Technion
Israel Institute of Technology

Coordination and Control of Multi-Robot Systems

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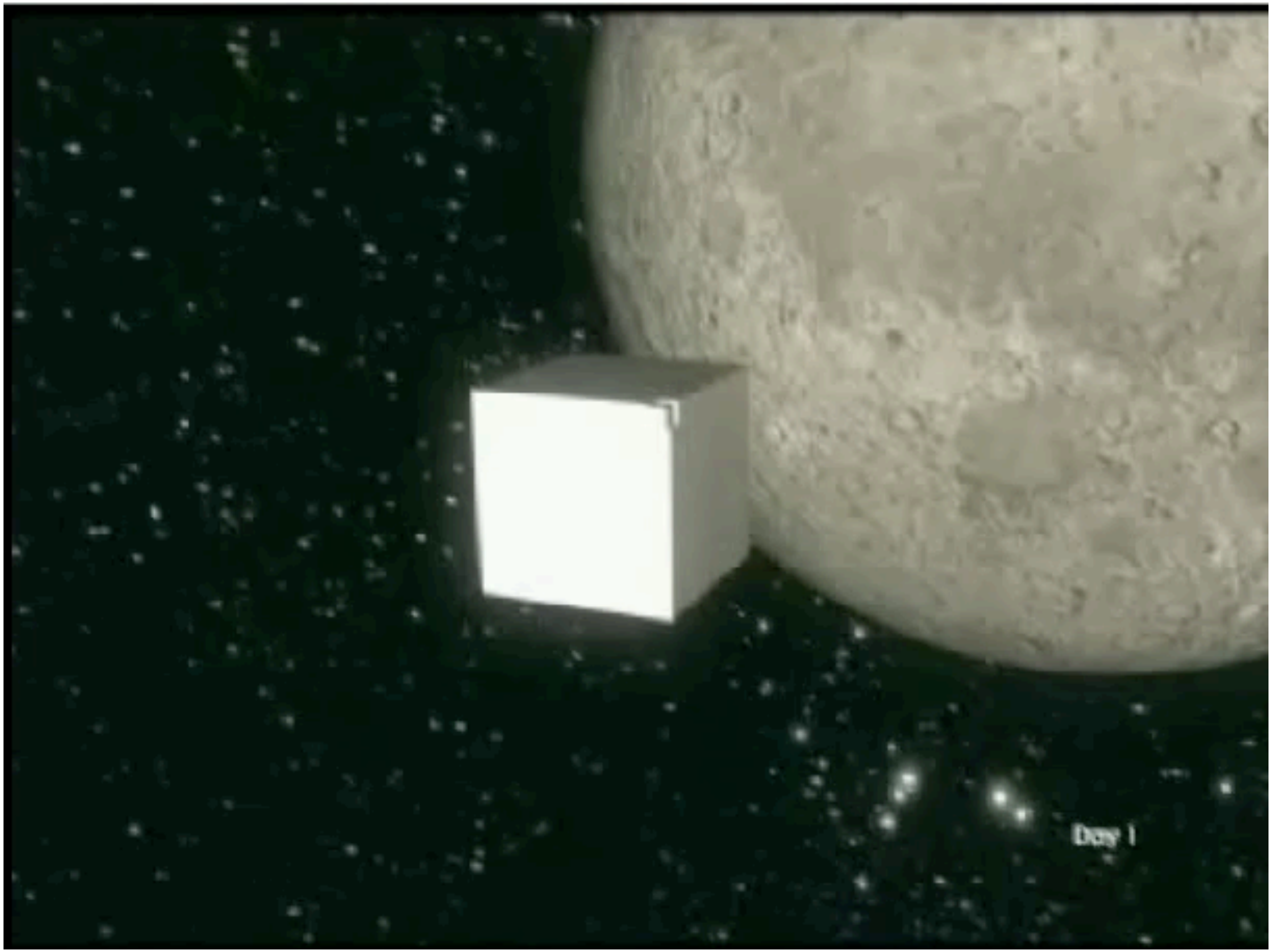
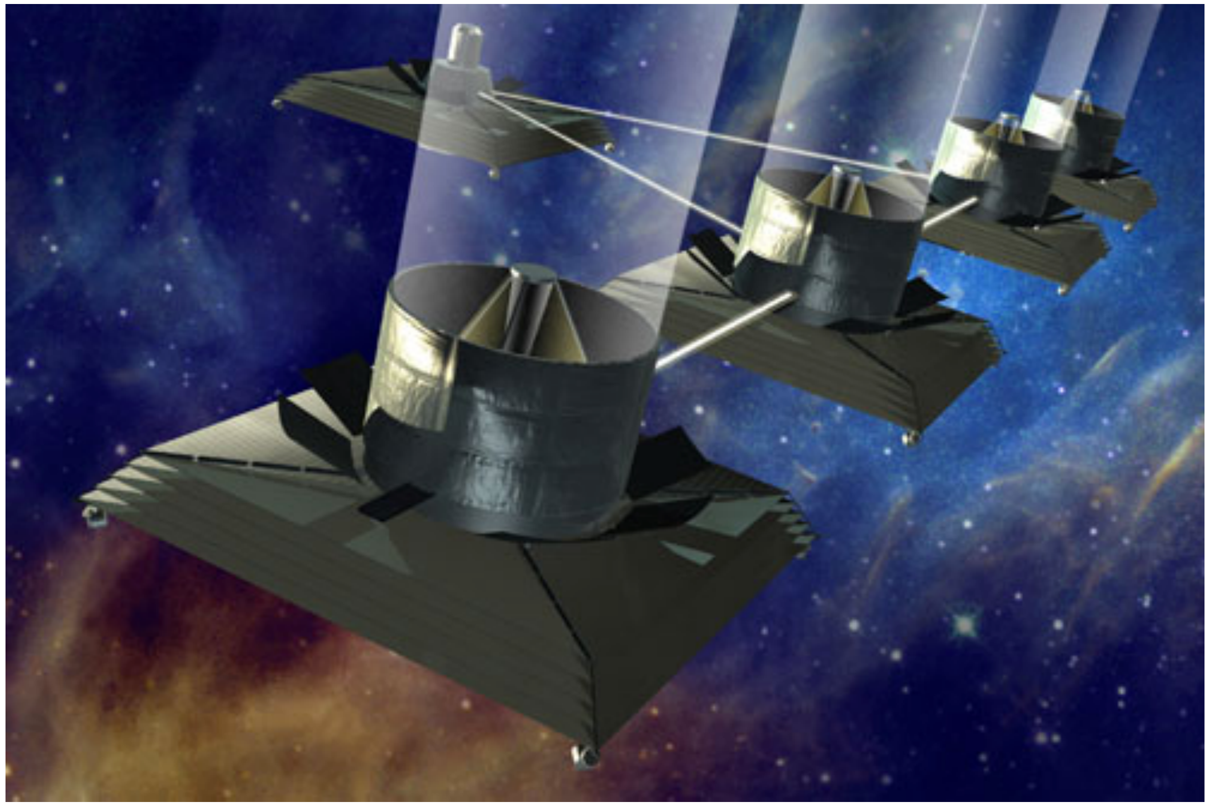
Control in Aerospace Systems

