ARCHITECTURES OF MULTI-AGENT SYSTEMS:

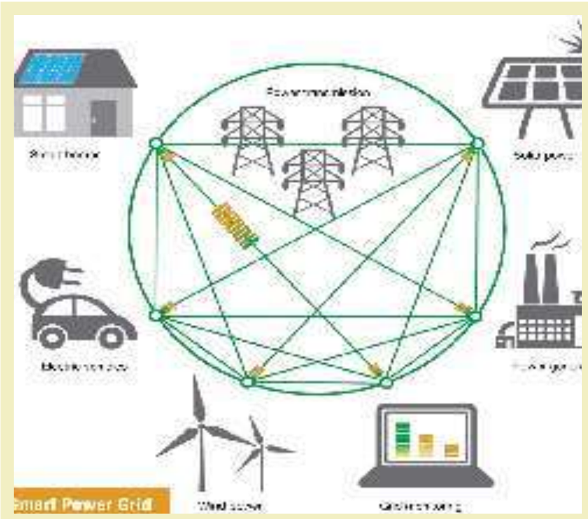
DYNAMIC PROPERTIES AND INFORMATION EXCHANGE NETWORKS

Daniel Zelazo
Faculty of Aerospace Engineering

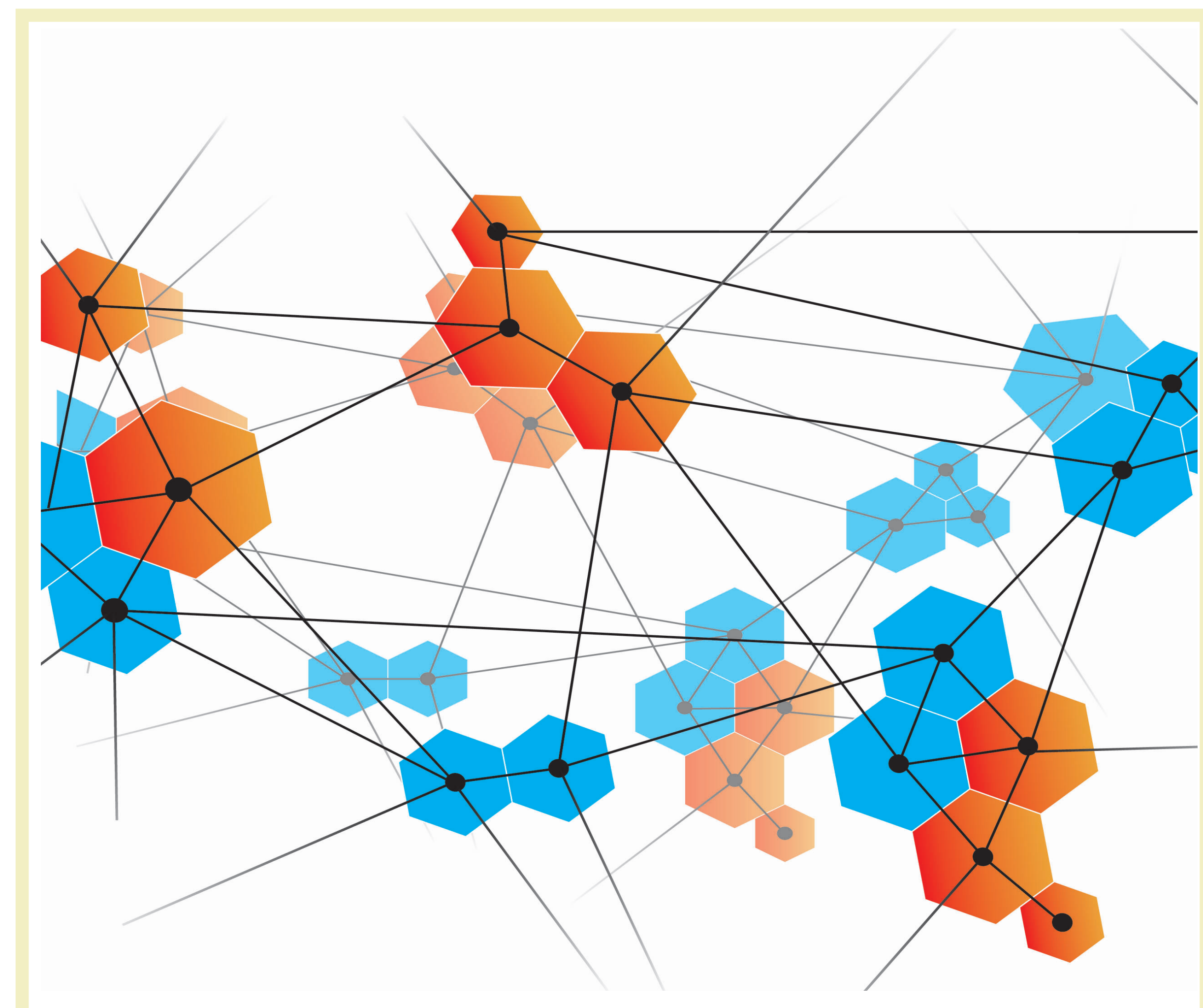
University of Colorado - Boulder
April 3, 2018

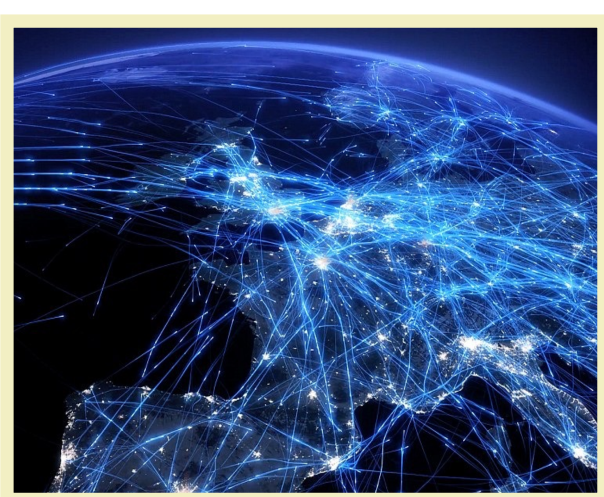
CoNeCt
Cooperative Networks and Controls Lab

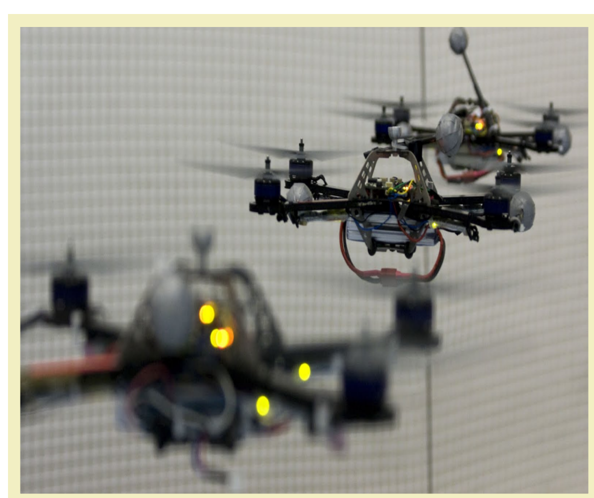
TECHNION
Israel Institute of Technology

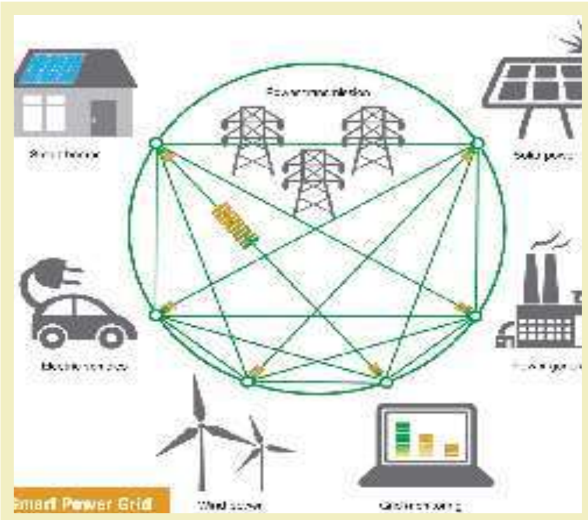


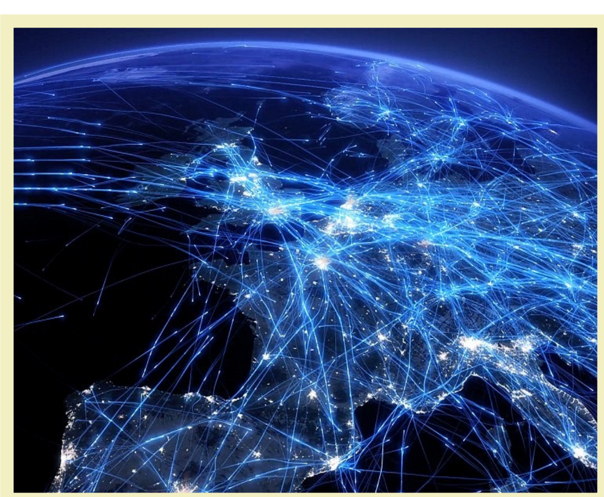
NETWORKS OF DYNAMICAL SYSTEMS ARE ONE OF THE ENABLING TECHNOLOGIES OF THE FUTURE

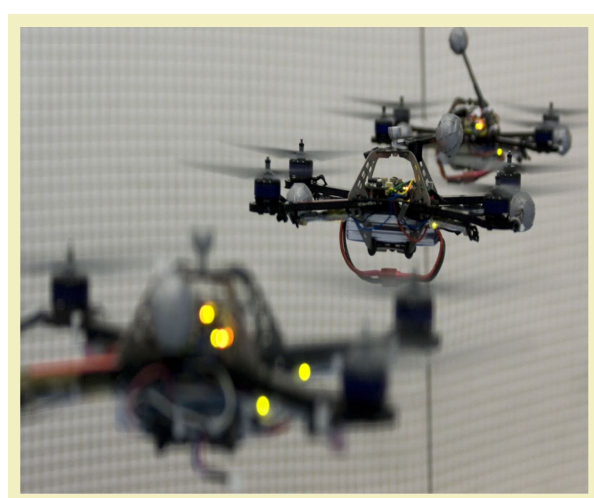








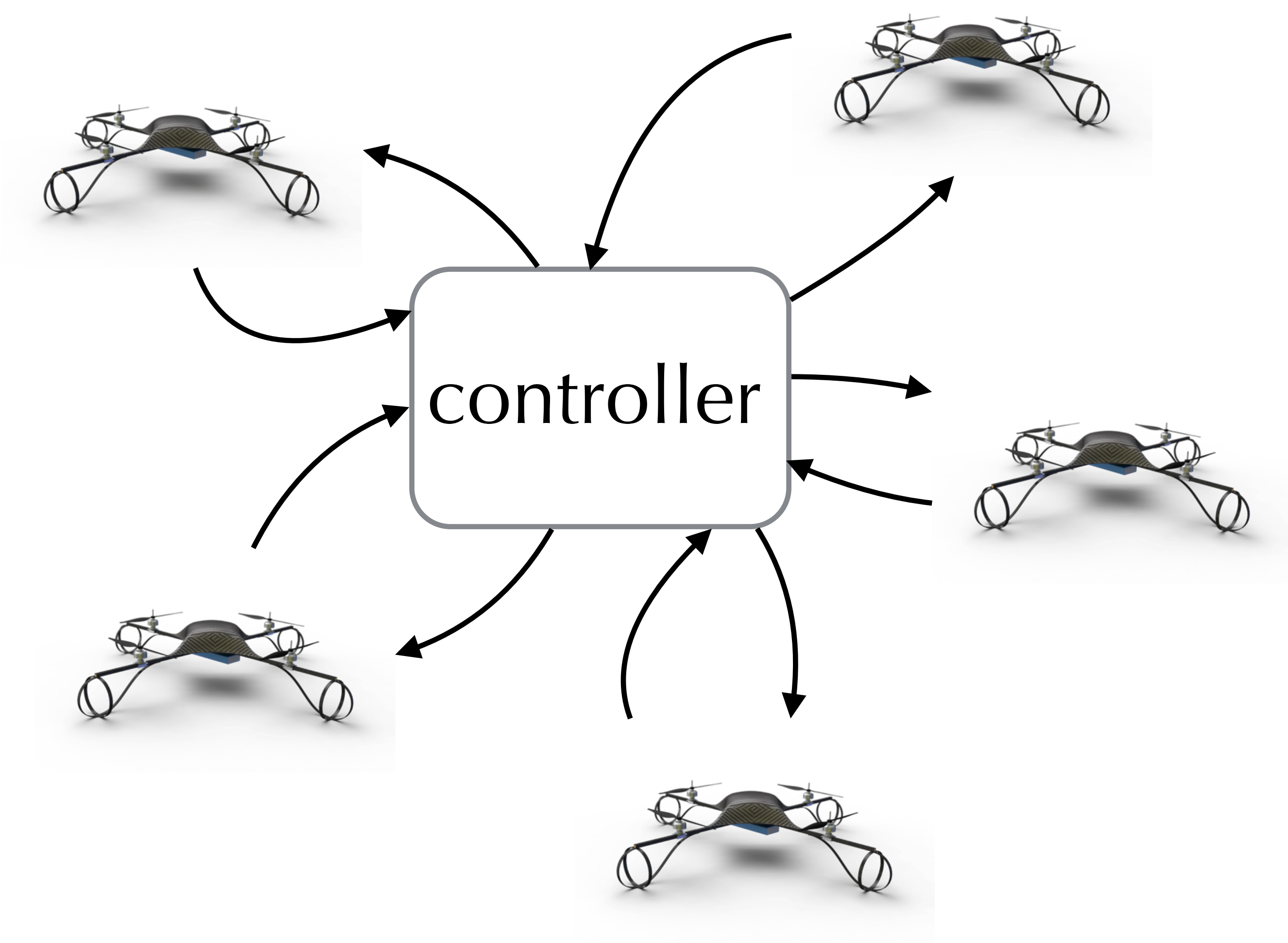




- how do we *analyze* these systems?
- how do we *design* these systems?

HOW DO WE CONTROL MULTI-AGENT SYSTEMS?

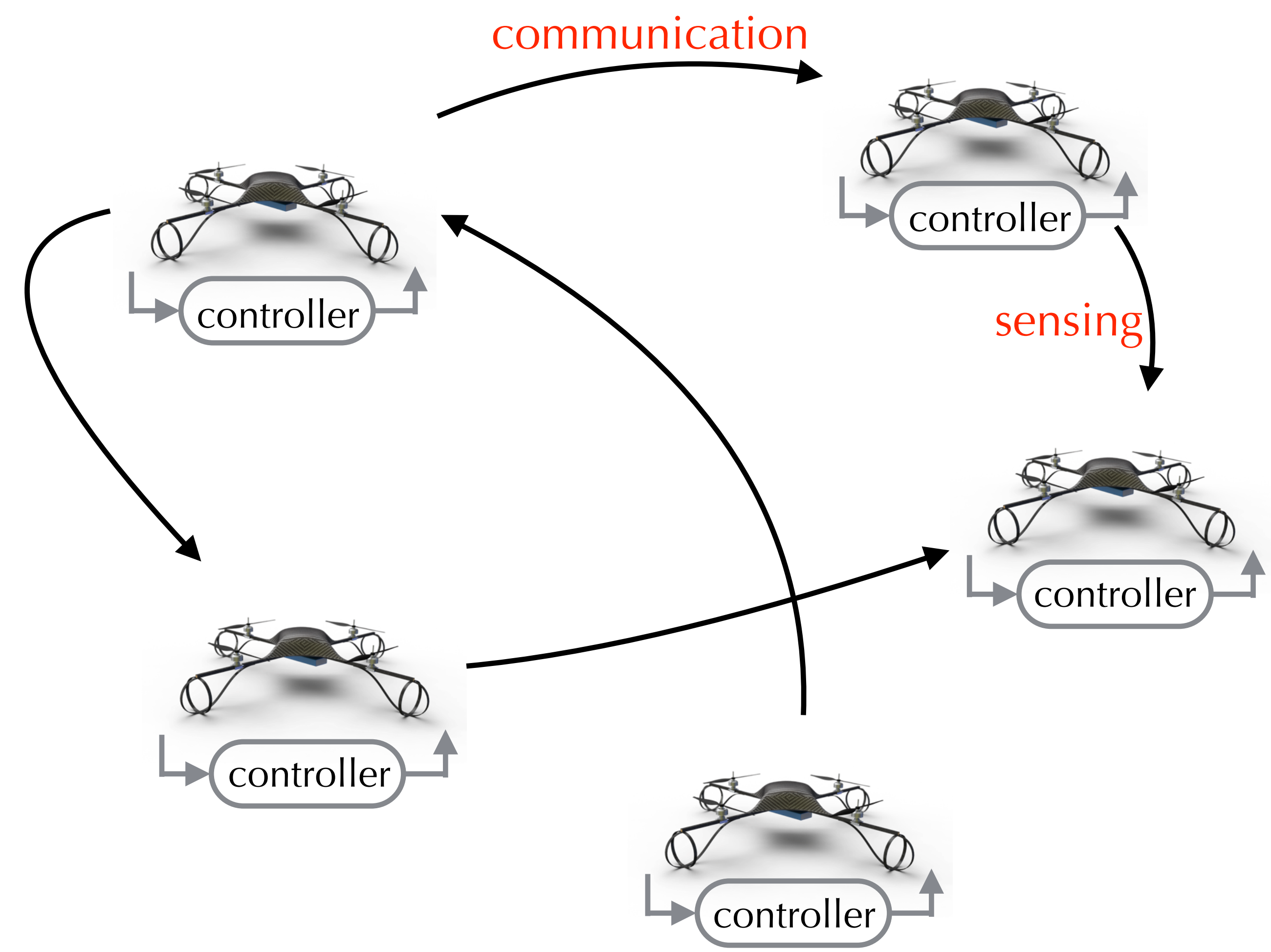
centralized approach



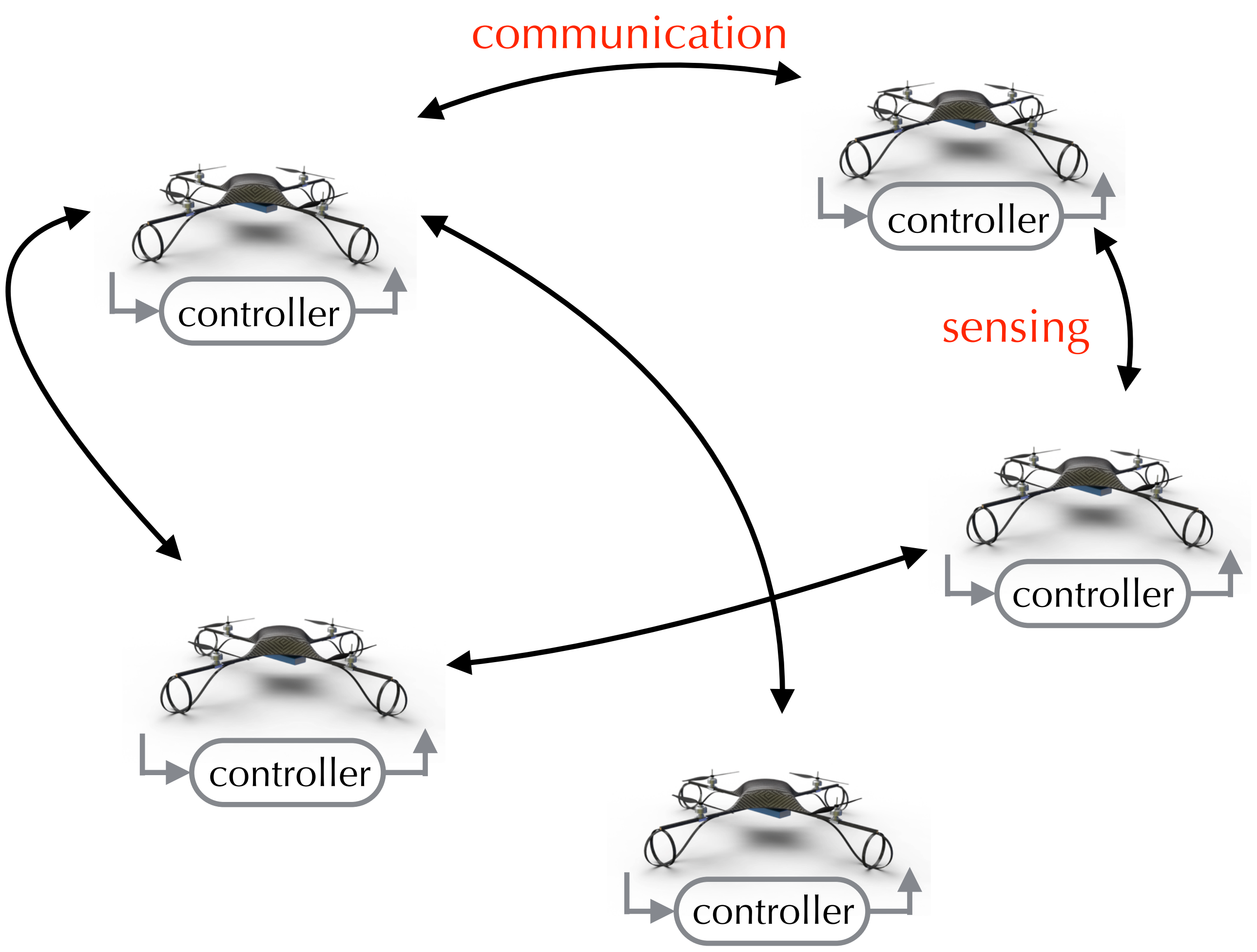
not scalable

not robust

decentralized/distributed approach

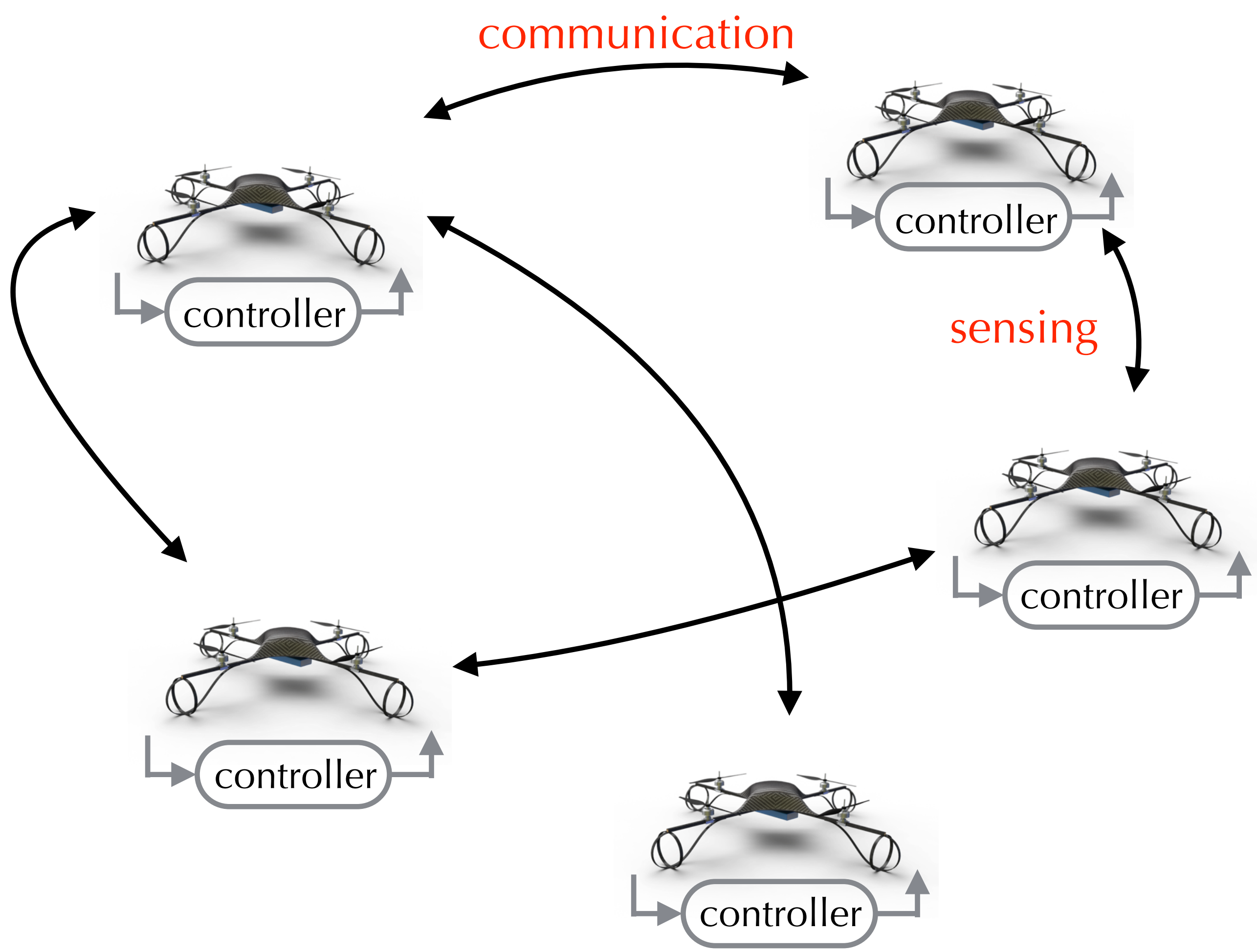


HOW DO WE CONTROL MULTI-AGENT SYSTEMS?



