

Coordination of multi-robot systems with bearing measurements

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CDC 2015 - Workshop on Taxonomies of Interconnected Systems: Partial and Imperfect Information in Multi-Agent Networks

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Formation Control is one of the canonical problems in multi-agent and multi-robot coordination

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Challenges in Multi-Robot Systems

- GPS
- Relative Position Sensing
- Range Sensing
- Bearing Sensing

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Sensing Communication

- Internet
- Radio
- Sonar
- MANet

Solutions to formation control problems in multirobot systems are *highly* dependent on the sensing and communication mediums available!

selection criteria depends on mission requirements, cost, environment…

Challenges in Multi-Robot Systems

Solutions to coordination problems in multi-robot systems are *highly* dependent on the sensing and communication mediums available!

selection criteria depends on mission requirements, cost, environment…

In real-world implementations, formation control must be achieved with *impartial or imperfect information* about the state of the entire formation

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Formation Control Strategies

- formation specified in a global coordinate system
- each agent assigned to a point in formation
- assumes GPS-type measurements
- formation specified by inter-agent *distances*
- agents tasked at maintaining distances to certain neighbors
- assumes distance sensing and relative-position information in a common reference frame

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Distance-Based Formation Control Law

$$
\dot{p}_i = u_i \nu_i = -\sum_{j \sim i} (||p_i - p_j||^2 - d_{ij}^2) (p_i - p_j)
$$

[Krick2007, Anderson2008, Dimarogonas2008, Dörfler2010]

- convergence to desired formation shape depends on the structure of the underlying sensing/ communication network
- local stability analysis

Rigidity Theory

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Graph Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

A rigid graph can only rotate and translate to ensure all distances between all nodes are preserved (i.e., preserve the shape)!

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Graph Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

NOT rigid!

There is a motion that preserves distances between nodes in the graph but the shape is *not* preserved!

A *rigid* graph can only *rotate* and *translate* to ensure all distances between all nodes are preserved (i.e., preserve the shape)!

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Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

Distance Rigidity

- maintain distance pairs
- rigid body rotations and translations
- rigidity for undirected graphs
- directed graph extensions persistence [Hendrickx, Anderson, Yu]
- distance-only extensions [Cao]

requires range sensing

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Rigidity Theory - Bearing Extensions **Technion**

TurtleBotII with Kinect Sensor

recently, there is an interest in *bearing-based* formation control

- (relatively) cheaper sensing
	- ‣ vision-based sensors
	- angle-of-arrival sensors

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Bearing Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

A *bearing rigid* graph can *scale* and *translate* to ensure bearings between all nodes are preserved (i.e., preserve the shape)!

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Bearing Rigidity Theory

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CDC 2015 - Workshop on Taxonomies of Interconnected Systems Dec. 14, 2015 Osaka, Japan *R*(*p*) decays and *Register Beet 16* 2015 Osaka Japan

- $\text{Null}(P_x) = \text{span}\{x\} \Longleftrightarrow P_x y = 0$ iff $x \parallel y$.
- $P_x^T = P_x$ and $P_x^2 = P_x$.
- $\frac{1}{x} \frac{1}{x}$.
 Five semi-definite *• P ^T* = *P^x* and *P*² *• P^x* is positive semi-definite.
- *•* Vectors nave the *position* semi-- "parallel" vectors have the same relative bearing vectors
- *arbitrary dimensions*

Bearing Rigidity Theory

 $\sqrt{2}$

...

 \overline{a} 6 $\overline{}$ 4

A framework is **infinitesimally rigid** if all the infinitesimal motions are *trivial* (i.e., translations and scalings).

Theorem

A framework is **bearing infinitesimally rigid** if and only if the rank of the bearing rigidity matrix is *dn-d-1*.

Bearing Rigidity Matrix

$$
R(p(\mathcal{V})) = \frac{\partial F_{\mathcal{B}}(\mathcal{G})}{\partial p(\mathcal{V})} =
$$

$$
\frac{P_{g_{ij}}}{\|p(v_i)-p(v_j)\|}
$$

$$
\left(E(\mathcal{G})^T\otimes I\right)\in\mathbb{R}^{md\times nd}
$$

[Zhao and Zelazo, TAC2015]

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...