Aerospace Systems of the Future



NASA Prospecting Asteroid Mission
http://attic.gsfc.nasa.gov/ants/pam.html

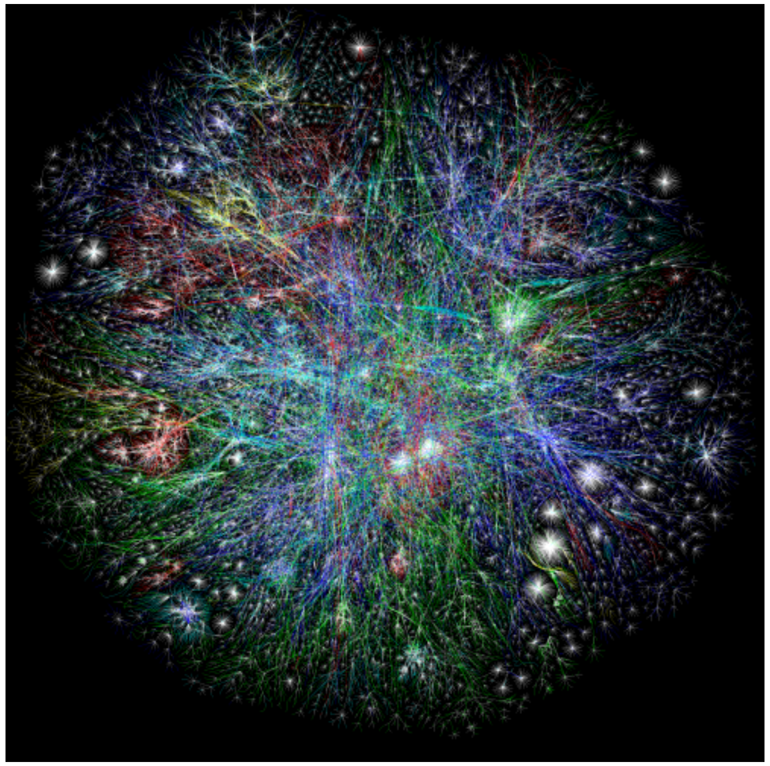
Networked Systems *are* the Future



Deep Space Interferometry

Power Distribution Networks - “smart grid”

Transportation Networks



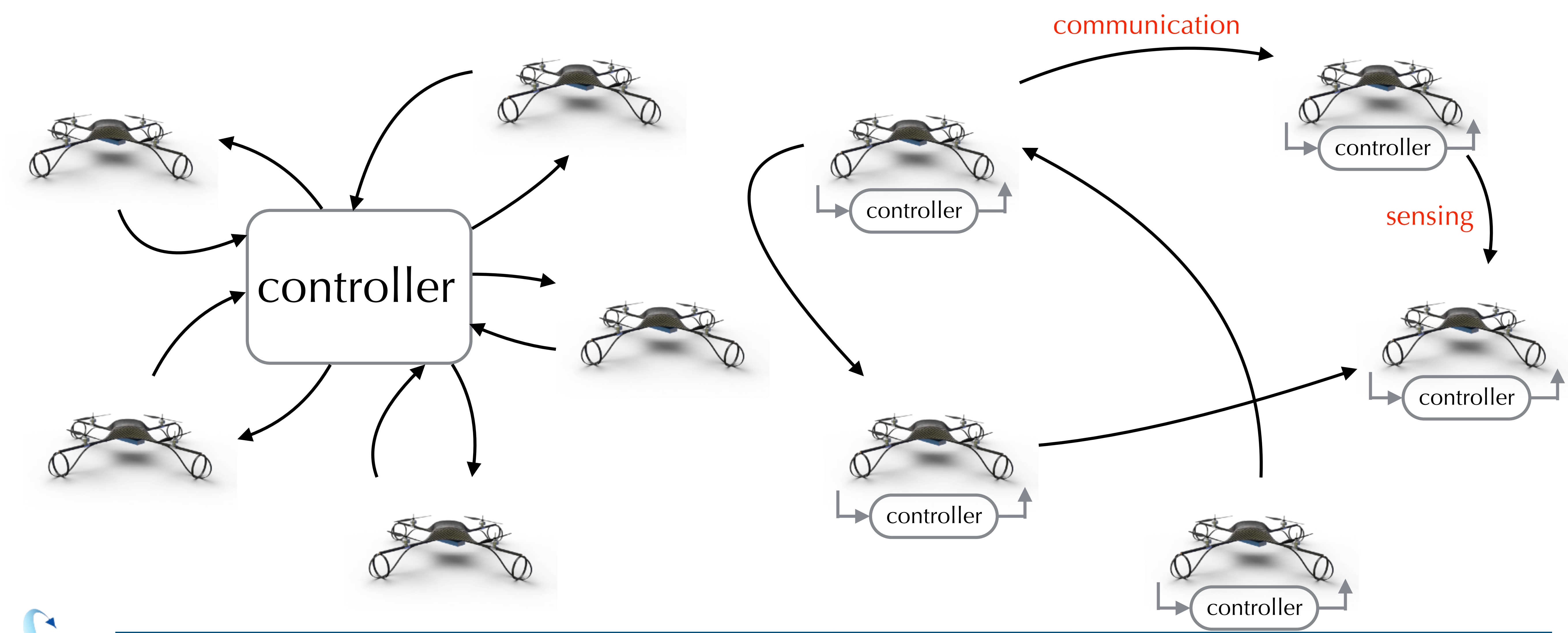
The Internet

Control of Networked Systems

How do we control multi-agent systems?