What is the right control architecture?

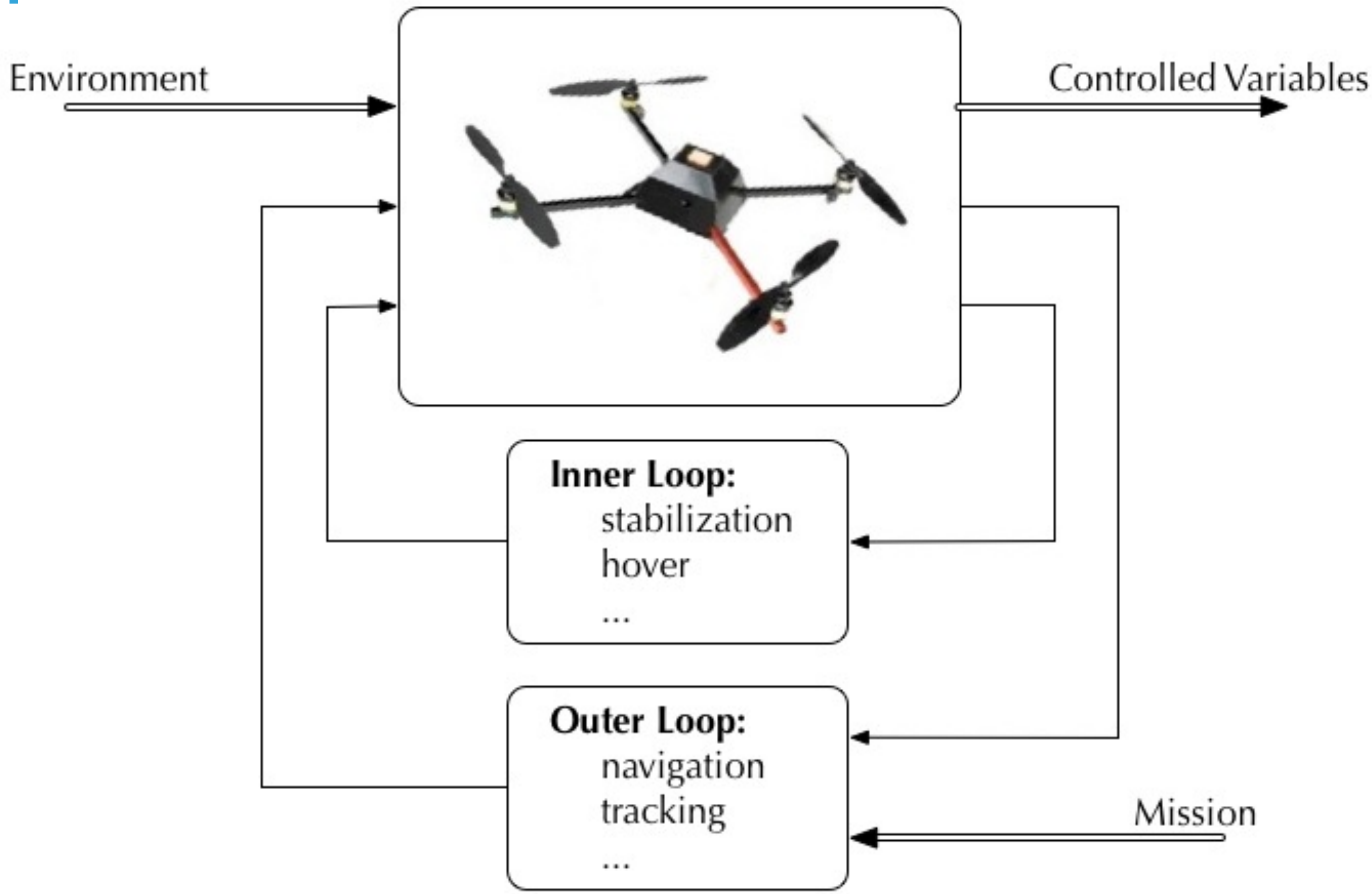
HOW DO WE CONTROL MULTI-AGENT SYSTEMS?



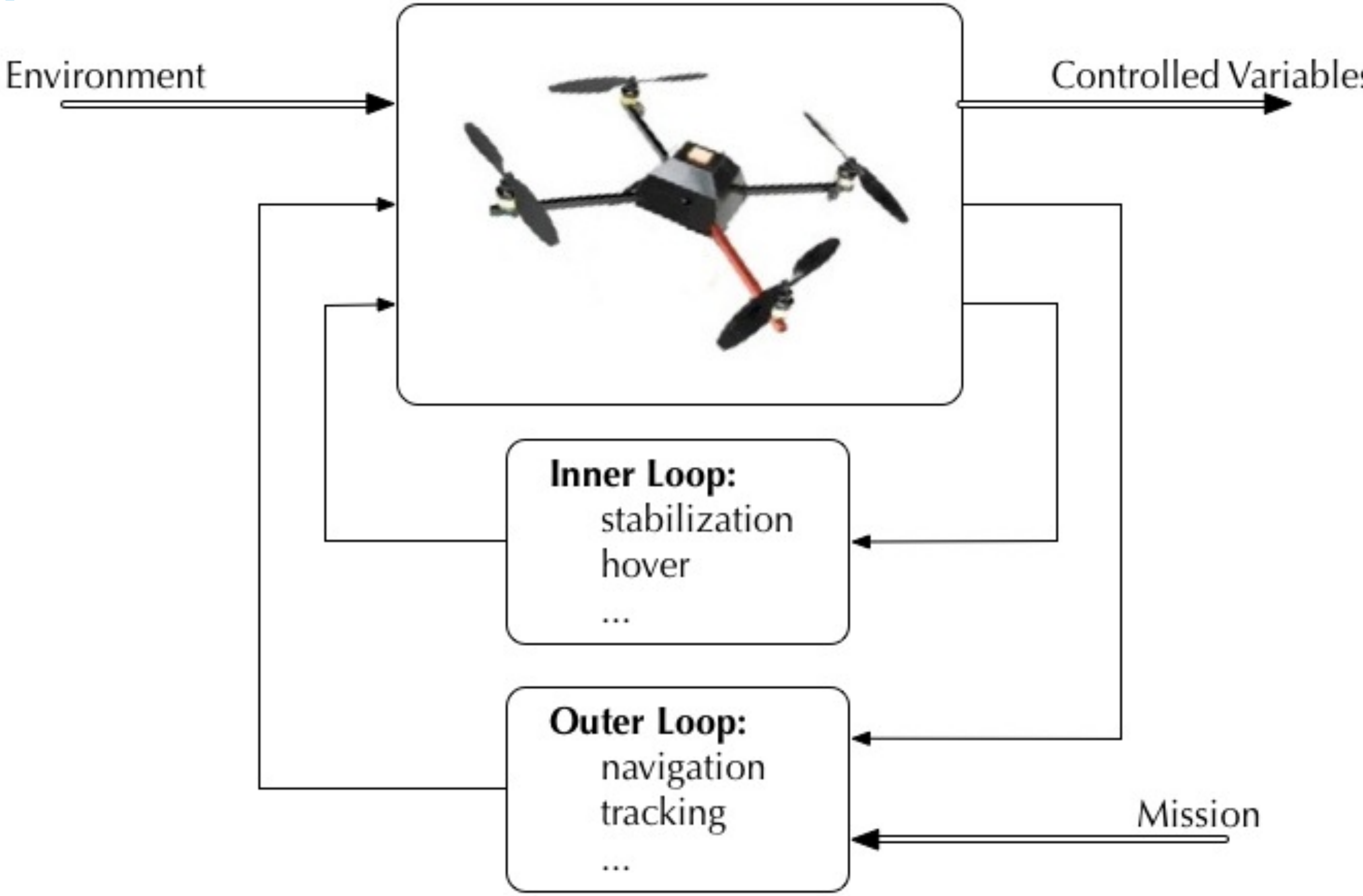
What is the right control architecture?

- of each agent
- of the information exchange layer

1 ROBOT

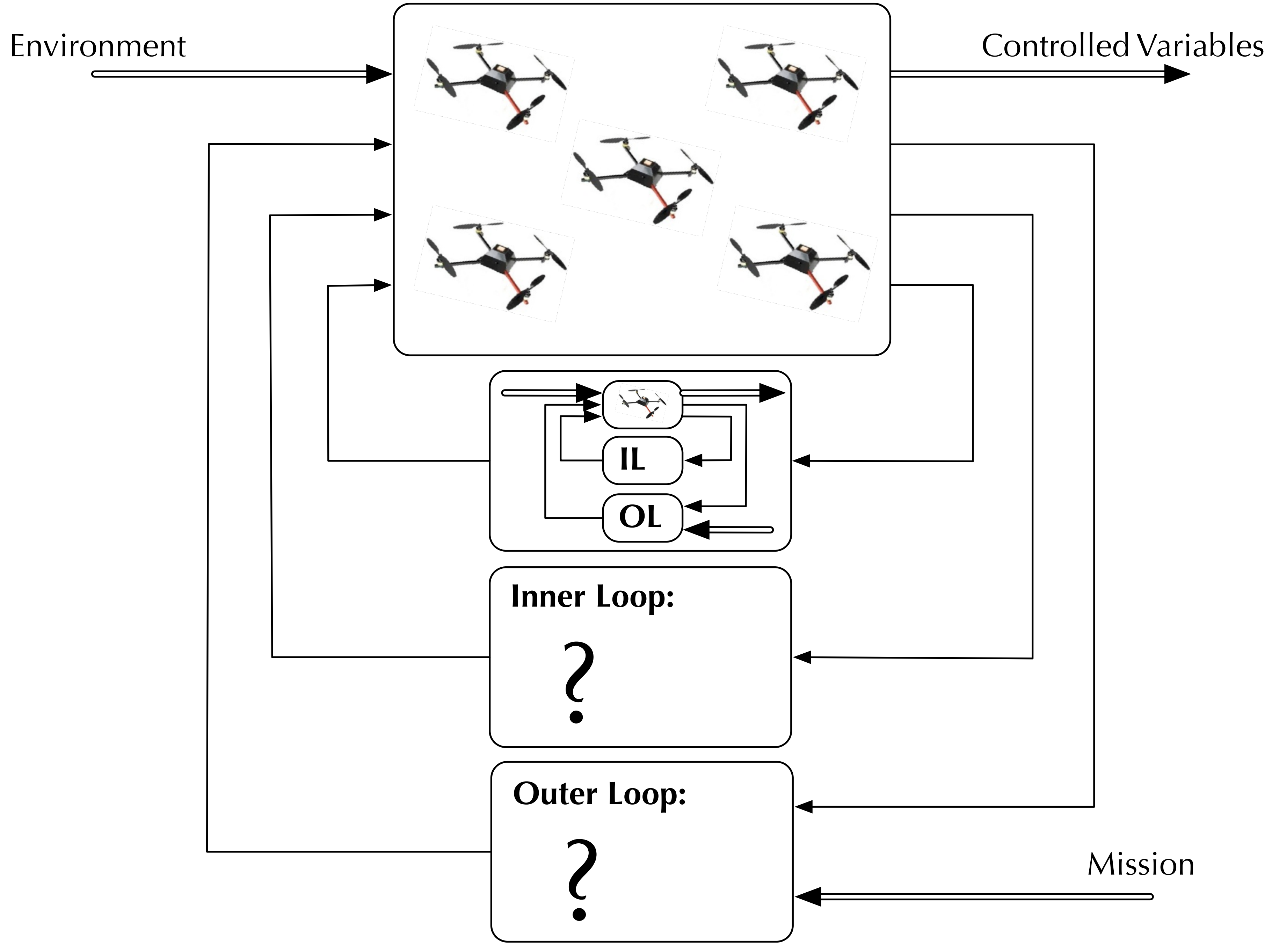

Environment
Controlled Variables
Inner Loop:
stabilization
hover
...
Outer Loop:
navigation
tracking
...
Mission

1 ROBOT

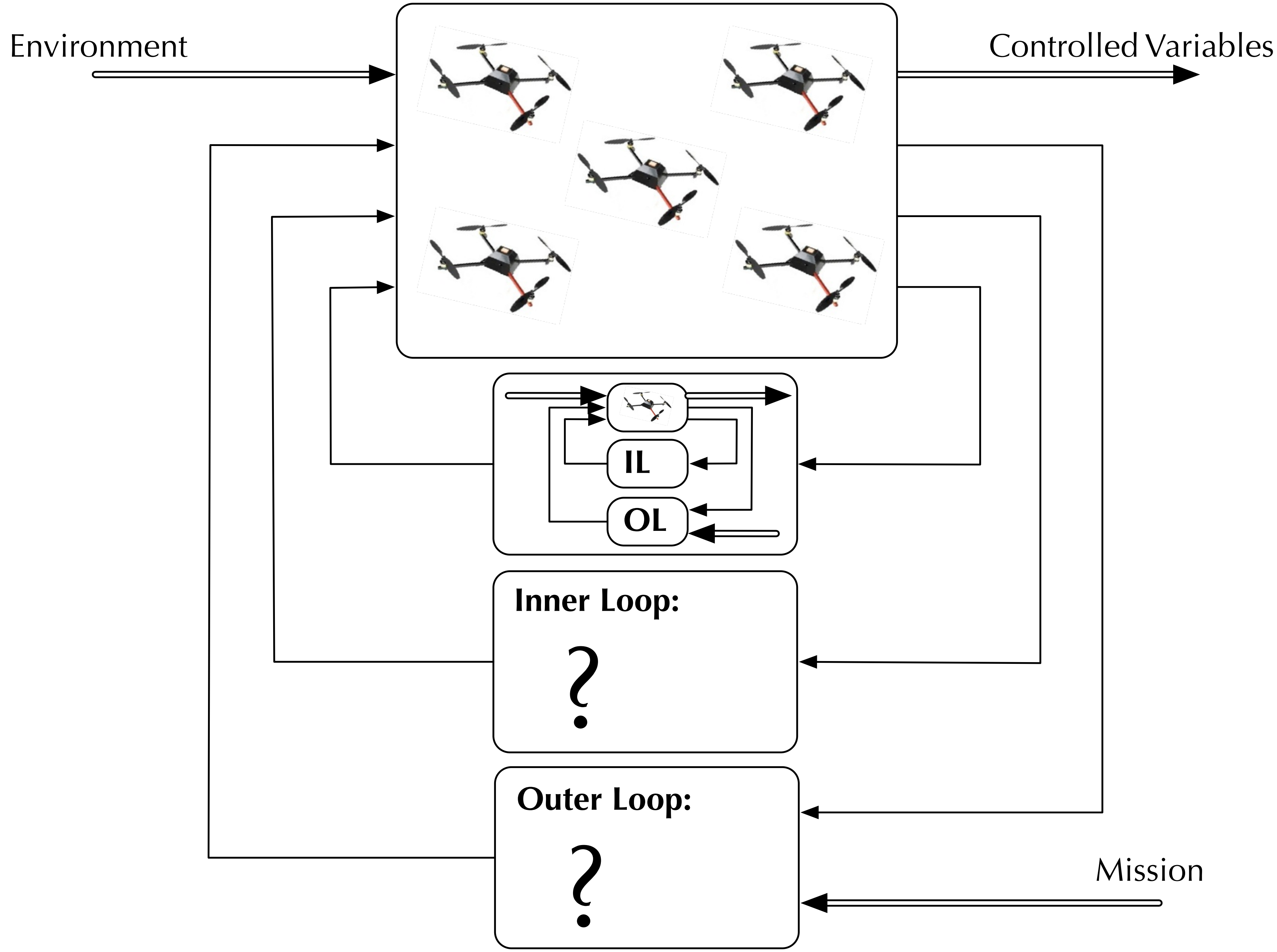


- dynamics

MULTI-ROBOT SYSTEM

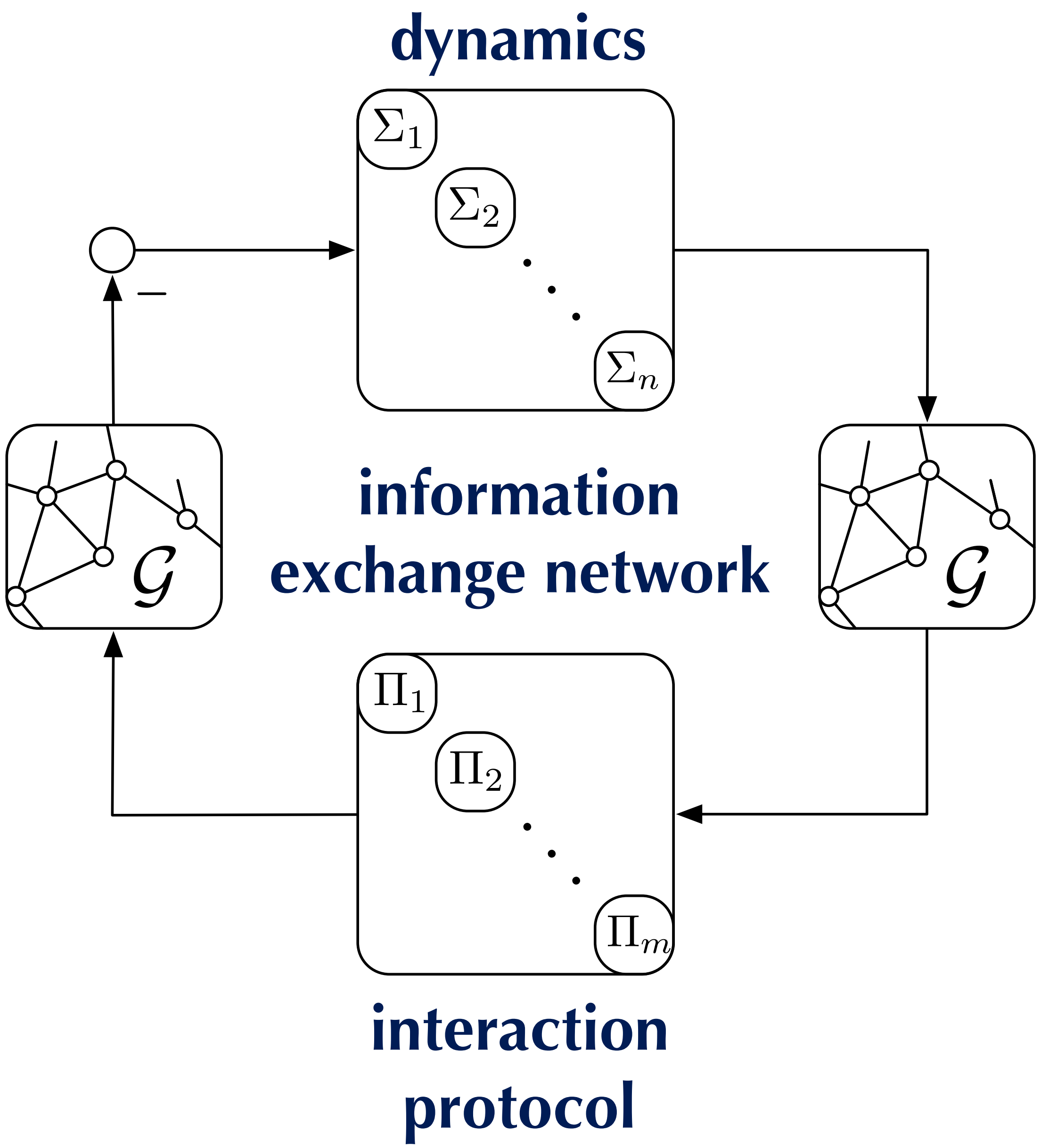


MULTI-ROBOT SYSTEM



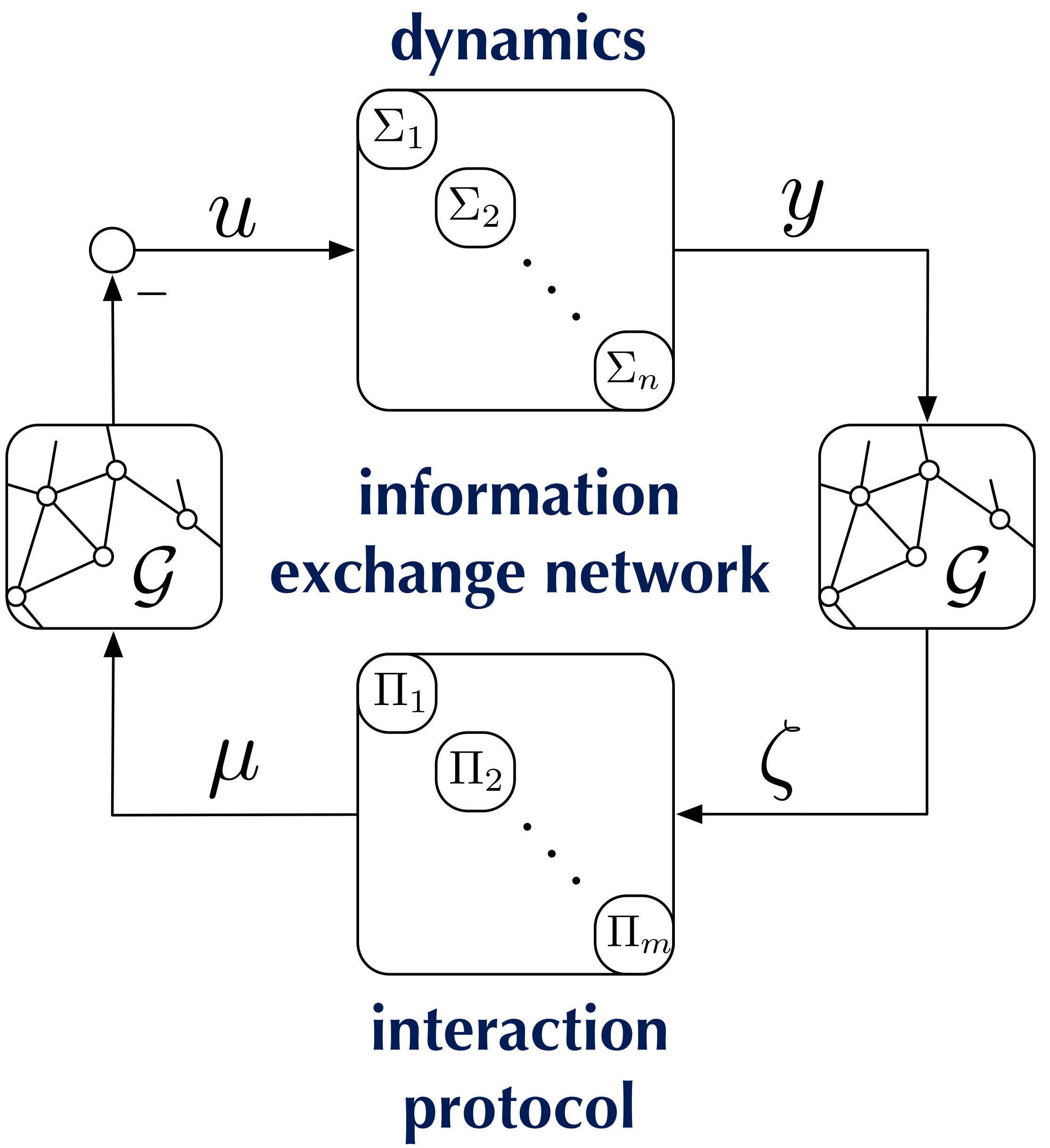
- **dynamics and the information exchange layer**

MULTI-AGENT SYSTEM ARCHITECTURES

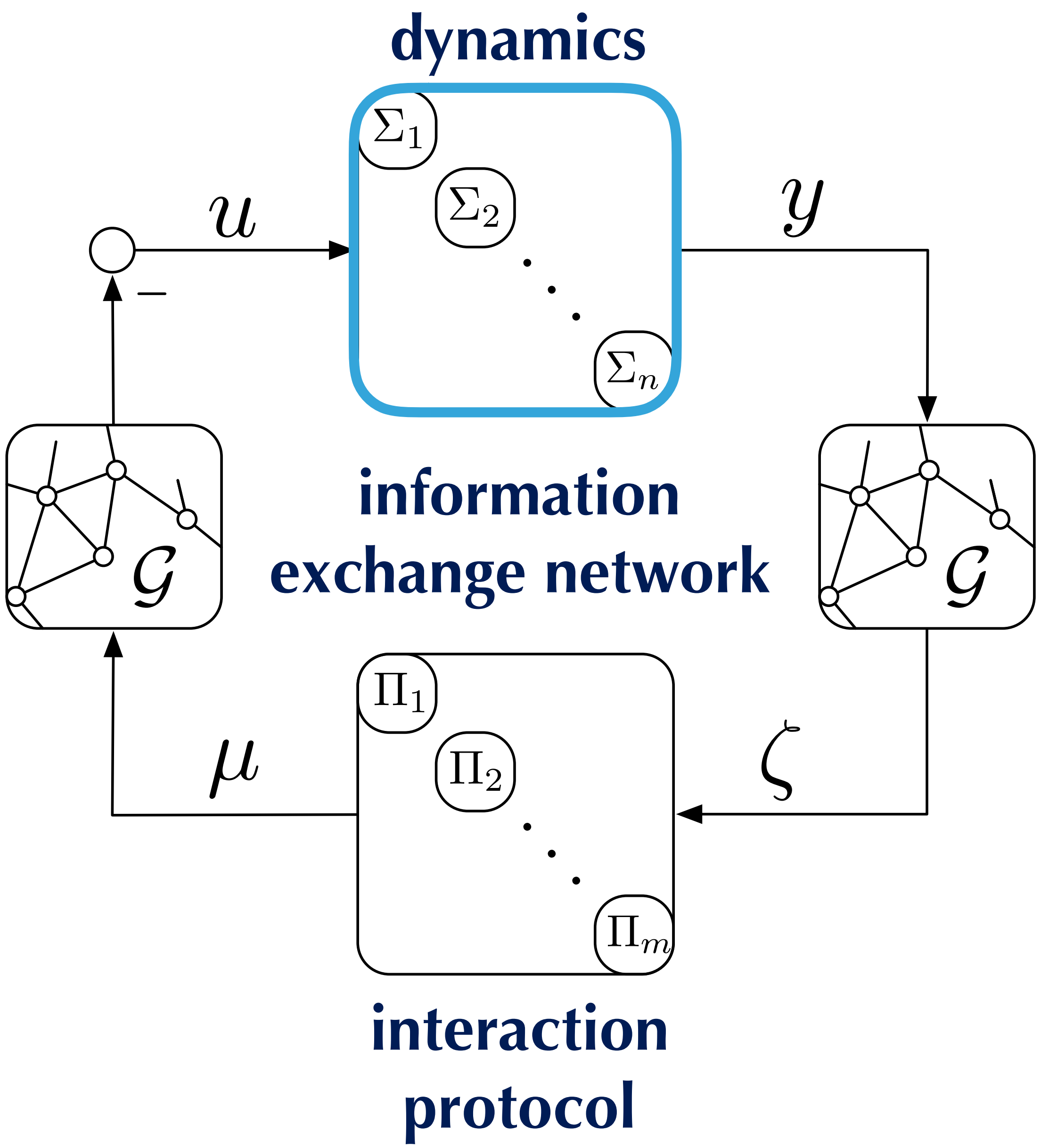


- the networked system
- dynamics for coordination
- information exchange architectures

