Distance-Constrained Formation Tracking Control

Oshri Rozenheck
Technion Autonomous Systems Program

Shiyu Zhao, Daniel Zelazo
Faculty of Aerospace Engineering

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Motivation

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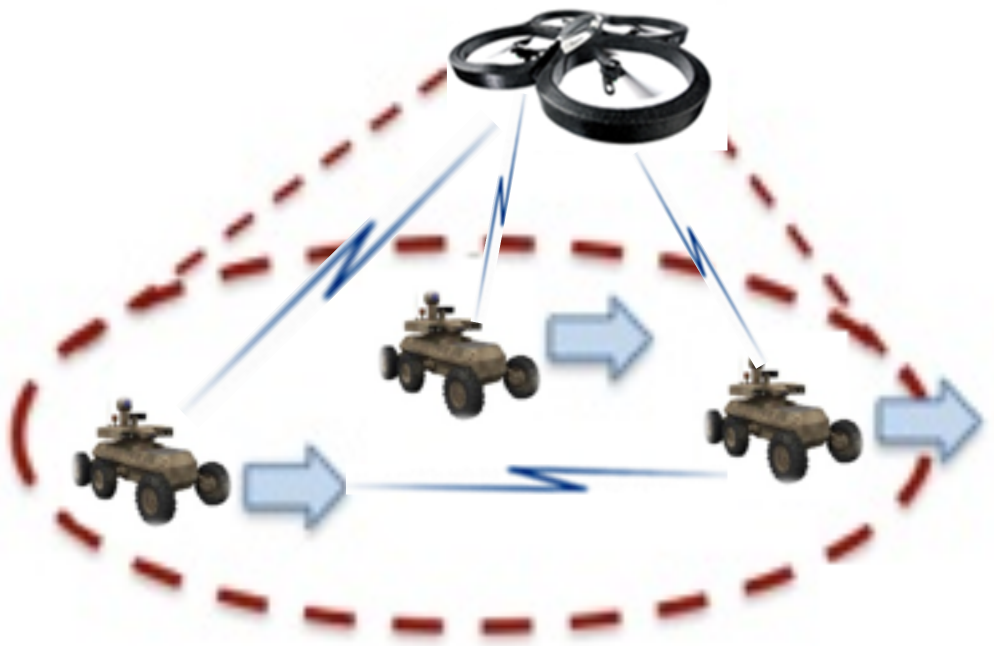
- A team of autonomous surface vehicles (ASV) tasked with exploring unknown terrain
- A scout drone provides team leader with waypoints of interest
- Vehicles should maintain a specified spatial formation

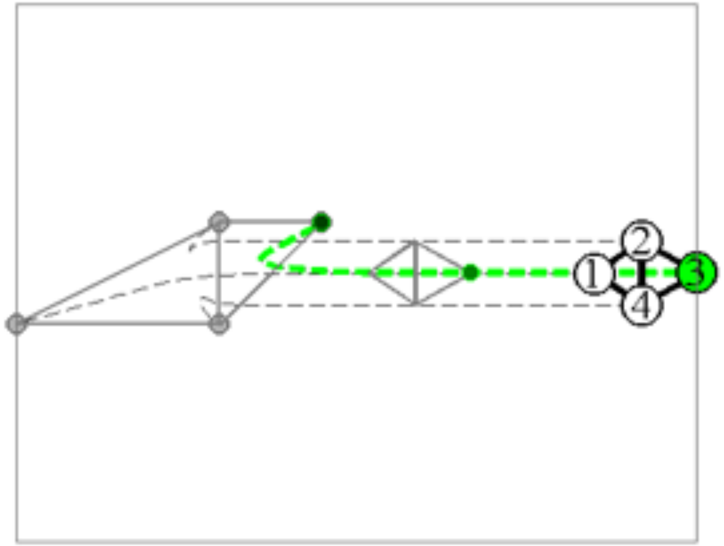


Formation Objective: Follow the leader while maintaining the desired formation

Distance-Constrained Formation Tracking Control

Formation Objective: Follow the leader while maintaining the desired formation

- Formation Stabilization
- Formation Tracking



Formation Stabilization

- a team of kinematic point masses

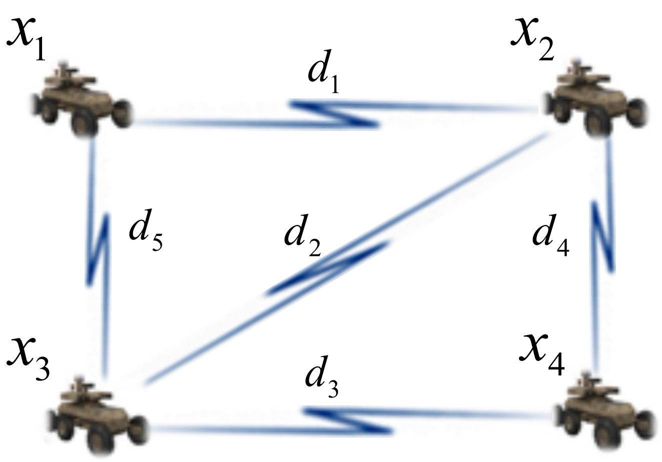
$$\dot{x}_i = u_i,\ i = 1, \ldots, n$$
$$x_i, u_i \in \mathbb{R}^2$$