centralized approach

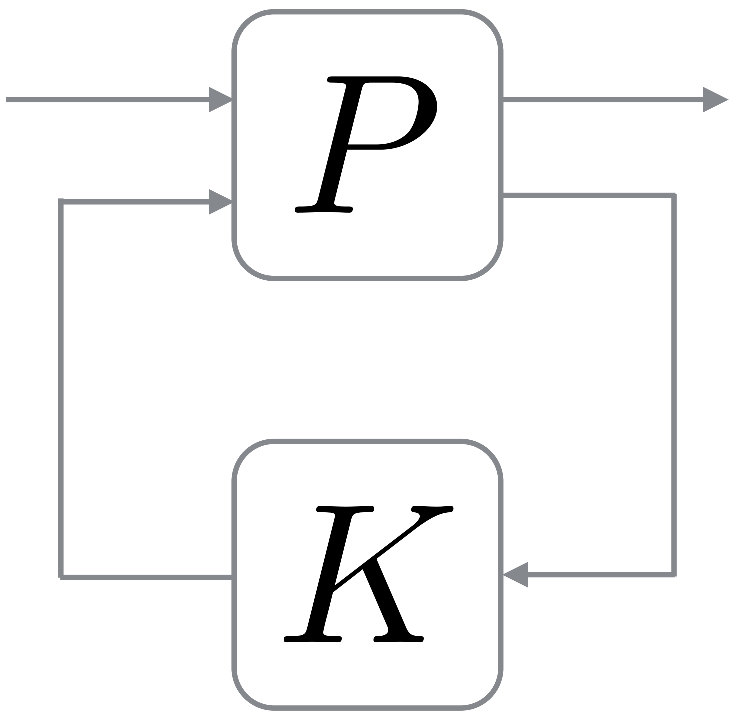
decentralized/distributed approach



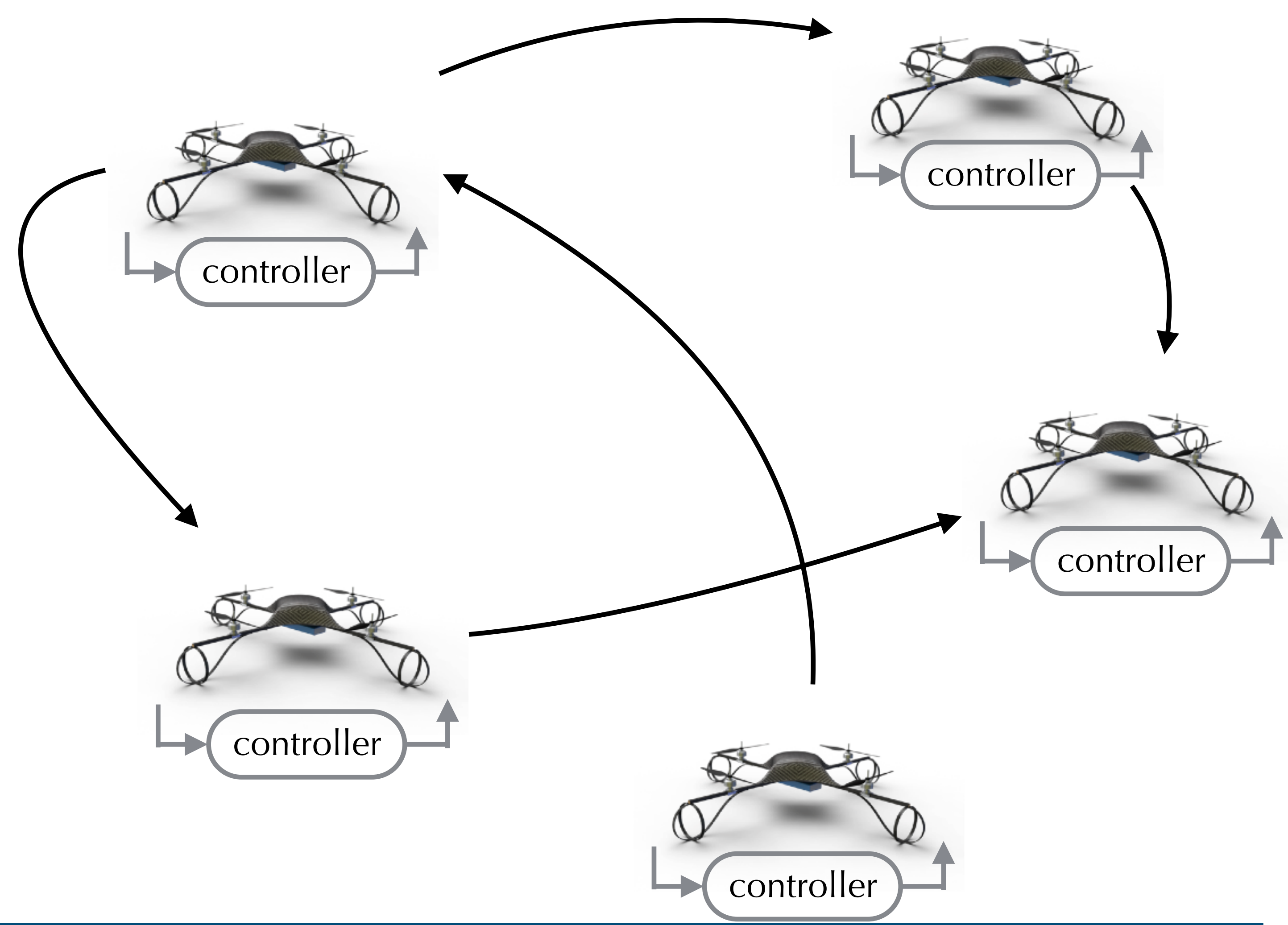
Control of Networked Systems

How do we control multi-agent systems?

