

# A CHARACTERIZATION OF ALL LINEAR PASSIVIZING INPUT-OUTPUT TRANSFORMATIONS OF A PASSIVE-SHORT SYSTEM: THE SISO CASE

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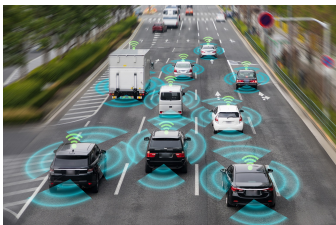
Miel Sharf (Jether Energy) and **Daniel Zelazo**

December 19, 2024

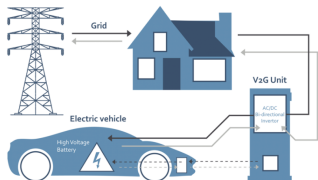
IEEE Conference on Decision and Control



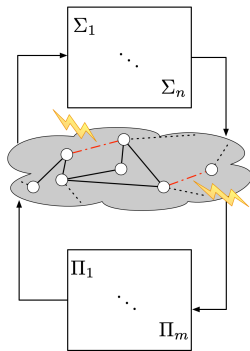
# OPEN MULTI-AGENT SYSTEMS



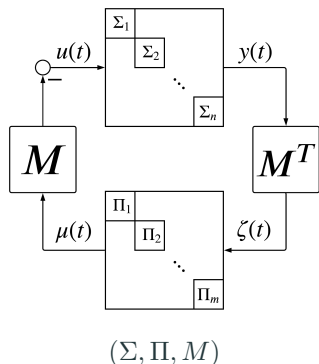
network of self-driving cars



smart-grid with EV integration



Resilience and robustness of network systems required for safe operations



## Network Interconnection

- Network is encoded by a matrix

$$M \in \mathbb{R}^{n \times m}$$

- $[M]_{ij} = \begin{cases} \star, & \text{controller } j \text{ access to agent } i \\ 0, & \text{otherwise} \end{cases}$

## A Stability Result

The stability of the dynamic network  $(\Sigma, \Pi, M)$  can be guaranteed for output-strictly passive agent dynamics  $\Sigma_i$  and passive controller dynamics  $\Pi_e$ .

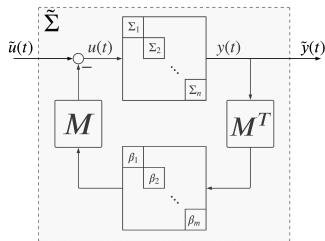
[Corollary of B&Z 2014]

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- ▶ what if this cannot be guaranteed?

# PASSIVATION BY THE NETWORK

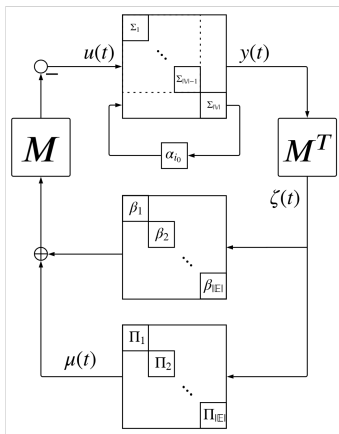
- ▶ stability result requires a **passivity** property to hold
- ▶ what if this cannot be guaranteed?



- ▶  $\rho_i$  is **passivity index** of each agent
  - $\rho_i = 0$  : passive
  - $\rho_i > 0$  : strictly output-passive
  - $\rho_i < 0$  : **output passive short**
- ▶  $R = \text{diag}(\rho_1, \dots, \rho_n)$

- ▶ network can be used to passivify agents [Belabbas, Chen, Z 2023]

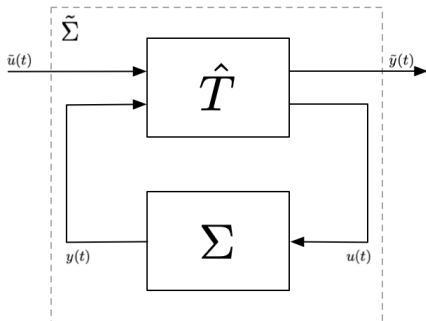
# PASSIVATION BY THE NETWORK



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- ▶  $R = \text{diag}(\rho_1, \dots, \rho_n)$

▶ a single agent can be used to passively entire network [Sharf, Z 2019]

## PASSIVATION GOALS



- ▶ how do we passivise a dynamical system?
  - feedback passivation
  - loop-transformations (classic)
- ▶ can we passivise a system to achieve arbitrary passivity indices?
- ▶ can we characterize **all transformations** that map a system with given passivity index to a system with prescribed passivity index?



