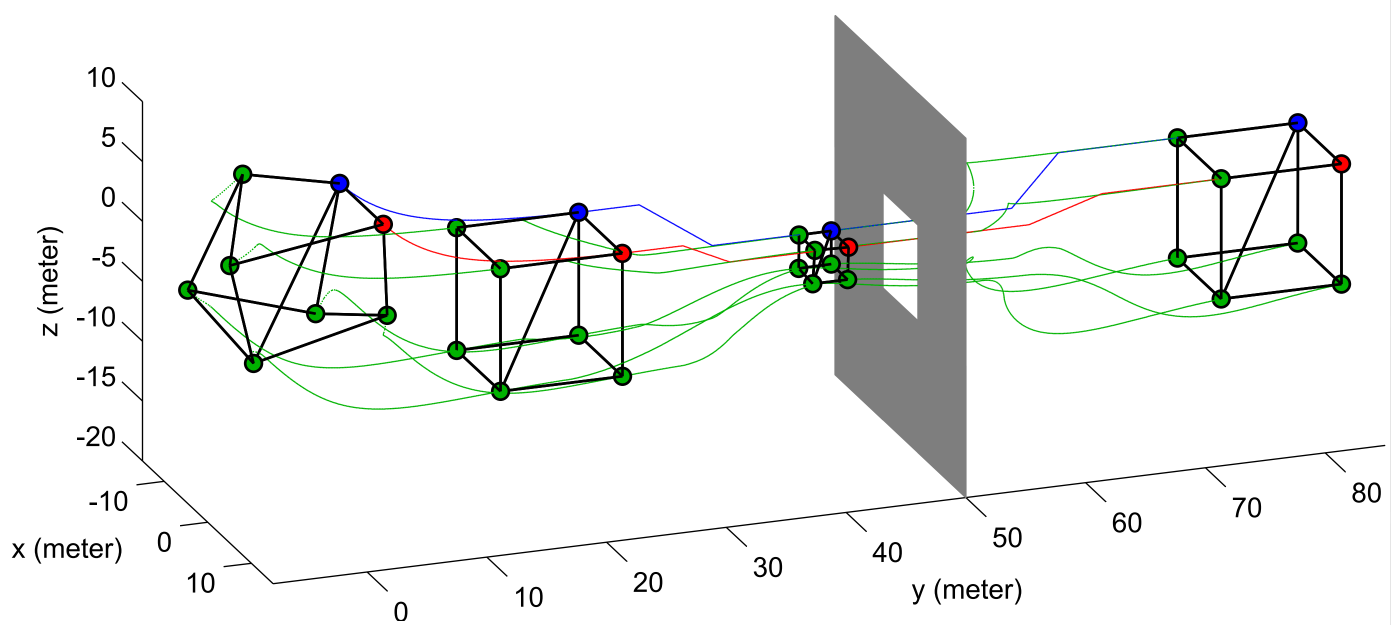
Technion
Israel Institute of Technology

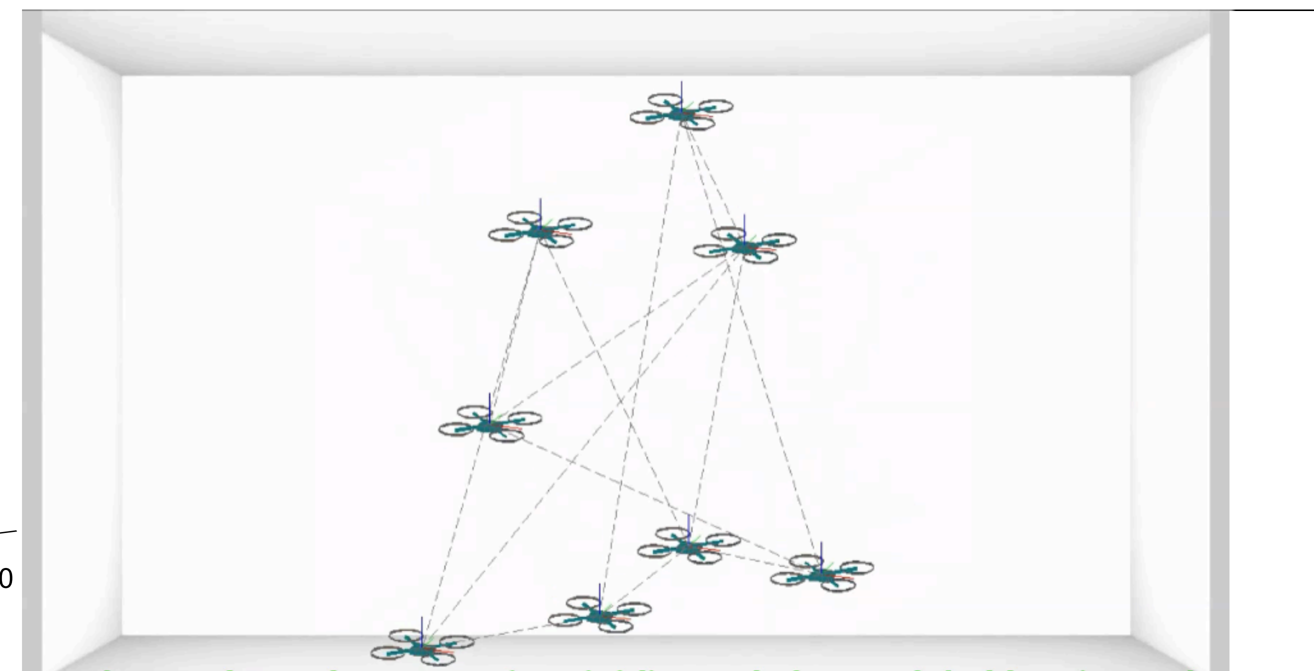
Coordination of multi-robot systems with bearing measurements

Daniel Zelazo
Faculty of Aerospace Engineering
Technion-Israel Institute of Technology

CDC 2015 - Workshop on Taxonomies of Interconnected Systems:
Partial and Imperfect Information in Multi-Agent Networks

Formation Control is one of the canonical problems in multi-agent and multi-robot coordination





S. Zhao and D. Zelazo, "Bearing rigidity and almost global bearing-only formation stabilization," IEEE Transactions on Automatic Control, 2015

Challenges in Multi-Robot Systems

Sensing

- GPS
- Relative Position Sensing
- Range Sensing
- Bearing Sensing

Communication

- Internet
- Radio
- Sonar
- MANet

Solutions to formation control problems in multi-robot systems are *highly* dependent on the sensing and communication mediums available!

selection criteria depends on mission requirements, cost, environment…

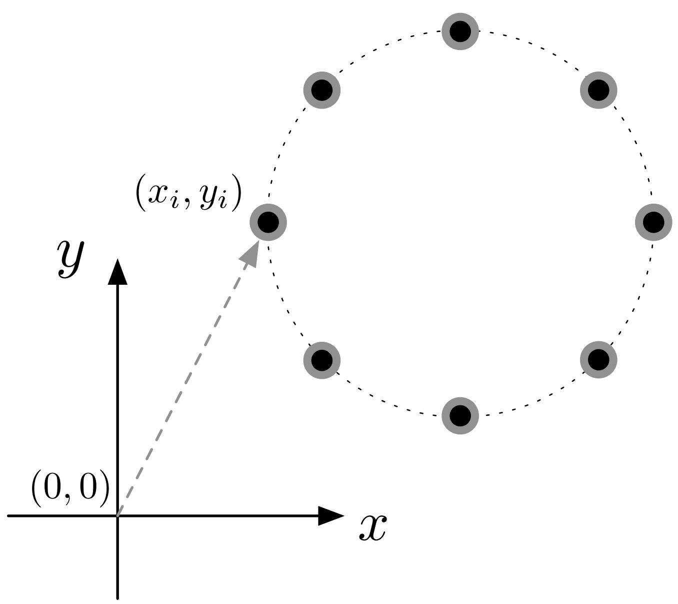
Challenges in Multi-Robot Systems

Solutions to coordination problems in multi-robot systems are *highly* dependent on the sensing and communication mediums available!