- a sensing and communication network

$$\mathcal{G} = (\mathcal{V}, \mathcal{E})$$

  - agents can sense relative positions and distances of neighbors

$$e_k = x_j - x_i \quad \text{edge k} = ij \in \mathcal{E}$$
$$\|e_k\|^2$$



the formation is specified by a *distance constraint vector*

$$d = \begin{bmatrix} d_1^2 & \cdots & d_{|\mathcal{E}|}^2 \end{bmatrix}^T$$

Formation Stabilization

Theorem [Krick 2007, Anderson 2008, Dimarogonas 2008, Dörfler 2010]

Under the distributed control

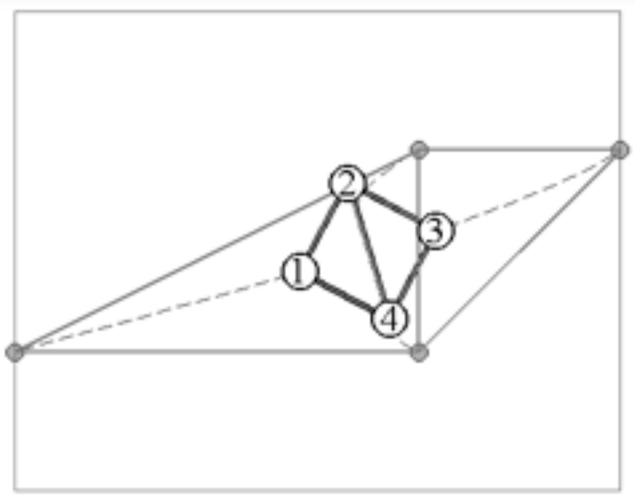
$$u_i = \sum_{j \sim i} \left(\|e_k\|^2 - d_k^2\right) e_k$$

$$e_k = x_j - x_i$$

the set

$$\mathcal{F}(x) = \left\{x \in \mathbb{R}^{2n} \mid \|e_k\|^2 - d_k^2 = 0, k = 1, \ldots, |\mathcal{E}|\right\}$$

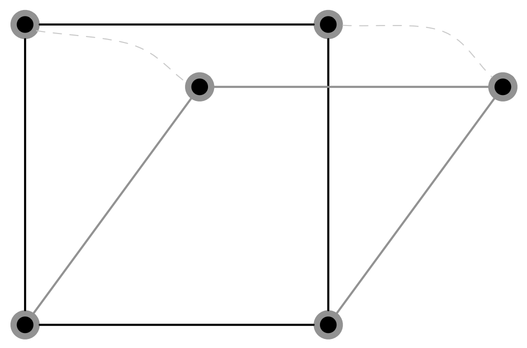
is locally and asymptotically stable.



Rigidity Theory

What are the minimum number of distance constraints required to ensure the *correct* formation shape?

$$\mathcal{F}(x) = \{x \in \mathbb{R}^{2n} \mid \|e_k\|^2 - d_k^2 = 0, k = 1, \dots, |\mathcal{E}|\}$$