dynamic systems
and control theory



graph theory



Control of Networked Systems

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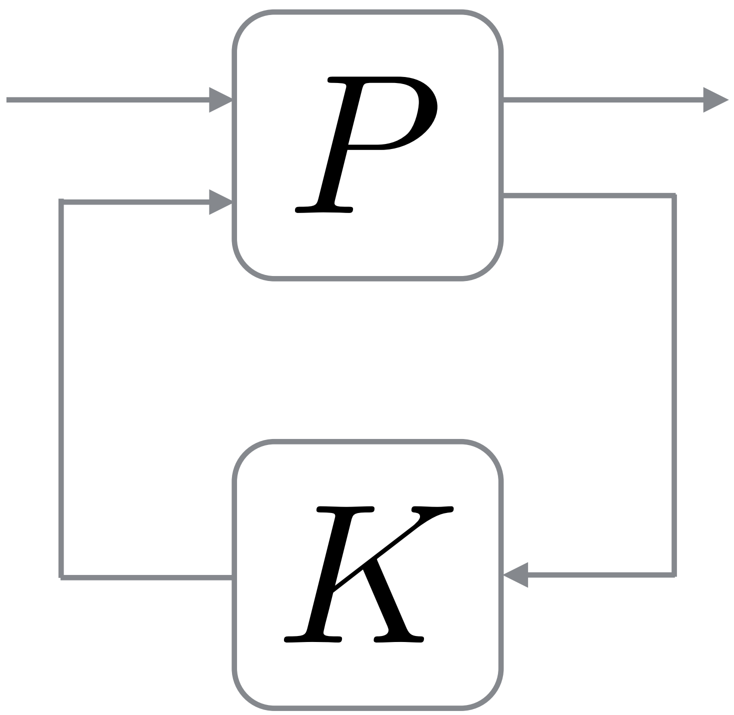
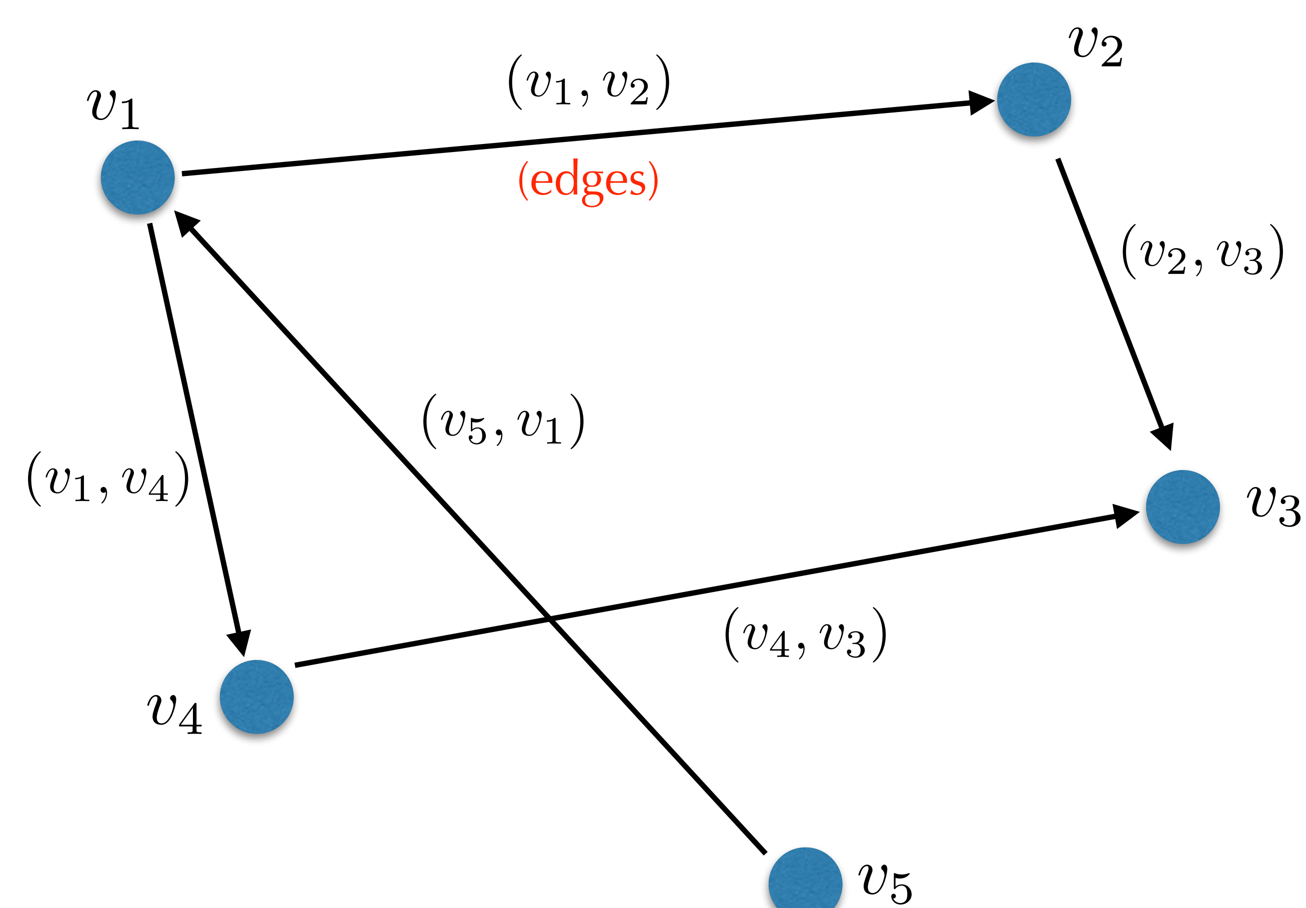


graph theory

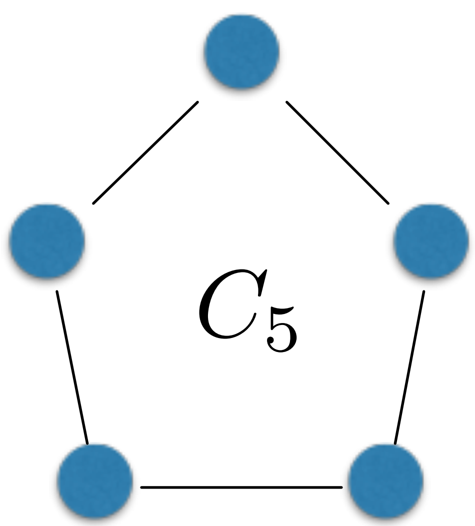


$$\mathcal{G} = (\mathcal{V}, \mathcal{E})$$

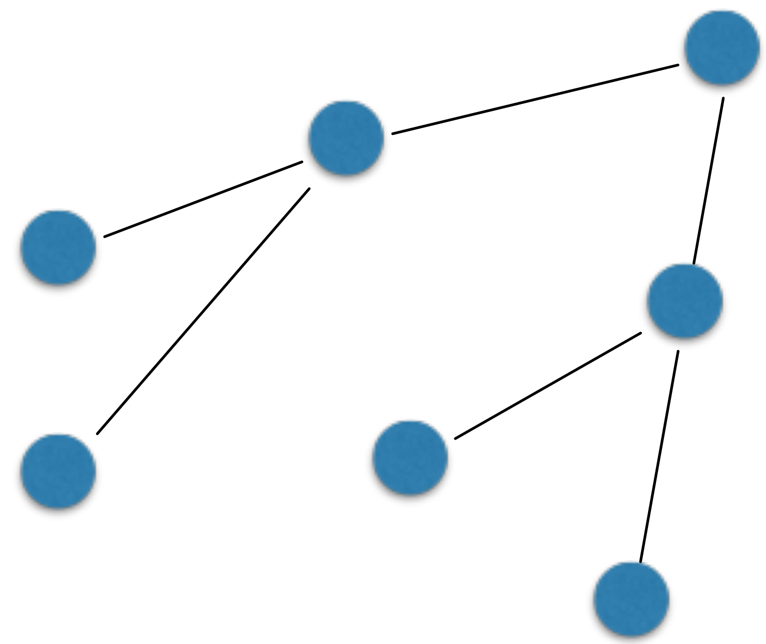
Graph Theory

Trees and Cycles

A *cycle* is a connected graph where each node has 2 neighbors



A *tree* is a connected graph containing no cycles



Graph Laplacian Matrix

$$L(\mathcal{G})$$

$$[L(\mathcal{G})]_{ii} = d_i$$ # of neighbors of node *i*

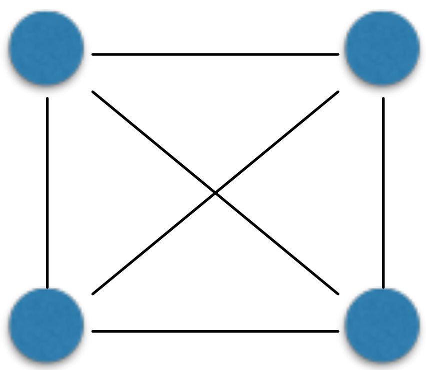
$$[L(\mathcal{G})]_{ij} = \begin{cases} 0 & \text{nodes } i \text{ and } j \text{ not neighbors} \\ -1 & \text{nodes } i \text{ and } j \text{ are neighbors} \end{cases}$$