## Definition

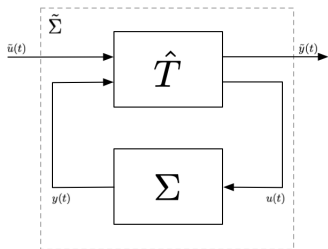
Let  $\Sigma$  be a SISO system with a constant input-output steady-state pair  $(u, y)$ . The system is said to be **input-output  $(\rho, \nu)$ -passive** wrt  $(u, y)$  if there exists a  $C^1$  positive semi-definite storage function  $S(x)$  and numbers  $\rho, \nu \in \mathbb{R}$ , such that  $\rho\nu < 1/4$  and

$$\dot{S} = \frac{\partial S}{\partial x} f(x, u) \leq (y - y)(u - u) - \rho(y - y)^2 - \nu(u - u)^2,$$

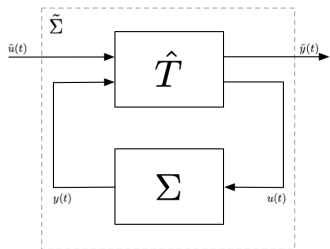
for any trajectory  $u, y$ .

- ▶  $\rho = \nu = 0 \Rightarrow$  **passivity**
- ▶  $\rho, \nu > 0 \Rightarrow$  **strict input/output passivity**
- ▶  $\rho, \nu < 0 \Rightarrow$  **passive short**

## FEEDBACK PASSIVATION



For a passive-short system  $\Sigma : u \mapsto y$ , we aim to find a map  $\hat{T}$  such that the closed-loop system  $\tilde{\Sigma} : \tilde{u} \mapsto \tilde{y}$  is passive. This is known as **feedback passivation**.



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## Problem Statement

Find **all** I/O transformations  $\hat{T}$  that map  $\Sigma$  from a I/O  $(\rho, \nu)$ -passive to a I/O  $(\rho_*, \nu_*)$ -passive system.

Consider the following system:

$$\begin{aligned}\dot{x} &= -\sqrt[3]{x} + 0.5x + 0.5u \\ y &= 0.5x - 0.5u\end{aligned}$$

the system is **passive-short**

$$S(x) = \frac{1}{6}x^2$$

$$\dot{S} = yu + \frac{2}{3}y^2 + \frac{1}{3}u^2 - \frac{1}{3}(2y + u)\sqrt[3]{2y + u} \leq yu + \frac{2}{3}y^2 + \frac{1}{3}u^2$$

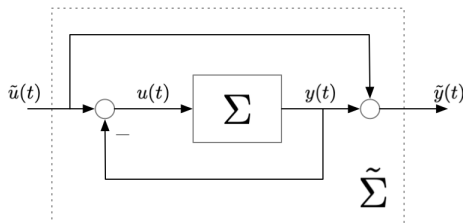
system has  $\rho = -2/3, \nu = -1/3$

## A RUNNING EXAMPLE

we can consider the following transformation:

$$\begin{cases} u(t) &= \tilde{u}(t) - y(t) \\ \tilde{y}(t) &= u(t) + y(t) \end{cases}$$

$$\Rightarrow \begin{bmatrix} u(t) \\ \tilde{y}(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y(t) \\ \tilde{u}(t) \end{bmatrix}$$



yields the transformed system

$$\dot{x} = -\sqrt[3]{x} + \tilde{u}$$

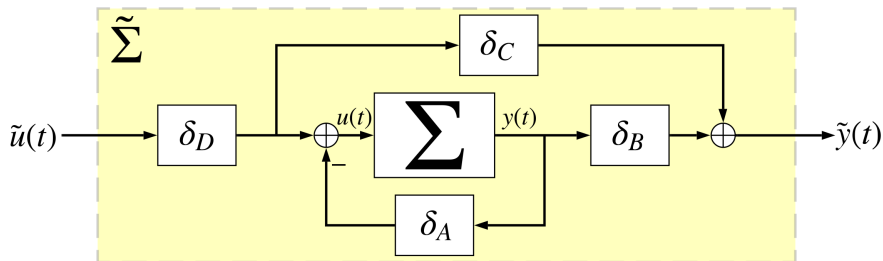
$$\tilde{y} = x$$

which is passive with storage function  $S(x) = \frac{1}{2}x^2$  satisfying

$$\dot{S}(x) = \tilde{y}\tilde{u} - \tilde{y}\sqrt[3]{\tilde{y}} \leq \tilde{y}\tilde{u}$$

