SYMMETRY-CONSTRAINED FORMATION MANEUVERING

WORKSHOP: GSC 2025

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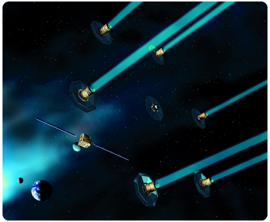
CONNECT LAB
Cooperative Networks and Controls

INTRODUCTION

Distributed coordination schemes have many practical applications:

- UAVs
  - Surveillance and reconnaissance
  - Mapping
  - Aerial transportation
  - Mobile communication networks
  - Coordinated maneuvering

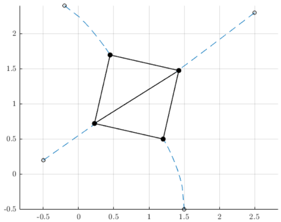
- Spacecraft
  - Interferometric arrays
  - Constellations for sensing



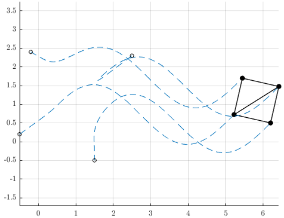
OBJECTIVES

Given a team of agents able to sense/communicate *only* with neighboring agents:

Formation Acquisition



Formation Maneuvering



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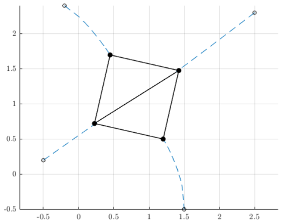
- Overview of classic distance constrained formation Control
- Introduction of a novel control strategy for symmetry constrained formations



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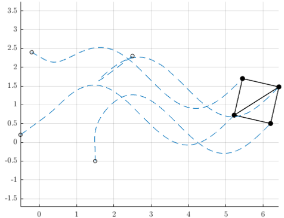

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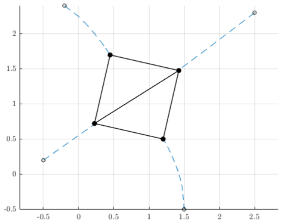
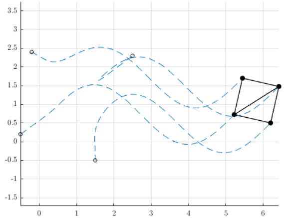


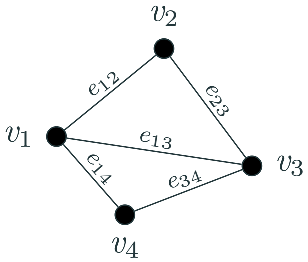
Formation Maneuvering

> Design a control strategy that enables symmetry-constrained formations to maneuver through space as a cohesive rigid body



FORMATION CONTROL - AGENT CONFIGURATION

- A team of n agents interact according to an information exchange graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$



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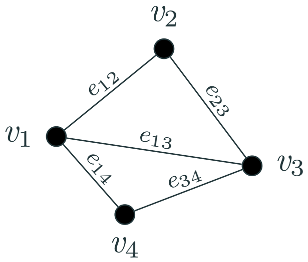
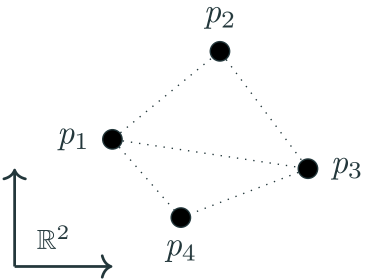
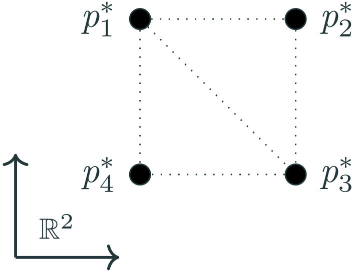