$$L(C_5) = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

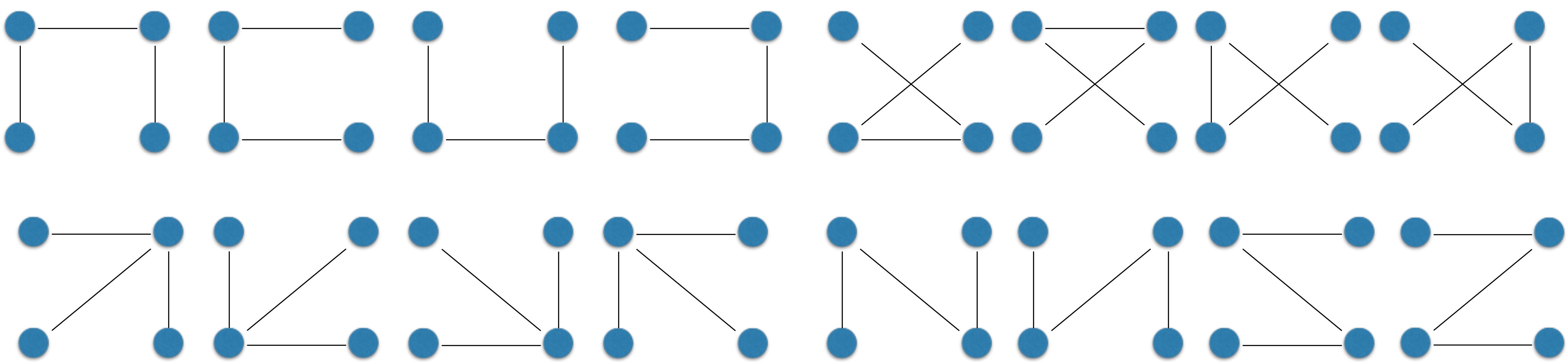
Graph Theory

Matrix-Tree Theorem

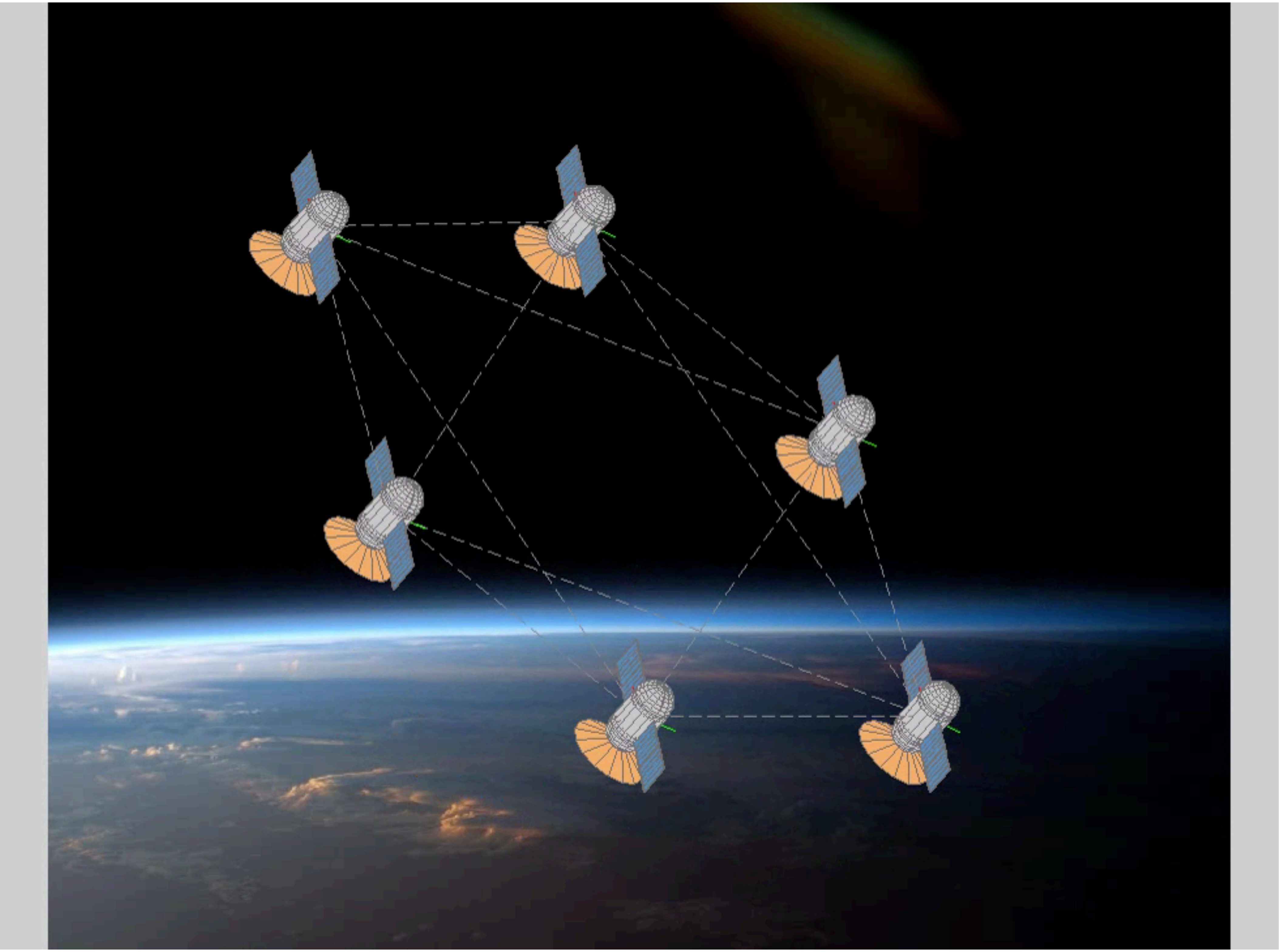
The number of trees in a graph is equal to the determinant of any sub-matrix of the graph Laplacian obtained by deleting one row and one column.



$$L(\mathcal{G}) = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \qquad \tau(\mathcal{G}) = 16$$