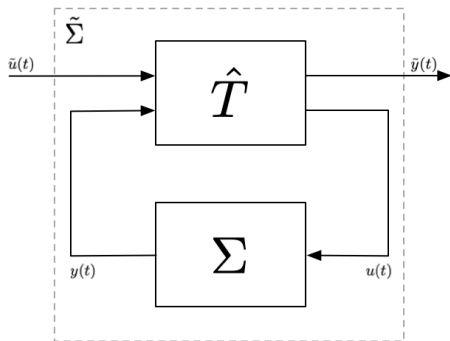
# LOOP TRANSFORMATIONS

The **loop transformation**, combination of feedback, feedforward, pre-, and post-multiplication is the classic approach to feedback passivation



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for this work, we prefer to consider the map  $T : \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \mapsto \begin{bmatrix} \tilde{u}(t) \\ \tilde{y}(t) \end{bmatrix}$

(always possible when  $\hat{T}_{21}$ -block is invertible)

A geometric approach to finding our map  $T$ ...

### Projective Quadratic Inequalities

A *projective quadratic inequality (PQI)* is an inequality with variables  $\xi, \chi \in \mathbb{R}$  of the form

$$0 \leq a\xi^2 + b\xi\chi + c\chi^2 = \mathbf{f}_{(a,b,c)}(\xi, \chi),$$

for some numbers  $a, b, c$ , not all zero.

- ▶ inequality is called *non-trivial* if  $b^2 - 4ac > 0$
- ▶  $\mathcal{C}_{\xi, \chi}$ : solution set of the PQI, all points  $(\xi, \chi) \in \mathbb{R}^2$  satisfying the inequality



PQI:

$$0 \leq a\xi^2 + b\xi\chi + c\chi^2 = \mathbf{f}_{(a,b,c)}(\xi, \chi),$$

recall our definition for I/O  $(\rho, \nu)$ -passivity

$$\dot{S} \leq yu - \rho y^2 - \nu u^2$$

PQI captures passivity

$$\dot{S} \leq \mathbf{f}_{(-\nu, 1, -\rho)}(u, y)$$

Solution set

$$\mathcal{C}_{\rho, \nu} = \{(\xi, \chi) \in \mathbb{R} \times \mathbb{R} : \mathbf{f}_{(-\nu, 1, -\rho)}(\xi, \chi) \geq 0\}$$

- ▶ we are interested in maps  $T : \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \mapsto \begin{bmatrix} \tilde{u}(t) \\ \tilde{y}(t) \end{bmatrix}$
- ▶ original system has a PQI solution set  $\mathcal{C}_{\rho, \nu}$  for some  $(\rho, \nu)$
- ▶ transformed system has PQI solution set  $\mathcal{C}_{\rho^*, \nu^*}$  for some  $(\rho^*, \nu^*)$

## GEOMETRIC UNDERSTANDING OF PQIS

- ▶ we are interested in maps  $T : \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \mapsto \begin{bmatrix} \tilde{u}(t) \\ \tilde{y}(t) \end{bmatrix}$
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- ▶ transformed system has PQI solution set  $\mathcal{C}_{\rho^*, \nu^*}$  for some  $(\rho^*, \nu^*)$

An I/O transformation  $T$  maps an I/O  $(\rho, \nu)$ -passive system to an I/O  $(\rho_*, \nu_*)$ -passive system if and only if it maps the PQI

$$0 \leq \mathbf{f}_{(-\nu, 1, -\rho)}(\xi, \chi)$$

to the PQI

$$0 \leq \mathbf{f}_{(-\nu^*, 1, -\rho^*)}(\xi, \chi)$$

(or to a stricter inequality)

## EXAMPLE REVISITED

recall our earlier example...