 $\overline{}$

 $\mathbf{1}$ \mathcal{L} \mathcal{L} $\mathbf{1}$

Distance and Bearing Rigidity

Bearing-Based Formation Control

The **bearing-based formation control** problem is to design a (distributed) control law that drives the agents to a desired spatial configuration determined by interagent bearings.

A gradient controller $\left(1 \right)$ $\left(2 \right)$ $\left(2 \right)$ $\left(3 \right)$ $\left(4 \right)$ 1 $\overline{2}$ 3 4 g_{13}^* target formation $\Phi(p) = \sum$ $\{i,j\} \in \mathcal{E}$ $||g_{ij} - g_{ij}^*||^2$ $u = -\nabla_p \Phi(p) = R^T(p) g^*$ $\dot{p}_{i}= \sum$ $j \sim i$ 1 $||p_j - p_i||$ $P_{g_{ij}}g_{ij}^{\ast}$

- control requires bearings and *distances*!

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A Bearing-Only Formation Controller M Technion

a bearing-only approach

$$
\dot{p}_i(t) = -\sum_{j \sim i} P_{g_{ij}(t)} g^*_{ij}
$$

- ^a distributed protocol

- almost-global stability exponential stability
- centroid and scale invariance
- works for arbitrary dimension - collision avoidance

stability analysis depends on the **bearing rigidity** of the formation!

x assumes undirected graph

x assumes common inertial frame

[Zhao and Zelazo, TAC2015]

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A Bearing-Only Formation Controller Mechnion

formation stabilization," IEEE Transactions on Automatic Control, 2015

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A Bearing-Only Formation Controller W **Technion**

S. Zhao and D. Zelazo, "Bearing-Based Formation Maneuvering", 2015 $10₁$ $\mathbf{0}$ z (meter) -10 $-20.$ 90 80 -10 70 60 50 x (meter) 0 40 30 20 10 10 y (meter) $\mathbf{0}$ -10

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a bearing-only approach

$$
\dot{p}_i(t) = -\sum_{j \sim i} P_{g_{ij}(t)} g^*_{ij}
$$

- formation maneuvering control (TCNS '15)
- leader-follower setups
- network localization problems (Automatica '15 (submitted))

Bearing-Based Formation Stabilization with Directed Interaction Topologies

Friday A07 9:30 - 9:50

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Zhao, Zelazo

Formation Control in Local Coordinates Marchine of Technion

SE(2) Rigidity Theory

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SE(2) Rigidity Theory

bar-and-joint frameworks in SE(2)

 (\mathcal{G}, p, ψ)

 $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ a directed graph $p: \mathcal{V} \to \mathbb{R}^2$ $\psi: \mathcal{V} \rightarrow \mathcal{S}^1$ $\scriptstyle v_3$

 (p, ψ) (*p*₁, *v*₁) (*p*(*v*₁), *v*(*v*₁)) v_1 *G* $\chi(v_3)$ v_2 $\chi(v_2)$

a directed edge indicates availability of relative bearing measurement

$$
b_{\mathcal{G}}(p,\psi) = \begin{bmatrix} r_{e_1}^T & \cdots & r_{e_{|\mathcal{E}|}} \end{bmatrix}^T
$$

Infinitesimal Motions in SE(2)

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

SE(2) Rigidity

- maintain bearings in *local* frame
- rigid body rotations and scaling + *coordinated rotations*

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Rigidity Theory

A framework is **infinitesimally rigid** if all the infinitesimal motions are *trivial* (i.e., translations, scalings, coordinated rotations).

Theorem

A framework is **SE(2) infinitesimally rigid** if and only if the rank of the directed bearing rigidity matrix is *3n-4*.