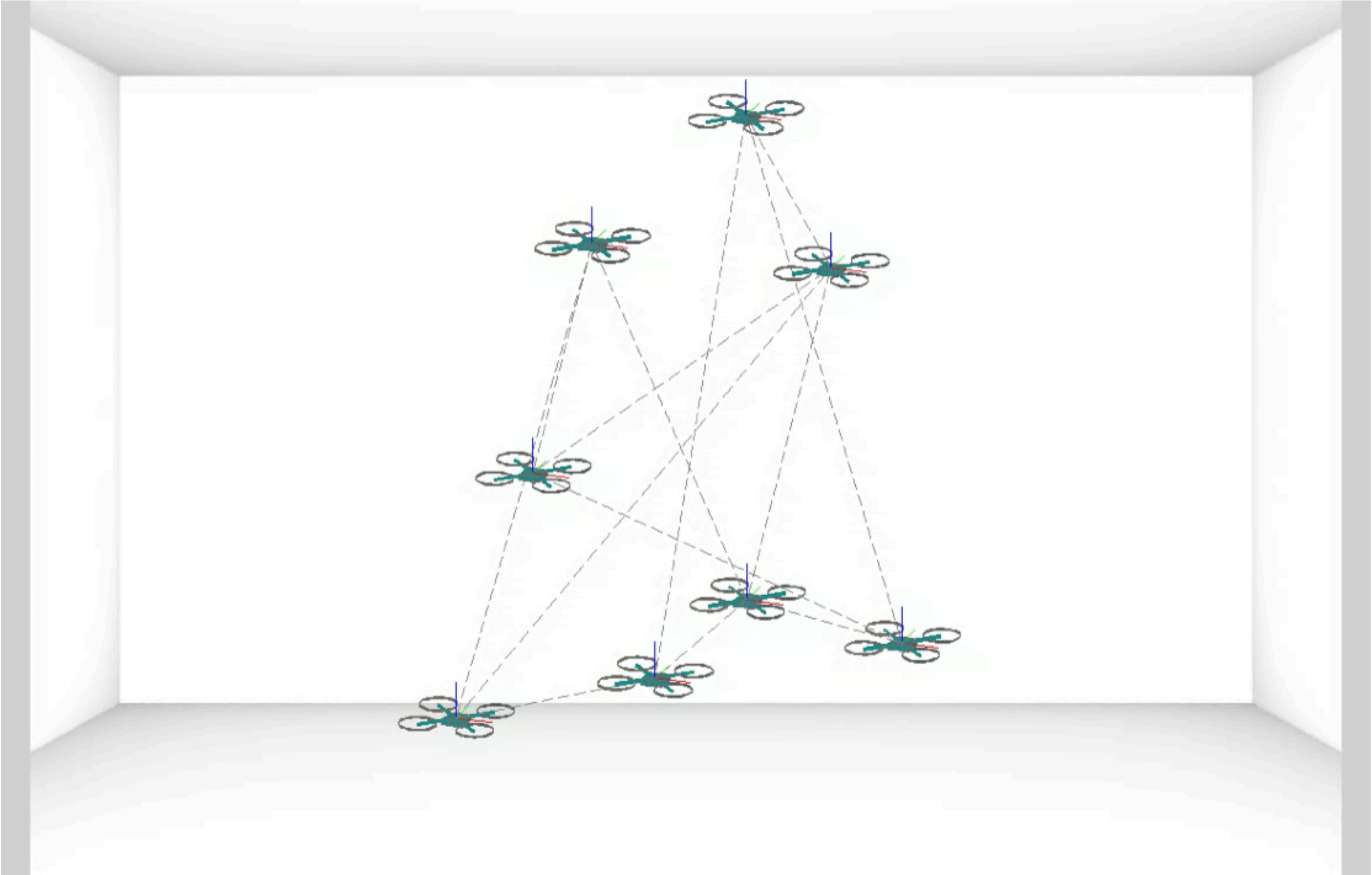
Formation Control



Formation Control

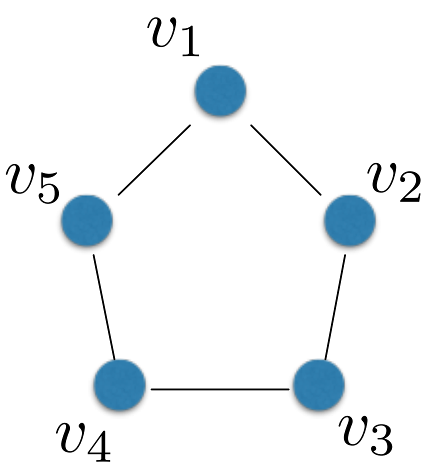
Formation Control Problem

Given a team of robots endowed with the ability to sense relative distance or direction information of neighboring robots, design a control for each robot using only *local information* that moves the team into a desired formation shape.



Formation Control

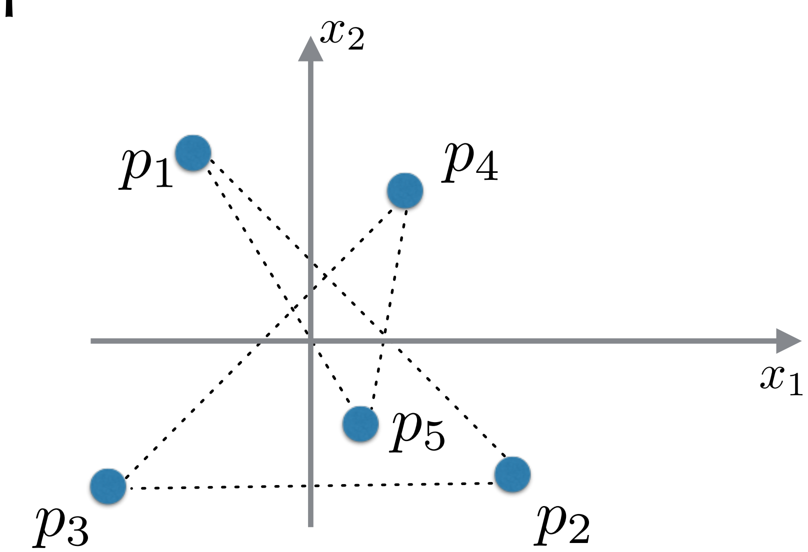
Assume the sensing graph is *fixed*



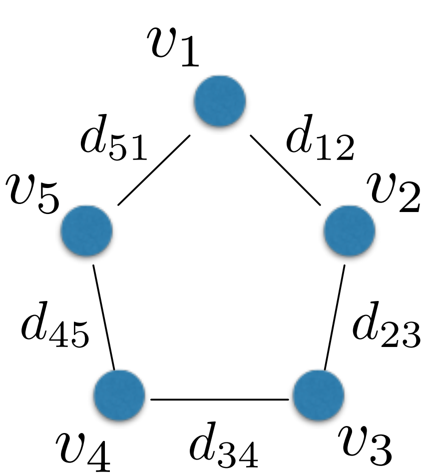
$$\mathcal{G} = (\mathcal{V}, \mathcal{E})$$

Each robot is modeled as a kinematic point mass in 2-D

$$\dot{p}_i = u_i$$



Specify the *formation* by specifying the distances between neighbors



$$d = \begin{bmatrix} d_{12} \\ d_{23} \\ d_{34} \\ d_{45} \\ d_{51} \end{bmatrix}$$

Control depends on distances and relative positions of neighbors

$$u_i = \sum_{j \sim i} f(d_{ij}, p_i - p_j)$$

(robot j is a neighbor of robot i)