$$\dot{x} = -\sqrt[3]{x} + 0.5x + 0.5u$$

$$y = 0.5x - 0.5u$$

satisfies

$$\frac{1}{3}\chi^2 + \chi\xi + \frac{2}{3}\xi^2 = \mathbf{f}_{(1/3,1,2/3)}(\xi, \chi) \geq 0$$

we considered the transformation  $\begin{bmatrix} \tilde{\chi} \\ \tilde{\xi} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \chi \\ \xi \end{bmatrix}$

transformed system satisfies some PQI

$$a\tilde{\chi}^2 + b\tilde{\chi}\tilde{\xi} + c\tilde{\xi}^2 \geq 0$$

## EXAMPLE REVISITED

we should recover original PQI by expressing in original coordinates

$$\begin{aligned}0 &\leq a\tilde{\chi}^2 + b\tilde{\chi}\tilde{\xi} + c\tilde{\xi}^2 \\ &= a(\chi + \xi)^2 + b(\chi + \xi)(\chi + 2\xi) + c(\chi + 2\xi)^2 \\ &= (a + b + c)\chi^2 + (2a + 3b + 4c)\chi\xi + (a + 2b + 4c)\xi^2\end{aligned}$$

solving for  $(a, b, c)$  using

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ 1 \\ \frac{2}{3} \end{bmatrix}$$

gives  $a = c = 0$ ,  $b = 1/3$  implying that

$$0 \leq \frac{1}{3}\tilde{\chi}\tilde{\xi}$$

i.e., the transformed system is passive

### main idea

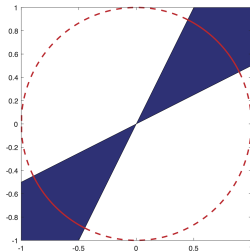
Let  $\mathcal{A}$  be the solution set of the original PQI. The solution set of the new PQI under the transformation  $T$  is

$$T(\mathcal{A}) = \{T(\chi, \xi) : (\chi, \xi) \in \mathcal{A}\}.$$

We can therefore study the effect of linear transformations on PQIs by studying their actions on the solution sets.

## A GEOMETRIC APPROACH

The solution set of any non-trivial PQI is a symmetric double-cone. Moreover, any symmetric double-cone is the solution set of some non-trivial PQI.



### Theorem\*

[Sharf, Jain, Z 2021]

Let  $(\xi_1, \chi_1), (\xi_2, \chi_2)$  be non-colinear solutions of  $a_1\xi^2 + \xi\chi + c_1\chi^2 = 0$ , and  $(\tilde{\xi}_1, \tilde{\chi}_1), (\tilde{\xi}_2, \tilde{\chi}_2)$  be non-colinear solutions of  $a_2\xi^2 + \xi\chi + c_2\chi^2 = 0$ .

Define

$$T_1 = \begin{bmatrix} \tilde{\xi}_1 & \tilde{\xi}_2 \\ \tilde{\chi}_1 & \tilde{\chi}_2 \end{bmatrix} \begin{bmatrix} \xi_1 & \xi_2 \\ \chi_1 & \chi_2 \end{bmatrix}^{-1}, T_2 = \begin{bmatrix} \tilde{\xi}_1 & -\tilde{\xi}_2 \\ \tilde{\chi}_1 & -\tilde{\chi}_2 \end{bmatrix} \begin{bmatrix} \xi_1 & \xi_2 \\ \chi_1 & \chi_2 \end{bmatrix}^{-1}.$$

Then one of  $T_1, T_2$  transforms the PQI  $a_1\xi^2 + \xi\chi + c_1\chi^2 \geq 0$  to the PQI  $\tau a_2\xi^2 + \tau\xi\chi + \tau c_2\chi^2 \geq 0$  for some  $\tau > 0$ .

## EXAMPLE CONTINUED

...back to our original system with PQI

$$\frac{1}{3}\chi^2 + \chi\xi + \frac{2}{3}\xi^2 = \mathbf{f}_{(1/3,1,2/3)}(\xi, \chi) \geq 0$$

can be rewritten as

$$\frac{1}{3}(\chi + \xi)(\chi + 2\xi) = 0$$

so two solutions are  $(2, -1), (-1, 1) \in \mathcal{C}_{1/3,2/3}$



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the new PQI satisfies

$$\frac{1}{3}\tilde{\chi}\tilde{\xi} \geq 0$$

with solutions  $(1, 0), (0, 1) \in \mathcal{C}_{0,0}$

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the new PQI satisfies

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with solutions  $(1, 0), (0, 1) \in \mathcal{C}_{0,0}$  applying theorem

$$T_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

i.e., the transformation we found earlier!