- The graph can be embedded in Euclidean space $\mathbb{R}^d$ as a framework $(\mathcal{G}, p)$. The position of the i-th agent is given by $p_i(t) \in \mathbb{R}^d$



- By implementing distance constraints, the desired formation can be defined as a framework $(\mathcal{G}, \mathbf{p}^\star)$



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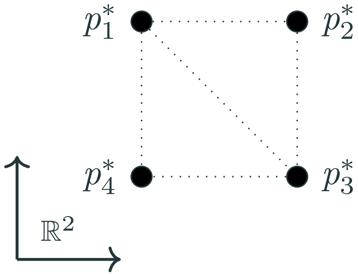
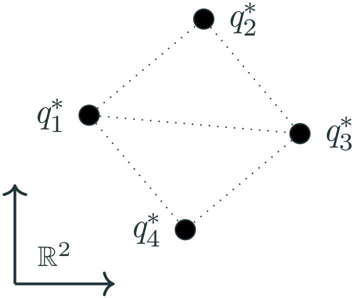


- Rotations and translations of this configuration result in some congruent framework $(\mathcal{G}, \mathbf{q}^\star)$ that also satisfies the constraints



- The desired formation is characterized by a set of M constraints, encoded in the function $F : \mathbb{R}^{nd} \to \mathbb{R}^M$, and a configuration $\mathbf{p}^\star$ satisfying the constraints
- The set of all feasible formations is

$$\mathcal{F}(p) = \{p \in \mathbb{R}^{nd} \mid F(p) = F(\mathbf{p}^\star)\}$$

Formation Control Objective

For an ensemble of n agents with dynamics

$$\dot{p}_i = u_i,$$

with $p_i(t) \in \mathbb{R}^d$, an information exchange graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, and formation constraint function $F : \mathbb{R}^{nd} \to \mathbb{R}^M$, design a distributed control law for each agent $i \in \{1, \ldots, n\}$ such that the set

$$\mathcal{F}(p) = \{p \in \mathbb{R}^{nd} \mid F(p) = F(\mathbf{p}^\star)\},$$

is asymptotically stable.

Theorem [Krick 2009]

Consider the potential function

$$F_f(p) = \frac{1}{4} \sum_{ij \in \mathcal{E}} \left(\|p_i(t) - p_j(t)\|^2 - (d_{ij}^\star)^2 \right)^2$$

and assume the desired distances $d_{ij}^\star$ correspond to a feasible formation. Then the gradient dynamical system

$$\dot{p}_i = u_i = -\nabla_{p_i} F_f(p) = \sum_{ij \in \mathcal{E}} \left(\|p_i - p_j\|^2 - (d_{ij}^\star)^2 \right) (p_j - p_i)$$

asymptotically converges to the critical points of the potential function, i.e., $\frac{\partial F_f(p)}{\partial p} = 0$.

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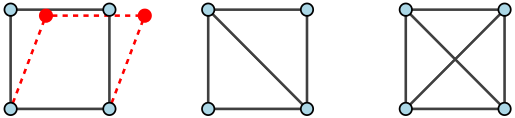
How do we define shapes ?



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- the number of constraints required to ensure the desired shape
- how the constraints should be distributed on the network

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Rigidity Matrix $R(\mathcal{G}, p)$

$$R(p) = \frac{\partial F(p)}{\partial p} = \mathrm{diag}(p_i - p_j)(E^T \otimes I_d)$$

The rigidity matrix helps us determine whether a framework $(\mathcal{G}, p)$ is **infinitesimally rigid**.

