Bearing-Based Distributed Control and Estimation of Multi-Agent Systems

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Outline

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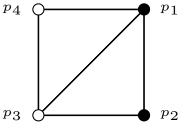
Technion Israel Institute of Technology

◦ Problem Statement



Notations:

- A network of n nodes in $\mathbb{R}^d$ ($n \geq 2, d \geq 2$)
- The underlying graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$
- The position of each node is $p_i \in \mathbb{R}^d$ ($i \in \mathcal{V}$)

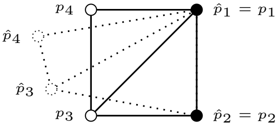
Anchors and followers:

- The first n_a nodes are anchors and the rest n_f nodes are followers ($n_a + n_f = n$)
- The position of each anchor is **known**, and the position of each follower is **unknown** and to be localized
- Denote $\mathcal{V}_a = \{1, \ldots, n_a\}$ and $\mathcal{V}_f = \{n_a + 1, \ldots, n\}$; $p_a = [p_1^T, \ldots, p_{n_a}^T]^T$ and $p_f = [p_{n_a+1}^T, \ldots, p_n^T]^T$

Follower $i \in \mathcal{V}_f$ can obtain

- Bearings of its neighbors: $\{g_{ij}\}_{j \in \mathcal{N}_i}$
- Estimates of its neighbors: $\{\hat{p}_j\}_{j \in \mathcal{N}_i}$

◦ Problem Statement



Problem statement:

- Each follower has an estimate $\hat{p}_i$ of its own position p_i. Design a distributed localization protocol based on $\{g_{ij}\}_{j\in\mathcal{N}_i}$ and $\{\hat{p}_j\}_{j\in\mathcal{N}_i}$ such that $\hat{p}_i(t) \to p_i$ as $t \to \infty$

Assumption:

- Undirected graph: $(i,j) \in \mathcal{E} \Leftrightarrow (j,i) \in \mathcal{E}$. Nodes i and j can measure the bearings of each other and communicate with each other
- Global orientation: each node can measure the bearings of their neighbors with respect to a global orientation

Two problems to solve:

- What kind of networks can be localized?
- How to distributedly localize a network?

◦ **Localizability Analysis: What kind of networks can be localized?**

Localizability is a fundamental property of a network. A network must be localizable in order to be localized in either centralized or distributed ways.

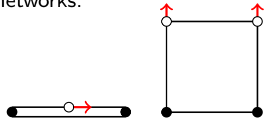
The bearing-only network localization problem is to retrieve p_f by solving

$$\begin{cases} \dfrac{\hat{p}_j - \hat{p}_i}{\|\hat{p}_j - \hat{p}_i\|} = g_{ij}, & \forall (i,j) \in \mathcal{E}, \\ \hat{p}_i = p_i, & \forall i \in \mathcal{V}_a. \end{cases} \tag{1}$$

Definition (Bearing-Only Network Localizability)

A network $\mathcal{G}(p)$ is called *localizable* if the true location p is the unique feasible solution to (1).

Examples of non-localizable networks:

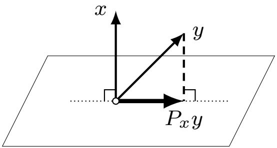


◦ Localizability Analysis: An orthogonal projection operator

For any nonzero vector $x \in \mathbb{R}^d$ $(d \geq 2)$, define the orthogonal projection operator $P : \mathbb{R}^d \to \mathbb{R}^{d\times d}$ as

$$P(x) \triangleq I_d - \frac{x}{\|x\|}\frac{x^T}{\|x\|}.$$

For notational simplicity, denote $P_x \triangleq P(x)$. Note that P_x is an orthogonal projection matrix that geometrically projects any vector onto the orthogonal compliment of x.



- $P_x^T = P_x$ and $P_x^2 = P_x$.
- P_x is positive semi-definite.
- $\mathrm{Null}(P_x) = \mathrm{span}\{x\}$ and the eigenvalues of P_x are $\{0, 1^{(d-1)}\}$.