### summary

A map  $T$  transforms an I/O  $(\rho, \nu)$ -passive system to an I/O  $(\rho_*, \nu_*)$ -passive system if and only if it sends  $\mathcal{C}_{\rho, \nu}$  into  $\mathcal{C}_{\rho_*, \nu_*}$ , which we denote by  $\mathcal{C}_{\rho, \nu} \hookrightarrow \mathcal{C}_{\rho_*, \nu_*}$

- ▶ earlier theorem gives a characterization for these maps - allows to find a map from one double cone to another double cone
- ▶ we would like to characterize all possible maps

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- ▶ earlier theorem gives a characterization for these maps - allows to find a map from one double cone to another double cone
- ▶ we would like to characterize all possible maps

### main idea

show that all maps from an arbitrary double cone into another arbitrary double cone can be built using maps from  $\mathcal{C}_{0,0}$  into itself

## Proposition

Let  $\rho, \nu, \rho_*, \nu_*$  be any four numbers such that  $\rho\nu, \rho_*\nu_* < 1/4$ , and let  $T : \mathcal{C}_{\rho,\nu} \hookrightarrow \mathcal{C}_{\rho_*,\nu_*}$ . Let  $S_{\rho,\nu} : \mathcal{C}_{0,0} \hookrightarrow \mathcal{C}_{\rho,\nu}$  and  $S_{\rho_*,\nu_*} : \mathcal{C}_{0,0} \hookrightarrow \mathcal{C}_{\rho_*,\nu_*}$  built using Theorem  $\star$ . Then there exists a matrix  $Q : \mathcal{C}_{0,0} \hookrightarrow \mathcal{C}_{0,0}$ , such that  $T = S_{\rho_*,\nu_*} Q S_{\rho,\nu}^{-1}$  holds.

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- ▶ by composition of maps we have

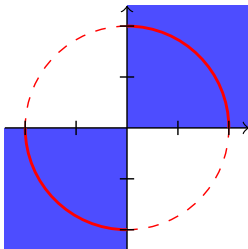
$$Q = S_{\rho_*,\nu_*}^{-1} T S_{\rho,\nu} \Leftrightarrow \mathcal{C}_{0,0} \xrightarrow{S_{\rho,\nu}} \mathcal{C}_{\rho,\nu} \xrightarrow{T} \mathcal{C}_{\rho_*,\nu_*} \xrightarrow{S_{\rho_*,\nu_*}^{-1}} \mathcal{C}_{0,0}$$

- ▶  $Q$  therefore maps  $\mathcal{C}_{0,0}$  into itself
- ▶ all maps are invertible, therefore

$$T = S_{\rho_*,\nu_*} Q S_{\rho,\nu}^{-1}$$

## Proposition

A matrix  $T \in GL_2(\mathbb{R})$  sends  $\mathcal{C}_{0,0}$  into itself if and only if all of the entries of  $T$  have the same sign, i.e.,  $T_{ij}T_{kl} \geq 0$  for every  $i, j, k, l \in \{1, 2\}$ .



## Proposition †

Let  $\mu, \tau$  be any two numbers such that  $\mu\tau < 1/4$ . Recall that  $S_{\mu,\tau}$  is a map  $\mathcal{C}_{0,0} \hookrightarrow \mathcal{C}_{\mu,\tau}$ , as constructed in Theorem  $\star$ . Define  $R = \sqrt{1 - 4\tau\mu}$ .

- i) If  $\tau < 0$ , we can choose  $S_{\mu,\tau} = \frac{1}{2\tau} \begin{bmatrix} -1-R & 1-R \\ -2\tau & 2\tau \end{bmatrix}$ .
- ii) If  $\tau > 0$ , we can choose  $S_{\mu,\tau} = \frac{1}{2\tau} \begin{bmatrix} 1+R & 1-R \\ 2\tau & 2\tau \end{bmatrix}$ .
- iii) If  $\tau = 0$ , we can choose  $S_{\mu,\tau} = \begin{bmatrix} 1 & \mu \\ 0 & 1 \end{bmatrix}$ .