NETWORKED DYNAMIC SYSTEMS



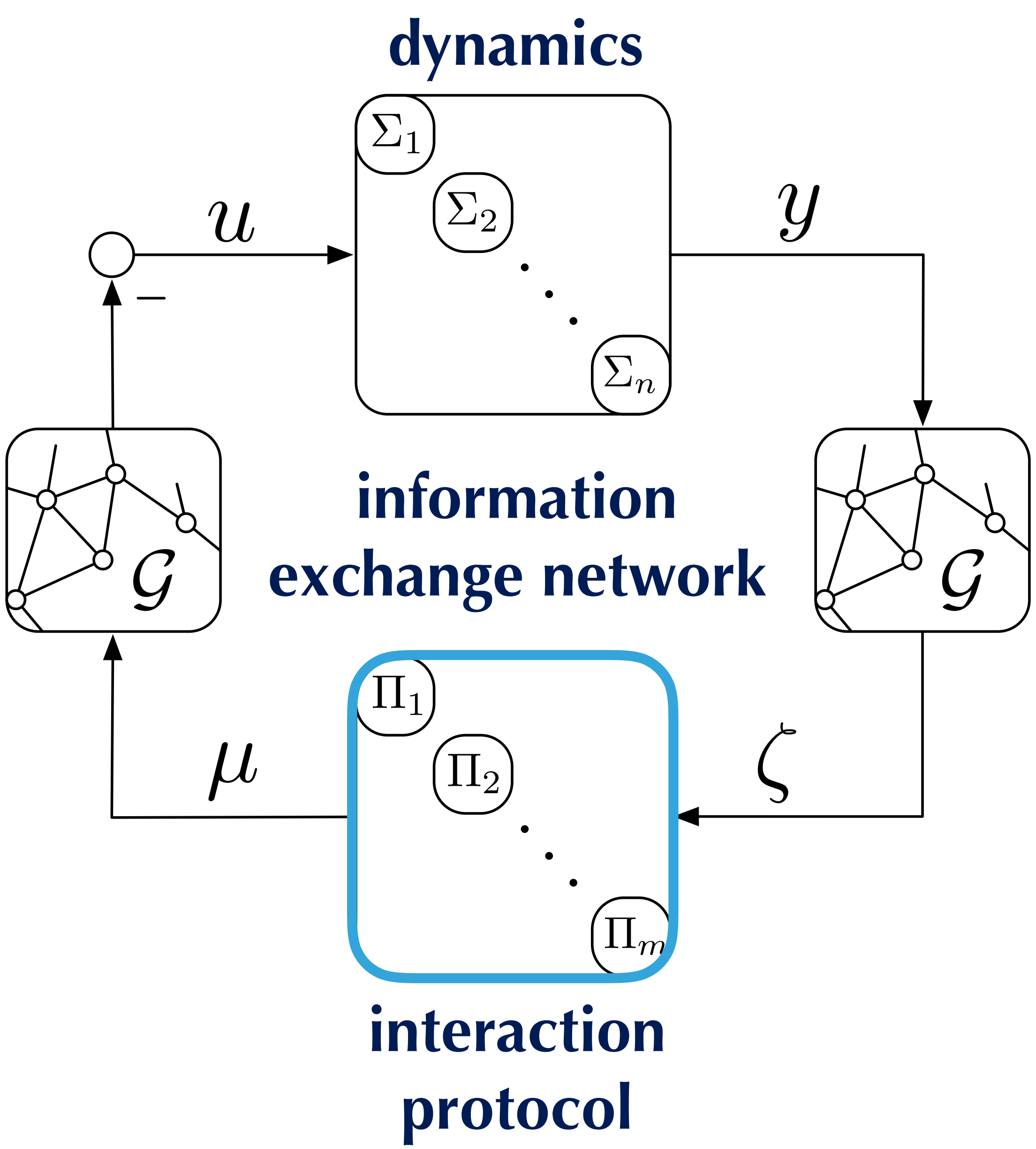
NETWORKED DYNAMIC SYSTEMS



Agent Dynamics

$$\Sigma_i : \begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = h_i(x_i, u_i) \end{cases}$$

NETWORKED DYNAMIC SYSTEMS



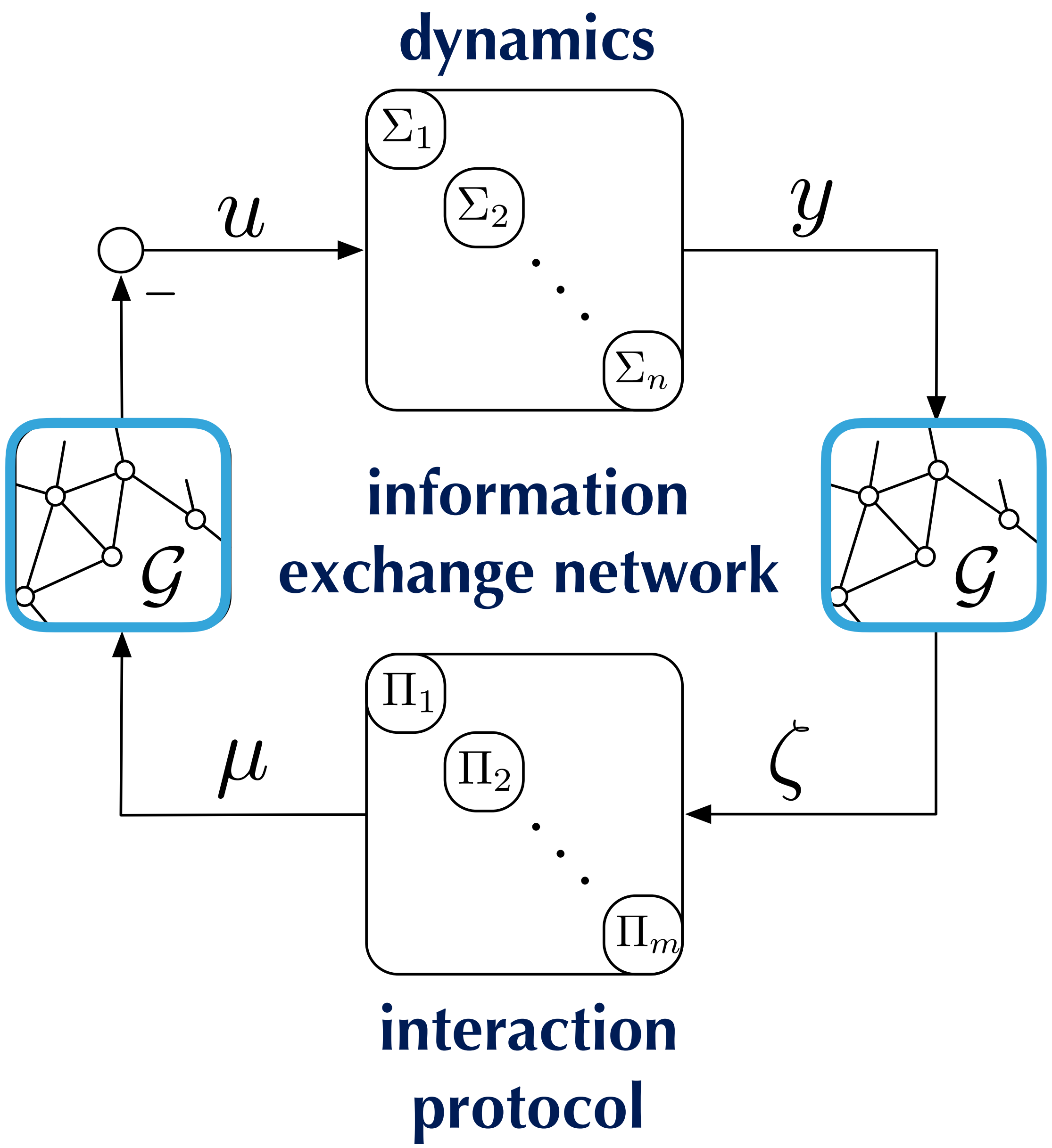
Agent Dynamics

$$\Sigma_i : \begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = h_i(x_i, u_i) \end{cases}$$

Controller Dynamics

$$\Pi_e : \begin{cases} \dot{\eta}_e = \phi_e(\eta_e, \zeta_e) \\ \mu_e = \psi_e(\eta_e, \zeta_e) \end{cases}$$

NETWORKED DYNAMIC SYSTEMS



Agent Dynamics

$$\Sigma_i : \begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = h_i(x_i, u_i) \end{cases}$$

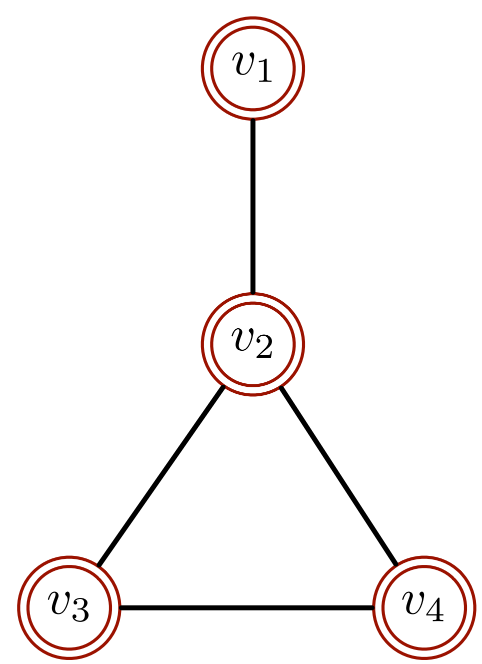
Controller Dynamics

$$\Pi_e : \begin{cases} \dot{\eta}_e = \phi_e(\eta_e, \zeta_e) \\ \mu_e = \psi_e(\eta_e, \zeta_e) \end{cases}$$

Information Exchange Network

A Graph

$$\mathcal{G} = (\mathcal{V}, \mathcal{E})$$

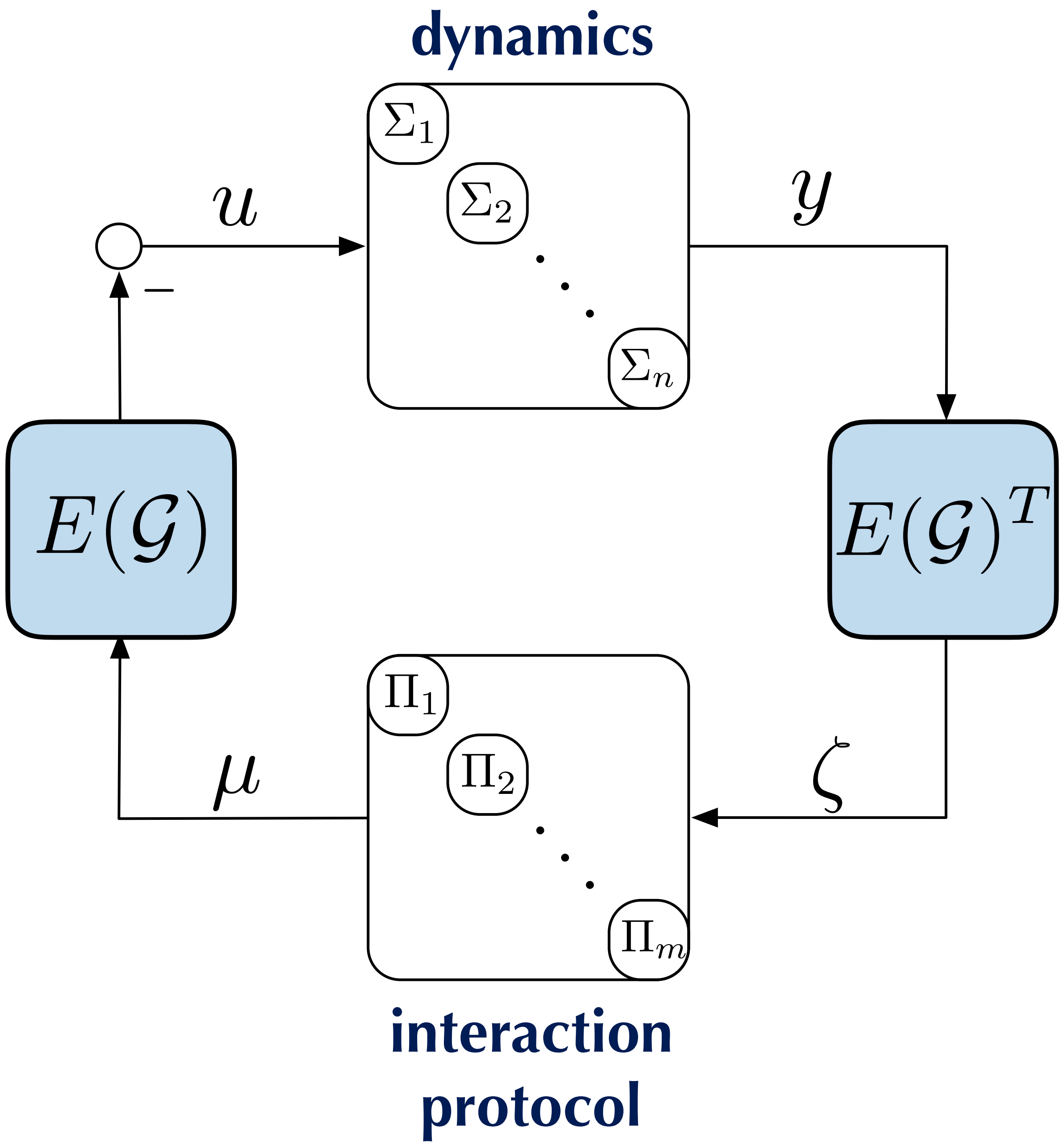


Incidence Matrix

$$E(\mathcal{G}) \in \mathbb{R}^{|\mathcal{V}| \times |\mathcal{E}|}$$

$$E(\mathcal{G}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

DIFFUSIVELY COUPLED NETWORKS



Kumamoto Model

$$\dot{\theta}_i = -k \sum_{i \sim j} \sin(\theta_i - \theta_j)$$

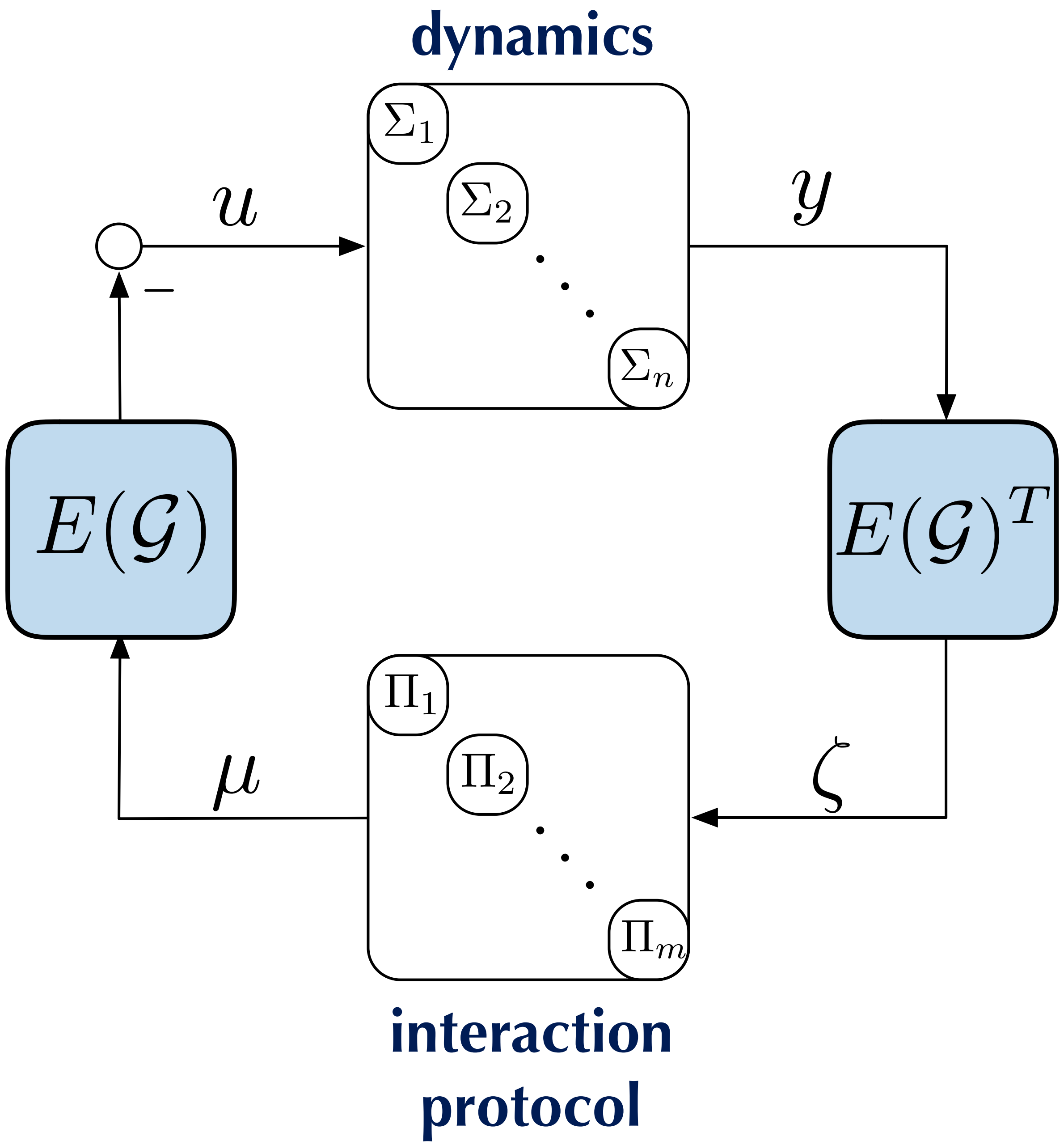
Traffic Dynamics Model

$$\dot{v}_i = \kappa_i \left(V_i^0 - v_i + V_i^1 \sum_{i \sim j} \tanh(p_j - p_i) \right)$$

Neural Network

$$\begin{aligned} C\dot{V}_i &= f(V_i, h_i) + \textstyle\sum_{i \sim j} g_{ij}(V_j - V_i) \\ \dot{h}_i &= g(V_i, h_i) \end{aligned}$$

DIFFUSIVELY COUPLED NETWORKS



Kumamoto Model

$$\dot{\theta}_i = -k \sum_{i \sim j} \sin(\theta_i - \theta_j)$$

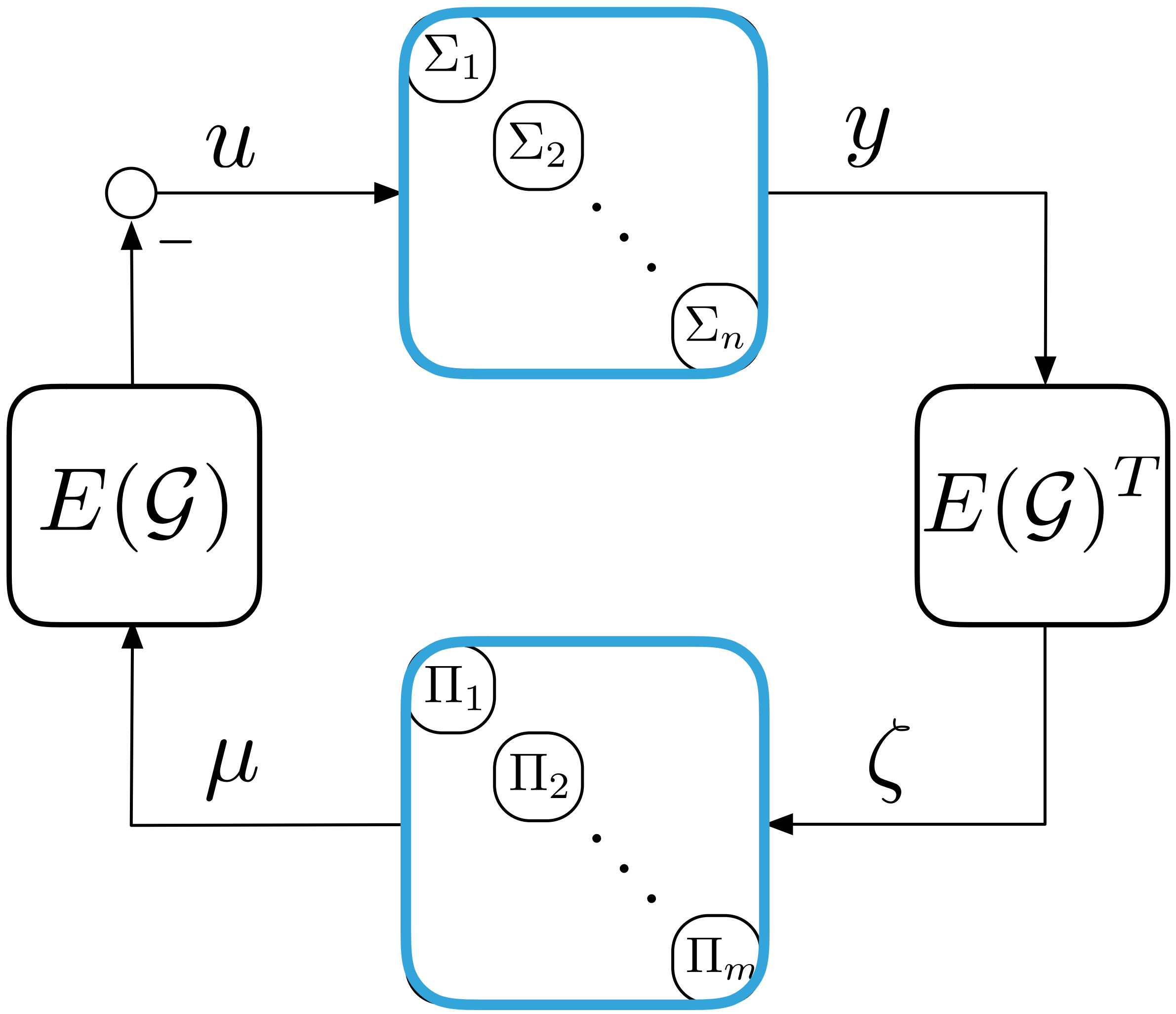
Traffic Dynamics Model

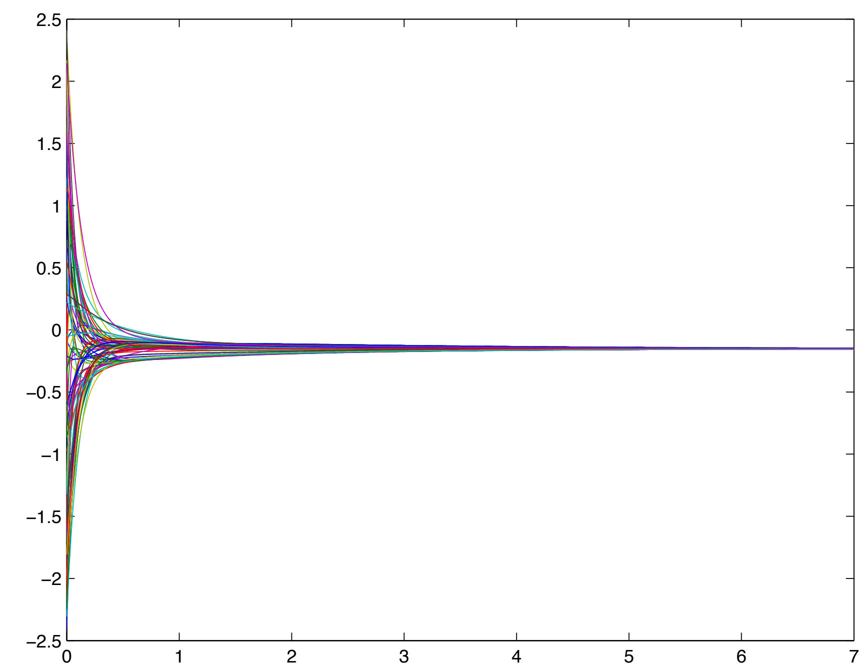
$$\dot{v}_i = \kappa_i \left(V_i^0 - v_i + V_i^1 \sum_{i \sim j} \tanh(p_j - p_i) \right)$$

Neural Network

$$\begin{aligned} C\dot{V}_i &= f(V_i, h_i) + \textstyle\sum_{i \sim j} g_{ij}(V_j - V_i) \\ \dot{h}_i &= g(V_i, h_i) \end{aligned}$$

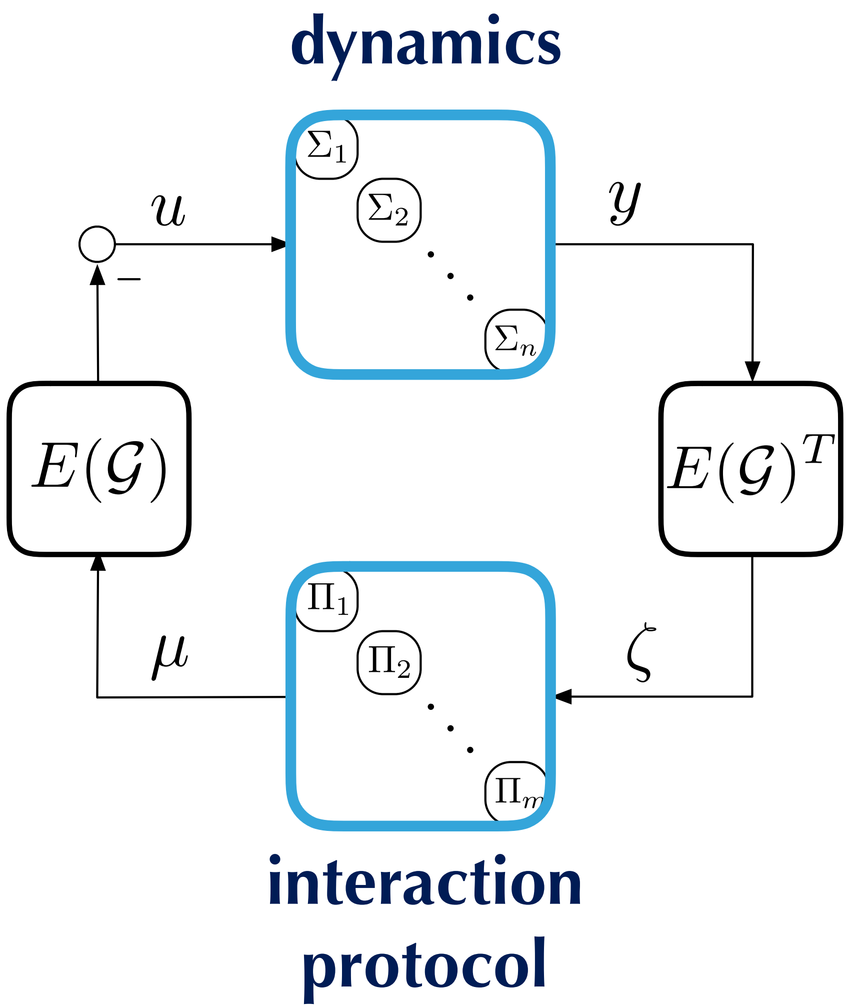
SYNCHRONIZATION – A NETWORK OPTIMIZATION PERSPECTIVE





What properties should the agent and controller dynamics posses to solve the synchronization problem?