- E is the incidence matrix of $\mathcal{G}$
- Infinitesimal rigidity ensures that the shape is uniquely determined in a local sense, except from translations and rotations
- A framework is infinitesimally rigid if and only if $\mathrm{rk} R(p) = 2n - 3$ in $\mathbb{R}^2$

The state-space representation of the distance constrained formation control:

$$\boxed{\dot{p} = -\nabla_p F_f(p) = -R^T(p)\left(R(p)p - (d^\star)^2\right)}$$

[Krick 2009]

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- Local convergence to the desired formation shape is guaranteed if and only if the framework is infinitesimally rigid
- This leads to a minimal architectural requirement that ensures convergence to the correct formation. Equivalent to:

$$\boxed{\operatorname{rk} R(p) = 2|\mathcal{V}| - 3 \text{ and } |\mathcal{E}| = 2|\mathcal{V}| - 3} \qquad \text{(in } \mathbb{R}^2\text{)}$$

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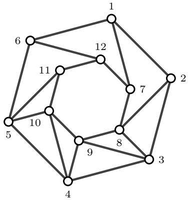
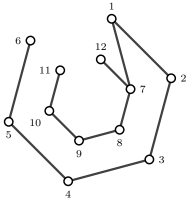
Q: Can the problem be solved with fewer constraints?

A: Yes, by leveraging the inherent symmetry in certain formations!

EXAMPLE

Rotation symmetry





- The ”classic” distance based formation control strategy requires at least 21 edges

- Incorporating symmetry constraints lowers the number of required edges to 11

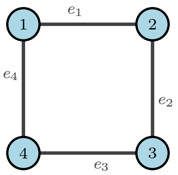
SYMMETRY AND GRAPH AUTOMORPHISMS

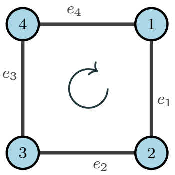
Automorphisms encode graph symmetries

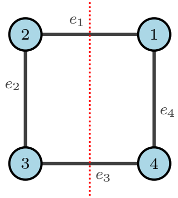
Graph Automorphism

An automorphism of the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a permutation $\psi : \mathcal{V} \to \mathcal{V}$ of its vertex set such that

$$\{v_i, v_j\} \in \mathcal{E} \Leftrightarrow \{\psi(v_i), \psi(v_j)\} \in \mathcal{E}$$







Identity:

$$\mathrm{Id} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

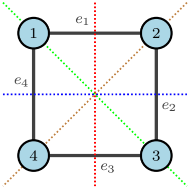
clock-wise 90° rotation:

$$\psi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

reflection:

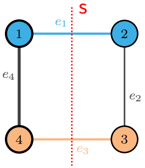
$$\psi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

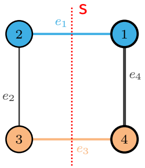
- Additional permutations can be found for the given graph considering all possible reflections and rotations



- The set of all automorphisms of $\mathcal{G}$ form a *group* - $\mathrm{Aut}(\mathcal{G})$
  - $\mathrm{Aut}(\mathcal{G}) = \{\mathrm{Id}, \psi_1, \psi_2, \ldots\}$
- For any subgroup $\Gamma \subseteq \mathrm{Aut}(\mathcal{G})$, we say that $\mathcal{G}$ is Γ-symmetric, which define specific symmetries in $\mathcal{G}$

Certain nodes are equivalent to each other and can be grouped together.





consider $\Gamma = \{\mathrm{Id}, \psi_2\}$ (reflection about mirror S)

- **Vertex Orbit:**
  $\Gamma_1 = \Gamma_2 = \{1, 2\},\ \Gamma_3 = \Gamma_4 = \{3, 4\}$
- **Edge Orbit:**
  $\Gamma_{e_1} = \{e_1\},\ \Gamma_{e_3} = \{e_2\},\ \Gamma_{e_2} = \Gamma_{e_4} = \{e_2, e_4\}$

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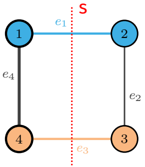

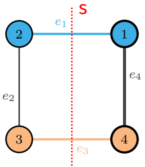

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  representative vertex set: $\mathcal{V}_0 = \{1, 4\}$
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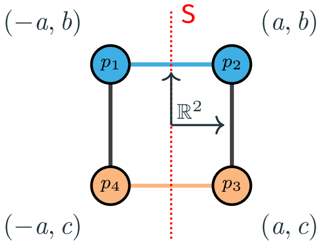
Graph symmetries can be realized in Euclidean space by assigning to each element of Γ an orthogonal matrix τ representing a point group isometry.