Directed Bearing Rigidity Matrix

$$
\mathcal{B}_{\mathcal{G}}(p,\psi) = \nabla_{(p,\psi)} b_{\mathcal{G}}(p,\psi)
$$

= $\left[-\text{diag}\left(\frac{P_{rwu}}{\|p_u - p_v\|} T(\psi_v)^T \right) (E^T \otimes I) - \text{diag}(r_{vu}^{\perp}) E_{out}^T \right]$

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Rigidity Theory

bearing congruent to (*G, p,*)*.*

A framework is **infinitesimally rigid** if all the infinitesimal motions are *trivial* (i.e., translations, scalings, coordinated rotations). Definition II.4 (Global rigidity of *SE*(2) Frameworks). *A framework* is **inimitesimally rigid** if all the infinitesimal motions are *trivial* (i.e., translations, scalings, coordinated re $\frac{1}{2}$ mations Proposition II.7. *For an infinitesimally rigid SE*(2) *frame-*

Theorem Theorem Theorem *notion of <i>infinitesimal* **rigidity is characterized by the chara**

A framework is **SE(2) infinitesimally rigid** if and only if the rank of the directed bearing rigidity matrix is *3n-4*. Δ framount is $\mathsf{CE}(2)$ infinitesimally rigid if and an function, *r*_p *G*(*C*) in the *directed* hearing rigidity matrix is $3n-4$ are directed bearing rigidity in Φ the rank of W are also able to define the notion of \mathcal{M}

Directed Bearing Rigidity Matrix *B***G** Bearing Rigidity Matrix

$$
\mathcal{B}_\mathcal{G}(p,\psi)=\nabla_{(p,\psi)}b_\mathcal{G}(p,\psi)
$$

$$
\frac{\partial r_{vu}}{\partial \chi_v} = \begin{bmatrix} -\frac{r_{vu}^{\perp} (r_{vu}^{\perp})^T}{\|p_u - p_v\|} T(\psi_v)^T & -r_{vu}^{\perp} \\ \frac{\partial r_{vu}}{\partial \chi_u} & = \begin{bmatrix} \frac{r_{vu}^{\perp} (r_{vu}^{\perp})^T}{\|p_u - p_v\|} T(\psi_v)^T & \mathbf{0} \end{bmatrix} \end{bmatrix}
$$

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GDC 2015 - Workshop on Tax **Faculty** of Aerospace Engineering

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work,

SE(2) Formation Control which proves that *^v*2*v*³ , and therefore *r^v*2*v*³ , can be com- \lim \lim \lim \lim \lim = *^v*3*v*² + *^v*1*v*³ *^v*3*v*¹ *^v*1*v*² + *^v*2*v*¹ ⇡*,* (9) ⁼ *kB*˜*G*((*V*))*^T ^bG*() ^b*^d G ,* (11)

Here, *k >* 0 is a scalar gain used to improve the rate of

The **SE(2) bearing-based formation control** problem is to design a (distributed) control law that drives the agents to a desired spatial configuration determined by interagent bearings measured in the local body frame of each agent. τ of α the system. For an algorithm α *k* = 1. Observe that by considering that by construction \mathbf{f} $\$ acternated by a ⁼ *^B*˜*G*((*V*))*^T* ^b*^d* which proves that *^v*2*v*³ , and therefore *r^v*2*v*³ , can be computed from the five available bearings. Therefore, measuring fixed bearing $\overline{}$ having a complete measurement \overline{a} part of the proof we observe that \mathcal{L} $\vert \vert$ infinitesimal rigidity \vert $E(2)$ heaving hased formation control $C_1(z)$ beaming-based formation control figurati ˙ \mathcal{L} ⁼ *^B*˜*G*((*V*))*^T* ^b*^d G.*

A gradient controller σ is diant controller **Staurent controller** for each agent expressed in the global frame. The global frame and global frame. The global fra $\mathbf A$ τ

$$
\Phi(p,\psi) = \sum_{(i,j)\in \mathcal{E}} ||r_{ij} - r_{ij}^*||^2
$$

$$
\begin{bmatrix} \dot{p} \\ \dot{\psi} \end{bmatrix} = -\nabla_{(p,\psi)}\Phi(p,\psi) = \mathcal{B}_{\mathcal{G}}(p,\psi)^T b_{\mathcal{G}}^*
$$

$$
\dot{p}_i = \sum_{(i,j) \in \mathcal{E}} \frac{P_{r_{ij}}}{\|p_j - p_i\|} r_{ij}^d + \sum_{(j,i) \in \mathcal{E}} T(\psi_j - \psi_i) \frac{P_{r_{ji}}}{\|p_i - p_j\|} r_{ji}^d
$$