SYNCHRONIZATION - A NETWORK OPTIMIZATION PERSPECTIVE



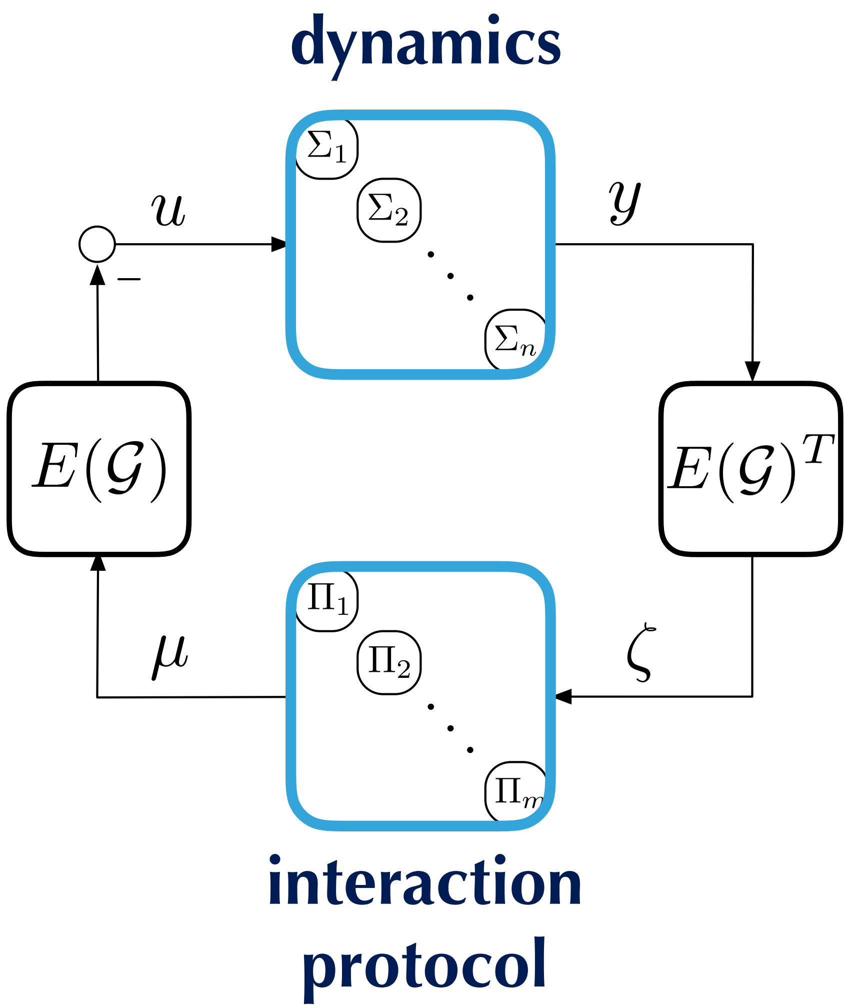
Synchronization

$$\lim_{t\to\infty} y_i(t) - y_j(t) = 0,\ \forall i, j$$

“Formation"

$$\lim_{t\to\infty} y(t) = \mathbf{y}$$

SYNCHRONIZATION – A NETWORK OPTIMIZATION PERSPECTIVE



Synchronization

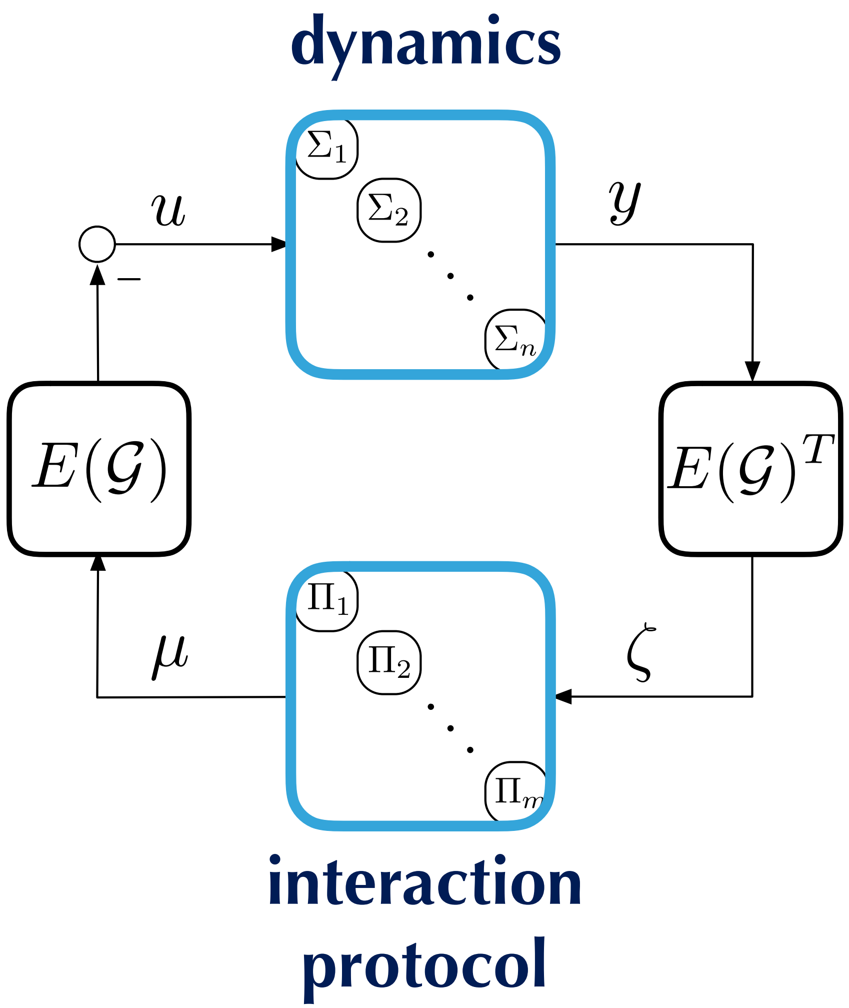
$$\lim_{t\to\infty} y_i(t) - y_j(t) = 0, \ \forall i, j$$

"Formation"

$$\lim_{t\to\infty} y(t) = \mathbf{y}$$

- **assume agents and controllers admit steady-state solutions**

SYNCHRONIZATION – A NETWORK OPTIMIZATION PERSPECTIVE



Synchronization

$$\lim_{t\to\infty} y_i(t) - y_j(t) = 0,\ \forall i,j$$

"Formation"

$$\lim_{t\to\infty} y(t) = \mathbf{y}$$

- **assume agents and controllers admit steady-state solutions**

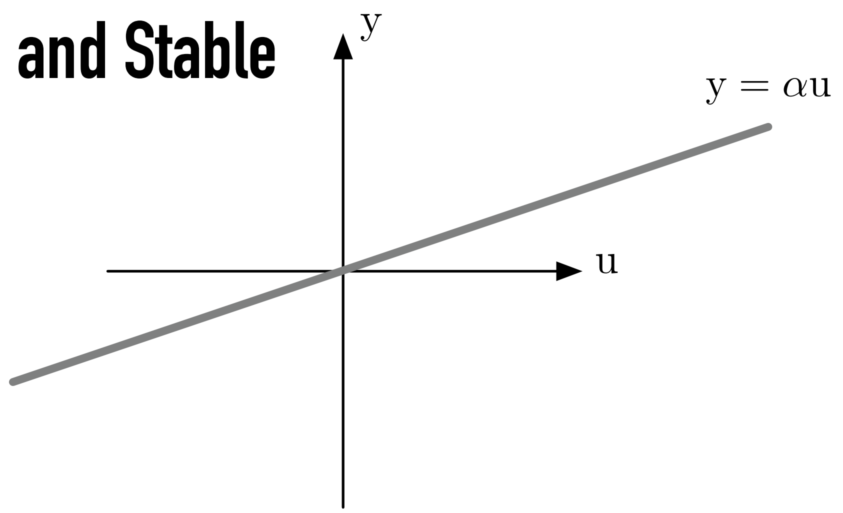
STEADY-STATE INPUT-OUTPUT RELATIONS

agents	**controllers**
$k_i(\mathrm{u}_i) = \{\mathrm{y}_i \,\vert\, (\mathrm{u}_i, \mathrm{y}_i) \in k_i\}$	$\gamma_e(\zeta_e) = \{\mu_e \,\vert\, (\zeta_e, \mu_e) \in \gamma_e\}$
$k_i^{-1}(\mathrm{y}_i) = \{\mathrm{u}_i \,\vert\, (\mathrm{u}_i, \mathrm{y}_i) \in k_i\}$	$\gamma_e^{-1}(\mu_e) = \{\mu_e \,\vert\, (\zeta_e, \mu_e) \in \gamma_e\}$

INPUT-OUTPUT RELATIONS

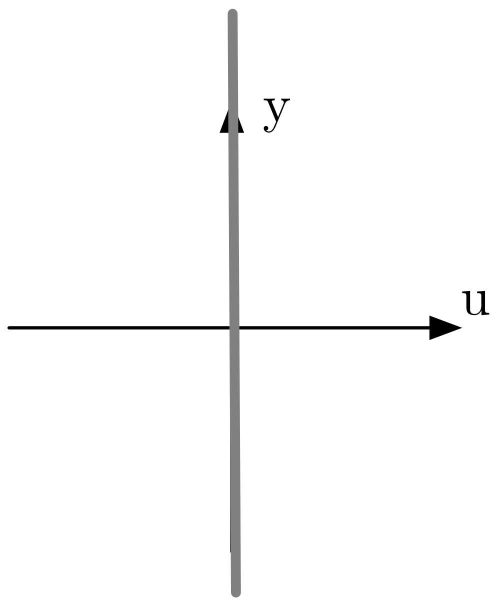
$$\Sigma : \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad \Rightarrow k(\mathrm{u}) = \{\mathrm{y} \,|\, \mathrm{y} = (-CA^{-1}B + D)\mathrm{u}\}$$

SISO and Stable



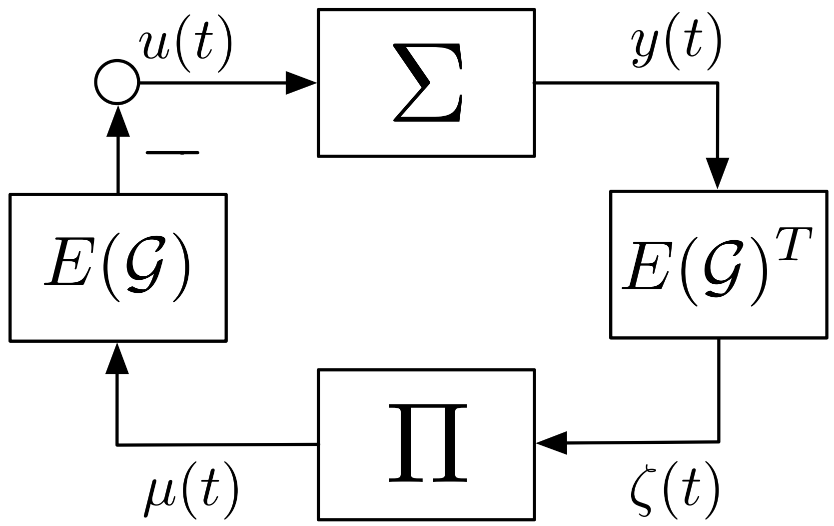
$$\Sigma : \begin{cases} \dot{x} = u \\ y = x \end{cases} \quad \Rightarrow k = \{(0, \mathrm{y}),\ \mathrm{y} \in \mathbb{R}\}$$

Integrator



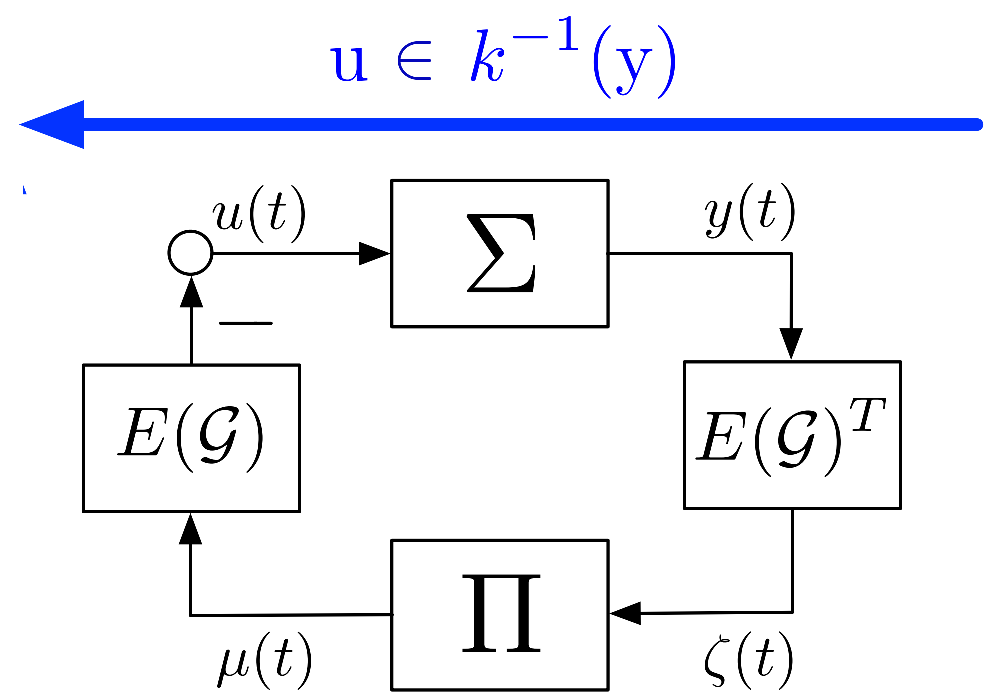
CONSISTENCY OF STEADY-STATES

The network enforces a relation on the steady-state



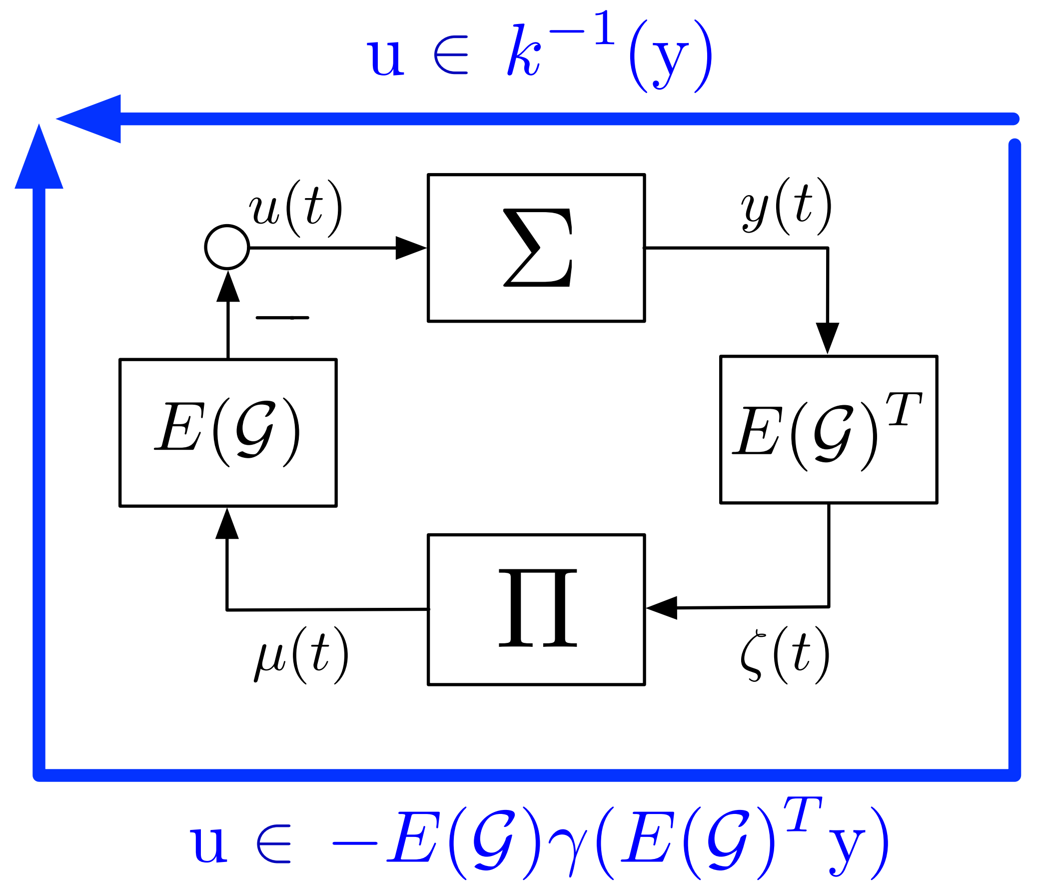
CONSISTENCY OF STEADY-STATES

The network enforces a relation on the steady-state



CONSISTENCY OF STEADY-STATES

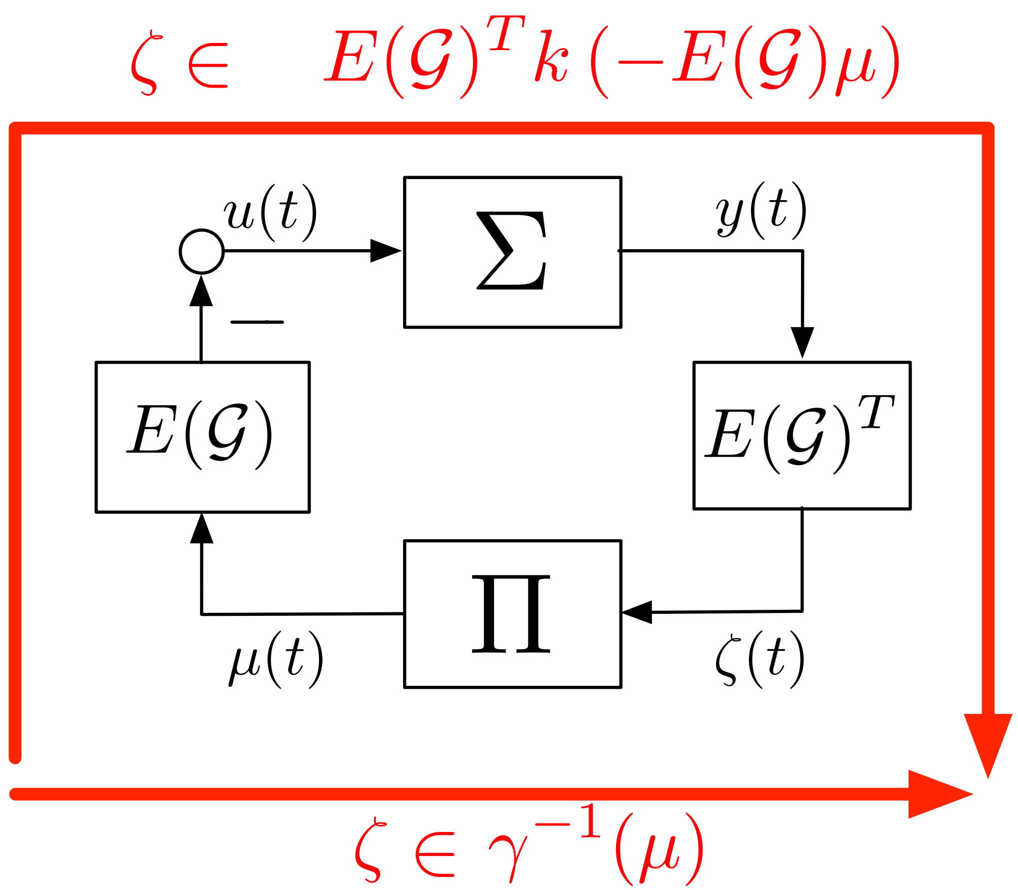
The network enforces a relation on the steady-state

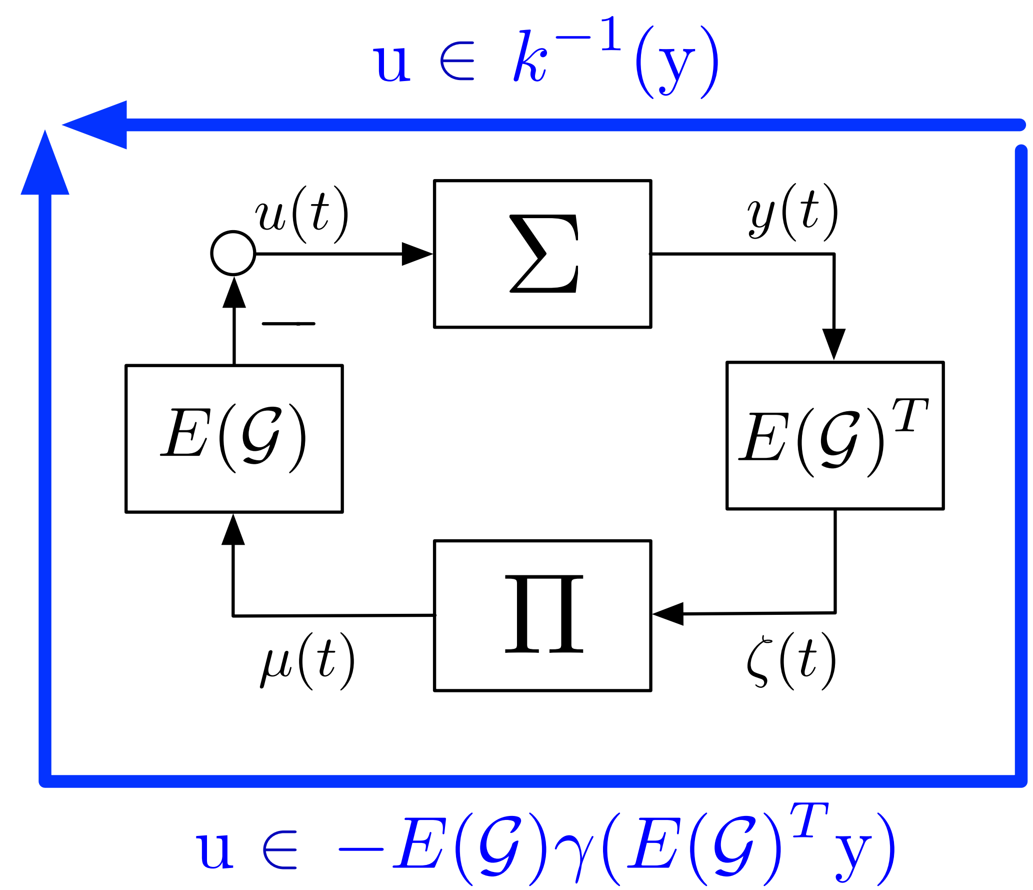


$$0 \in k^{-1}(\mathrm{y}) + E(\mathcal{G})\gamma\left(E(\mathcal{G})^T\mathrm{y}\right)$$