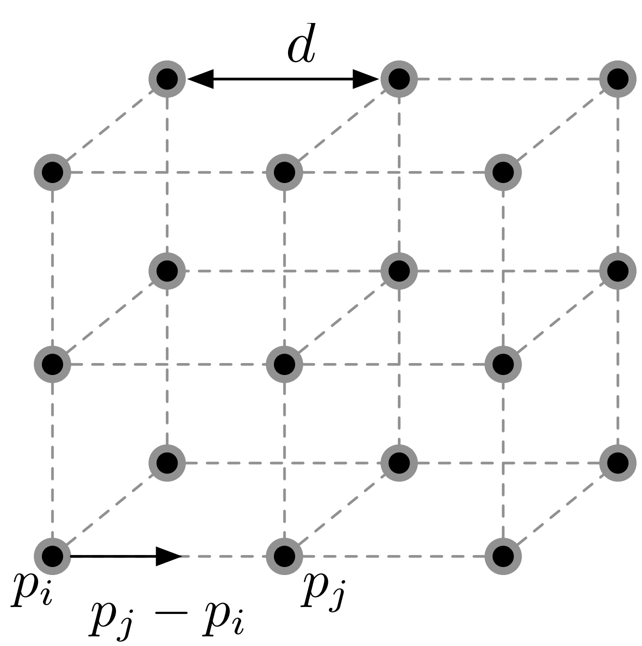
selection criteria depends on mission requirements, cost, environment…

In real-world implementations, formation control must be achieved with *impartial or imperfect information* about the state of the entire formation

Formation Control Strategies



- formation specified in a global coordinate system
- each agent assigned to a point in formation
- assumes GPS-type measurements



- formation specified by inter-agent *distances*
- agents tasked at maintaining distances to certain neighbors
- assumes distance sensing and relative-position information in a common reference frame

Distance-Based Formation Control

Distance-Based Formation Control Law

$$\dot{p}_i = u_i$$

$$u_i = -\sum_{j\sim i}\left(\|p_i - p_j\|^2 - d_{ij}^2\right)(p_i - p_j)$$

[Krick2007, Anderson2008, Dimarogonas2008, Dörfler2010]

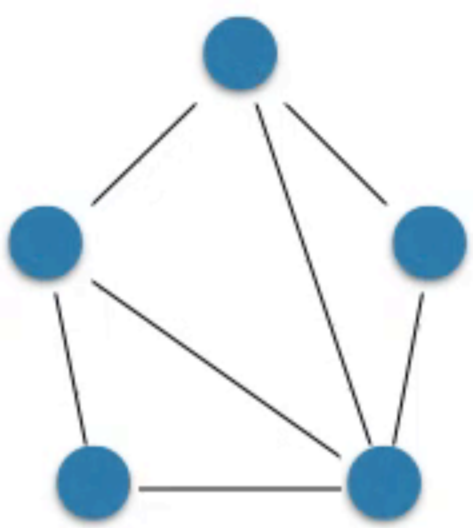
- convergence to desired formation shape depends on the structure of the underlying sensing/communication network
- local stability analysis

Rigidity Theory

Graph Rigidity Theory

Technion Israel Institute of Technology

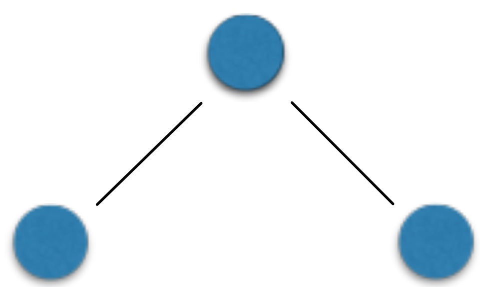
Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.



A *rigid* graph can only *rotate* and *translate* to ensure all distances between all nodes are preserved (i.e., preserve the shape)!

Graph Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.



NOT rigid!

There is a motion that preserves distances between nodes in the graph but the shape is *not* preserved!

A *rigid* graph can only *rotate* and *translate* to ensure all distances between all nodes are preserved (i.e., preserve the shape)!

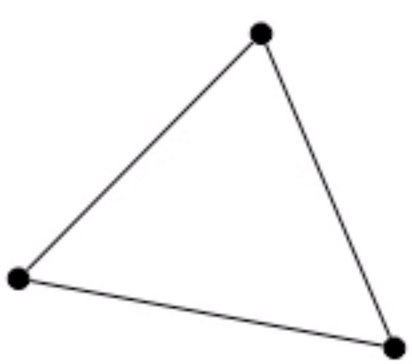
Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

Distance Rigidity

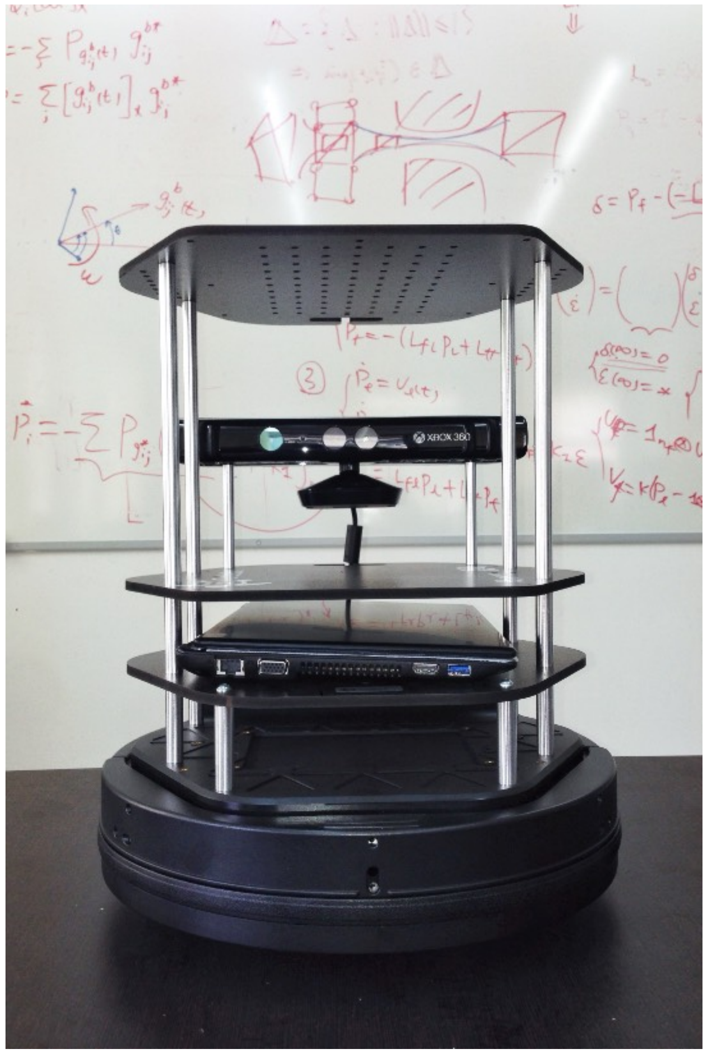
- maintain distance pairs
- rigid body rotations and translations

- rigidity for undirected graphs
- directed graph extensions - persistence [Hendrickx, Anderson, Yu]
- distance-only extensions [Cao]



- requires range sensing

Rigidity Theory - Bearing Extensions



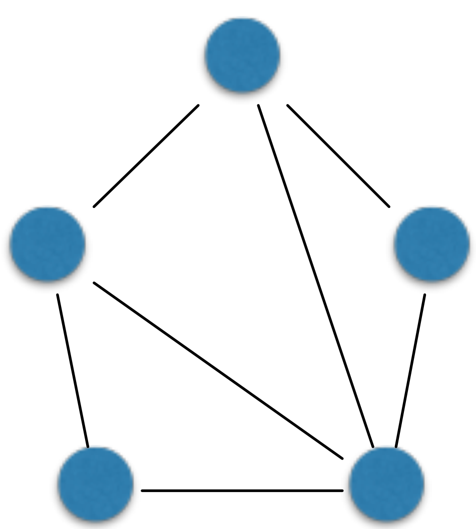
TurtleBotII with Kinect Sensor

recently, there is an interest in *bearing-based* formation control

- (relatively) cheaper sensing
  - vision-based sensors
  - angle-of-arrival sensors

Bearing Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.



A *bearing rigid* graph can *scale* and *translate* to ensure bearings between all nodes are preserved (i.e., preserve the shape)!