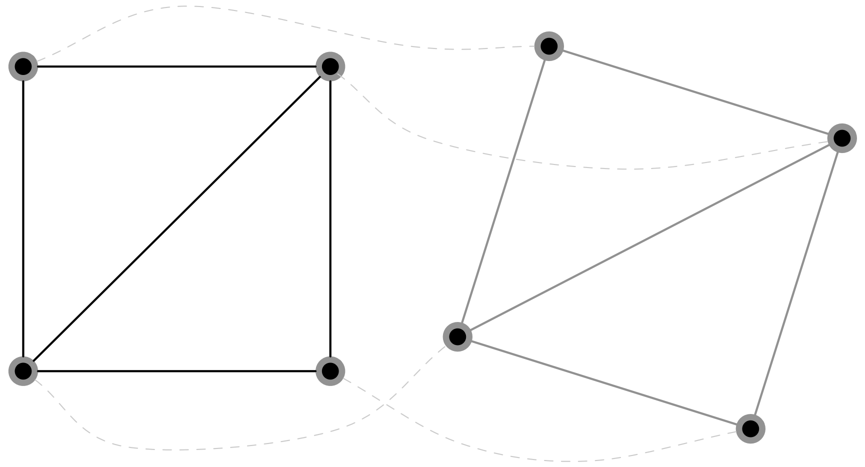


Edge function

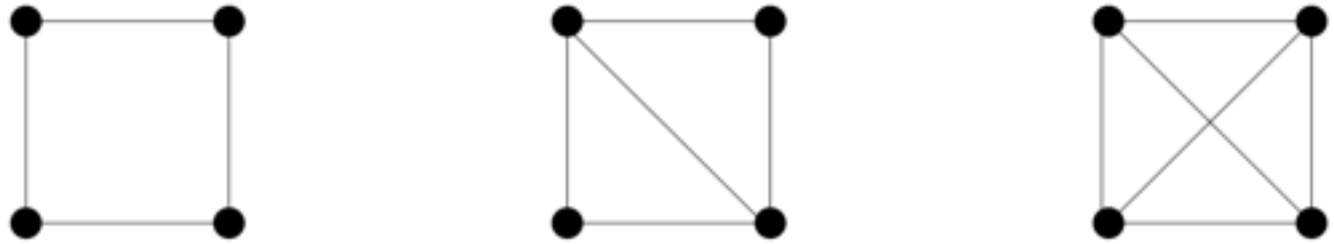
$$F(x) = \frac{1}{2}\begin{bmatrix} \|e_1\|^2 \\ \vdots \\ \|e_{|\mathcal{E}|}\|^2 \end{bmatrix}$$

Rigidity Matrix

$$R(x) = \frac{\partial F(x)}{\partial x} \in \mathbb{R}^{|\mathcal{E}| \times 2n}$$



Rigidity Theory



Not Rigid **Minimally Rigid** **Rigid**

- A framework is *infinitesimally rigid* if and only if $\mathrm{rk}[R(x)] = 2n - 3$
- A framework is *minimally rigid* if the removal of any edge leads to a non-rigid framework (i.e., $|\mathcal{E}| = 2n - 3$)
- A framework is *minimally infinitesimally rigid (MIR)* if it is both minimally rigid and infinitesimally rigid (i.e., $\mathrm{rk}[R(x)] = |\mathcal{E}| = 2n - 3$)

Formation Stabilization

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distributed control

$$u_i = \sum_{j \sim i} \left(\|e_k\|^2 - d_k^2\right) e_k$$

$$e_k = x_j - x_i$$

closed-loop dynamics

$$\dot{x} = -R(x)^T \underbrace{\left(R(x)x - d\right)}_{\delta}$$

distance error vector

$$\delta_k = \|e_k\|^2 - d_k^2$$

$$R(x) = \underbrace{\begin{bmatrix} \ddots & & \\ & e_k^T & \\ & & \ddots \end{bmatrix}}_{\text{diag}(e_k^T)} (E^T(\mathcal{G}) \otimes I_2)x$$

Distance Error Dynamics

$$\dot{\delta} = -2R(x)R(x)^T\delta$$

Formation Stabilization

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Theorem

For any minimally infinitesimally rigid framework, the formation error dynamics is locally exponentially stable.

Proof (sketch)

$$\delta = 0 \rightarrow x = x^*$$

Linearization of error dynamics

MIR → $R(x)$ full row rank

$$\dot{\tilde{\delta}} = -2\,\underbrace{R(x^*)R(x^*)^T}_{M(x^*)}\,\tilde{\delta}$$