Example



- Consider $\Gamma = \{\mathrm{Id}, \psi_2\}$ (Reflection about mirror S)
- Isometry $\tau(\psi_2) = \tau_s = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$:

$$\tau_s p_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} = p_2$$

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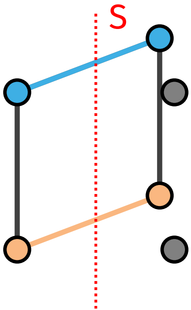
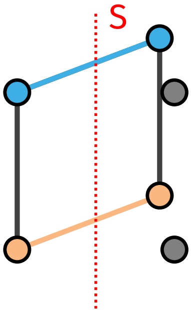


The symmetric relationship of $\tau(\Gamma)$-symmetric frameworks is only satisfied for special configurations

$\tau(\Gamma)$-SYMMETRIC FRAMEWORKS



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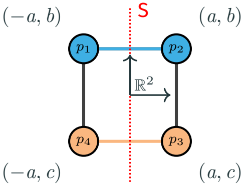
Isometries of the desired configuration coincide with symmetries of the automorphisms of $\mathcal{G}$



$$R(p) = \begin{bmatrix} (-2a \quad 0) & (2a \quad 0) & (0 \quad 0) & (0 \quad 0) \\ (0 \quad b-c) & (0 \quad 0) & (0 \quad 0) & (0 \quad c-b) \\ (0 \quad 0) & (0 \quad b-c) & (0 \quad c-b) & (0 \quad 0) \\ (0 \quad 0) & (0 \quad 0) & (-2a \quad 0) & (2a \quad 0) \end{bmatrix}$$

Due to symmetry, certain rows and columns of the rigidity matrix are redundant.

Orbit Rigidity Matrix $\mathcal{O}(\mathcal{G}_0, p)$ [Schulze 2011]

$$\mathcal{O}(\mathcal{G}_0, p) = \begin{bmatrix} (2p_1 - \tau_s p_1 - \tau_s{}^{-1} p_1)^T & (0 \quad 0) \\ (p_1 - p_4)^T & (p_4 - p_1)^T \\ (0 \quad 0) & (2p_4 - \tau_s p_4 - \tau_s{}^{-1} p_4)^T \end{bmatrix} = \begin{bmatrix} (-2a \quad 0) & (0 \quad 0) \\ (b-c) & (c-b) \\ (0 \quad 0) & (-2a \quad 0) \end{bmatrix}$$

Describes the $\tau(\Gamma)$-symmetric infinitesimal rigidity properties of $\tau(\Gamma)$-symmetric frameworks.

The introduction of the orbit rigidity matrix suggests a further way to exploit symmetries in formation control:

- Only representative edges are required to maintain distances
- Symmetries within vertex orbits have no need for distance constraints

Define a *symmetric formation potential*

$$F_f(p(t)) = F_e(p(t)) + F_s(p(t))$$

where

- The representative edge formation potential:

$$F_e(p(t)) = \frac{1}{4} \sum_{ij \in \mathcal{E}_0} \left(\|p_i(t) - \tau(\gamma) p_j(t)\|^2 - (d^{\star}_{i\gamma(j)})^2 \right)^2$$

- The symmetry potential:

$$F_s(p(t)) = \frac{1}{2} \sum_{i \in \mathcal{V}_0} \sum_{\substack{u,v \in \Gamma_i \\ uv \in \mathcal{E}}} \|p_u(t) - \tau(\gamma_{vu}) p_v(t)\|^2$$

[Zelazo 25]

FORCED SYMMETRIC FORMATION CONTROL

The states are defined as $\tilde{p}(t) = Pp(t) = \begin{bmatrix} p_0^T(t) & p_f^T(t) \end{bmatrix}^T$, for some permutation matrix P.