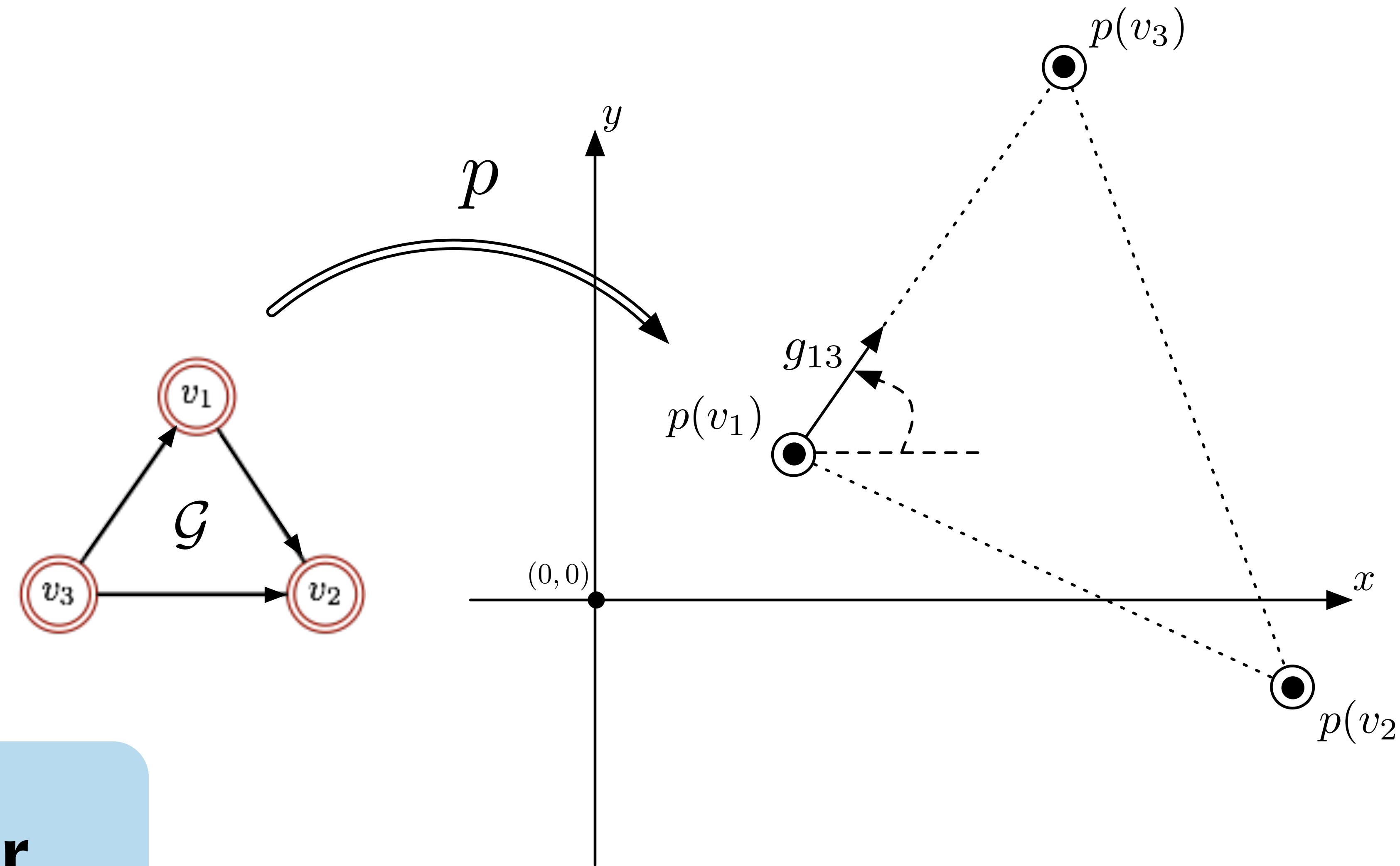
Bearing Rigidity Theory

bar-and-joint frameworks

$(\mathcal{G}, p)$

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ a graph

$p : \mathcal{V} \to \mathbb{R}^2$



relative bearing vector

$$g_{ij} = \frac{p(v_j) - p(v_i)}{\|p(v_j) - p(v_i)\|}$$

$$F_B(p) \triangleq \begin{bmatrix} g_1^T & \cdots & g_m^T \end{bmatrix}^T \in \mathbb{R}^{dm}$$

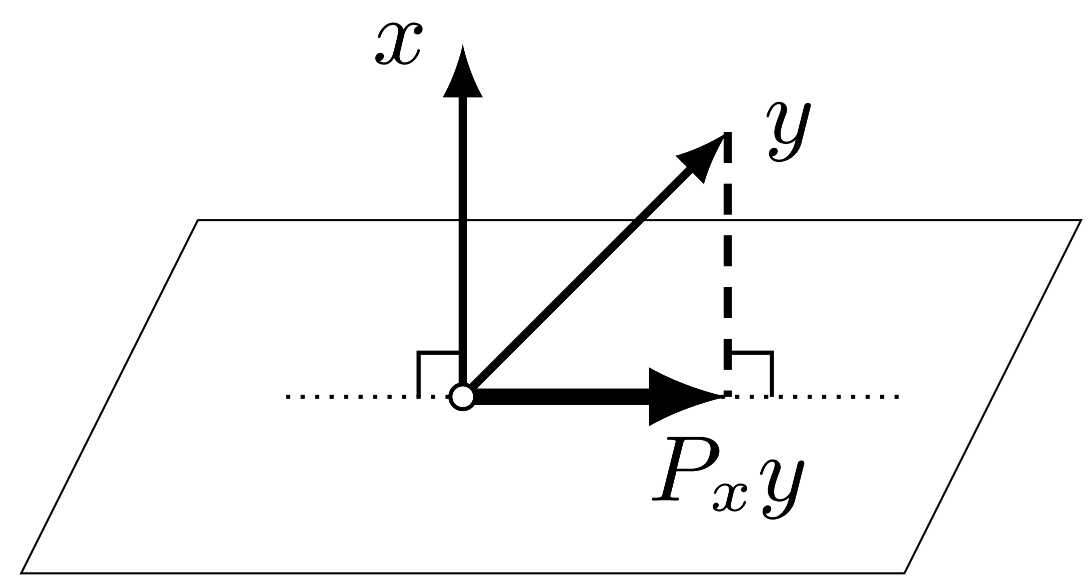
When is a framework bearing rigid?

Bearing Rigidity Theory

Technion Israel Institute of Technology

orthogonal projection operator

$$P_x = I - \frac{1}{\|x\|^2} x x^T$$



- $\mathrm{Null}(P_x) = \mathrm{span}\{x\} \iff P_x y = 0$ iff $x \parallel y$.
- $P_x^T = P_x$ and $P_x^2 = P_x$.
- P_x is positive semi-definite.

- "parallel" vectors have the same relative bearing vectors
- *arbitrary dimensions*

Bearing Rigidity Theory

A framework is **infinitesimally rigid** if all the infinitesimal motions are *trivial* (i.e., translations and scalings).

Theorem

A framework is **bearing infinitesimally rigid** if and only if the rank of the bearing rigidity matrix is *dn-d-1*.

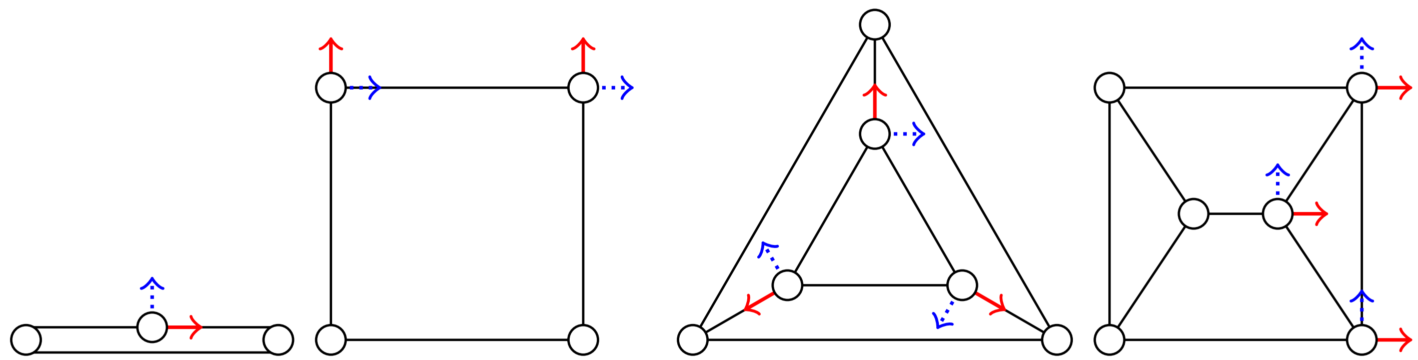
Bearing Rigidity Matrix

$$R(p(\mathcal{V})) = \frac{\partial F_{\mathcal{B}}(\mathcal{G})}{\partial p(\mathcal{V})} = \begin{bmatrix} \ddots & & \\ & \frac{P_{g_{ij}}}{\|p(v_i)-p(v_j)\|} & \\ & & \ddots \end{bmatrix} \left(E(\mathcal{G})^T \otimes I\right) \in \mathbb{R}^{md \times nd}$$