→ $M(x^*)$ positive definite

Distance-Constrained Formation Tracking Control

Formation Objective: Follow the leader while maintaining the desired formation

- One agent is injected with external velocity reference command

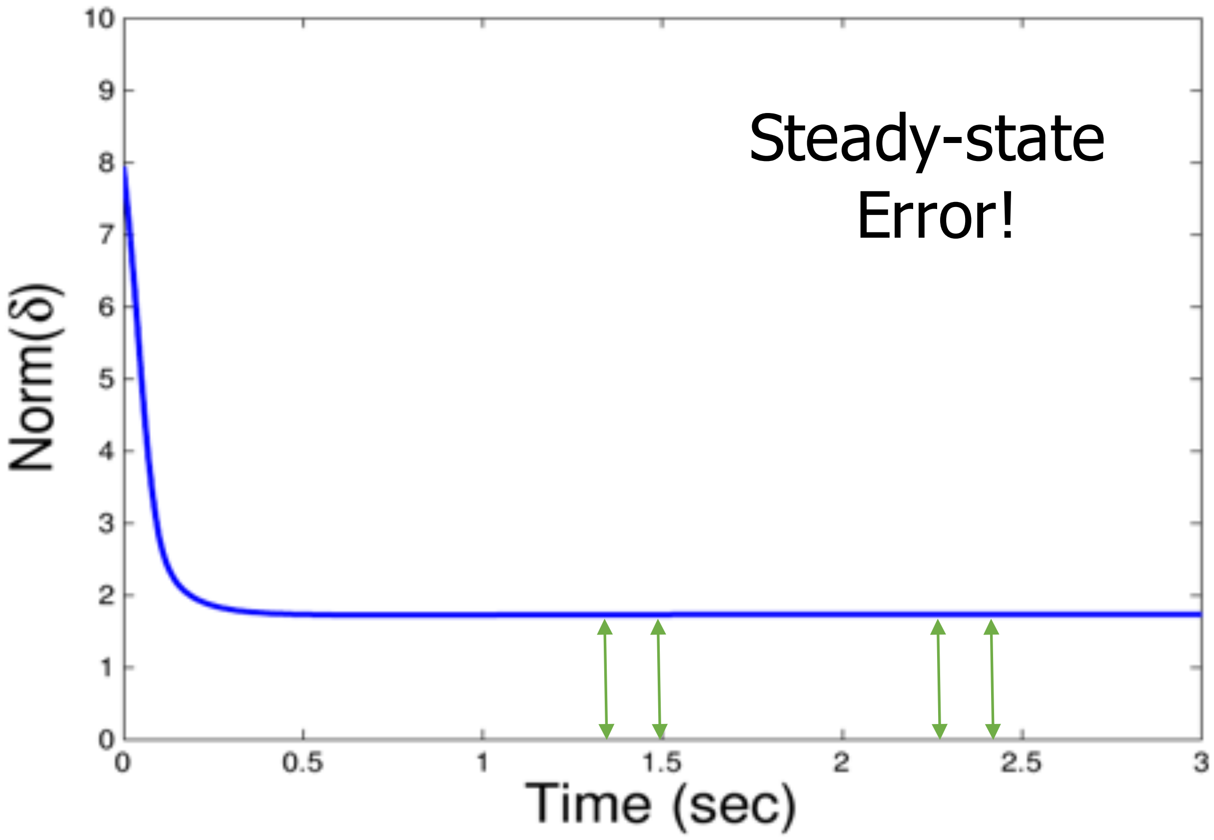
$$\dot{x}_1 = u_1 + v_{ref}$$

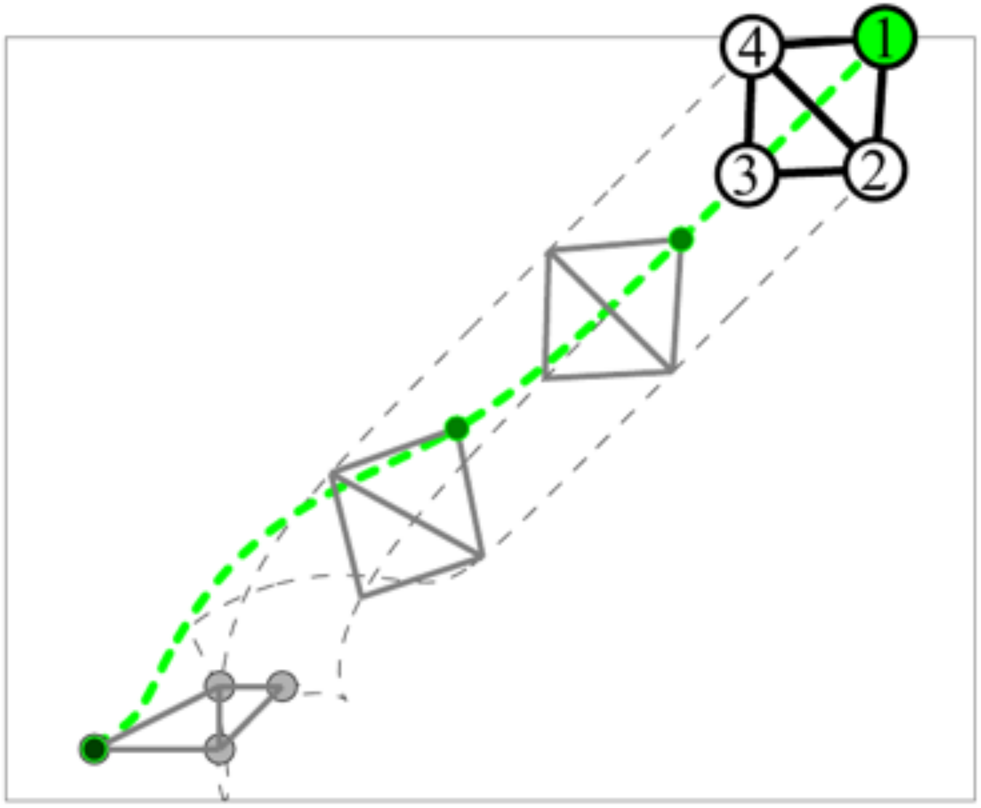
- Closed-loop dynamics

$$\dot{x} = -R(x)^T \left(R(x)x - d\right) + Bv_{ref}$$

$$B = \begin{bmatrix} I \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Distance-Constrained Formation Tracking Control





$$\dot{x} = -R(x)^T \left(R(x)x - d\right) + Bv_{ref}$$

Steady-state Formation Error

Theorem

For a constant leader reference velocity, the steady-state formation error of the linearized error dynamics,

$$\dot{\tilde{\delta}} = -2R(x^*)R(x^*)^T\tilde{\delta} + 2R(x^*)B\mathrm{v}$$

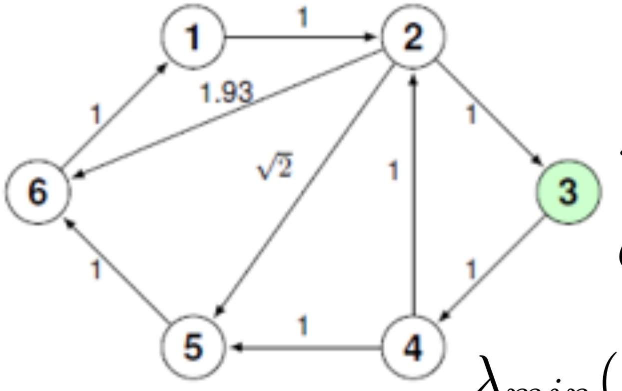
is

$$\lim_{t\to\infty}\tilde{\delta}(t) = \left(R(x^*)R(x^*)^T\right)^{-1}R(x^*)B\mathrm{v}$$