CONSISTENCY OF STEADY-STATES

The network enforces a relation on the steady-state



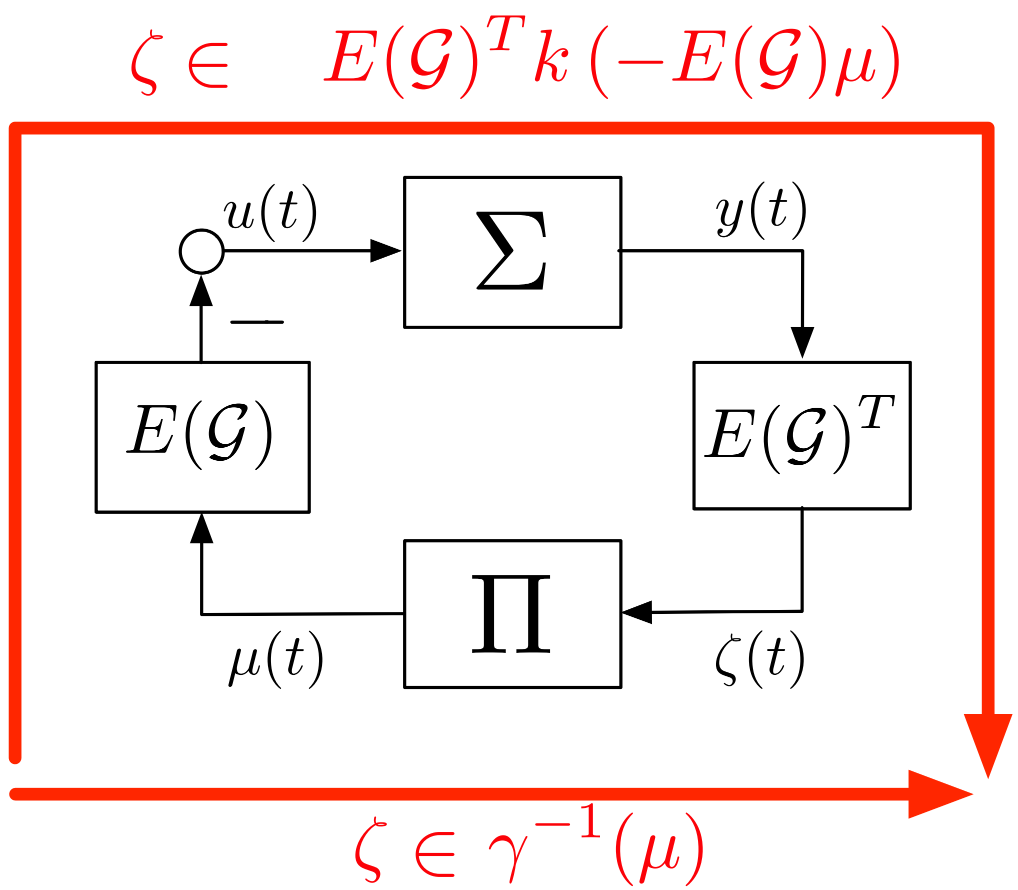


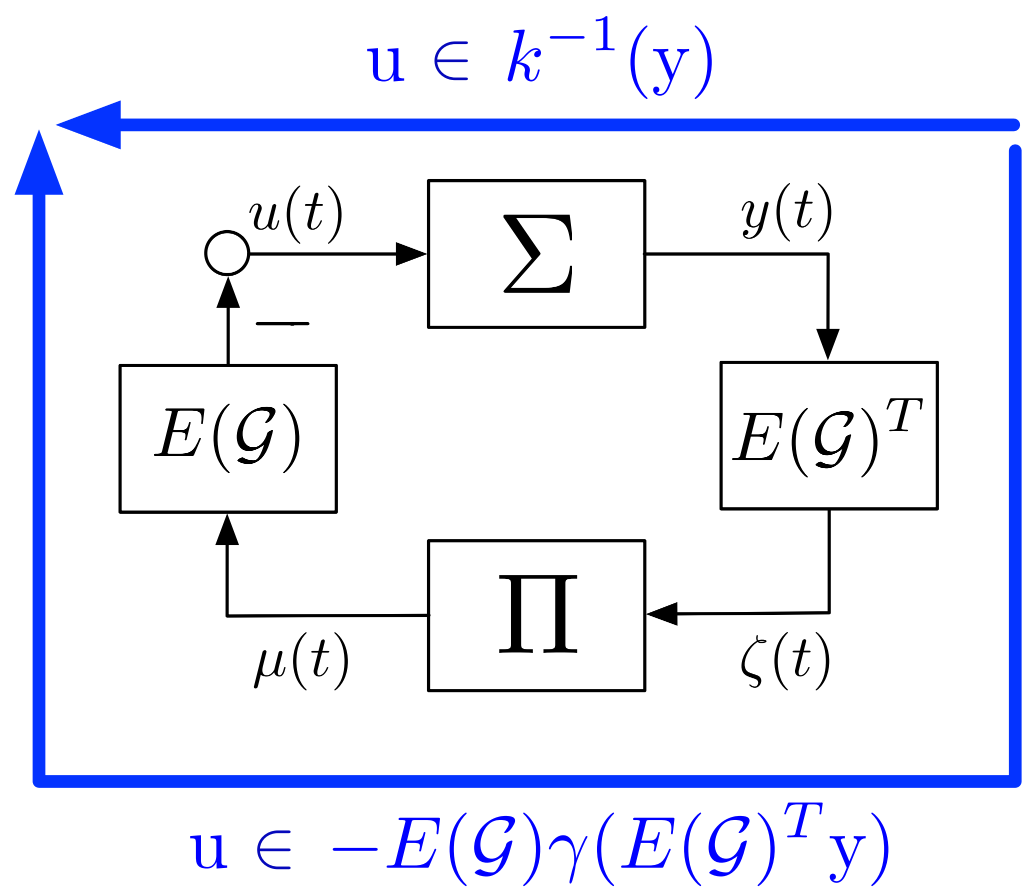
$$0 \in \gamma^{-1}(\mu) - E(\mathcal{G})^T k\left(-E(\mathcal{G})\mu\right)$$

$$0 \in k^{-1}(\mathrm{y}) + E(\mathcal{G})\gamma\left(E(\mathcal{G})^T \mathrm{y}\right)$$

CONSISTENCY OF STEADY-STATES

The network enforces a relation on the steady-state





$$0 \in \gamma^{-1}(\mu) - E(\mathcal{G})^T k\left(-E(\mathcal{G})\mu\right)$$

$$0 \in k^{-1}(\mathrm{y}) + E(\mathcal{G})\gamma\left(E(\mathcal{G})^T\mathrm{y}\right)$$

What are the solutions, if they exist, of this system of non-linear inclusions?

INTEGRATING THE CONSISTENCY EQUATIONS

INTEGRAL FUNCTIONS OF STEADY-STATE I/O RELATIONS

agents

$$\partial K_i = k_i \qquad K = \sum_{i=1}^{|\mathcal{V}|} K_i$$

$$\partial K_i^\star = k_i^{-1} \qquad K^\star = \sum_{i=1}^{|\mathcal{V}|} K_i^\star$$

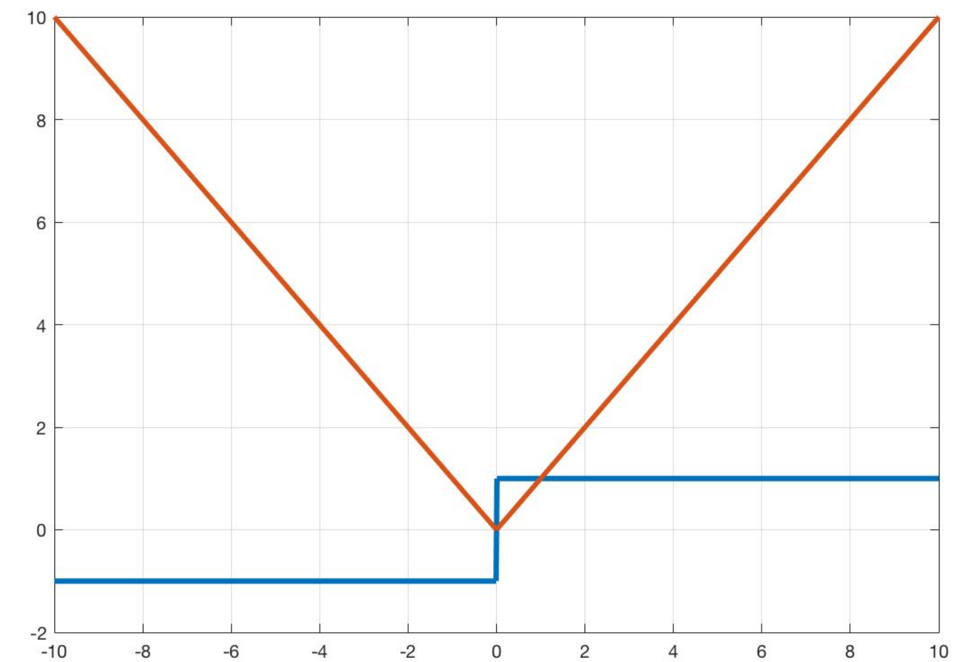
controllers

$$\partial \Gamma_e = \gamma_e \qquad \Gamma = \sum_{e=1}^{|\mathcal{E}|} \Gamma_e$$

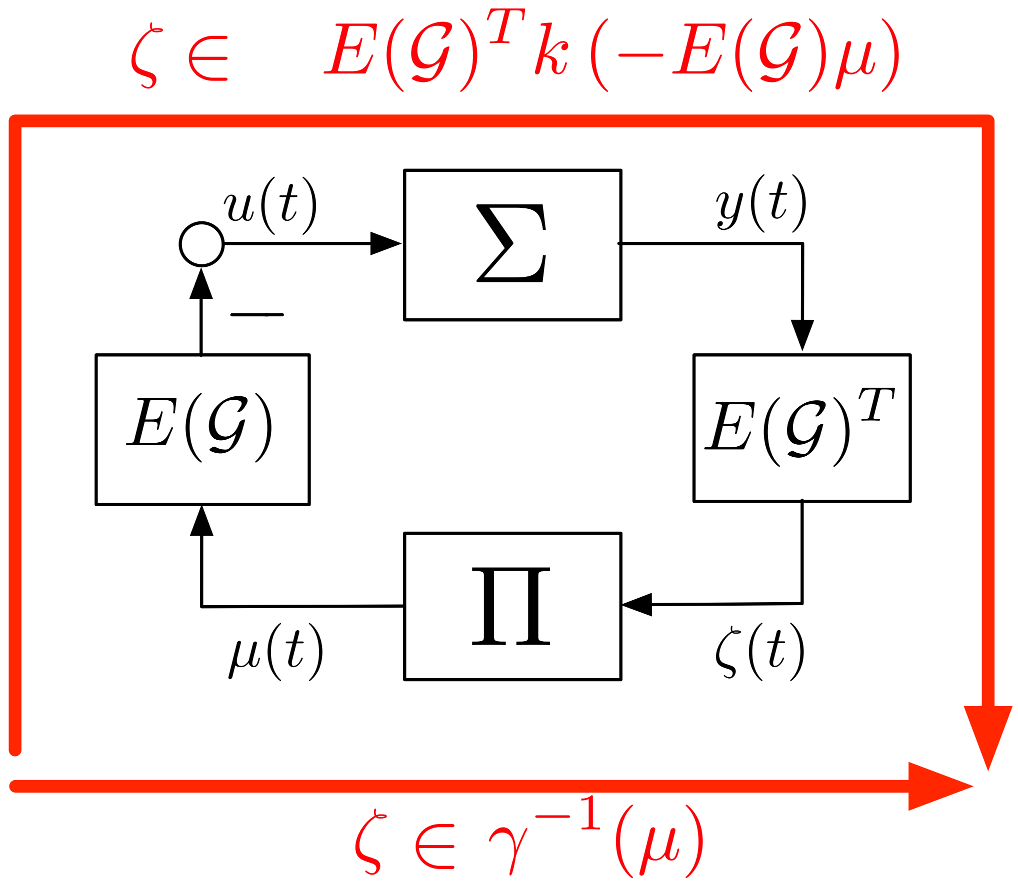
$$\partial \Gamma_e^\star = \gamma_e^{-1} \qquad \Gamma^\star = \sum_{e=1}^{|\mathcal{E}|} \Gamma_e^\star$$

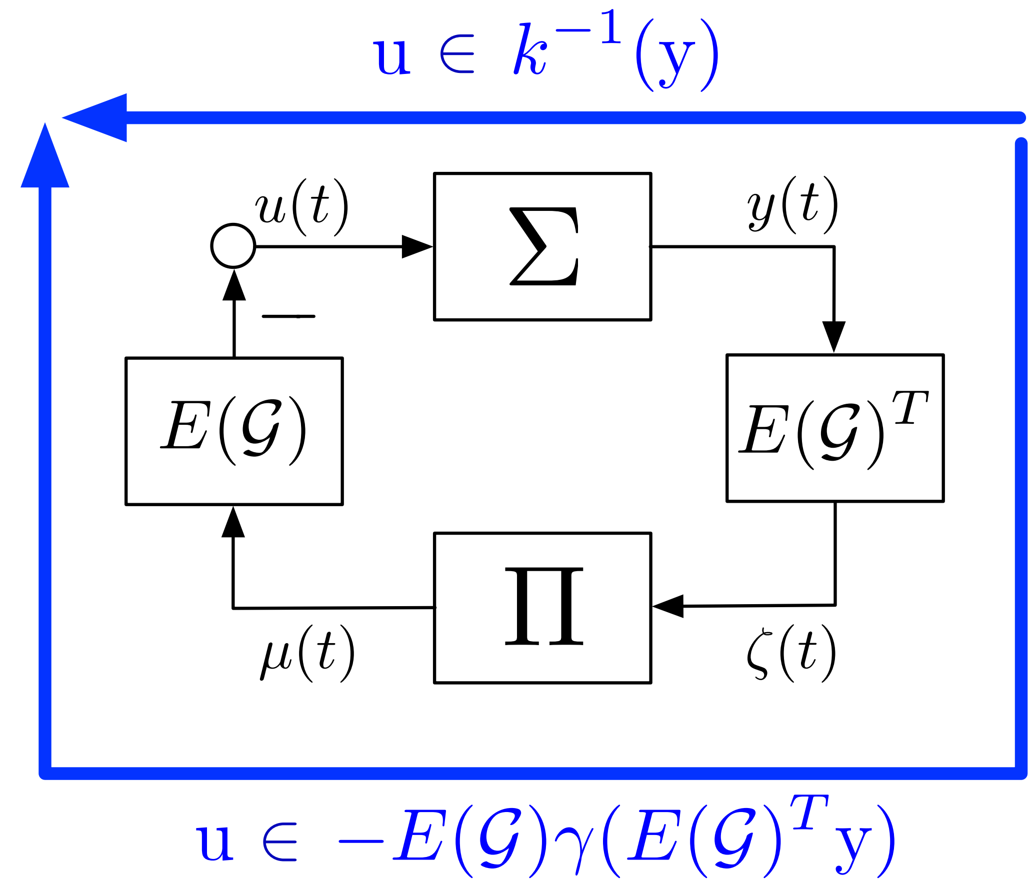
example

- $\mathrm{y} = k(\mathrm{u}) = sgn(\mathrm{u})$
- $K(\mathrm{u}) = |\mathrm{u}|$



OPTIMIZATION PERSPECTIVE





$$0 \in \gamma^{-1}(\mu) - E(\mathcal{G})^T k\left(-E(\mathcal{G})\mu\right)$$

$$0 \in k^{-1}(\mathrm{y}) + E(\mathcal{G})\gamma\left(E(\mathcal{G})^T\mathrm{y}\right)$$

$$\min_{\mathrm{u},\mu} \quad \textstyle\sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e) \qquad s.t. \quad \mathrm{u} + E(\mathcal{G})\mu = 0$$

||

$$\min_{\mathrm{y},\zeta} \quad \textstyle\sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e) \qquad s.t. \quad E(\mathcal{G})^T\mathrm{y} = \zeta$$

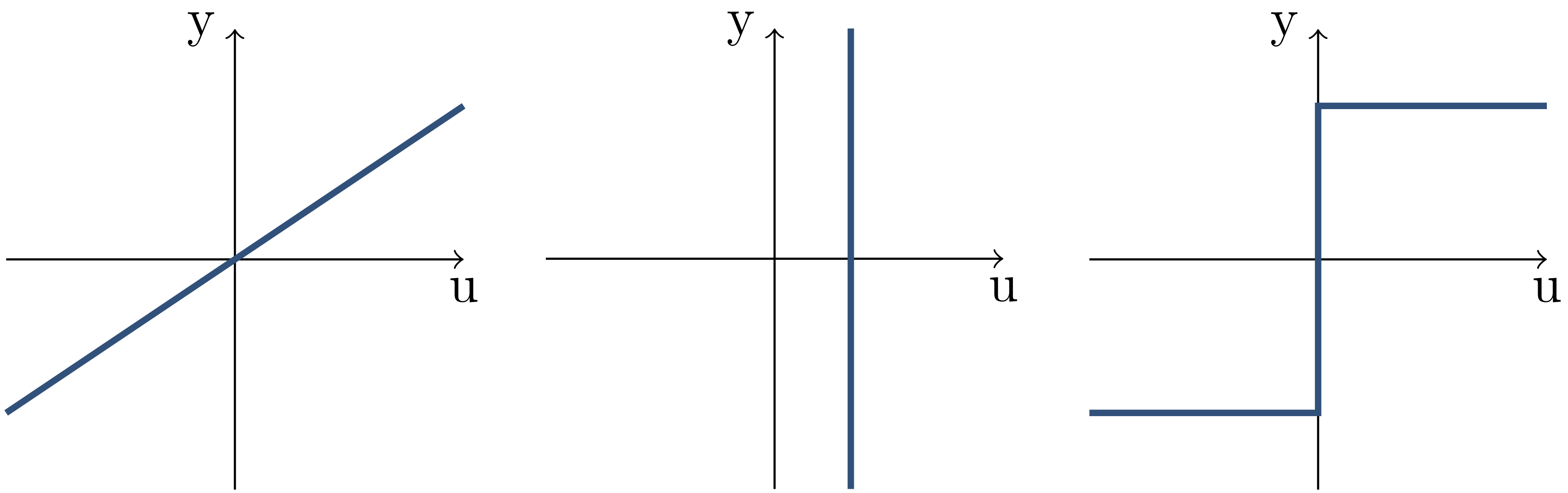
MONOTONE RELATIONS AND CONVEXITY

Theorem [Rockafellar, *Convex Analysis*]

The sub-differential for the closed proper convex functions on $\mathbb{R}$ are the maximal monotone relations from $\mathbb{R}$ to $\mathbb{R}$.

Maximal Monotone Relations

complete non-decreasing curves in $\mathbb{R}^2$



"up" and "to the right"

INTEGRATING THE CONSISTENCY EQUATIONS

INTEGRAL FUNCTIONS OF STEADY-STATE I/O RELATIONS

agents

$$\partial K_i = k_i \qquad K = \sum_{i=1}^{|\mathcal{V}|} K_i$$

$$\partial K_i^\star = k_i^{-1} \qquad K^\star = \sum_{i=1}^{|\mathcal{V}|} K_i^\star$$

controllers

$$\partial \Gamma_e = \gamma_e \qquad \Gamma = \sum_{e=1}^{|\mathcal{E}|} \Gamma_e$$

$$\partial \Gamma_e^\star = \gamma_e^{-1} \qquad \Gamma^\star = \sum_{e=1}^{|\mathcal{E}|} \Gamma_e^\star$$

when steady-state I/O relations are *maximally monotone*, their integral functions are *convex!*

$$K \underset{\texttt{convex dual}}{\Leftrightarrow} K^\star \qquad\qquad \Gamma \underset{\texttt{convex dual}}{\Leftrightarrow} \Gamma^\star$$

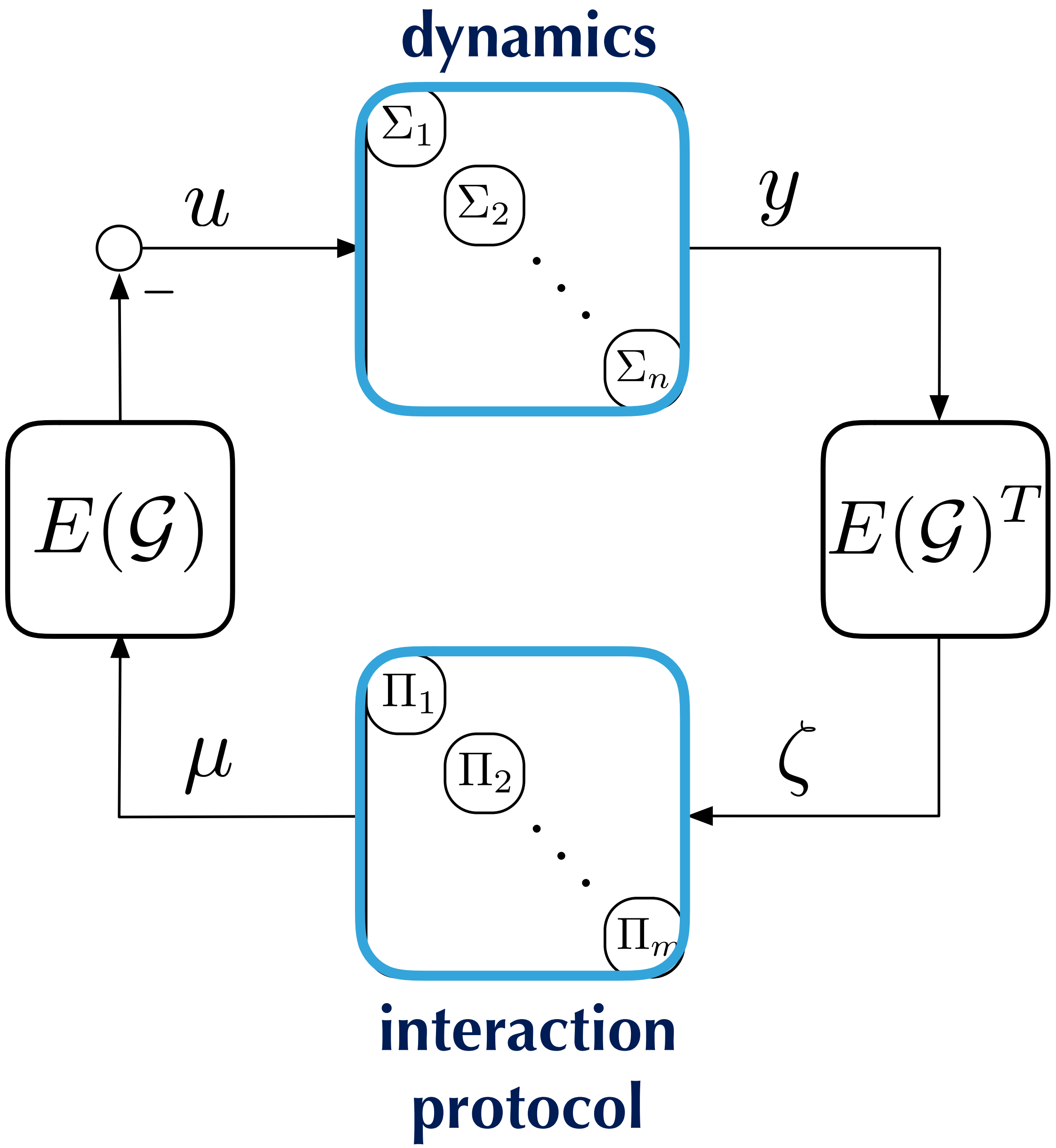
NETWORK OPTIMIZATION PERSPECTIVE

Optimal Potential Problem		**Optimal Flow Problem**
$\min_{\mathrm{y},\zeta} \quad \sum_i K_i^\star(y_i) + \sum_e \Gamma_e(\zeta_e)$ $s.t. \quad E(\mathcal{G})^T \mathrm{y} = \zeta$	$\Vert$	$\min_{u,\mu} \quad \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e)$ $s.t. \quad \mathrm{u} + E(\mathcal{G})\mu = 0.$

$$\mathrm{OPP} \underset{\texttt{convex dual}}{\Leftrightarrow} \mathrm{OFP}$$

when the steady-state input-output relations a *maximally monotone*, the solutions of network consistency equations are the optimal solutions of the *convex dual network optimization problems*!