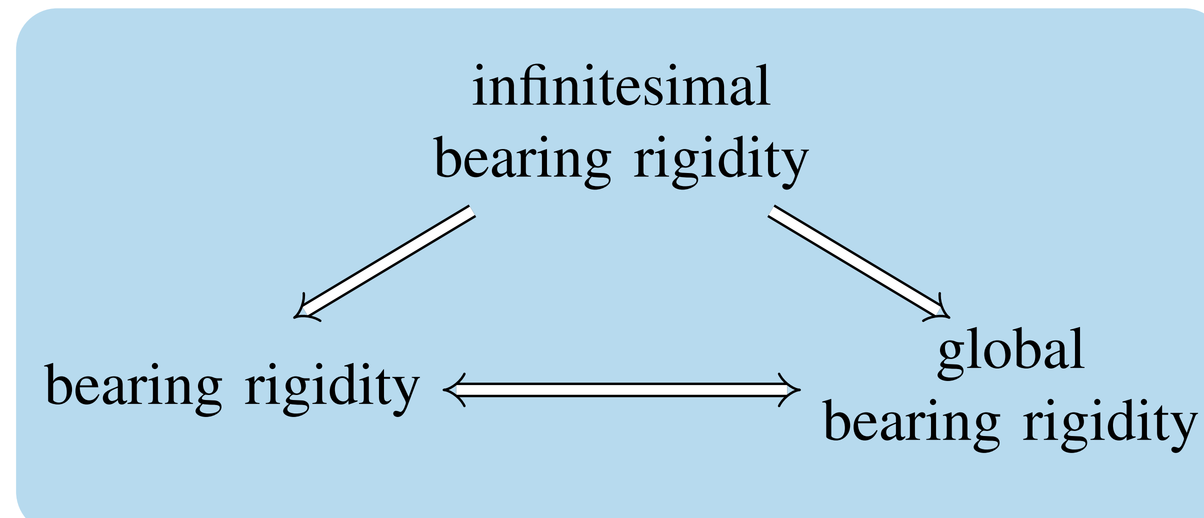
[Zhao and Zelazo, TAC2015]

Distance and Bearing Rigidity

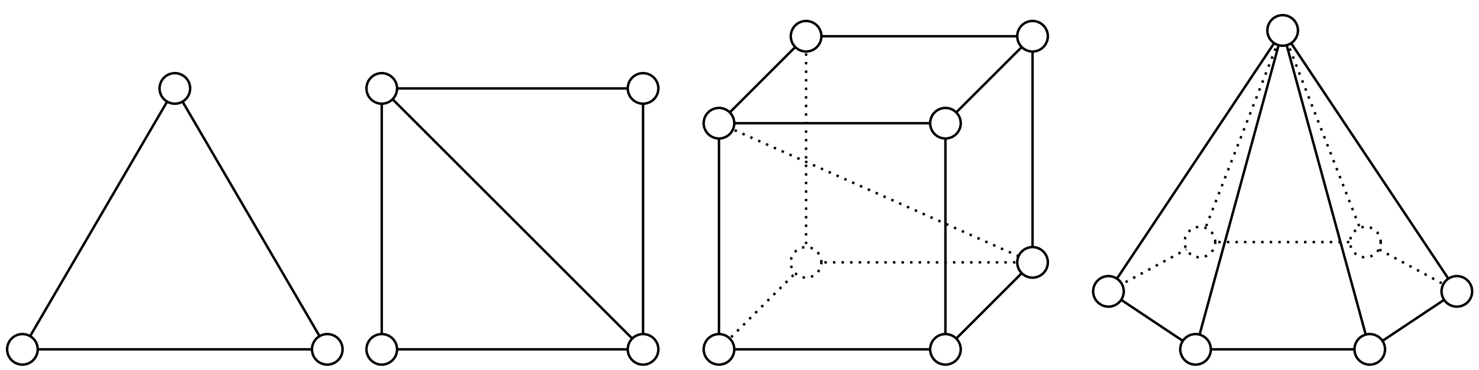
non-infinitesimally bearing rigid



(a) (b) (c) (d)



infinitesimally bearing rigid



(a) (b) (c) (d)

*this relation does *not* hold for distance rigidity

[Zhao and Zelazo, TAC2015]

Bearing-Based Formation Control

The **bearing-based formation control** problem is to design a (distributed) control law that drives the agents to a desired spatial configuration determined by interagent bearings.

A gradient controller

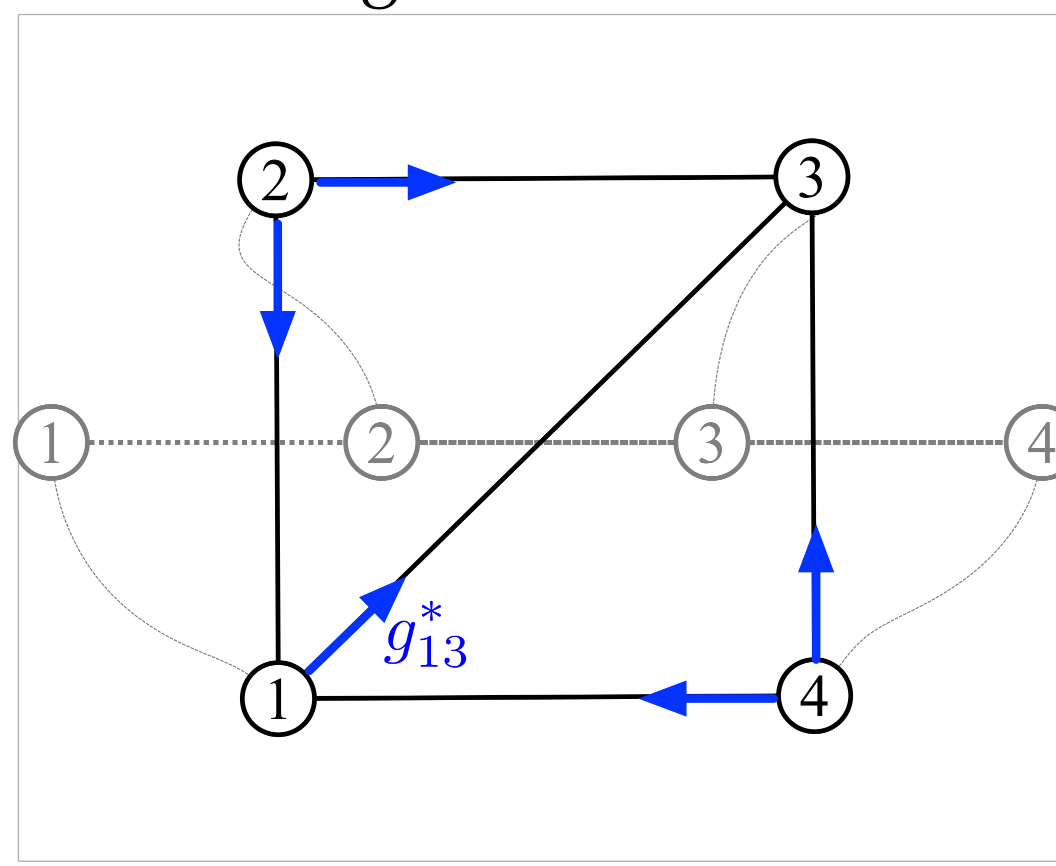
$$\Phi(p) = \sum_{\{i,j\}\in\mathcal{E}} \|g_{ij} - g_{ij}^*\|^2$$

$$u = -\nabla_p \Phi(p) = R^T(p) g^*$$

$$\dot{p}_i = -\sum_{j\sim i} \frac{1}{\|p_j - p_i\|} P_{g_{ij}} g_{ij}^*$$

- control requires bearings and *distances*!

target formation



A Bearing-Only Formation Controller

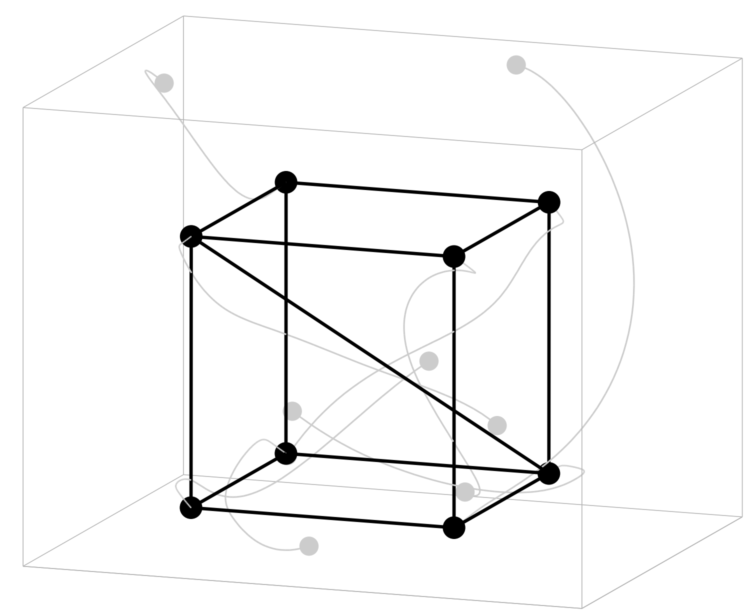
a bearing-only approach

$$\dot{p}_i(t) = -\sum_{j\sim i} P_{g_{ij}(t)} g_{ij}^*$$

stability analysis depends on the **bearing rigidity** of the formation!