and is bounded as

$$\|\tilde{\delta}(\infty)\| \leq \frac{\sqrt{d_{max}\lambda_{max}(L(\mathcal{G}))}}{\lambda_{min}(R(x^*)R(x^*)^T)}\|\mathrm{v}\|$$

$$v = 0.1 m/sec$$

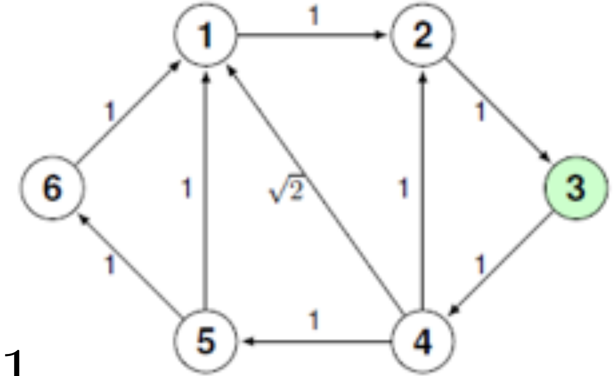


(a) Graph 1.

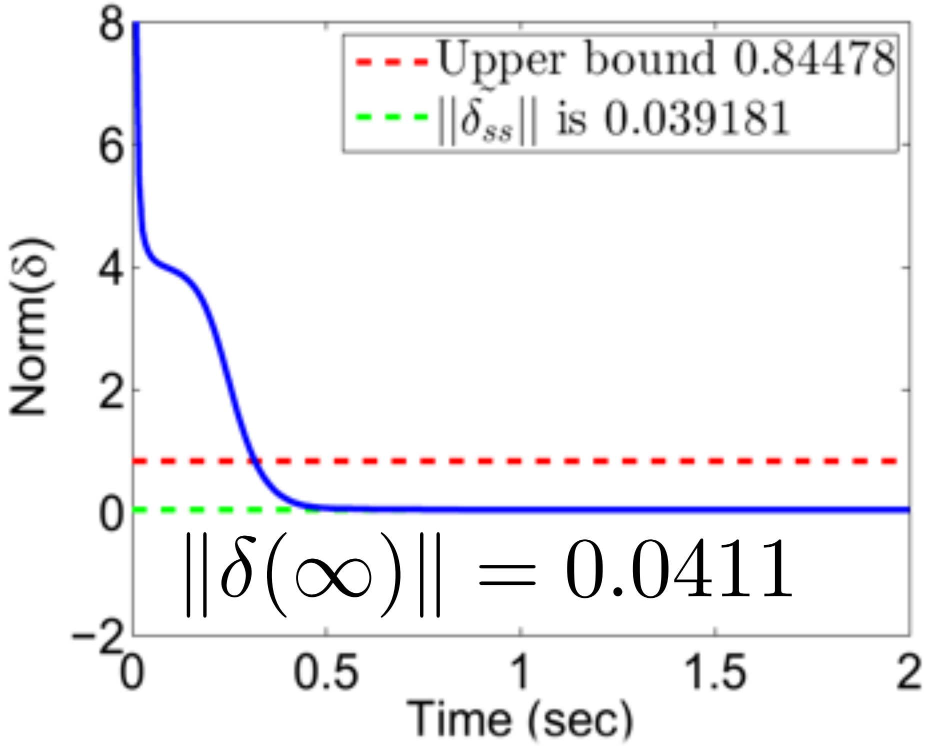
$$\lambda_{max}(\mathcal{G}_1) = 6 \qquad \lambda_{max}(\mathcal{G}_2) = 5.343$$