- $p_0(t)$ - the restriction of the configuration vector $p(t)$ to agents in the representative vertex set $\mathcal{V}_0$
- $p_f(t)$ - The remaining agents

Propose the gradient control

$$u(t) = -\nabla F_f(p(t))$$

The dynamics in state-space form become

$$\boxed{\begin{bmatrix} \dot{p}_0(t) \\ \dot{p}_f(t) \end{bmatrix} = \begin{bmatrix} -\mathcal{O}^T(\mathcal{G}_0, p_0(t))\left(\mathcal{O}(\mathcal{G}_0, p_0(t))p_0(t) - \mathbf{d}_0^2\right) \\ 0 \end{bmatrix} - PQP^T \begin{bmatrix} p_0(t) \\ p_f(t) \end{bmatrix}}$$

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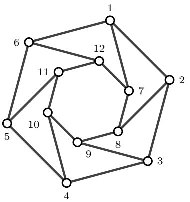
Compare to

$$\boxed{\dot{p} = -R^T(p)\left(R(p)p - (d^\star)^2\right)}$$

Example



- $\tau(\Gamma)$-symmetric framework with $2\pi/6$ rotational symmetry

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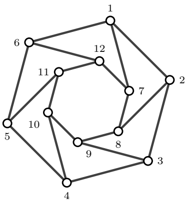

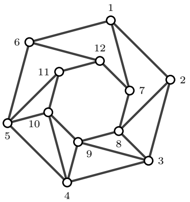

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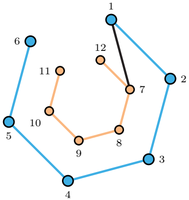

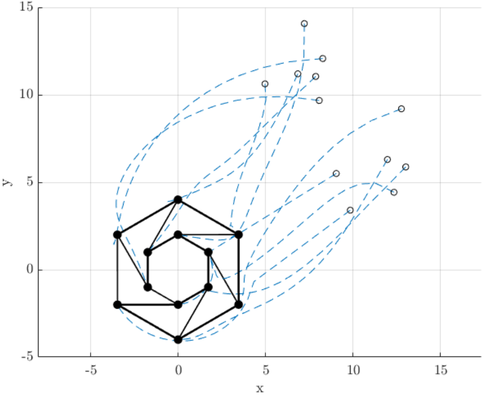

- The forced symmetric formation control strategy requires only 11 edges

FORMATION MANEUVERING

- Formation maneuvering aims to satisfy the formation control objective while simultaneously moving the formation through space as a rigid body

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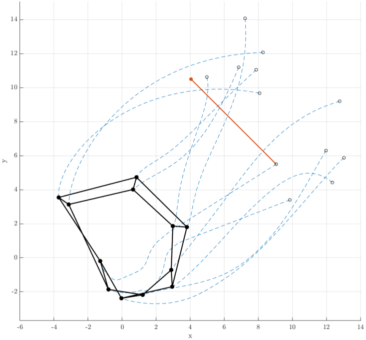
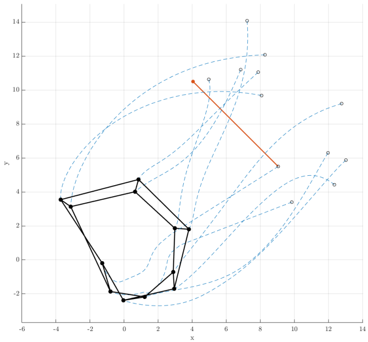


- $\tau(\Gamma)$-symmetric frameworks by definition have point-group symmetries defined with respect to some fixed inertial point

- Formation maneuvering aims to satisfy the formation control objective while simultaneously moving the formation through space as a rigid body
- Secondary objective:

$$\lim_{x \to \infty} ||\dot{p}_i(t) - v_i(t)|| = 0$$

  where $v_i \in \mathbb{R}^d$ is the desired rigid body velocity for each agent



- $\tau(\Gamma)$-symmetric frameworks by definition have point-group symmetries defined with respect to some fixed inertial point

Idea: Introduce a trajectory defined by a virtual state $r(t) \in \mathbb{R}^d$ and a time-varying rotation matrix $R(t) \in SO(d)$.