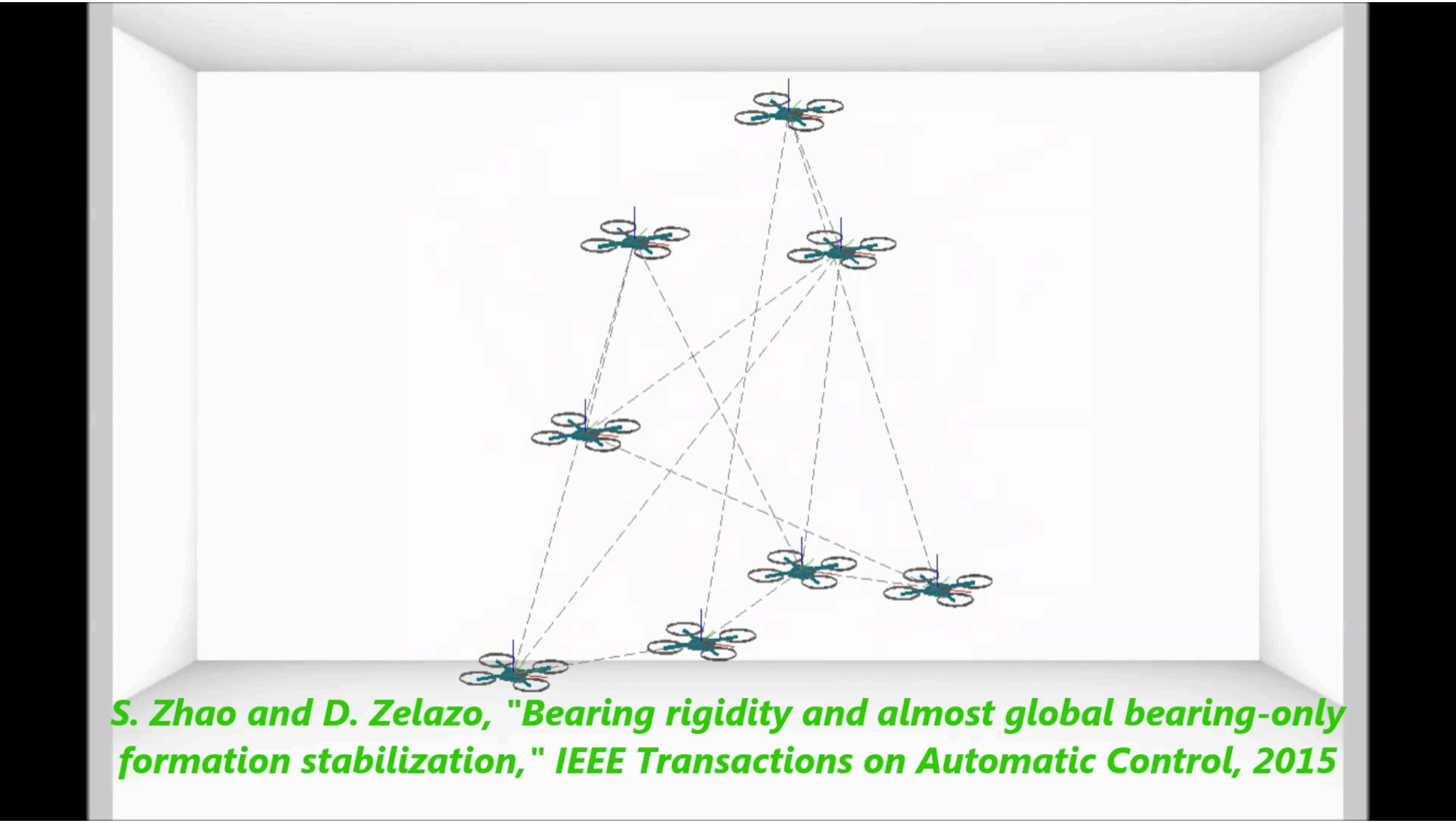
- a distributed protocol
- almost-global stability
  exponential stability
- centroid and scale invariance
- works for arbitrary dimension
- collision avoidance

x assumes undirected graph
x assumes common inertial frame

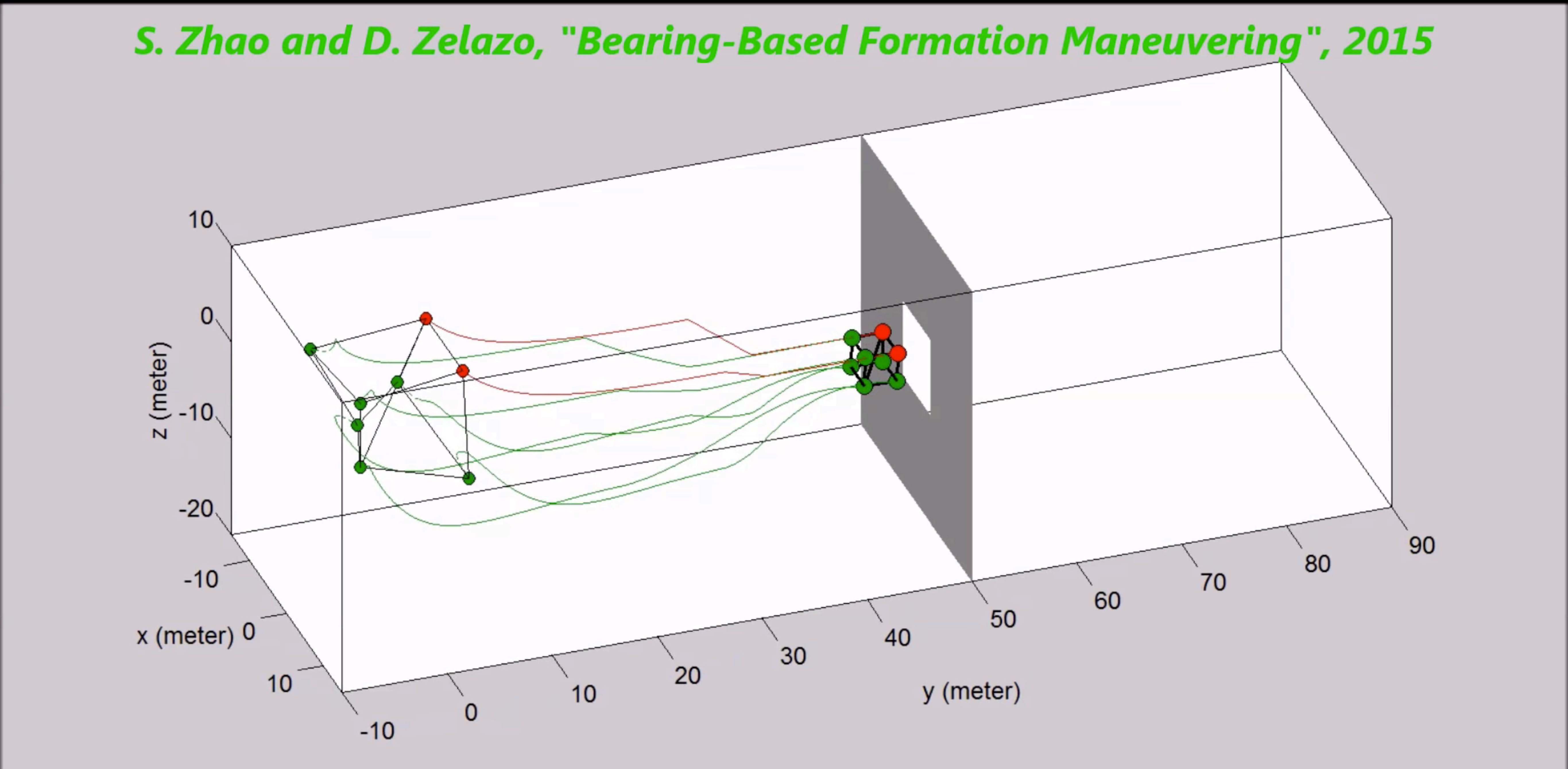


[Zhao and Zelazo, TAC2015]