Formation Control

Formation Control Law

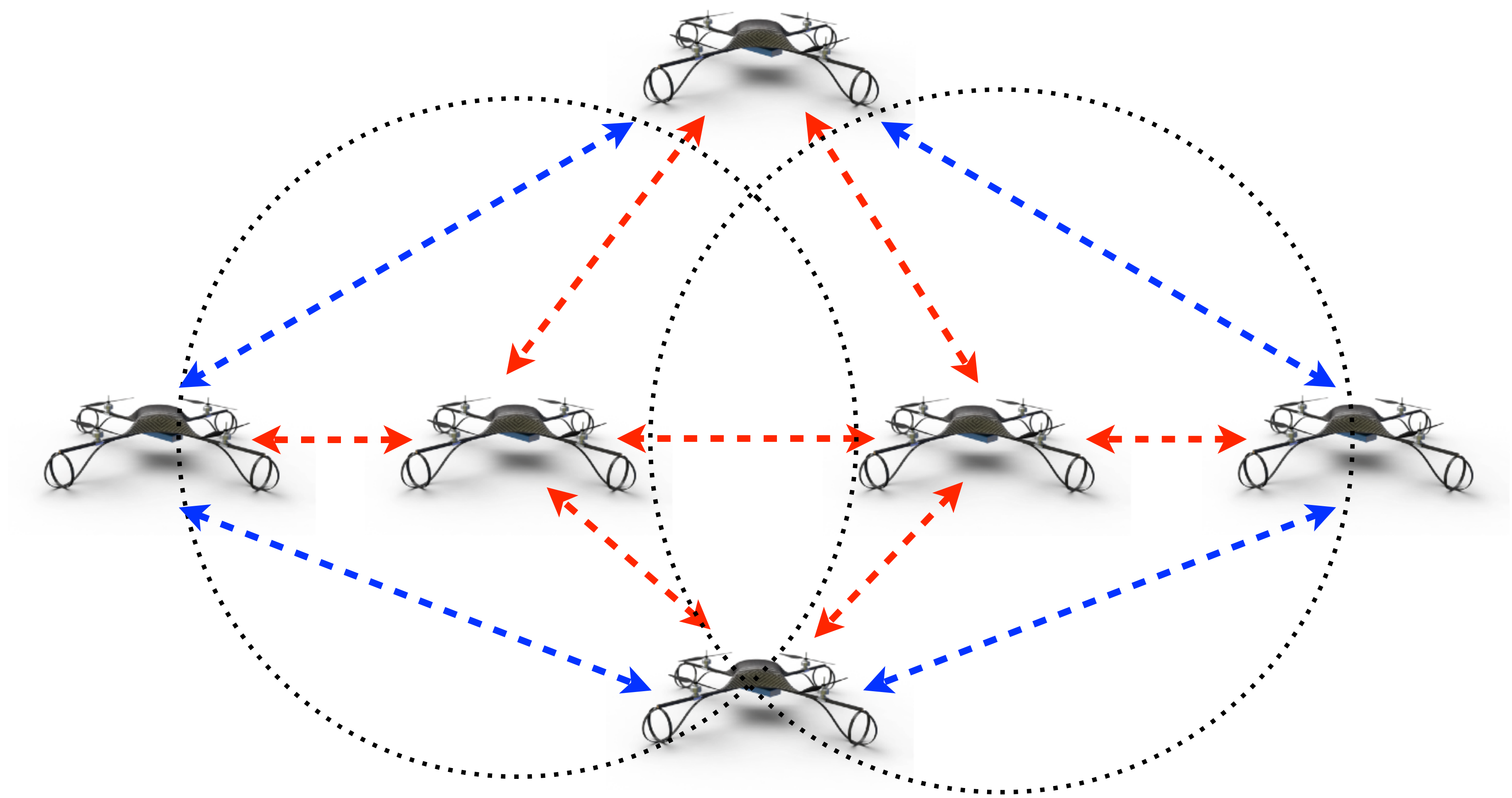
$$\dot{p}_i = u_i$$

$$u_i = -\sum_{j\sim i}\left(\|p_i - p_j\|^2 - d_{ij}^2\right)(p_i - p_j)$$

The control tries to minimize the error between the *desired* robot distances and the *measured* robot distances

Will this always work?

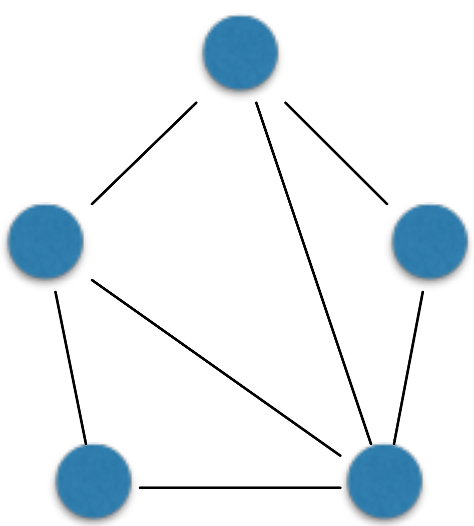
Formation Control and Graph Rigidity



A *minimum* number of distance measurements are required to *uniquely* determine the desired formation!

Graph Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.



A *rigid* graph can only *rotate* and *translate* to ensure all distances between all nodes are preserved (i.e., preserve the shape)!

Graph Rigidity Theory

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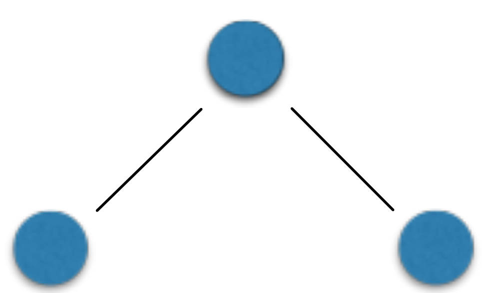


NOT rigid!
There is a motion that preserves distances between nodes in the graph but the shape is *not* preserved!

A *rigid* graph can only *rotate* and *translate* to ensure all distances between all nodes are preserved (i.e., preserve the shape)!

Formation Control and Graph Rigidity

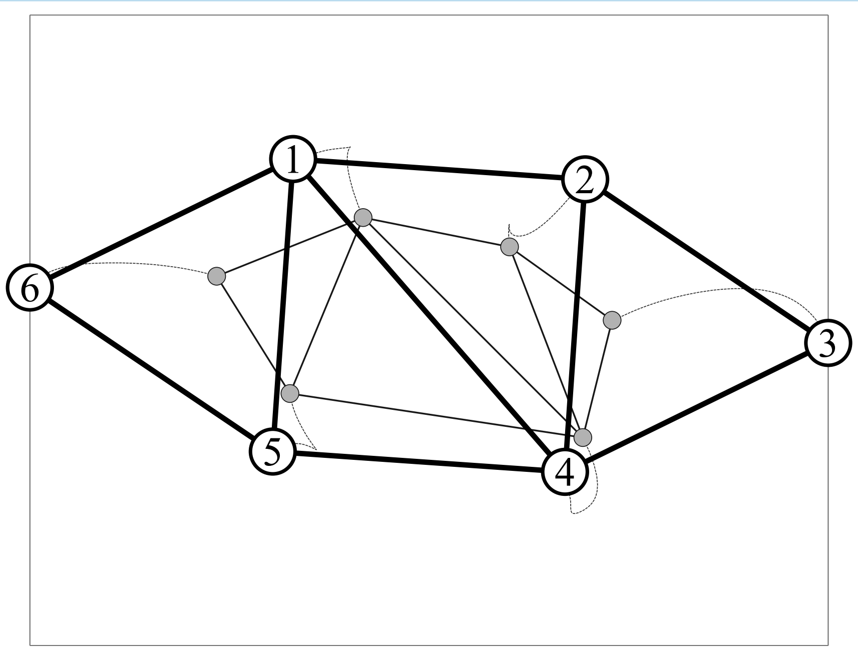
Theorem [Krick 2009]