Proposition

- The shifted state

$$\bar{c}(t) = \begin{bmatrix} c_0^T(t) & c_f^T(t) \end{bmatrix}^T = P(p(t) - \mathbb{1} \otimes r(t))$$

allows the agents to agree on a different origin defined by $r(t)$.

- For an angular velocity $\omega(t) \in \mathbb{R}^3$, describing the rotational dynamics of the trajectory, the time-varying rotation matrix $R(t)$ satisfies $\dot{R}(t) = R(t)\omega(t)^\wedge$, and the corresponding isometry is defined by the similarity transformation:

$$\tau_\gamma(t) = R(t)\tau(\gamma)R(t)^T$$

CENTRALIZED APPROACH

Proposition

- The shifted state

$$\bar{c}(t) = \begin{bmatrix} c_0^T(t) & c_f^T(t) \end{bmatrix}^T = P(p(t) - \mathbb{1} \otimes r(t))$$

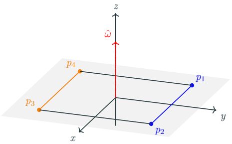
  allows the agents to agree on a different origin defined by $r(t)$.

- For an angular velocity $\omega(t) \in \mathbb{R}^3$, describing the rotational dynamics of the trajectory, the time-varying rotation matrix $R(t)$ satisfies $\dot{R}(t) = R(t)\omega(t)^\wedge$, and the corresponding isometry is defined by the similarity transformation:

$$\tau_\gamma(t) = R(t)\tau(\gamma)R(t)^T$$

Assumption:

- The desired configuration rotates about an axis $\hat{\omega}$ that passes through both the shifter formation's centroid and the origin



Define:

- Formation Control

$$u(t) = \begin{bmatrix} -\mathcal{O}^T(\mathcal{G}_0, c_0(t), \tau_\gamma(t)) \left(\mathcal{O}(\mathcal{G}_0, c_0(t), \tau_\gamma(t)) c_0(t) - \mathbf{d}_0^2 \right) \\ 0 \end{bmatrix} - PQ(\tau_\gamma(t)) P^T \begin{bmatrix} c_0(t) \\ c_f(t) \end{bmatrix}$$

- Virtual trajectory dynamics

$$v(t) = \mathbb{1} \otimes \dot{r}(t) + \begin{bmatrix} \cdots & \omega \times c_i(t) & \cdots \end{bmatrix}^T$$

Preposition

The modified control

$$\begin{bmatrix} \dot{p}_0(t) & \dot{p}_f(t) \end{bmatrix}^T = u(t) + v(t)$$

solves the formation maneuvering problem, ensuring (local) exponential stability to the desired symmetric formation shape.

Define the error system

$$\bar{e} = [\bar{\sigma}(t)^T \quad \bar{q}(t)^T]^T = \begin{bmatrix} \mathcal{O}(\mathcal{G}_0, c_0(t), \tau_\gamma(t))c_0(t) - \mathbf{d}_0^2 \\ Q(\tau_\gamma(t))\bar{c}(t) \end{bmatrix}$$

where $\bar{\sigma}(t)$ and $\bar{q}(t)$ represent the distance and symmetry errors, respectively.