A Bearing-Only Formation Controller



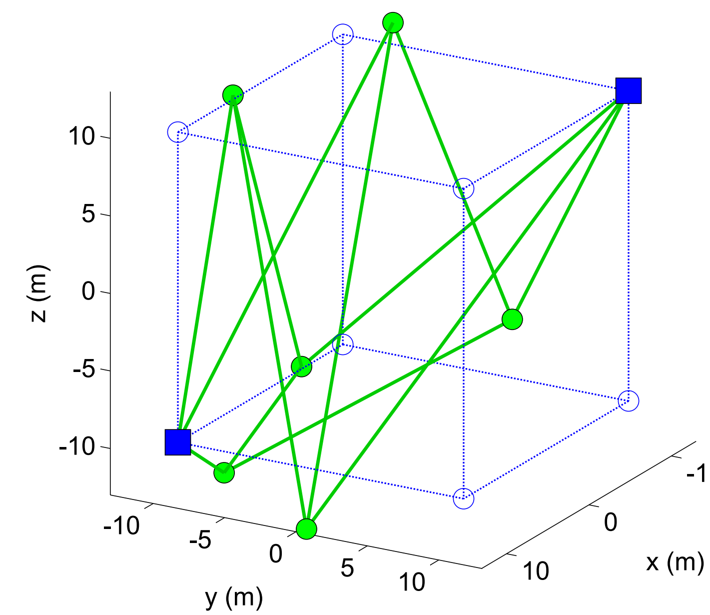
A Bearing-Only Formation Controller



Bearing Rigidity Theory

a bearing-only approach

$$\dot{p}_i(t) = -\sum_{j \sim i} P_{g_{ij}(t)} g_{ij}^*$$



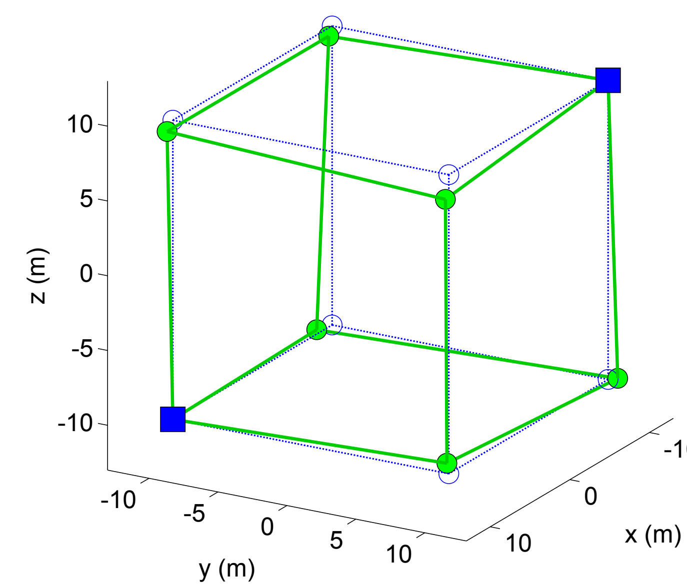
- formation maneuvering control (TCNS '15)
- leader-follower setups
- network localization problems (Automatica '15 (submitted))

Bearing-Based Formation Stabilization with Directed Interaction Topologies

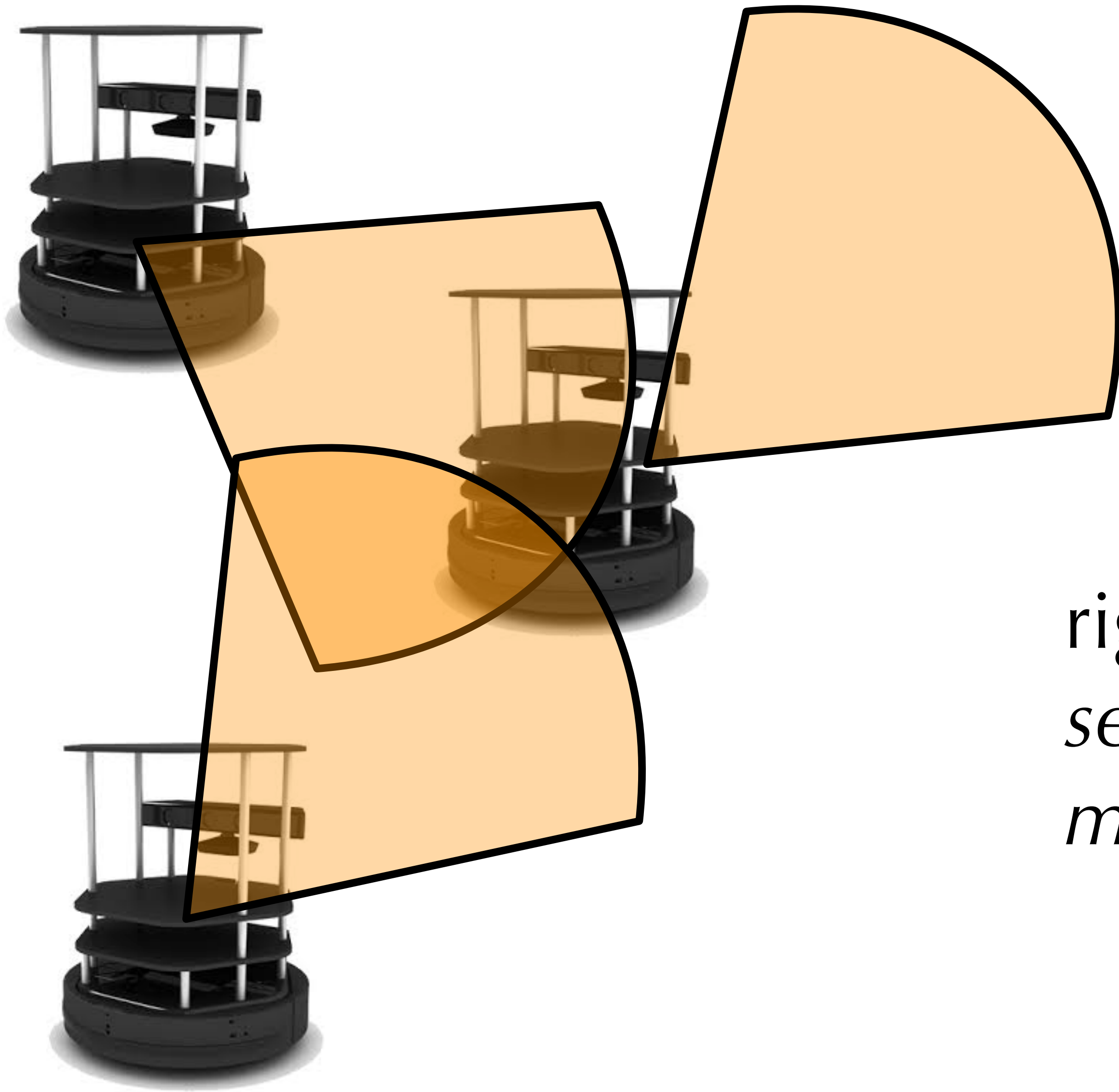
Friday A07

9:30 - 9:50

Zhao, Zelazo



Formation Control in Local Coordinates



- sensing is typically *physically attached to the body frame* of the robot
- sensing is inherently directed
- knowledge of common inertial frame is *not* a realistic assumption

rigidity theory extensions for *directed sensing graphs* and *local (body-frame) measurements*

SE(2) Rigidity Theory

SE(2) Rigidity Theory

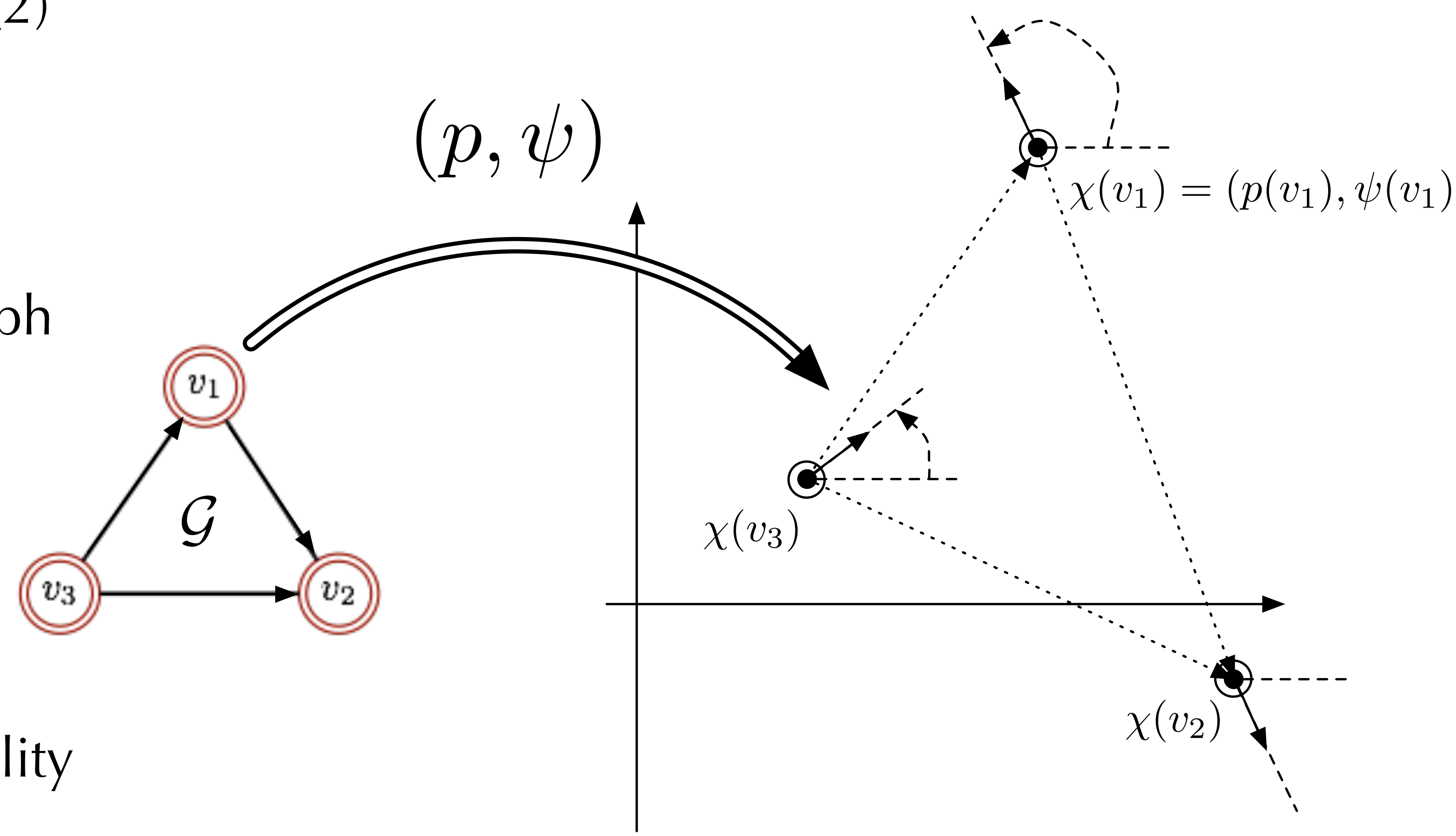
bar-and-joint frameworks in SE(2)