SYNCHRONIZATION – A NETWORK OPTIMIZATION PERSPECTIVE



- **assume agents and controllers admit steady-state solutions**
- **assume steady-state input-output maps are maximally monotone**
- **if the network system has a steady-state, it is an optimal solution of the OPP and OFP problems**

Under what conditions does the network system actually *converge* to these steady states?

PASSIVITY FOR COOPERATIVE CONTROL

a “classic” result…

- assume there exists constant signals $\mathbf{u}, \mathbf{y}, \boldsymbol{\mu}, \boldsymbol{\zeta}$ s.t. $\mathbf{u} = -E\boldsymbol{\mu}, , \boldsymbol{\zeta} = E^T\mathbf{y}$
- each dynamic system is output strictly passive with respect to $\mathrm{u}_i, \mathrm{y}_i$

$$\frac{d}{dt}S_i(x_i(t)) \leq (y_i(t) - \mathrm{y}_i)(u_i(t) - \mathrm{u}_i) - \rho_i\|y_i(t) - \mathrm{y}_i\|^2$$

- each controller is passive with respect to ζ_k, μ_k

$$\frac{d}{dt}W_k(\eta_k(t)) \leq (\mu_k(t) - \mu_k)(\zeta_k(t) - \zeta_k)$$

Theorem [Arcak 2007]

Suppose the above assumptions are satisfied. Then the network output converges to the constant value $\mathbf{y}$, i.e,

$$\lim_{t\to\infty} y(t) = \mathbf{y}$$

A PASSIVITY REFINEMENT FOR MONOTONE RELATIONS

MEIP Systems [Burger, Z, Allgower 2014]

The dynamical SISO system

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), \mathrm{w}) \\ y(t) &= h(x(t), u(t), \mathrm{w}) \end{aligned}$$

is *maximal equilibrium independent passive* if there exists a maximal monotone relation $k_y \subset \mathbb{R}^2$ such that for all $(\mathrm{u}, \mathrm{y}) \in k_y$ there exists a positive semi-definite storage function $S(x(t))$ satisfying

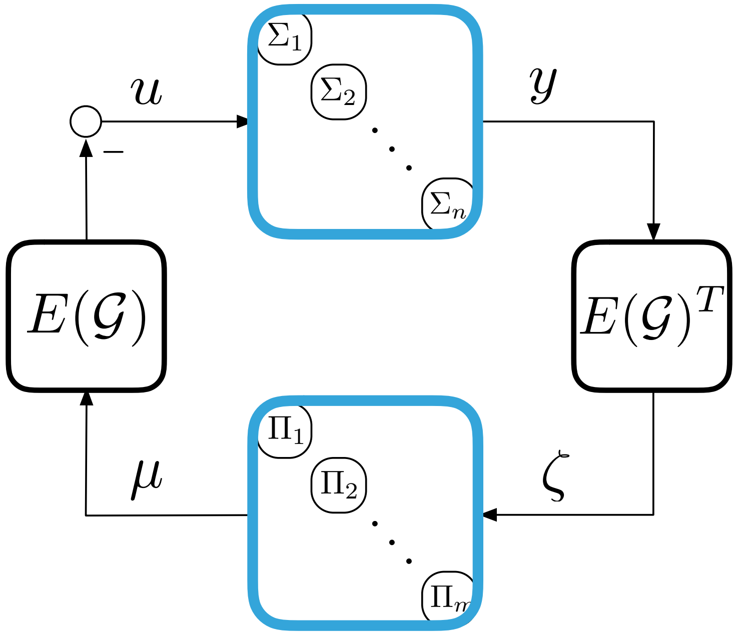
$$\frac{d}{dt} S(x(t)) \leq (y(t) - \mathrm{y})(u(t) - \mathrm{u}).$$

Furthermore, it is *output-strictly maximal equilibrium independent passive* if additionally there is a constant $\rho > 0$ such that

$$\frac{d}{dt} S(x(t)) \leq (y(t) - \mathrm{y})(u(t) - \mathrm{u}) - \rho \|y(t) - \mathrm{y}\|^2.$$

- **an extension of *Equilibrium Independent Passivity*** [Hines et. al. Automatica 2011]

NETWORKED MEIP SYSTEMS



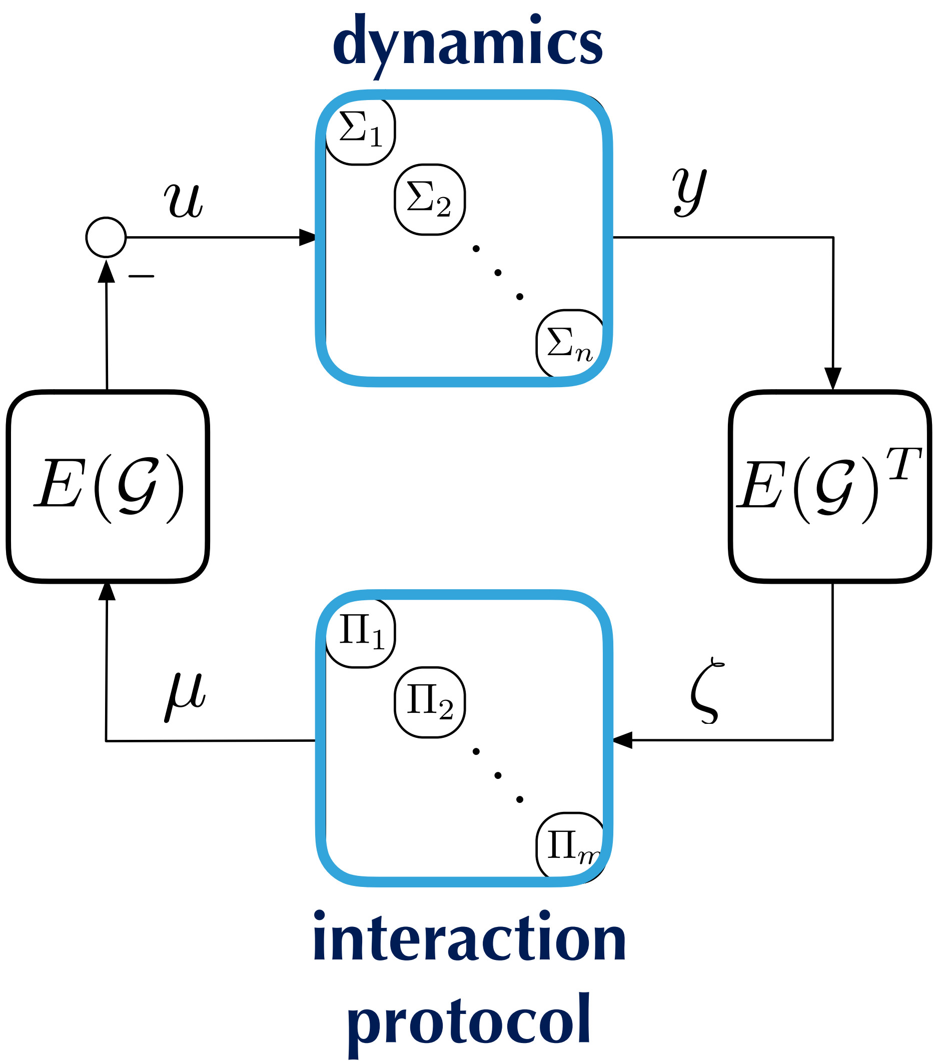
- **assume agents are output strictly MEIP**
- **assume controllers are MEIP**

Theorem [Burger, Z, Allgower 2014]

Assume the above assumptions hold. Then the signals $u(t), y(t), \zeta(t)$ and $\mu(t)$ converge to the constant signals $\hat{u}, \hat{y}, \hat{\zeta}$ and $\hat{\mu}$ which are optimal solutions to the problems (OFP) and (OPP):

Optimal Potential Problem	**Optimal Flow Problem**
$\min_{y,\zeta} \quad \sum_i K_i^\star(y_i) + \sum_e \Gamma_e(\zeta_e)$	$\min_{u,\mu} \quad \sum_i K_i(u_i) + \sum_e \Gamma_e^\star(\mu_e)$
$s.t. \quad E^T y = \zeta$	$s.t. \quad u + E\mu = 0.$

MONOTONICITY AND PASSIVITY-BASED COOPERATIVE CONTROL

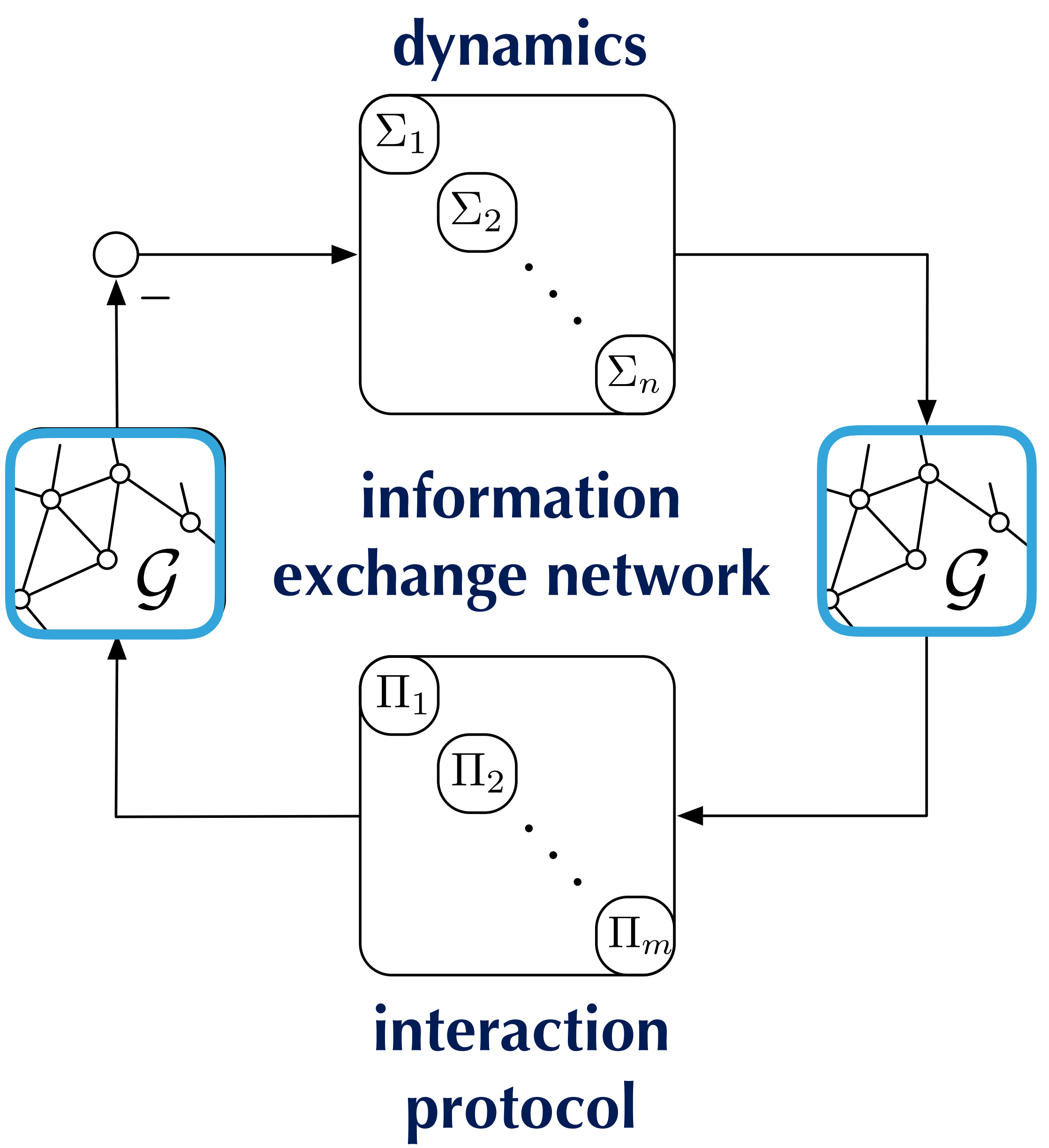


- **an** analysis **result - convergence of network system and solutions of a pair of network optimization problems**
  [Automatica '14, TAC '17 (under review)]

- **a** synthesis **result - it is possible to** ***design*** **the controllers to achieve a desired steady by** ***shaping*** **the network optimization problems**
  [L-CSS '17]

- **cooperative control of** passivity-short systems **- optimization framework relates** ***regularization*** **to output-feedback passivation of the agents**
  [L-CSS '18 (under review)]

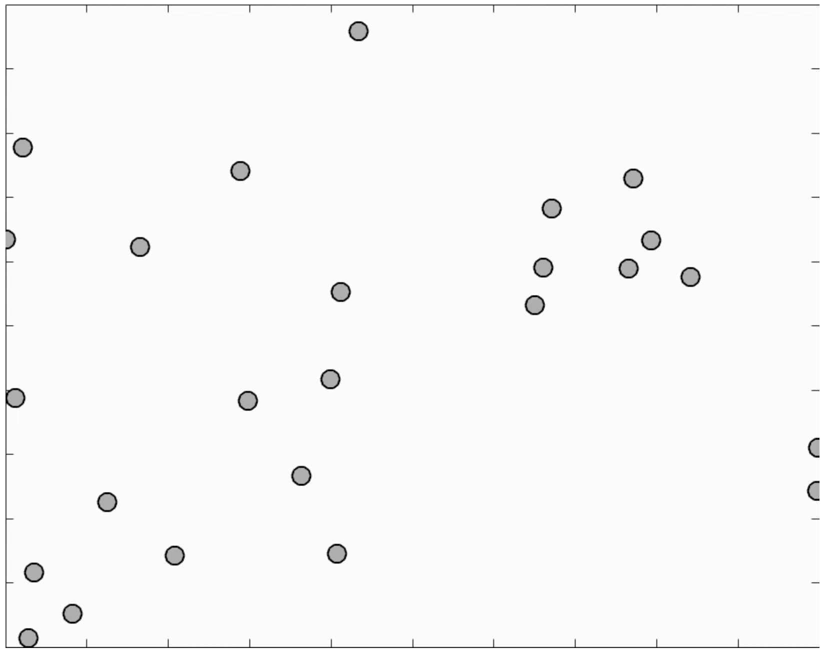
MULTI-AGENT SYSTEM ARCHITECTURES



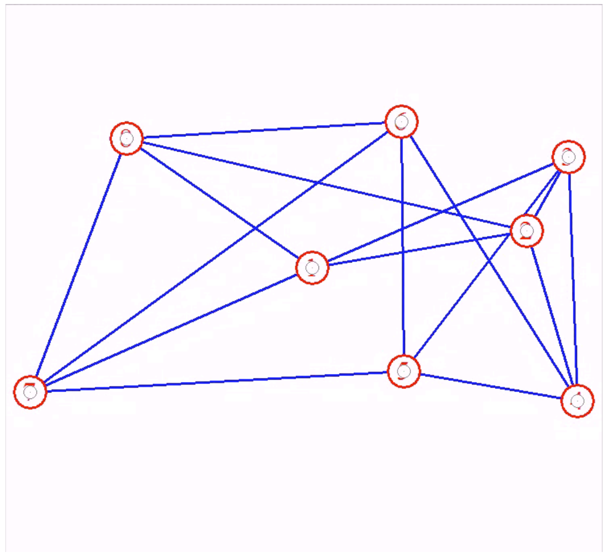
- the networked system
- dynamics for coordination
- information exchange architectures

COORDINATION OBJECTIVES

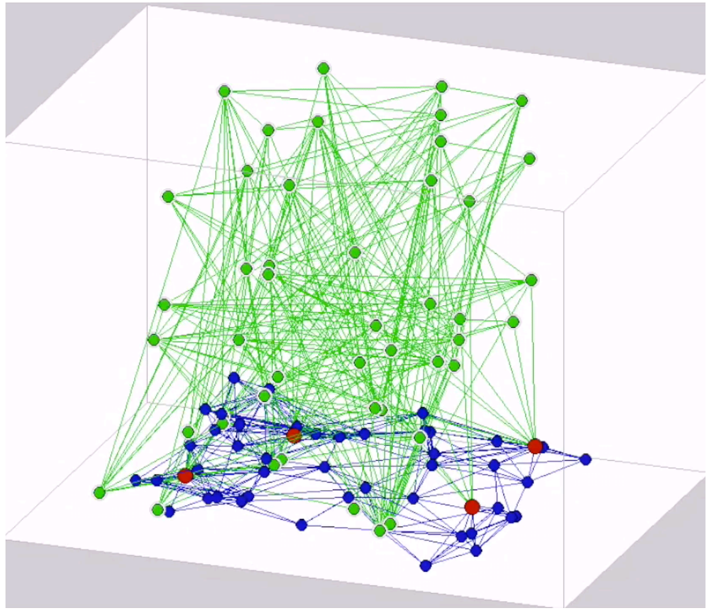
rendezvous



formation control



localization

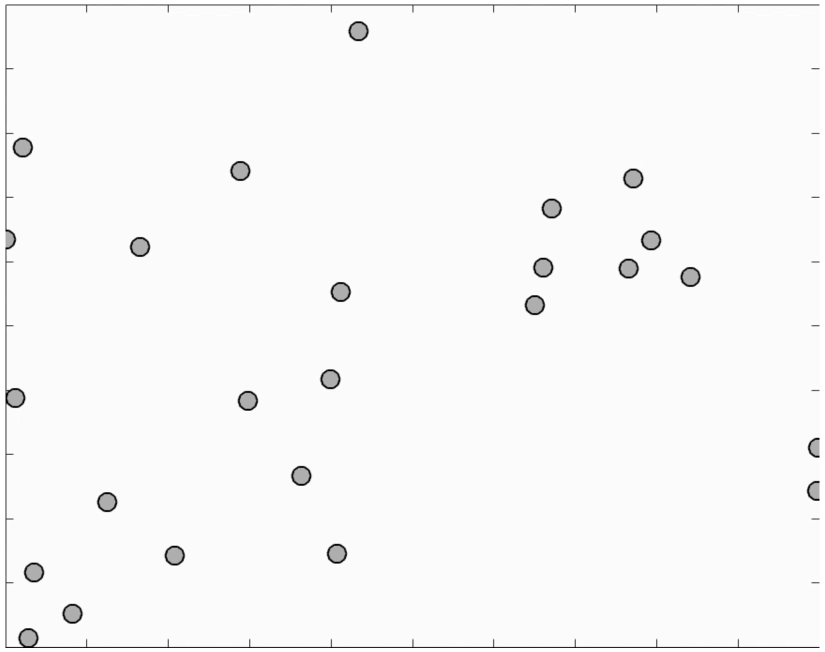


Does the control strategy need to change with different sensing/communication?

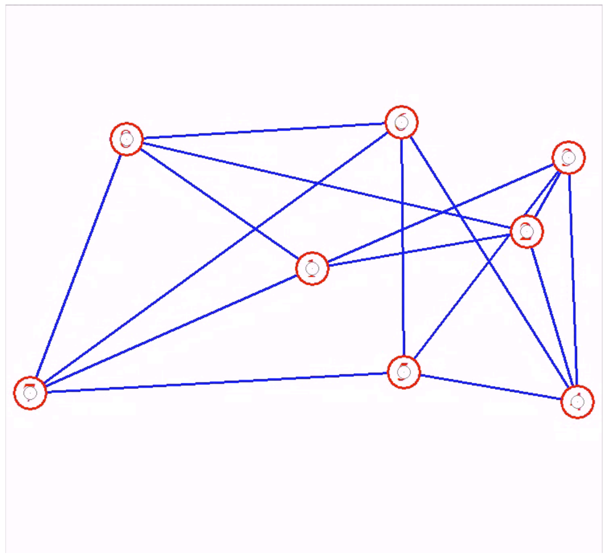
Are there common architectural requirements for information exchange that do not depend on the choice of sensing?

COORDINATION OBJECTIVES

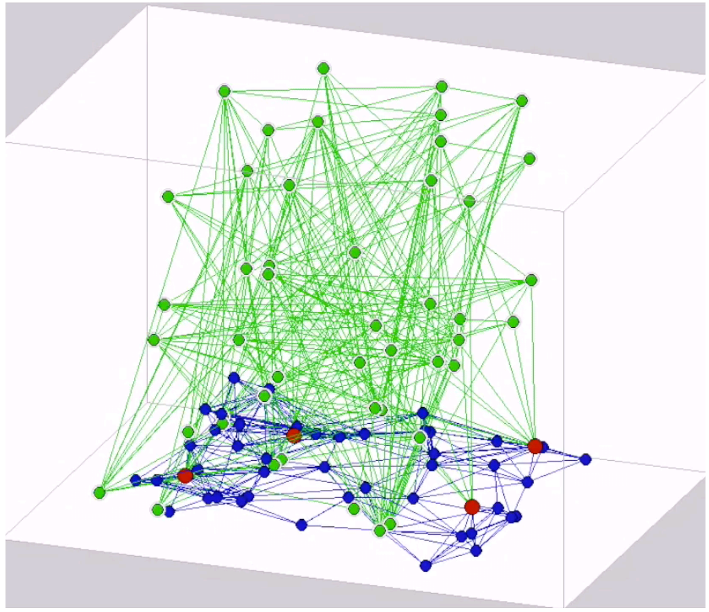
rendezvous



formation control



localization

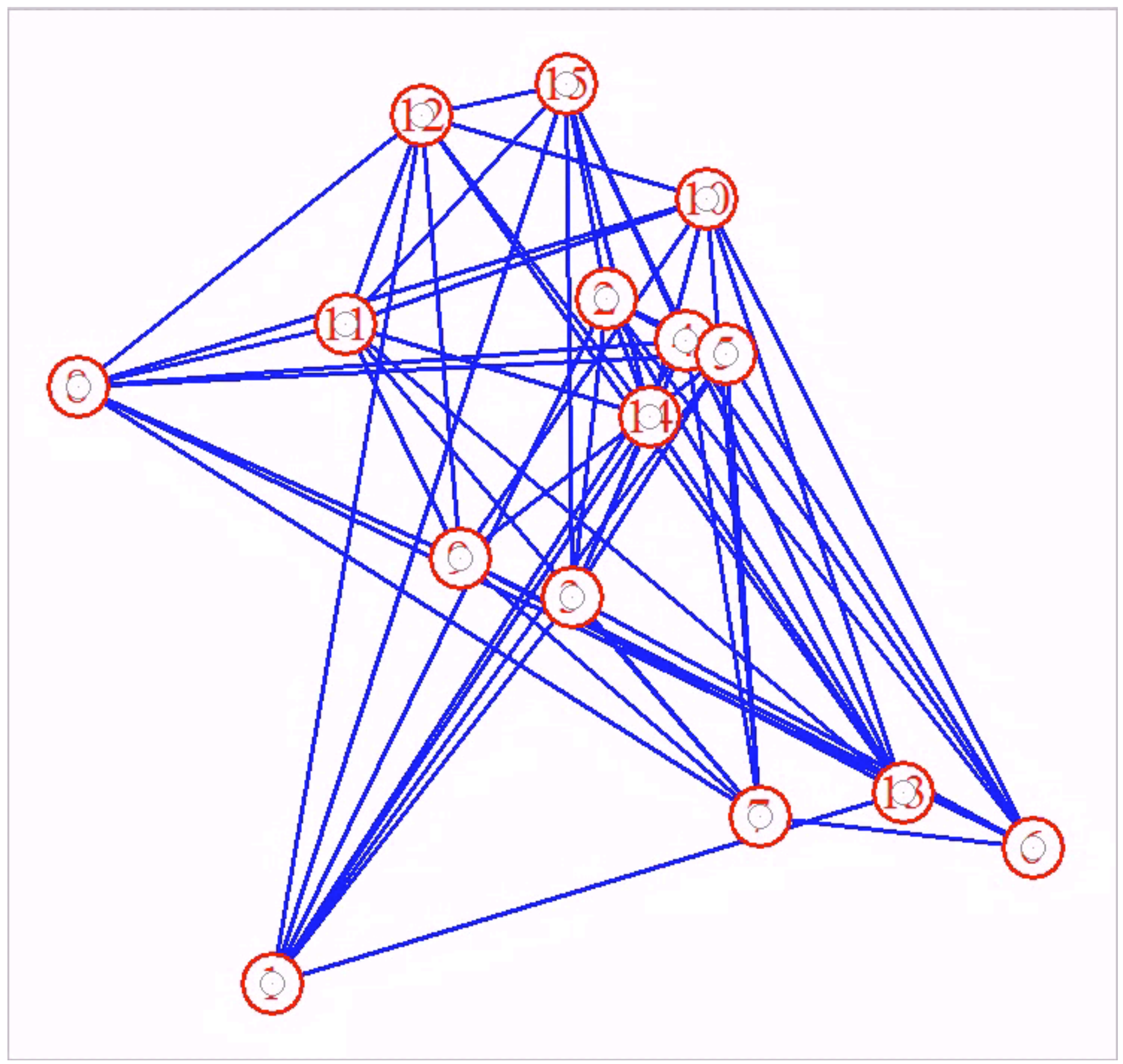


Does the control strategy need to change with different sensing/communication?

Are there common architectural requirements for information exchange that do not depend on the choice of sensing?