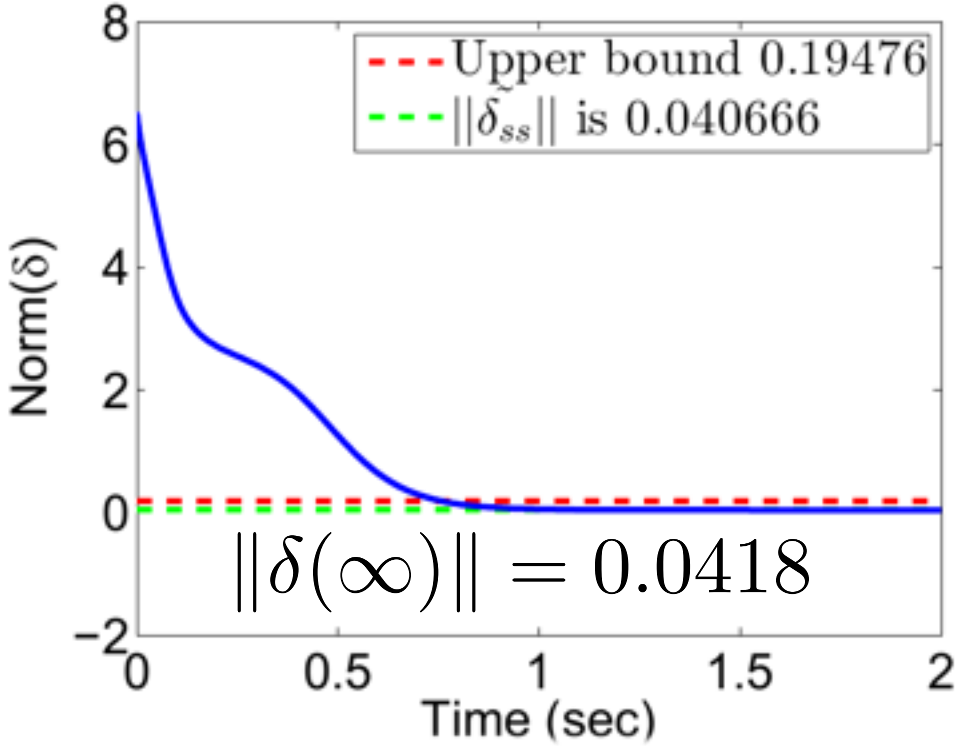
$$d_{max} = 1.93 \qquad d_{max} = \sqrt{2}$$

$$\lambda_{min}(M(x^*)) = 0.402 \qquad \lambda_{min}(M(x^*)) = 1.411$$

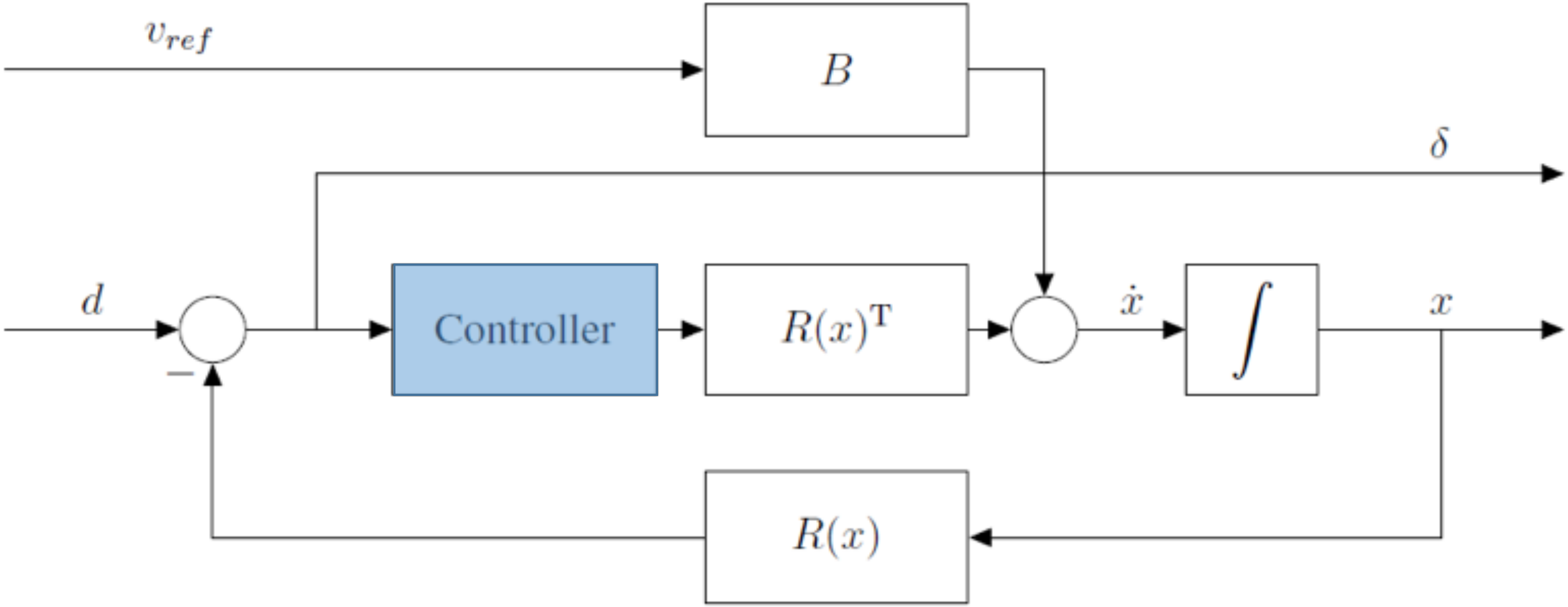


(b) Graph 2.