$$(\mathcal{G}, p, \psi)$$

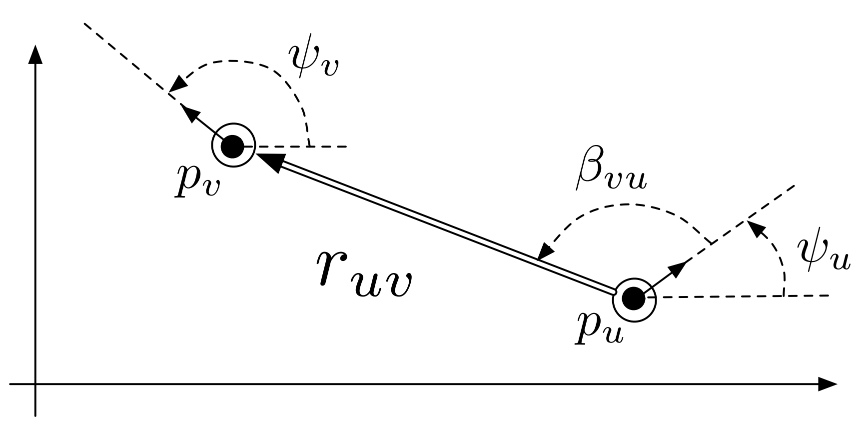
$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ a *directed* graph

$$p : \mathcal{V} \rightarrow \mathbb{R}^2$$

$$\psi : \mathcal{V} \rightarrow \mathcal{S}^1$$



a directed edge indicates availability of relative bearing measurement



$$b_{\mathcal{G}}(p, \psi) = \left[\begin{array}{ccc} r_{e_1}^T & \cdots & r_{e_{|\mathcal{E}|}} \end{array} \right]^T$$

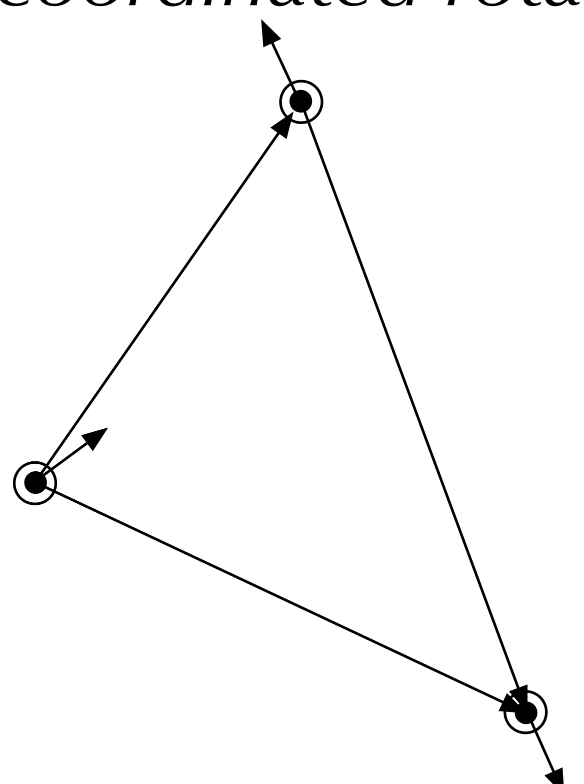
$$r_{uv} = \left[\begin{array}{cc} \cos(\psi_u) & \sin(\psi_u) \\ -\sin(\psi_u) & \cos(\psi_u) \end{array} \right] \frac{p_v - p_u}{\|p_v - p_u\|}$$

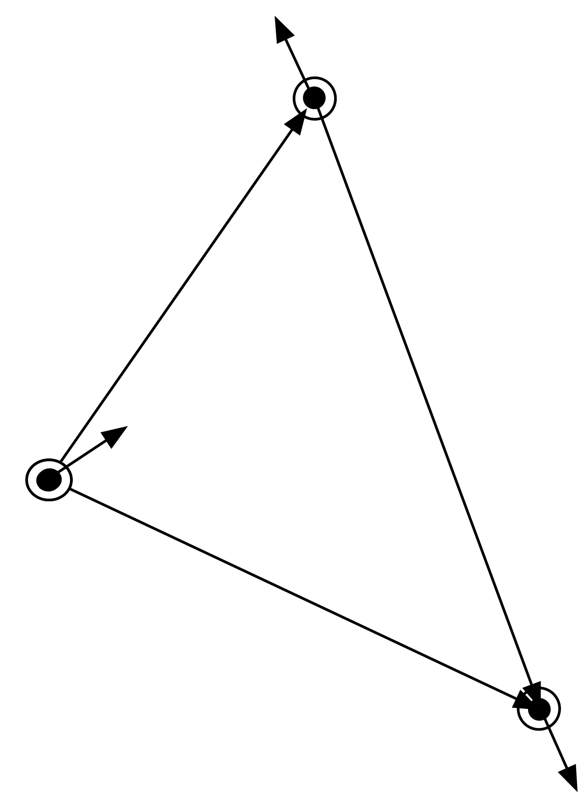
Infinitesimal Motions in SE(2)

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

SE(2) Rigidity

- maintain bearings in *local* frame
- rigid body rotations and scaling + *coordinated rotations*





Rigidity Theory

A framework is **infinitesimally rigid** if all the infinitesimal motions are *trivial* (i.e., translations, scalings, coordinated rotations).

Theorem

A framework is **SE(2) infinitesimally rigid** if and only if the rank of the directed bearing rigidity matrix is *3n-4*.

Directed Bearing Rigidity Matrix

$$\mathcal{B}_{\mathcal{G}}(p,\psi) = \nabla_{(p,\psi)} b_{\mathcal{G}}(p,\psi)$$

$$= \left[\begin{array}{cc} -\mathrm{diag}\left(\frac{P_{r_{vu}}}{\|p_u - p_v\|} T(\psi_v)^T\right)(E^T \otimes I) & -\mathrm{diag}(r_{vu}^{\perp}) E_{out}^T \end{array} \right]$$

Rigidity Theory

A framework is **infinitesimally rigid** if all the infinitesimal motions are *trivial* (i.e., translations, scalings, coordinated rotations).

Theorem

A framework is **SE(2) infinitesimally rigid** if and only if the rank of the directed bearing rigidity matrix is *3n-4*.

Directed Bearing Rigidity Matrix

$$\mathcal{B}_{\mathcal{G}}(p,\psi) = \nabla_{(p,\psi)} b_{\mathcal{G}}(p,\psi)$$

$$\frac{\partial r_{vu}}{\partial \chi_v} = \begin{bmatrix} -\frac{r_{vu}^{\perp}(r_{vu}^{\perp})^T}{\|p_u - p_v\|} T(\psi_v)^T & -r_{vu}^{\perp} \end{bmatrix}$$

$$\frac{\partial r_{vu}}{\partial \chi_u} = \begin{bmatrix} \frac{r_{vu}^{\perp}(r_{vu}^{\perp})^T}{\|p_u - p_v\|} T(\psi_v)^T & \mathbf{0} \end{bmatrix}$$

SE(2) Formation Control

The **SE(2) bearing-based formation control** problem is to design a (distributed) control law that drives the agents to a desired spatial configuration determined by interagent bearings measured in the local body frame of each agent.