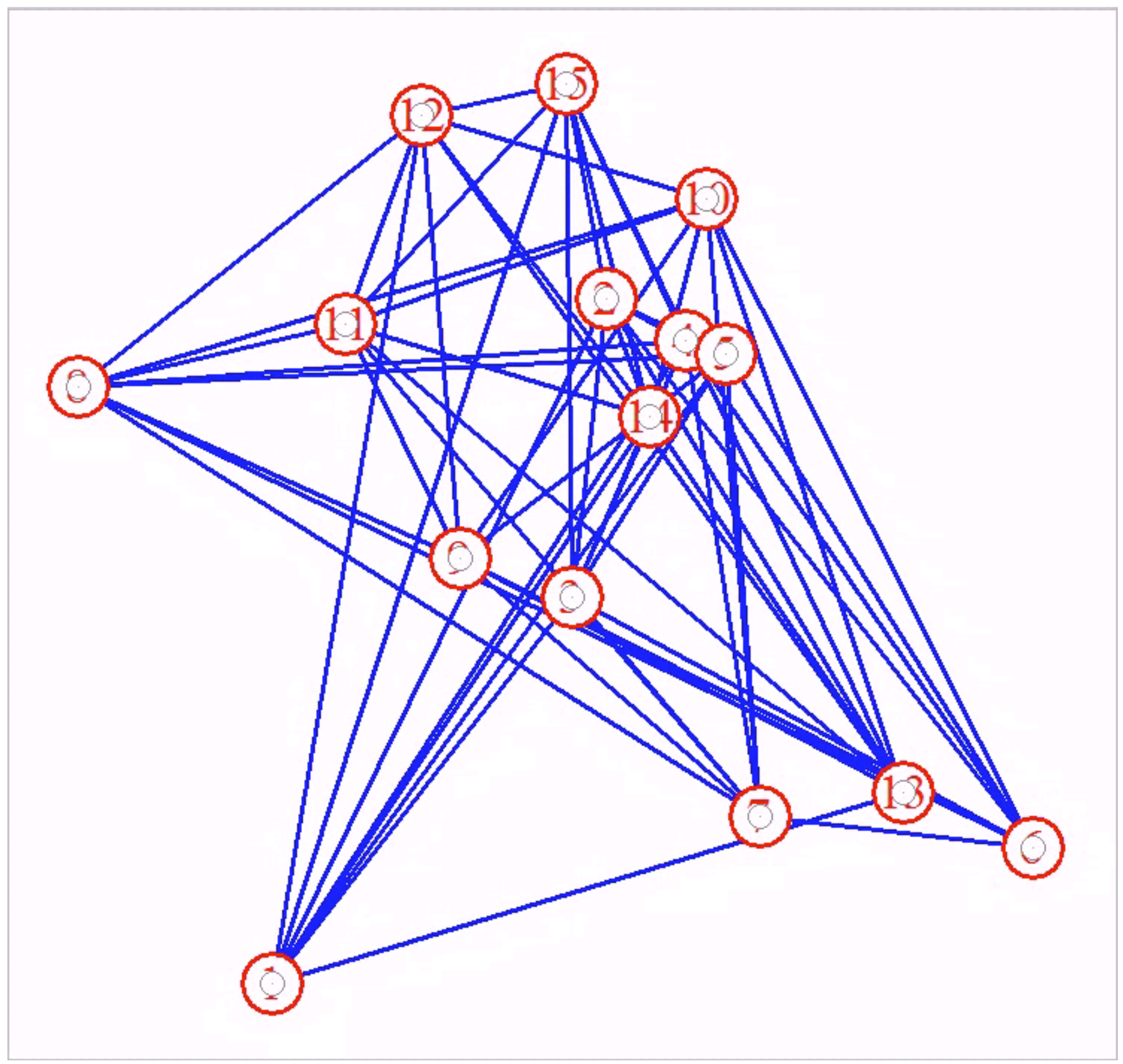
FORMATION CONTROL

Given a team of robots endowed with the ability to sense/communicate with neighboring robots, design a control for each robot using only *local information* that moves the team to a desired geometric pattern.



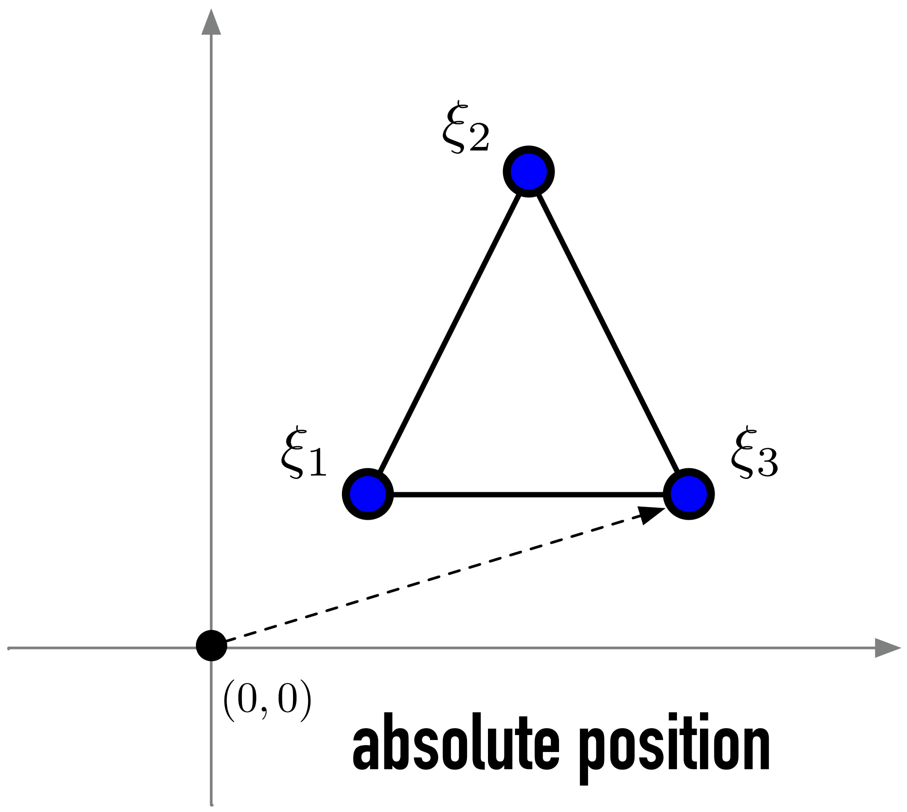
FORMATION CONTROL

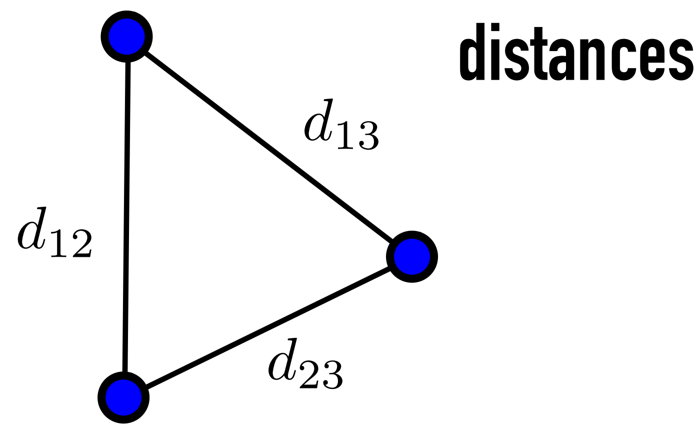
Given a team of robots endowed with the ability to sense/ communicate with neighboring robots, design a control for each robot using only *local information* that moves the team to a desired geometric pattern.

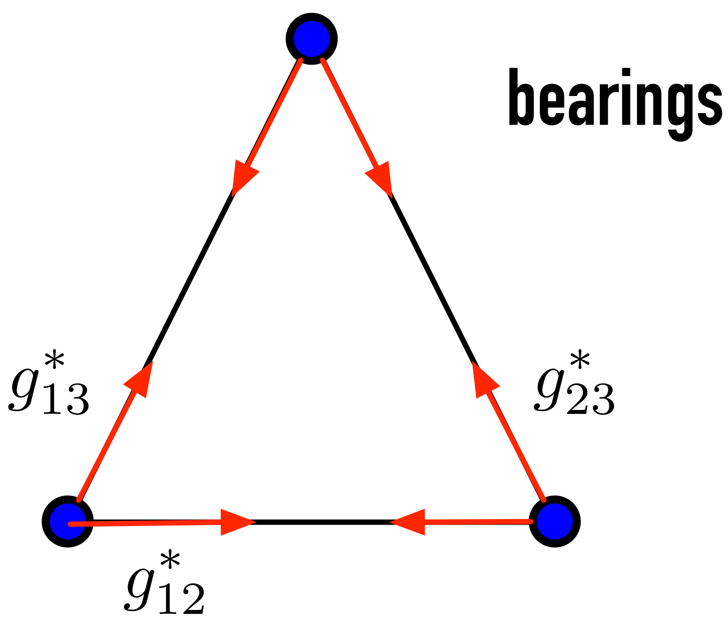


FORMATION DETERMINATION = SENSOR SELECTION

HOW TO DEFINE A SHAPE







DISTANCE CONSTRAINED

Formation

- SPECIFIED BY DISTANCES BETWEEN PAIRS OF ROBOTS

$$d_{ij} \in \mathbb{R}$$

Control

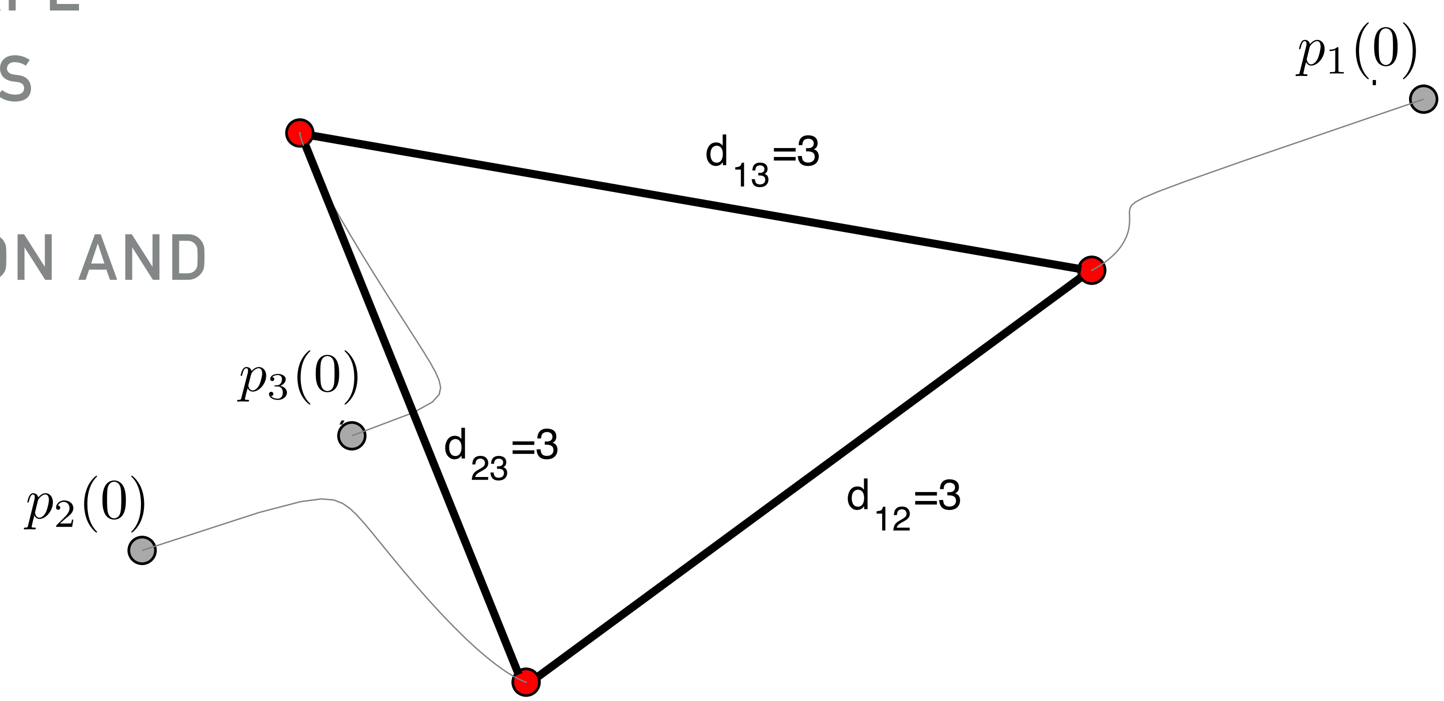
$$u_i = \sum_{i \sim j} (\|p_i - p_j\|^2 - d_{ij}^2)(p_j - p_i)$$

[Krick2009]

- FINAL FORMATION WILL BE A TRANSLATION OR ROTATION OF SHAPE SATISFYING DISTANCE CONSTRAINTS

- AGENTS REQUIRE RELATIVE POSITION AND DISTANCES

$$p_j - p_i$$



BEARING ONLY

Formation

- SPECIFIED BY BEARING VECTORS

$$g_{ij}^* \in \mathbb{R}^2, \ \|g_{ij}^*\| = 1$$

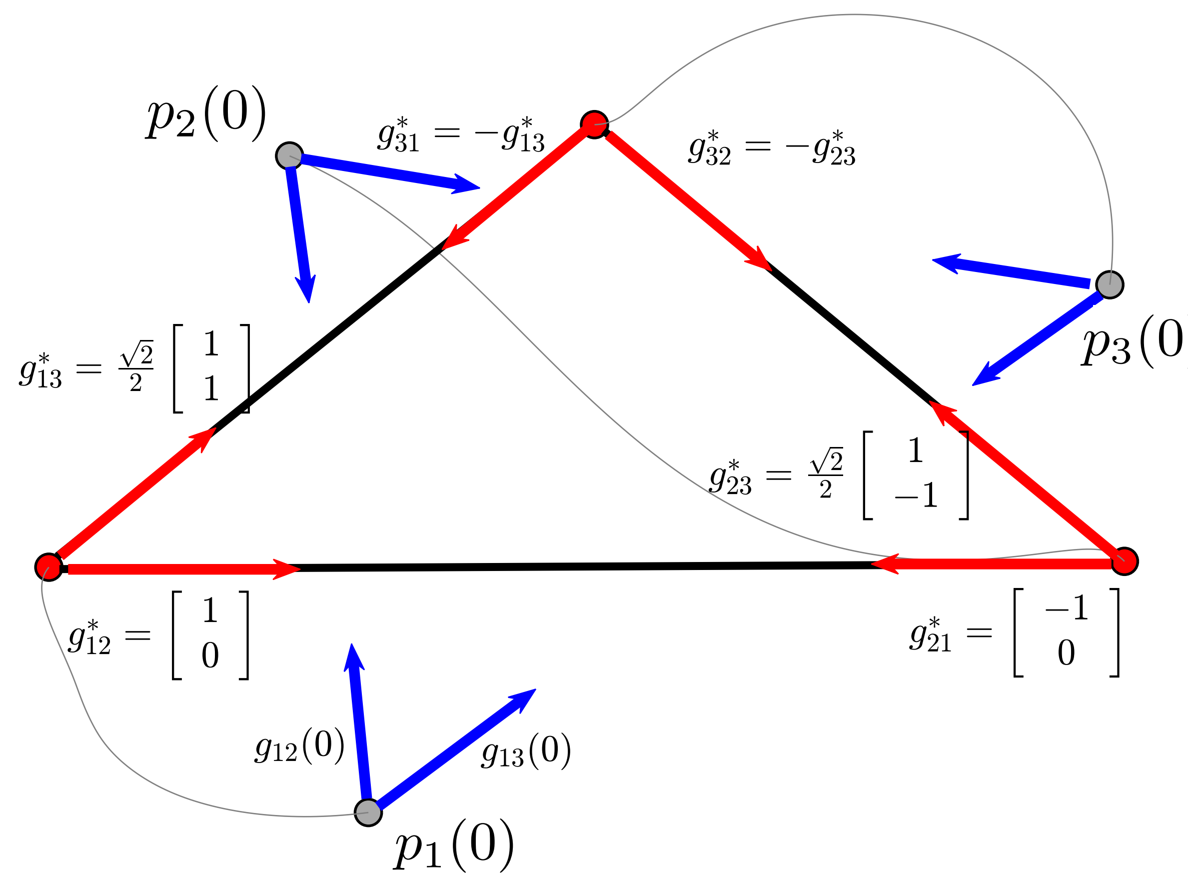
- FINAL FORMATION WILL BE A TRANSLATION OR SCALING OF SHAPE SATISFYING BEARING CONSTRAINTS
- AGENTS REQUIRE BEARING MEASUREMENTS

$$g_{ij} = \frac{p_j - p_i}{\|p_i - p_j\|}$$

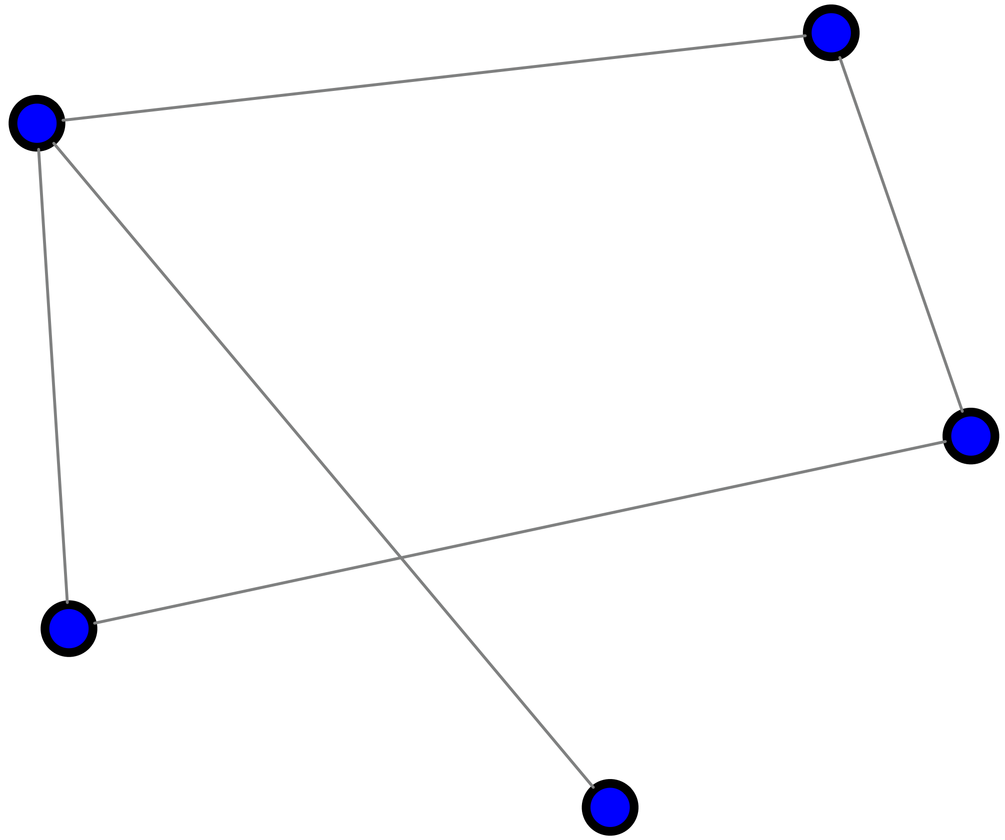
Control

$$u_i = -\sum_{i \sim j} (I - g_{ij} g_{ij}^T) g_{ij}^*$$

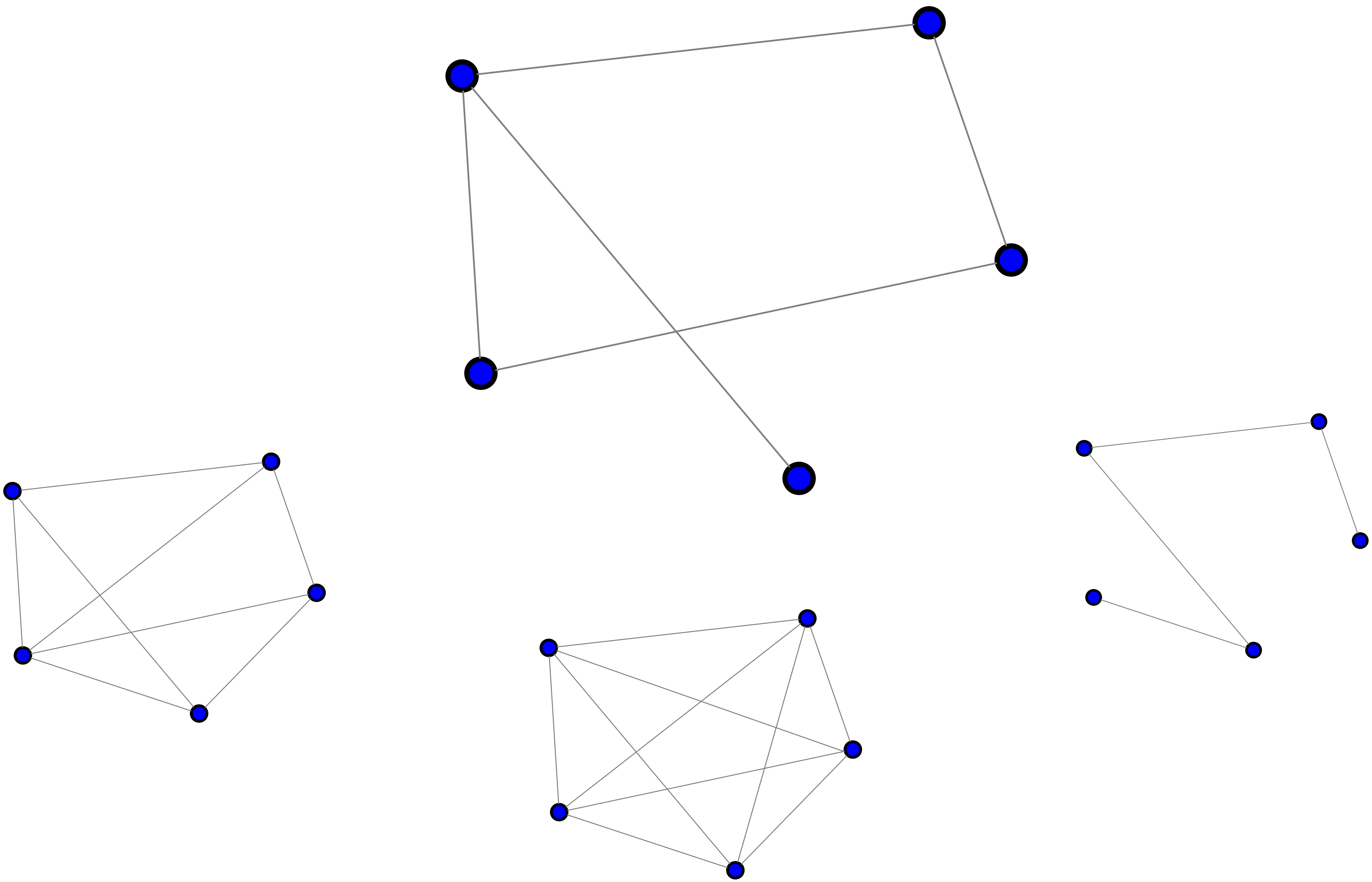
[Zhao, Z 2016]



INFORMATION EXCHANGE NETWORK AND FORMATION DETERMINATION



INFORMATION EXCHANGE NETWORK AND FORMATION DETERMINATION



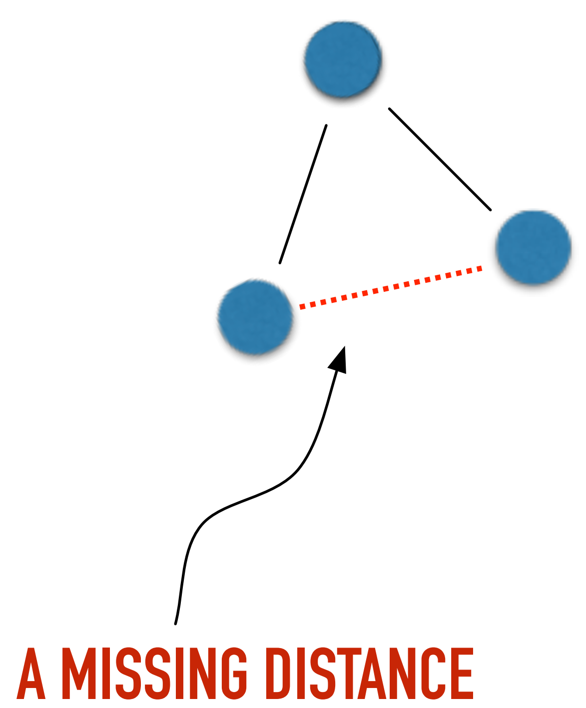
SENSORS, GRAPHS, AND SHAPES

Given a desired formation shape, a sensing modality and its corresponding formation controller, will all information exchange networks (graphs) solve the formation control problem?