 $\dot{\psi}_i = -\sum_{i,j} (r_{ij}^\perp)^T r_{ij}^d$ $(i,j) \in \mathcal{E}$ $\frac{d}{ij}$ $-\sum_{(i,j)\in S} (r_{ij}^{\perp})^T r_{ij}^a$ (*i,j*)2*E*

Consider a team of *n* agents (*n* 2) in *SE*(2) where there is

no knowledge of a common reference frame. The dynamics

הפקולטה להנדסת אוירונוטיקה וחלל Faculty of Aerospace Engineering A few comments regarding the above control strategy are Γ above control strategy and Γ $\frac{1}{2}$ acuny or $\frac{1}{2}$ has pace $\frac{1}{2}$ incerning **A few comments regarding the above control strategy areas for the above c** $\mathbf v$ independent order. In the control independent of $\mathbf v$ x requires distances

x requires communication

x requires relative orientation

A Bearing-Only Formation Controller The confidential continuation. For the continuation of the continuation. For the continuation of the c

*^B*ˆ*G*()

a scale-free SE(2) formation control stability analysis depends

$$
T(\psi_i)^T \dot{p}_i = -\sum_{(i,j)\in\mathcal{E}} P_{r_{ij}} r_{ij}^d + \sum_{(j,i)\in\mathcal{E}} T(\psi_i - \psi_j)^T P_{r_{ji}} r_{ji}^d
$$

$$
\dot{\psi}_i = -\sum_{(i,j)\in\mathcal{E}} (r_{ij}^{\perp})^T r_{ij}^d,
$$

and we assume for the purpose of analysis that the agents are

on the **SE(2)** bearing **rigidity** of the formation! \mathbf{U} : \mathbf{u} and \mathbf{v} , \mathbf{v} as found in \mathbf{v} the new variable *^p* ⁼ *^pp^d* and ⁼ *^d*. Differentiating **Transport in the Formation:**

Bearing-Only Formation Control
Using an SE(2) Rigidity Theory ˙ $\frac{1}{2}$ $I_{\rm I}$ is a model is the this control is in fact different that the this in fact different than $I_{\rm I}$ the one proposed in [15]. In particular, in [15] a consensus-9:50 - 10:10 **Using an SE(2) Rigidity Theory** Friday A07

type algorithm is used to align all agents to align all agents to align all agents to all agents to align all a
The algorithm is used to all agents to all agents to all agents to all all agents to all all all all all all a

הפקולטה להנדסת אוירונוטיקה וחלל *frame*, while the control action in (13) does not enforce any **Faculty of Aerospace Engineering**

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ⁱ) = sin(*ⁱ*). Let *V* = P*ⁿ*

An SE(2) Formation Controller

The formation reaches the desired bearings

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- coordination methods for multi-agent systems depend on sensing and communication mediums
- *• rigidity theory* is a powerful framework for handling high-level multi-agent objectives under different sensing and communication constraints
	- *-* bearing rigidity
	- *-* SE(2) rigidity
	- *-* SE(n) rigidity
- directed sensing still has many open challenges

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Invited Session Advertisement

Rigidity Theory for Problems in Multi-Agent Coordination

Friday A07 8:30 - 10:30

Organizers Daniel Zelazo Paolo Robuffo-Giordano Antonio Franchi

Speakers

- Z. Sun, U. Helmke, B.D.O. Anderson *Rigid Formation Shape Control in General Dimensions: An Invariance Principle and Open Problems*
- R. Williams, A. Gasparri, M. Soffietti, G. Sukhatme *Redundantly Rigid Topologies in Decentralized Multi-Agent Networks* - T. Eren
	- *Combinatorial Measures of Rigidity in Wireless Sensors and Robot Networks*
- S. Zhao, D. Zelazo *Bearing-based Formation Stabilization with Directed Interaction Topologies*
- D. Zelazo, P. Robuffo Giordano, A. Franchi *Bearing-only Formation Control Using an SE(2) Rigidity Theory*
- I. Shames, T. Summers, F. Farokhi, R.C. Shekhar *Conditions and Strategies for Uniqueness of the Solutions to Cooperative Localization and Mapping Using Rigidity Theory*

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Questions?

Fabrizio Schiano

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