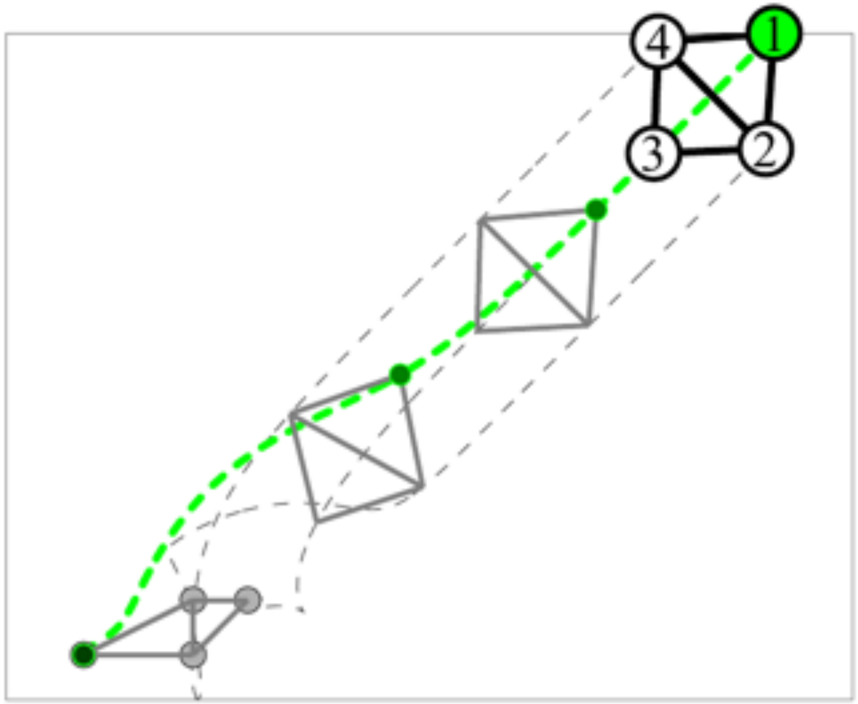
Eliminate the Steady-State Error

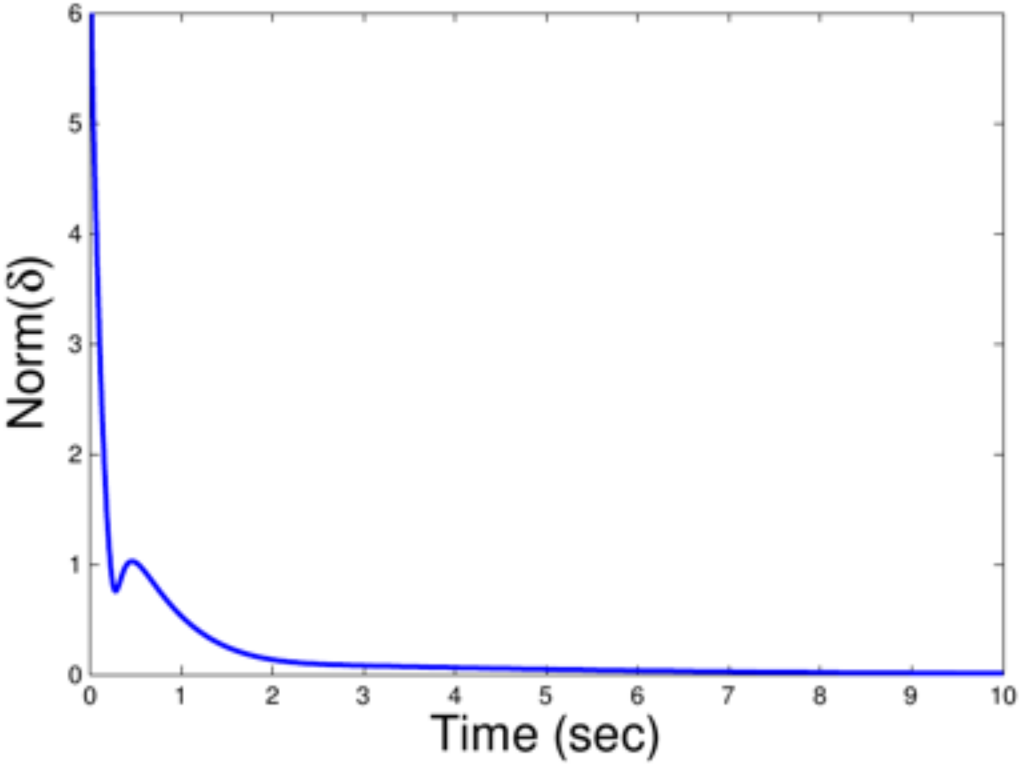


A Proportional-Integral Control

$$\dot{x} = -\kappa_p R(x)^T \left(R(x)x - d\right) - \kappa_I R(x)^T \int_0^t \left(R(x)x - d\right) d\tau$$

$$\kappa_P, \kappa_I > 0$$





A Proportional-Integral Control

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Distance Error Dynamics with PI Control