A gradient controller

$$\Phi(p,\psi) = \sum_{(i,j)\in\mathcal{E}} \|r_{ij} - r_{ij}^*\|^2$$

$$\begin{bmatrix} \dot{p} \\ \dot{\psi} \end{bmatrix} = -\nabla_{(p,\psi)}\Phi(p,\psi) = \mathcal{B}_{\mathcal{G}}(p,\psi)^T b_{\mathcal{G}}^*$$

$$\dot{p}_i = \sum_{(i,j)\in\mathcal{E}} \frac{P_{r_{ij}}}{\|p_j - p_i\|} r_{ij}^d + \sum_{(j,i)\in\mathcal{E}} T(\psi_j - \psi_i)\frac{P_{r_{ji}}}{\|p_i - p_j\|} r_{ji}^d$$

$$\dot{\psi}_i = -\sum_{(i,j)\in\mathcal{E}} (r_{ij}^{\perp})^T r_{ij}^d$$

x requires distances
x requires communication
x requires relative orientation

A Bearing-Only Formation Controller

a scale-free SE(2) formation control

$$T(\psi_i)^T \dot{p}_i = -\sum_{(i,j)\in\mathcal{E}} P_{r_{ij}} r_{ij}^d + \sum_{(j,i)\in\mathcal{E}} T(\psi_i - \psi_j)^T P_{r_{ji}} r_{ji}^d$$

$$\dot{\psi}_i = -\sum_{(i,j)\in\mathcal{E}} (r_{ij}^{\perp})^T r_{ij}^d,$$

stability analysis depends on the **SE(2) bearing rigidity** of the formation!

Bearing-Only Formation Control Using an SE(2) Rigidity Theory

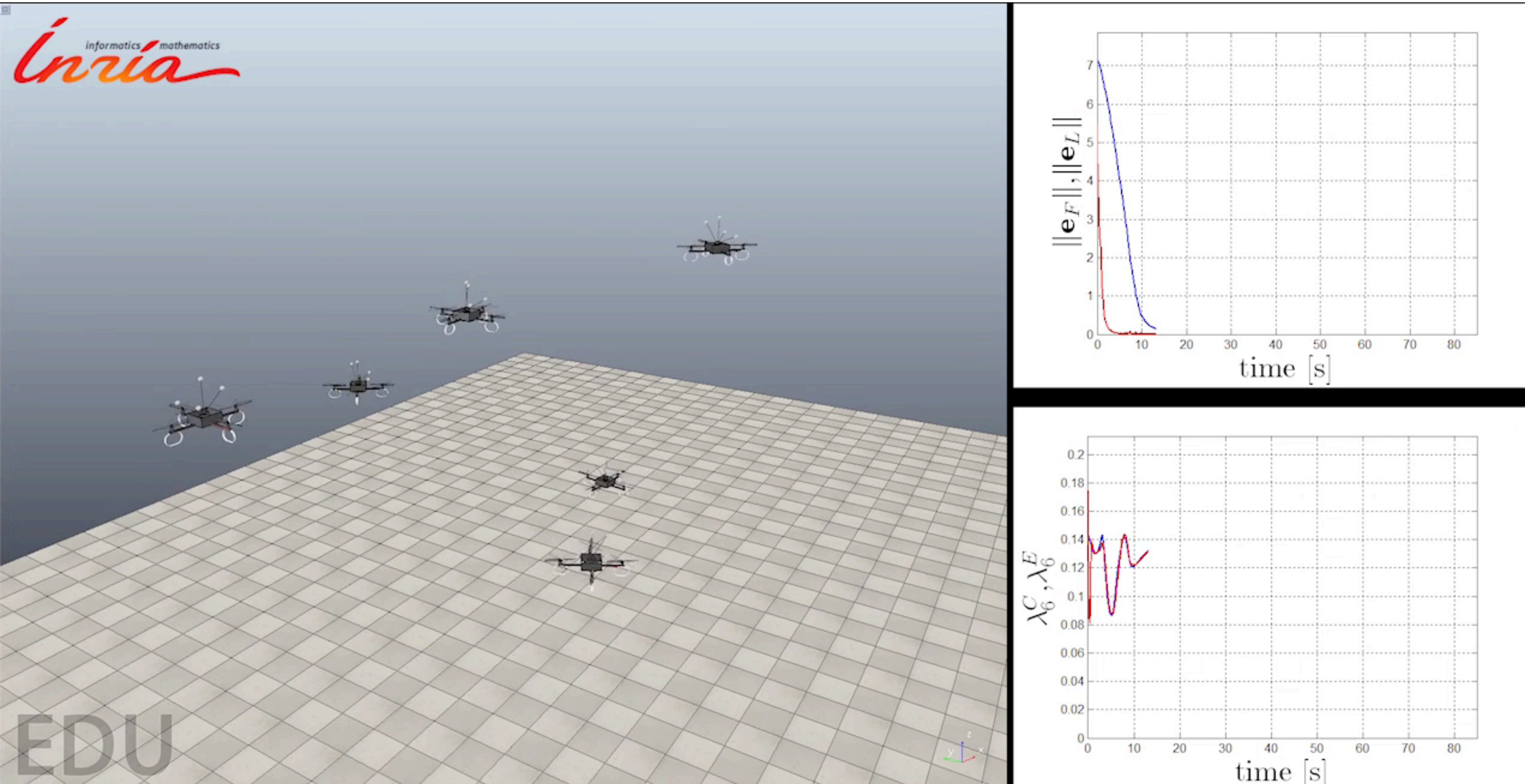
Friday A07

9:50 - 10:10

Zelazo, Robuffo Giordano, Franchi

An SE(2) Formation Controller

Technion
Israel Institute of Technology



The formation reaches the desired bearings

Conclusions and Outlook

- coordination methods for multi-agent systems depend on sensing and communication mediums

- *rigidity theory* is a powerful framework for handling high-level multi-agent objectives under different sensing and communication constraints
  - bearing rigidity
  - SE(2) rigidity
  - SE(n) rigidity

- directed sensing still has many open challenges

Invited Session Advertisement

Rigidity Theory for Problems in Multi-Agent Coordination

Friday A07
8:30 - 10:30

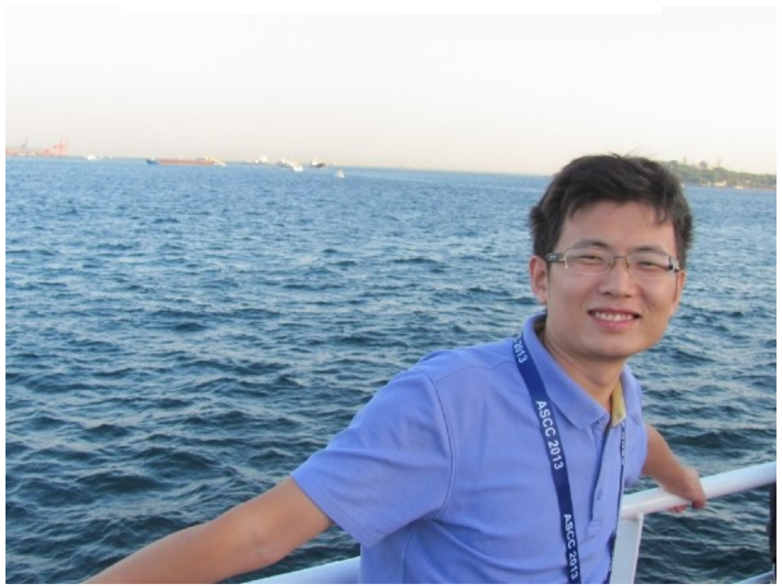
Organizers
Daniel Zelazo
Paolo Robuffo-Giordano
Antonio Franchi

Speakers

- Z. Sun, U. Helmke, B.D.O. Anderson
  Rigid Formation Shape Control in General Dimensions: An Invariance Principle and Open Problems
- R. Williams, A. Gasparri, M. Soffietti, G. Sukhatme
  Redundantly Rigid Topologies in Decentralized Multi-Agent Networks
- T. Eren
  Combinatorial Measures of Rigidity in Wireless Sensors and Robot Networks
- S. Zhao, D. Zelazo
  Bearing-based Formation Stabilization with Directed Interaction Topologies
- D. Zelazo, P. Robuffo Giordano, A. Franchi
  Bearing-only Formation Control Using an SE(2) Rigidity Theory
- I. Shames, T. Summers, F. Farokhi, R.C. Shekhar
  Conditions and Strategies for Uniqueness of the Solutions to Cooperative Localization and Mapping Using Rigidity Theory

Acknowledgements

UC Riverside — University of California



Dr. Shiyu Zhao

lagadic

Dr. Paolo Robuffo Giordano

LAAS-CNRS

Dr. Antonio Franchi

Fabrizio Schiano

Questions?