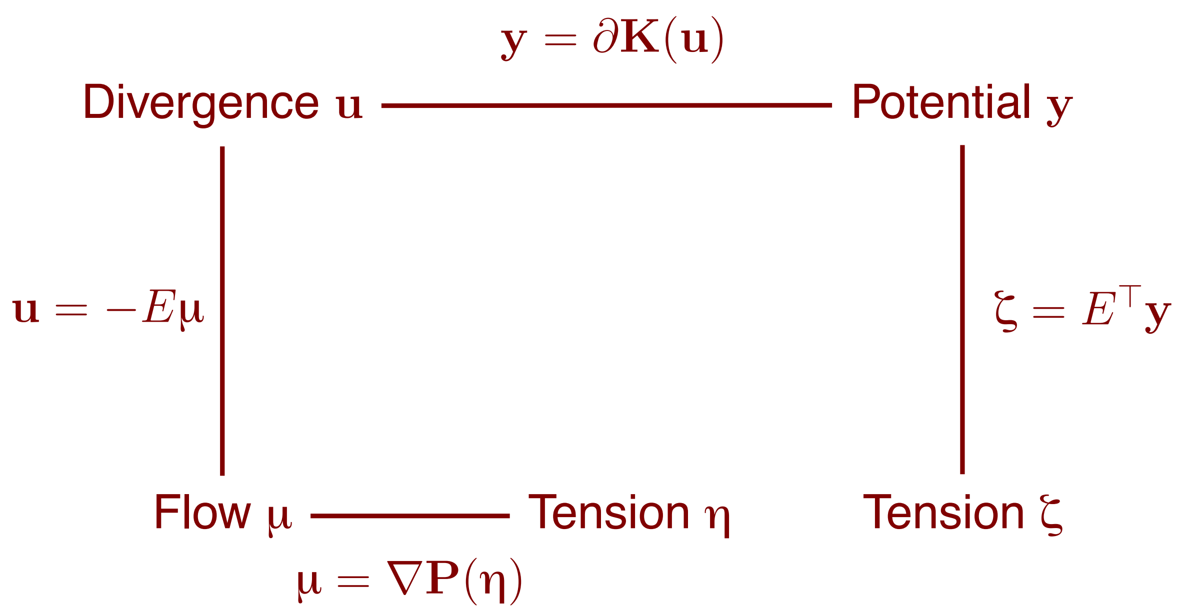




- maximal EIP systems
- connection to dual network optimization problems
- maximal EIP implies agreement solution is *inverse optimal*
- duality relation exists for cooperative control problems!

# Outlook

Passivity based cooperative control




- further extensions to MIMO systems
- controller synthesis
- "Duality" as a systems property

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Technion

Thank-you!

Questions?

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[1] M. Bürger, D. Zelazo, and F. Allgöwer, "Duality and network theory in passivity-based cooperative control," Automatica, 50:2051-2061, 2014.

[2] M. Sharf and D. Zelazo, "Cyclically-Monotone Relations and their use in Passivity-Based Cooperative Control," *in preparation*, 2017.