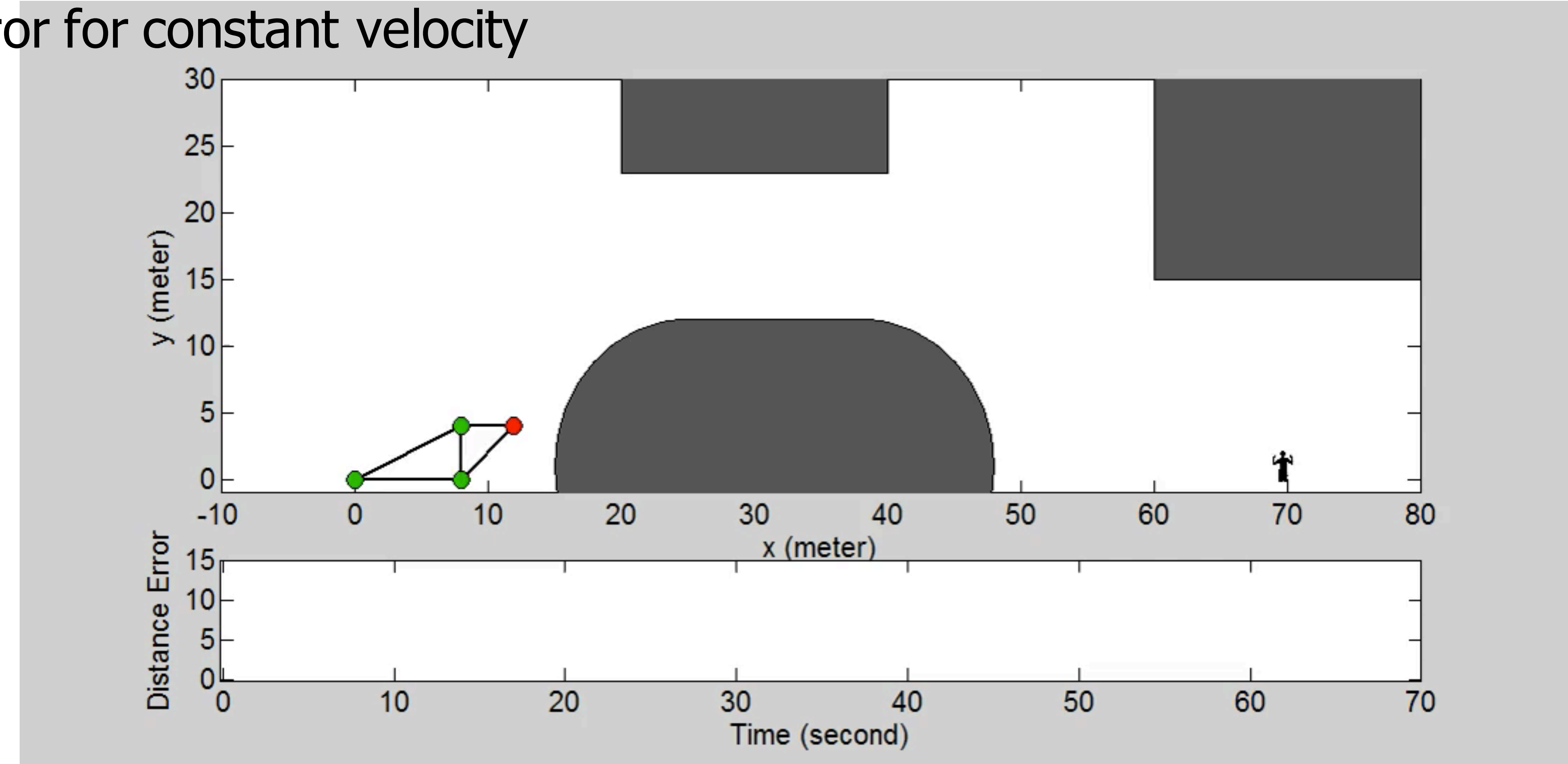
$$\begin{bmatrix} \dot{\delta} \\ \dot{\zeta} \end{bmatrix} = \begin{bmatrix} -2\kappa_p M(x) & 2I - M(x) \\ \kappa_I I & 0 \end{bmatrix} \begin{bmatrix} \delta \\ \zeta \end{bmatrix} + \begin{bmatrix} -2R(x)B \\ 0 \end{bmatrix} v_{ref}$$

Theorem

For any minimally infinitesimally rigid framework, and for $\kappa_P, \kappa_I > 0$ the origin of the zero-input error dynamics is locally asymptotically stable

A Proportional-Integral Control

The PI control scheme can ensure zero steady-state error for constant velocity references



Conclusions

- Velocity tracking problem for formations
- Steady-state error bounds
- A PI control solution

Acknowledgements

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Oshri Rozenheck

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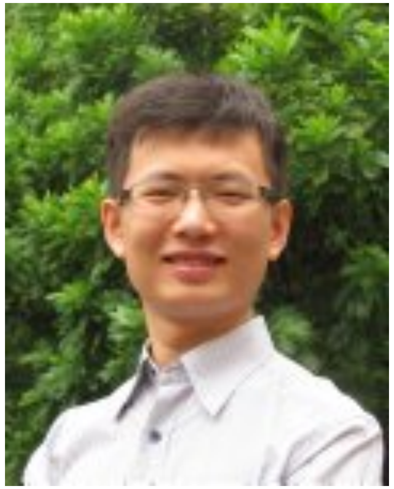
ISRAEL SCIENCE FOUNDATION



Shiyu Zhao

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