direct construction



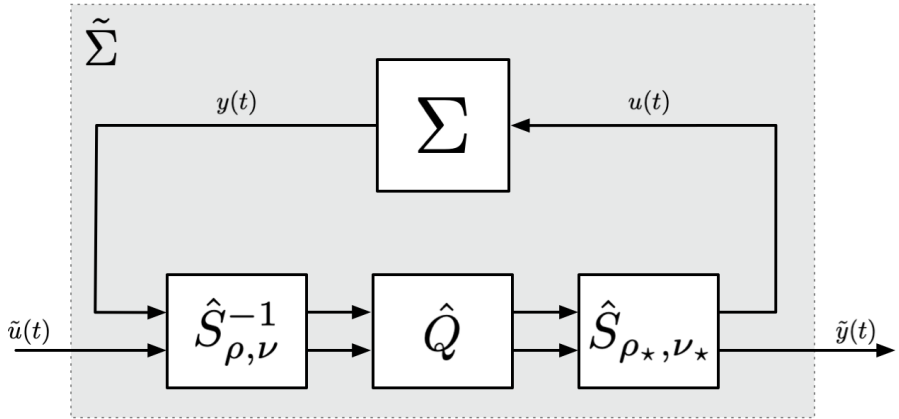
## Theorem

Let  $\Sigma$  be a SISO I/O  $(\rho, \nu)$ -passive system, and let  $T \in GL_2(\mathbb{R})$  be an invertible matrix I/O transformation. The transformed system  $\tilde{\Sigma}$  is I/O  $(\rho_*, \nu_*)$ -passive if and only if there exists a matrix  $M \in GL_2(\mathbb{R})$  such that

- i)  $M_{ij} \geq 0$  for all  $i, j \in \{1, 2\}$ ;
- ii) some  $\theta \in \{\pm 1\}$  such that  $T = S_{\rho_*, \nu_*}(\theta M)S_{\rho, \nu}^{-1}$ , where  $S_{\rho, \nu}, S_{\rho_*, \nu_*}$  are given in Proposition †.

In other words, the transformed system  $\tilde{\Sigma}$  is I/O  $(\rho_*, \nu_*)$ -passive if and only if all of the entries of the matrix  $S_{\rho_*, \nu_*}^{-1} T S_{\rho, \nu}$  have the same sign.

# MAIN RESULT



results can be generalized to MIMO systems

### Theorem

Sharf, Z 2024

Let  $\Sigma$  be an I/O  $(\rho, \nu)$ -passive system with input and output dimension equal to  $d$ , and let  $T \in GL_{2d}(\mathbb{R})$  be an invertible matrix inducing an I/O transformation. The transformed system  $\tilde{\Sigma}$  is I/O  $(\rho_*, \nu_*)$ -passive if and only if there exists a matrix  $M \in GL_{2d}(\mathbb{R})$  and some positive  $\lambda > 0$  such that:

$$T = (S_{\rho_*, \nu_*} \otimes \text{Id}_d)M(S_{\rho, \nu}^{-1} \otimes \text{Id}_d), \quad M^\top JM - \lambda J \geq 0,$$

where  $J = \begin{bmatrix} 0 & 0.5\text{Id}_d \\ 0.5\text{Id}_d & 0 \end{bmatrix}$ , i.e.,  $\tilde{\Sigma}$  is I/O  $(\rho_*, \nu_*)$ -passive if and only if there exists  $\lambda > 0$  such that  $X = (S_{\rho_*, \nu_*}^{-1} \otimes \text{Id}_d)T(S_{\rho, \nu} \otimes \text{Id}_d)$  satisfies  $X^\top JX - \lambda J \geq 0$ .

- ▶ framework can allow us to consider **optimal** passivizing transformations

$$\min_T \Phi(T)$$

s.t.  $T$  maps I/O  $(\rho, \nu)$  systems to I/O  $(\rho_*, \nu_*)$ -systems.

- ▶ framework can allow us to consider **optimal** passivizing transformations

$$\begin{aligned} \min_{T, \lambda, M} \quad & \Phi(T) \\ \text{s.t.} \quad & M = (S_{\rho^*, \nu^*} \otimes \text{Id}_d)^{-1} T (S_{\rho, \nu} \otimes \text{Id}_d) \\ & M^\top J M - \lambda J \geq 0 \\ & \lambda \geq 0, \end{aligned}$$

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- ▶ extend to different passivity variations (incremental, equilibrium independent, etc.)

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- ▶ extend to different passivity variations (incremental, equilibrium independent, etc.)
- ▶ applications to plug-and-play networks

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- ▶ M. Sharf, A. Jain and D. Zelazo, "A Geometric Method for Passivation and Cooperative Control of Equilibrium-Independent Passivity-Short Systems", *IEEE Transactions on Automatic Control*, 66(12):5877-5892, 2021.
- ▶ M. Sharf and D. Zelazo, "A Characterization of All Linear Passivizing Input-Output Transformations of a Passive-Short System: The SISO Case," *IEEE Control Systems Letters*, 8:532:537, 2024.
- ▶ M. Sharf and D. Zelazo, "A Characterization of Passivizing Input-Output Transformations of Nonlinear MIMO Systems," *IEEE Control Systems Letters*, 8:2733-2738, 2024.



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