◦ **Localizability Analysis: A Least-Squares formulation**

Nonlinear algebraic problem:

$$\begin{cases} \dfrac{\hat{p}_j - \hat{p}_i}{\|\hat{p}_j - \hat{p}_i\|} = g_{ij}, & \forall (i,j) \in \mathcal{E}, \\ \hat{p}_i = p_i, & \forall i \in \mathcal{V}_a. \end{cases}$$

$\Longleftrightarrow$ Linear algebraic problem:

$$\begin{cases} P_{g_{ij}}(\hat{p}_j - \hat{p}_i) = 0, & \forall (i,j) \in \mathcal{E}, \\ \hat{p}_i = p_i, & \forall i \in \mathcal{V}_a. \end{cases}$$

$\Longleftrightarrow$ Least-squares problem:

$$\begin{aligned} \underset{\hat{p} \in \mathbb{R}^{dn}}{\text{minimize}} \quad & J(\hat{p}) = \frac{1}{2} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{N}_i} \|P_{g_{ij}}(\hat{p}_i - \hat{p}_j)\|^2, \\ \text{subject to} \quad & \hat{p}_i = p_i, \quad i \in \mathcal{V}_a. \end{aligned}$$

◦ **Localizability Analysis: A Least-Squares formulation**

$\Longleftrightarrow$ Least-squares problem:

$$\begin{aligned} \underset{\hat{p}\in\mathbb{R}^{dn}}{\text{minimize}} \quad & J(\hat{p}) = \hat{p}^T \mathcal{L} \hat{p} \\ \text{subject to} \quad & \hat{p}_a = p_a \end{aligned}$$

where $\mathcal{L} \in \mathbb{R}^{dn\times dn}$ and the ijth subblock matrix of $\mathcal{L}$ is

$$[\mathcal{L}]_{ij} = \begin{cases} \mathbf{0}_{d\times d}, & i \neq j, (i,j) \notin \mathcal{E}, \\ -P_{g_{ij}}, & i \neq j, (i,j) \in \mathcal{E}, \\ \sum_{k\in\mathcal{N}_i} P_{g_{ik}}, & i = j, i \in \mathcal{V}. \end{cases}$$

The matrix $\mathcal{L}$ can be viewed as a matrix-weighted Laplacian matrix. We call $\mathcal{L}$ the bearing Laplacian since it carries the information of both the bearings and the underlying graph of the network. The bearing Laplacian $\mathcal{L}$ can be partitioned into

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_{aa} & \mathcal{L}_{af} \\ \mathcal{L}_{fa} & \mathcal{L}_{ff} \end{bmatrix},$$

where $\mathcal{L}_{aa} \in \mathbb{R}^{dn_a\times dn_a}$, $\mathcal{L}_{af} = \mathcal{L}_{fa}^T \in \mathbb{R}^{dn_a\times dn_f}$, and $\mathcal{L}_{ff} \in \mathbb{R}^{dn_f\times dn_f}$.

◦ Localizability Analysis: necessary and sufficient conditions

$\iff$ Unconstrained Least-Squares problem:

$$\min_{\hat{p}_f \in \mathbb{R}^{dn_f}} \tilde{J}(\hat{p}_f) = \hat{p}_f^T \mathcal{L}_{ff} \hat{p}_f + 2 p_a^T \mathcal{L}_{af} \hat{p}_f + p_a^T \mathcal{L}_{aa} p_a.$$

The Least-Squares problem has a unique global minimizer if and only if $\mathcal{L}_{ff}$ is nonsingular. The global minimizer is

$$\hat{p}_f^* = -\mathcal{L}_{ff}^{-1} \mathcal{L}_{fa} p_a = p_f.$$

Theorem (Algebraic Condition for Localizability)

A network $\mathcal{G}(p)$ is localizable if and only if the matrix $\mathcal{L}_{ff}$ is nonsingular.

More results can be found in

- S. Zhao and D. Zelazo. Bearing rigidity and almost global bearing-only formation stabilization. *IEEE Transactions on Automatic Control*, 2015a. to appear (available at arXiv:1408.6552)
- S. Zhao and D. Zelazo. Bearing-only network localization: localizability, sensitivity, and distributed protocols. *Automatica*, 2015b. under review (available at arXiv:1502.00154)

◦ **Localizability Analysis: examples**

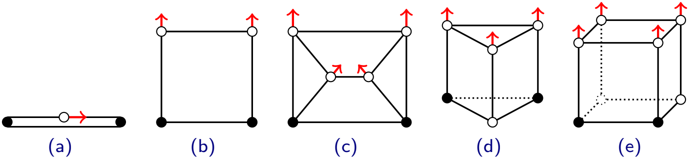


Figure: Examples of *non-localizable* networks.