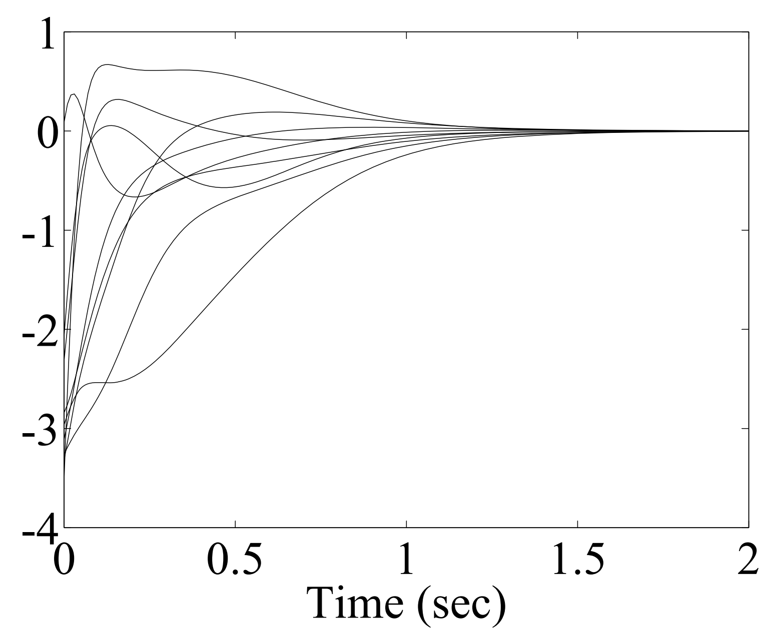
If the sensing graph is infinitesimally rigid, then the system

$$\dot{p}_i = u_i$$

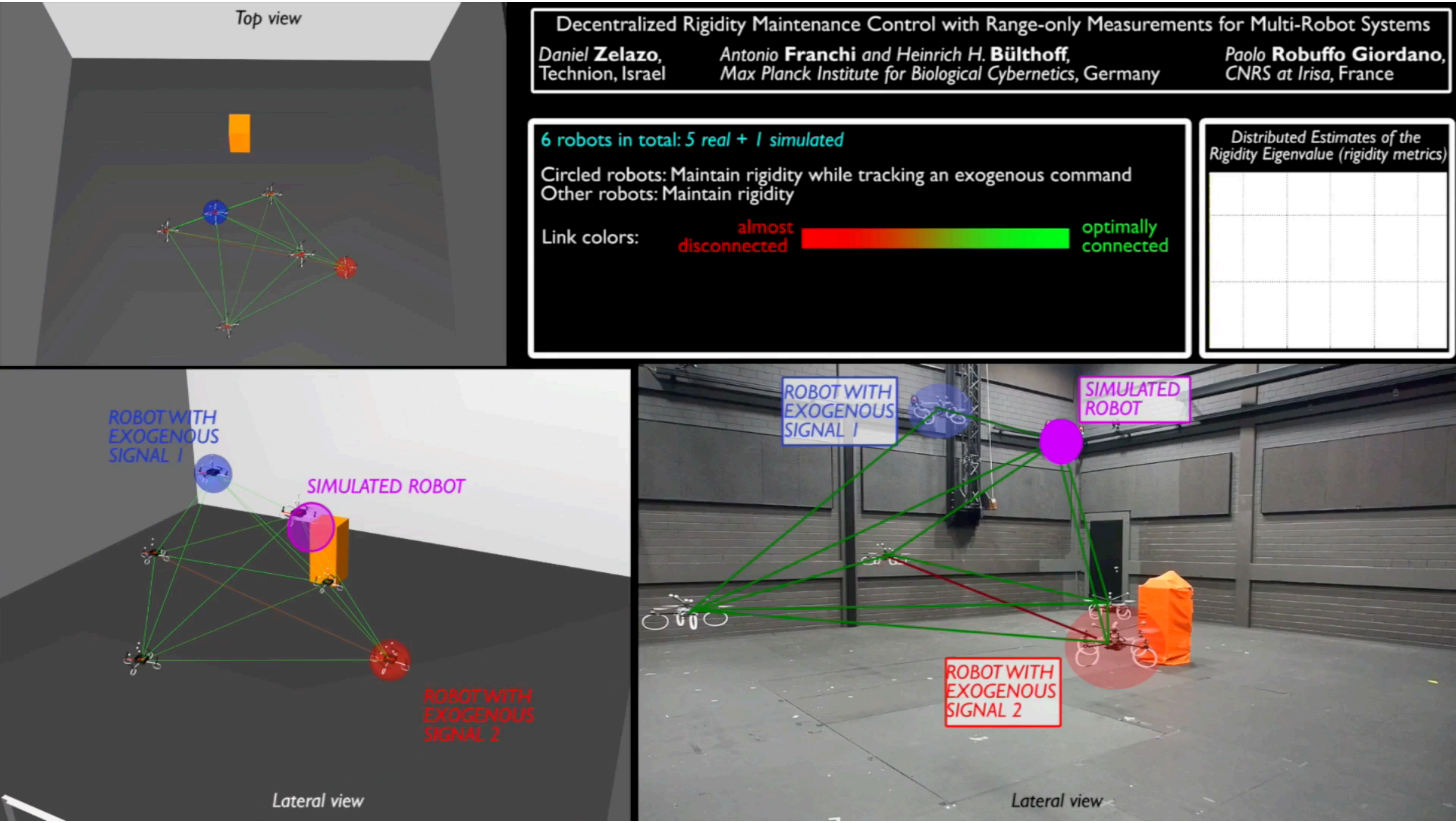
$$u_i = -\sum_{j\sim i} \left(\|p_i - p_j\|^2 - d_{ij}^2\right)(p_i - p_j)$$

(locally) asymptotically converges to the desired formation shape.





Rigidity Maintenance



Summary

- *multi-robot control and coordination* requires a blending of tools from control theory and graph theory
- formation control is an important problem in multi-agent systems with many challenges
- many open problems exist
  - coordination in harsh environments (no common reference frame, i.e., GPS)
  - coordination with "cheap" sensing and no communication (i.e., cameras)
  - dynamic sensing and communication

Acknowledgements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