Consider the Lyapunov candidate function

$$V(t) = \frac{1}{2}\bar{e}(t)^T\bar{e}(t)$$

Its time derivative satisfies

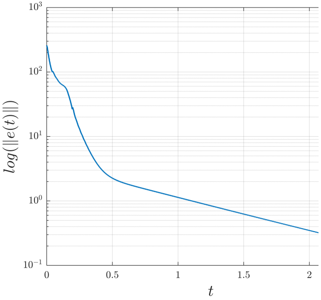
$$\dot{V}(t) = \bar{e}(t)^T \begin{bmatrix} \begin{bmatrix} \mathcal{O}(\mathcal{G}_0, c_0(t)) & 0 \end{bmatrix} \\ \bar{E}^T(\Gamma)P^T \end{bmatrix} \dot{\bar{c}}(t) \leq \alpha\|\bar{e}(t)\|^2, \quad \alpha < 0,$$

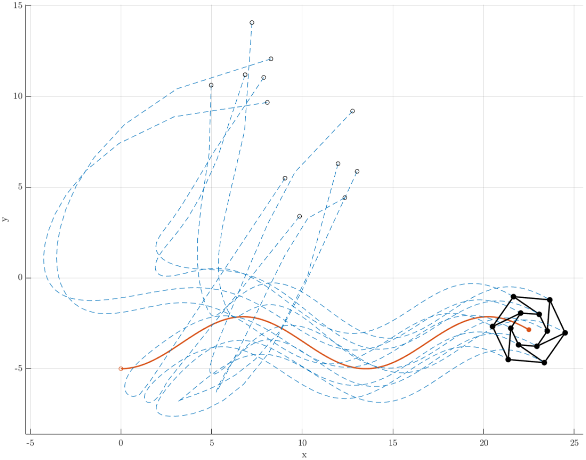
Since $\dot{V}(t)$ is negative definite in a neighborhood of the equilibrium, the error $\bar{e}(t)$ exponential converges to zero. Consequently, $u(t) \to v_m(t)$ as $e \to 0$.

Trajectory generated by:

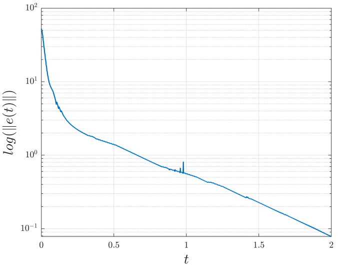
$$\dot{r}(t) = \begin{bmatrix} 4.5 & 3\sin(\tfrac{2}{3}\pi t) \end{bmatrix}^T,$$

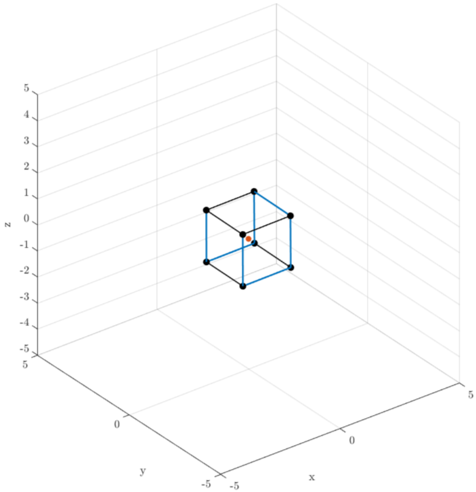
$$r(0) = -\begin{bmatrix} 5 & 5 \end{bmatrix}^T$$





- ”Classic” distance-based formation control needs a global reference agent and at least 21 edges
- The forced symmetric formation control strategy requires only 7 edges

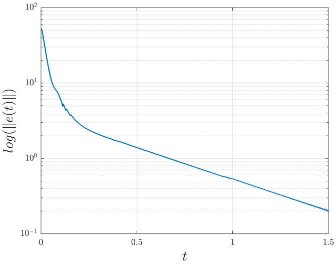


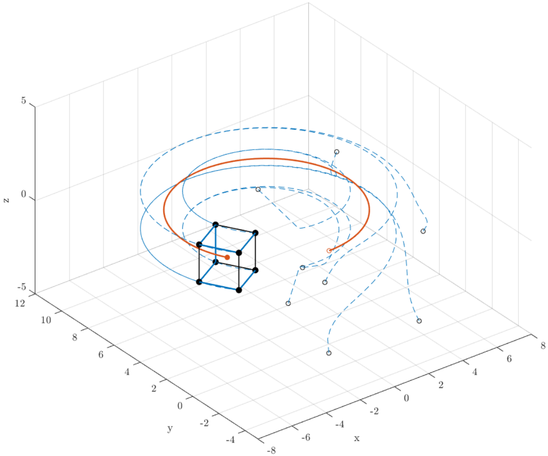


Trajectory generated by:

$$\dot{r}(t) = \begin{bmatrix} \frac{5}{3}\cos(\pi t) & \frac{5}{3}\sin(\pi t) & 0 \end{bmatrix}^T,$$

$$\begin{bmatrix} 0 & 0 & \frac{\pi}{3} \end{bmatrix}^T$$





A single agent is subjected to a reference velocity input $v_{ref}(t)$.

Proposition

The modified control strategy including a reference model takes the form:

$$\begin{bmatrix} \dot{p}_0(t) \\ \dot{p}_f(t) \end{bmatrix} = \begin{bmatrix} -\mathcal{O}^T(\mathcal{G}_0, c_0(t)) \left(\mathcal{O}(\mathcal{G}_0, c_0(t)) c_0(t) - \mathbf{d}_0^2 \right) \\ 0 \end{bmatrix} - PQP^T \begin{bmatrix} c_0(t) \\ c_f(t) \end{bmatrix} + \dot{\bar{r}}(t)$$

The trajectory is computed distributedly based on the consensus protocol:

$$\begin{cases} \dot{\bar{r}} & = -k_P \bar{L}(\mathcal{G}) \bar{r} - k_I \bar{\zeta} + nB \otimes v_{ref}(t) \\ \dot{\bar{\zeta}} & = \bar{L}(\mathcal{G}) \bar{r} \end{cases}$$

where:

- $L(\mathcal{G}) \in \mathbb{R}^{n \times n}$ is the Laplacian matrix of the information exchange graph $\mathcal{G}$
- $v_{ref} \in \mathbb{R}^d$ is the reference velocity input
- $B \in \mathbb{R}^n$ is a standard base vector denoting which agent is subjected to $v_{ref}(t)$

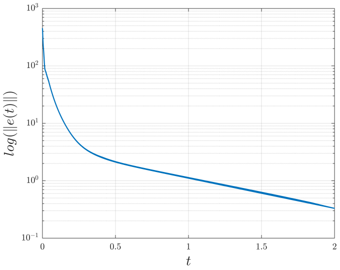
DISTRIBUTED APPROACH – FLOCKING EXAMPLE

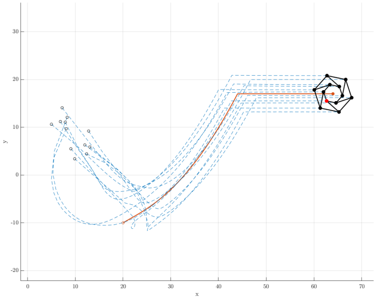
Trajectory generated by:

$$\dot{r}(t \leq 3) = \begin{bmatrix} 5 + 2t & 2t^2 + 3 \end{bmatrix}^T,$$

$$\dot{r}(t > 3) = \begin{bmatrix} 10 & 0 \end{bmatrix}^T,$$

$$r(0) = \begin{bmatrix} 10 & -10 \end{bmatrix}^T$$





Summary

- Rigid body translations and rotations can be executed while preserving point group symmetries in symmetry constrained formations
- A global velocity reference command can be applied to a single agent

Future Work

- Extend the distributed maneuvering approach to formations that undergo rotations
- Extend the approach to multi-agent systems with double integrator dynamics
- Investigate bearing rigidity extensions under symmetry constraints
- Explore distributed symmetry agreement to autonomously agree on a global symmetric configuration

Questions?