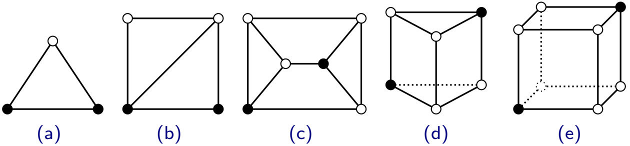


Figure: Examples of *localizable* networks.

◦ Distributed Localization Protocol

The global minimizer of $\tilde{J}(\hat{p}_f) = \hat{p}_f^T \mathcal{L}_{ff}\hat{p}_f + 2p_a^T \mathcal{L}_{af}\hat{p}_f + p_a^T \mathcal{L}_{aa}p_a$ can be solved by the gradient decent protocol

$$\dot{\hat{p}}_f(t) = -\nabla_{\hat{p}_f}\tilde{J}(\hat{p}_f) = -\mathcal{L}_{ff}\hat{p}_f(t) - \mathcal{L}_{fa}p_a,$$

whose elementwise expression is

$$\dot{\hat{p}}_i(t) = -\sum_{j\in\mathcal{N}_i} P_{g_{ij}}(\hat{p}_i(t) - \hat{p}_j(t)), \quad i \in \mathcal{V}_f.$$

where $P_{g_{ij}} = I_d - g_{ij}g_{ij}^T$.

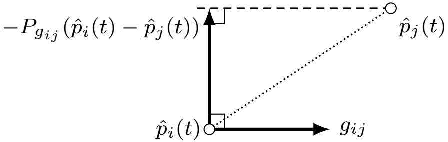


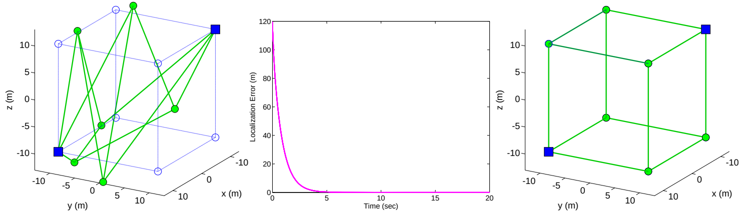
Figure: The geometric interpretation of the localization protocol.

Theorem

The protocol can globally localize a network if and only if the network is localizable.

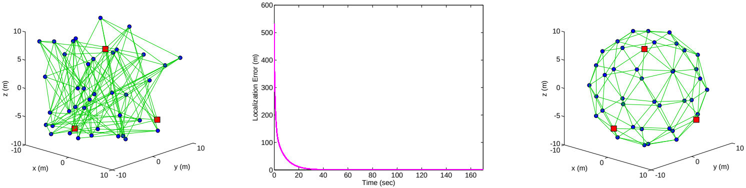
Bearing-Only Network Localization

◦ **Distributed Localization Protocol: simulation examples**



(a) Initial estimate (b) $\sum_{i\in\mathcal{V}_f} \|\hat{p}_i(t) - p_i\|$ (c) Final estimate

Figure: A simulation example to demonstrate the localization protocol.

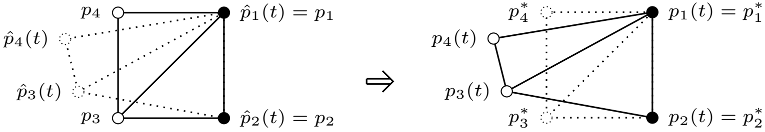


(a) Initial estimate (b) $\sum_{i\in\mathcal{V}_f} \|\hat{p}_i(t) - p_i\|$ (c) Final estimate

Figure: A simulation example to demonstrate the localization protocol.

Problem Statement

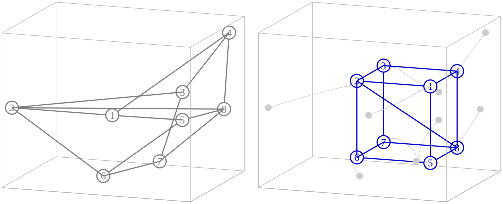
- The target formation is defined by the constant bearing constraints $\{g_{ij}^*\}_{(i,j)\in\mathcal{E}}$ and the leaders $\{p_i^*\}_{i\in\mathcal{V}_\ell}$ where $\mathcal{V}_\ell$ is the index set of the leaders
- The control objective is to steer $p_i(t) \to p_i^*$ where $i \in \mathcal{V}_f$ and p_i^* is the position in the target formation



(a) Network localization (b) Formation control

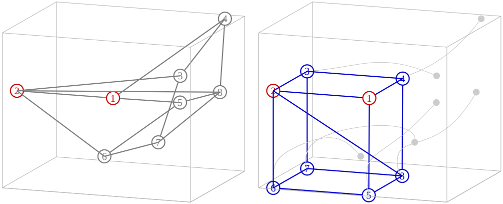
$$\dot{\hat{p}}_i(t) = -\sum_{j\in\mathcal{N}_i} P_{g_{ij}}(\hat{p}_i(t) - \hat{p}_j(t)) \quad\Longrightarrow\quad \dot{p}_i(t) = -\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}(p_i(t) - p_j(t))$$

◦ Simulation Examples



(a) Initial formation (b) Final formation

Figure: A 3D example for bearing-based formation control: leaderless case.



(a) Initial formation (b) Final formation

Figure: A 3D example for bearing-based formation control: leader-follower case. Agents in red are fixed leaders.

◦ **Latest Results:**

- S. Zhao and D. Zelazo. Translational and scaling formation maneuver control via a bearing-based approach. *IEEE Transactions on Control of Network Systems*, 2015c. under review (available at arXiv:1506.05636)
- S. Zhao and D. Zelazo. Bearing-based formation maneuvering. In *Proceedings of the 2015 IEEE Multi-Conference on Systems and Control*. under review (available at arXiv:1504.03517)

There are many formation control approaches. Why bearing-based formation control?

- Bearing-based formation control provides a natural solution to formation scale control

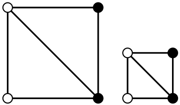


Figure: The bearing constraints are invariant to the formation scale.

- **Latest Results:**

- Single integrator dynamics:

$$\dot{p}_i(t) = -\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}\left[k_p(p_i(t)-p_j(t)) + k_I\int_0^t (p_i(\tau)-p_j(\tau))\mathrm{d}\tau\right]$$

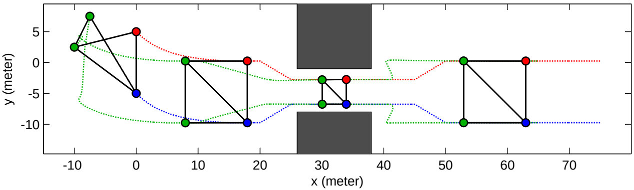
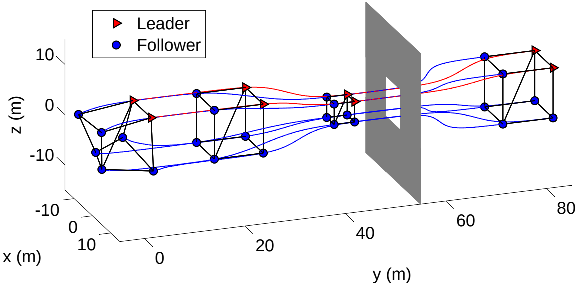


Figure: Leaders: red and blue circles; followers: circles

- **Latest Results:**

- Double integrator dynamics:

$$\dot{p}_i(t) = v_i(t), \quad \dot{v}_i(t) = -\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}\left[k_p(p_i(t)-p_j(t)) + k_v(v_i(t)-v_j(t))\right]$$



Conclusions

Main contributions:

- Bearing-Only Network Localization
  - When a network is localizable?
  - How to distributedly localize a network?
- Bearing-Based Formation Control

Compared to the existing studies, the proposed results are applicable to networks/formations in arbitrary dimensions.

Limitations and future work:

- The underlying graph is undirected/bidirectional. The directed case should be studied in the future.

Thank you!

Q & A

S. Zhao and D. Zelazo. Bearing-based formation maneuvering. In *Proceedings of the 2015 IEEE Multi-Conference on Systems and Control*. under review (available at arXiv:1504.03517).

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