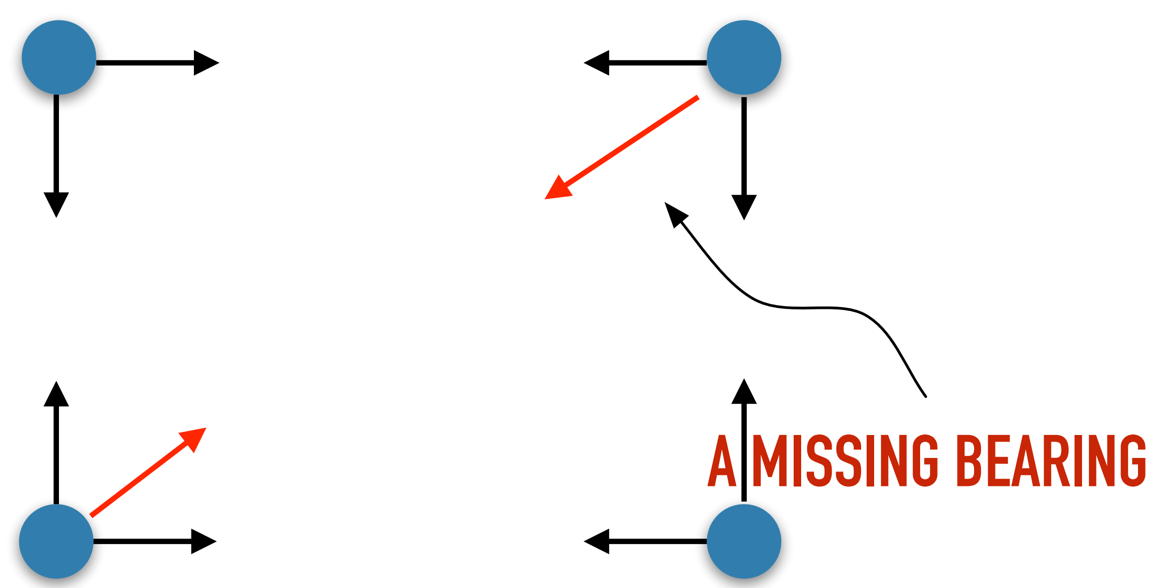
SENSORS, GRAPHS, AND SHAPES

Given a desired formation shape, a sensing modality and its corresponding formation controller, will all information exchange networks (graphs) solve the formation control problem?

The triangle (distance constrained)



the square (bearing only)



SENSORS, GRAPHS, AND SHAPES

For a given sensing modality, what kind of information exchange networks can (uniquely) determine a formation shape?

SENSORS, GRAPHS, AND SHAPES

For a given sensing modality, what kind of information exchange networks can (uniquely) determine a formation shape?

RIGIDITY THEORY

SENSORS, GRAPHS, AND SHAPES

For a given sensing modality, what kind of information exchange networks can (uniquely) determine a formation shape?

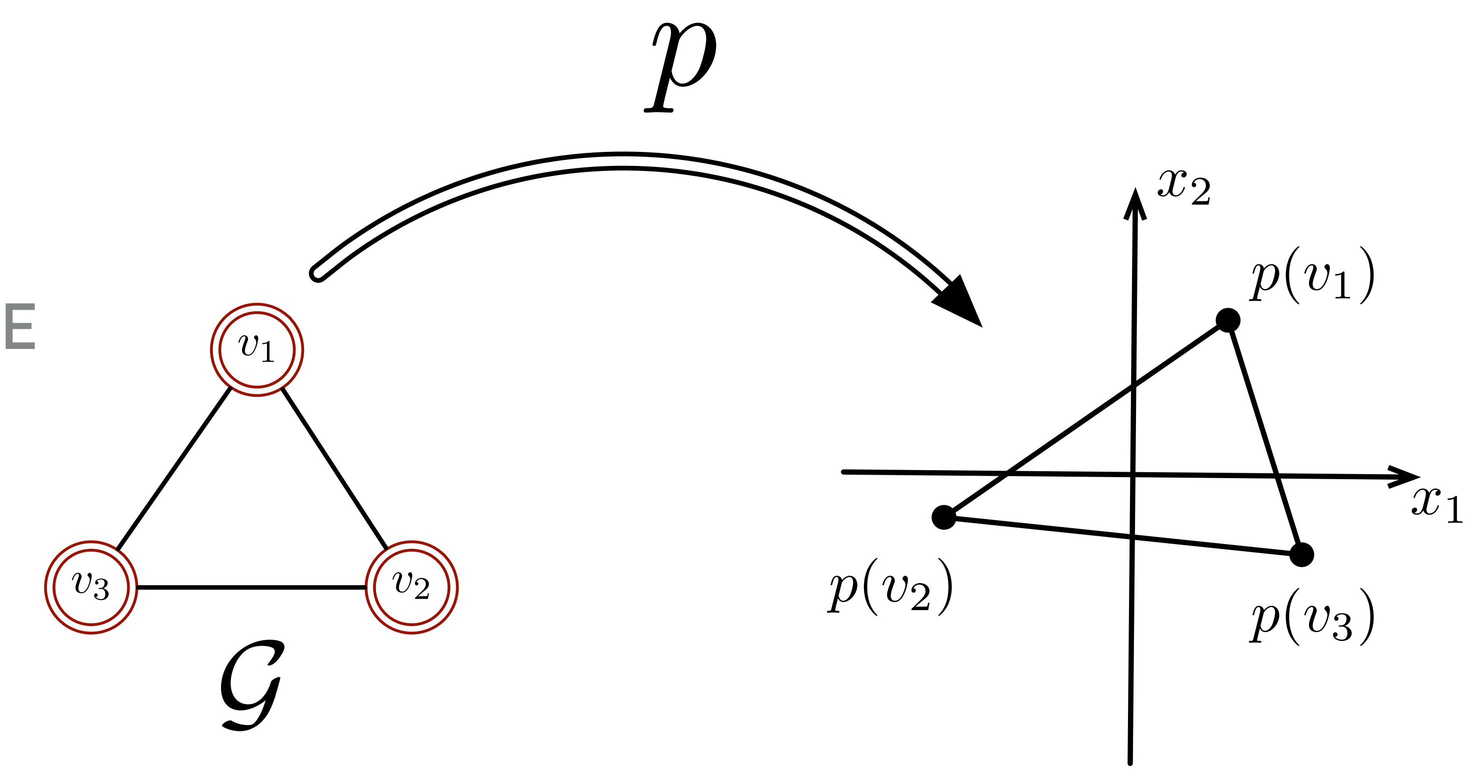
RIGIDITY THEORY

Rigidity is a combinatorial theory for characterizing the “stiffness” or “flexibility” of structures formed by rigid bodies connected by flexible linkages or hinges.

BEARING RIGIDITY THEORY

A *framework*

- A GRAPH
- A MAPPING TO A METRIC SPACE

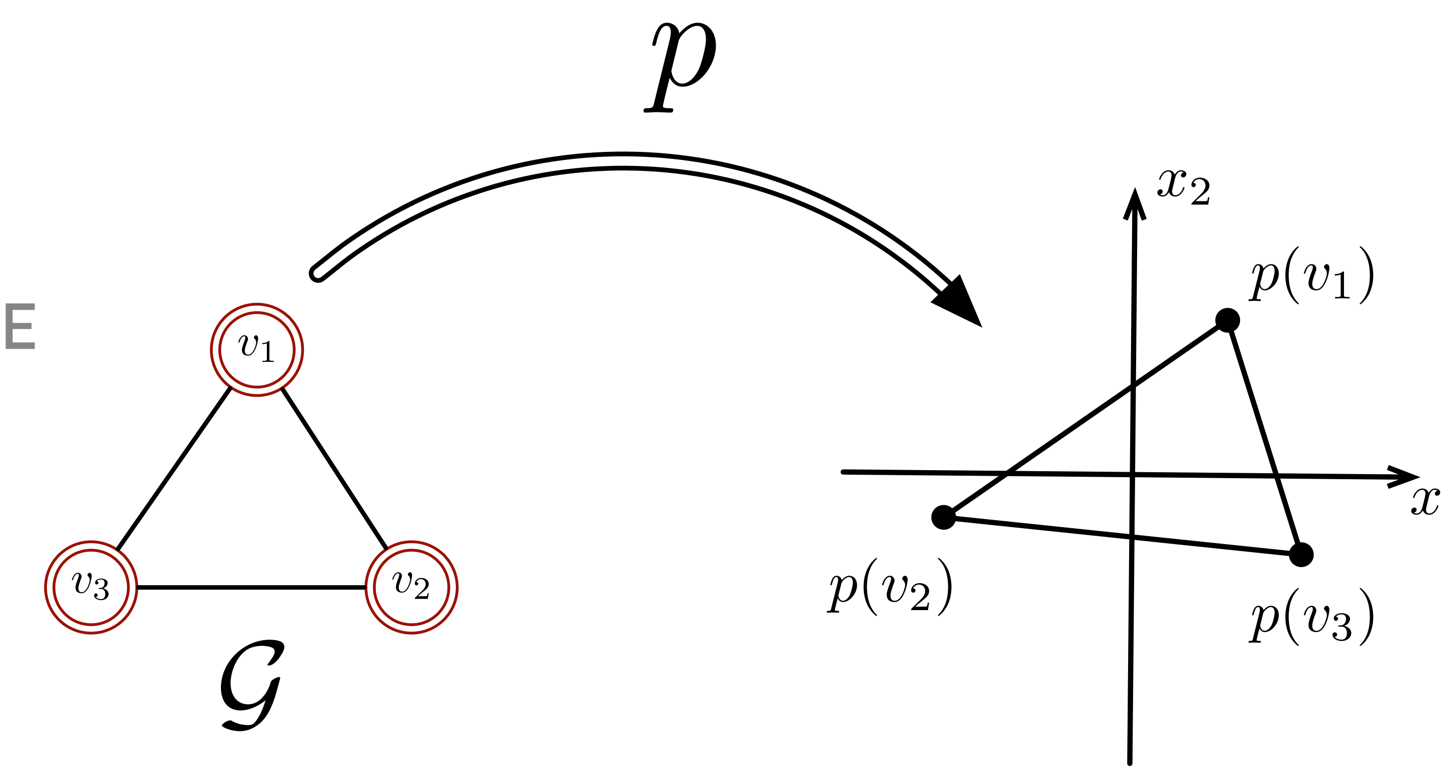


[Zhao, Z 2016]

BEARING RIGIDITY THEORY

A *framework*

- A GRAPH
- A MAPPING TO A METRIC SPACE



Two frameworks are *equivalent* if

$$(\mathcal{G}, p_0) \quad (\mathcal{G}, p_1)$$

$$\frac{p_0(v_j) - p_0(v_i)}{\|p_0(v_j) - p_0(v_i)\|} = \frac{p_1(v_j) - p_1(v_i)}{\|p_1(v_j) - p_1(v_i)\|}$$

$$\forall \{v_i, v_j\} \in \mathcal{E}$$

Two frameworks are *congruent* if

$$(\mathcal{G}, p_0) \quad (\mathcal{G}, p_1)$$

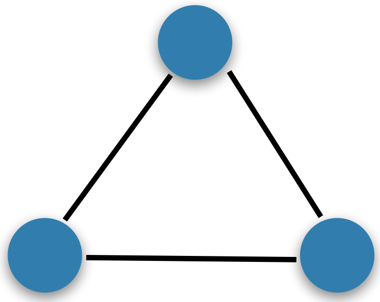
$$\frac{p_0(v_j) - p_0(v_i)}{\|p_0(v_j) - p_0(v_i)\|} = \frac{p_1(v_j) - p_1(v_i)}{\|p_1(v_j) - p_1(v_i)\|}$$

$$\forall v_i, v_j \in \mathcal{V}$$

[Zhao, Z 2016]

BEARING RIGIDITY THEORY

A framework is ***globally rigid*** if every framework that is equivalent to it is also congruent.

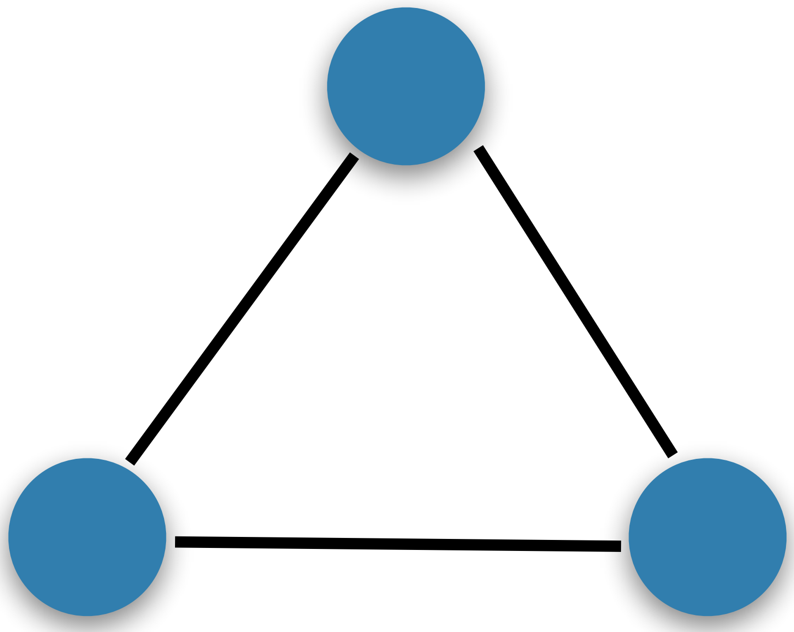


A bearing *rigid* graph can only *scale* and *translate* to ensure all bearings between all nodes are preserved (i.e., preserve the shape)!

[Zhao, Z 2016]

BEARING RIGIDITY THEORY

A framework is ***globally rigid*** if every framework that is equivalent to it is also congruent.



A bearing *rigid* graph can only *scale* and *translate* to ensure all bearings between all nodes are preserved (i.e., preserve the shape)!

[Zhao, Z 2016]

INFINITESIMAL RIGIDITY

A framework is ***infinitesimally rigid*** if every infinitesimal motion is *trivial*

Bearing Function

$$F_B(p) = \begin{bmatrix} \vdots \\ \frac{p(v_j)-p(v_i)}{\|p(v_i)-p(v_j)\|} \\ \vdots \end{bmatrix}$$

Distance Function

$$F_D(p) = \frac{1}{2}\begin{bmatrix} \vdots \\ \|p(v_i) - p(v_j)\|^2 \\ \vdots \end{bmatrix}$$

Bearing Rigidity Matrix

$$R_B(p) = \frac{\partial F_B(p)}{\partial p}$$

Distance Rigidity Matrix

$$R_D(p) = \frac{\partial F_D(p)}{\partial p}$$

Rigidity matrix is the linear term in the Taylor series expansion of the Distance/Bearing functions

$$F(p+\delta_p) = F(p) + \frac{\partial F(p)}{\partial p}\delta_p + h.o.t.$$

INFINITESIMAL RIGIDITY

A framework is ***infinitesimally rigid*** if every infinitesimal motion is *trivial*

Bearing Function

$$F_B(p) = \begin{bmatrix} \vdots \\ \frac{p(v_j)-p(v_i)}{\|p(v_i)-p(v_j)\|} \\ \vdots \end{bmatrix}$$

Distance Function

$$F_D(p) = \frac{1}{2}\begin{bmatrix} \vdots \\ \|p(v_i) - p(v_j)\|^2 \\ \vdots \end{bmatrix}$$

Bearing Rigidity Matrix

$$R_B(p) = \frac{\partial F_B(p)}{\partial p}$$

Distance Rigidity Matrix

$$R_D(p) = \frac{\partial F_D(p)}{\partial p}$$

infinitesimal motions are precisely the motions that satisfy

$$R(p)\delta_p = \frac{\partial F(p)}{\partial p}\delta_p = 0$$

INFINITESIMAL RIGIDITY

Bearing Function

$$F_B(p) = \begin{bmatrix} \vdots \\ \frac{p(v_j)-p(v_i)}{\|p(v_i)-p(v_j)\|} \\ \vdots \end{bmatrix}$$

Distance Function

$$F_D(p) = \frac{1}{2} \begin{bmatrix} \vdots \\ \|p(v_i) - p(v_j)\|^2 \\ \vdots \end{bmatrix}$$

Bearing Rigidity Matrix

$$R_B(p) = \frac{\partial F_B(p)}{\partial p}$$

Distance Rigidity Matrix

$$R_D(p) = \frac{\partial F_D(p)}{\partial p}$$

Theorem

A framework is infinitesimally (distance, bearing) rigid if and only if the rank of the rigidity matrix is 2n-3.

3 trivial motions in the plane

INFINITESIMAL RIGIDITY

For a given sensing modality, what kind of information exchange networks can (uniquely) determine a formation shape?

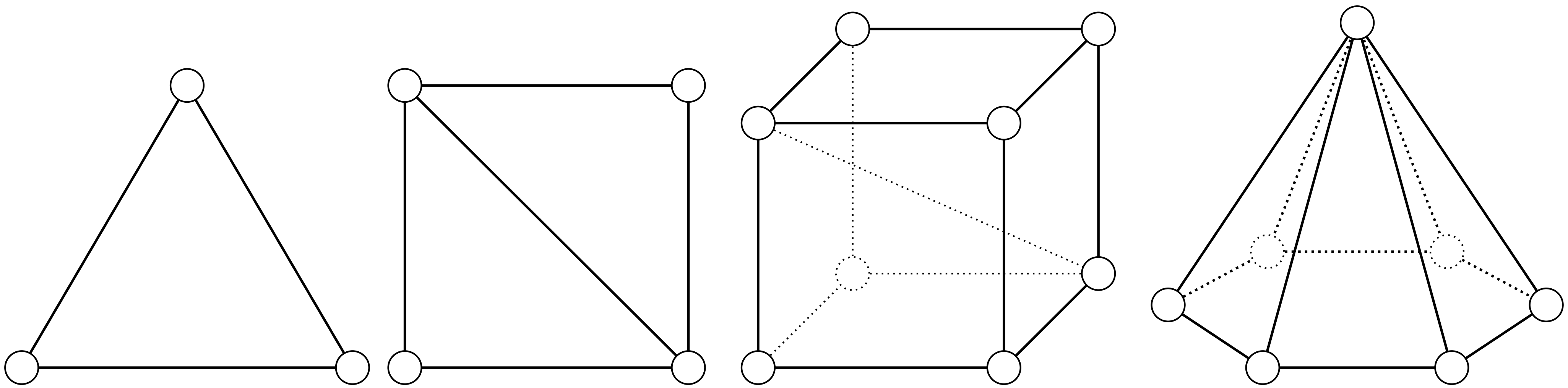
INFINITESIMALLY RIGID

Theorem [Zhao, Z 2016]

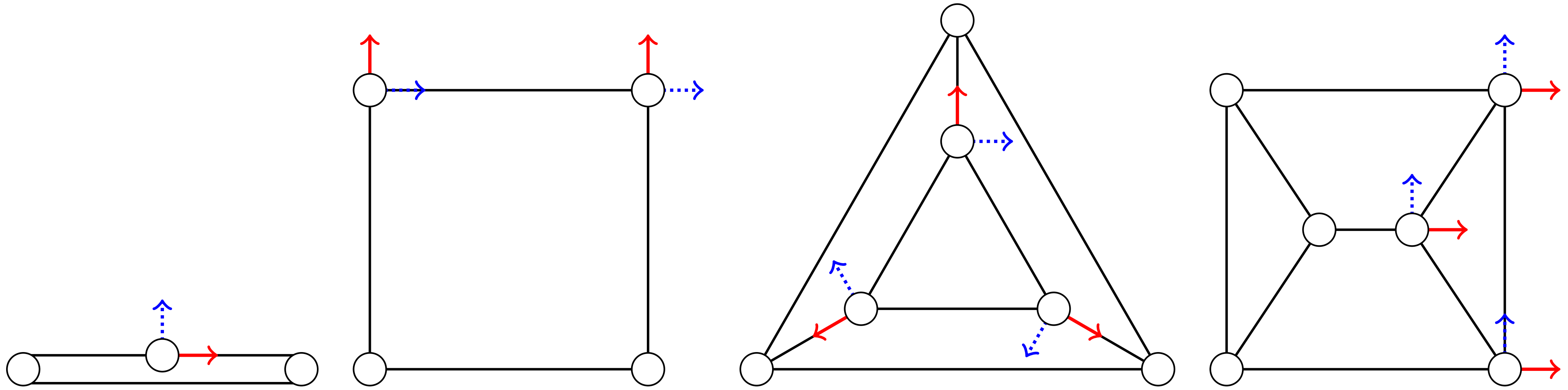
An infinitesimally bearing rigid framework can be *uniquely* determined up to a translation and scaling factor

INFINITESIMAL RIGIDITY

Infinitesimally bearing rigid frameworks



Non-Infinitesimally bearing rigid frameworks



"robots" - modeled as kinematic point mass

$$\dot{x}_i = u_i$$

Distance Control

$$u_i = \sum_{i\sim j}(\|p_i - p_j\|^2 - d_{ij}^2)(p_j - p_i)$$

$$\dot{x} = -R_D(p)^T R_D(p)p - R_D(p)^T d^2$$

[Krick2009]

Bearing Control

$$u_i = -\sum_{i\sim j}(I - g_{ij}g_{ij}^T)g_{ij}^*$$

$$\dot{x} = -R_B(p)^T g^*$$

[Zhao, Z 2016]

"robots" - modeled as kinematic point mass

$$\dot{x}_i = u_i$$

Distance Control

$$u_i = \sum_{i \sim j} (\|p_i - p_j\|^2 - d_{ij}^2)(p_j - p_i)$$

$$\dot{x} = -R_D(p)^T R_D(p) p - R_D(p)^T d^2$$

locally exponentially stable
undesirable equilibriums
[Krick2009]

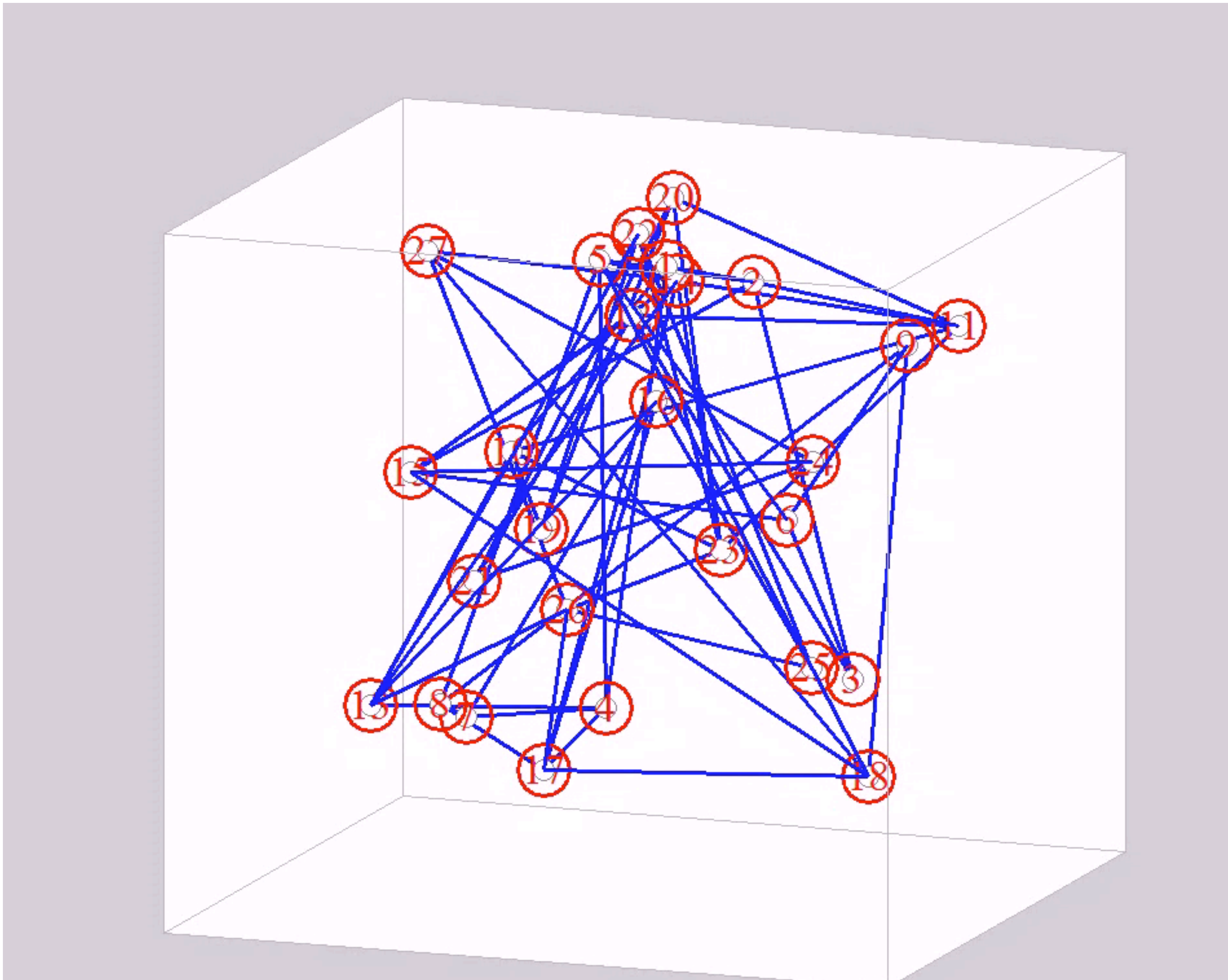
Bearing Control

$$u_i = -\sum_{i \sim j} (I - g_{ij} g_{ij}^T) g_{ij}^*$$

$$\dot{x} = -R_B(p)^T g^*$$

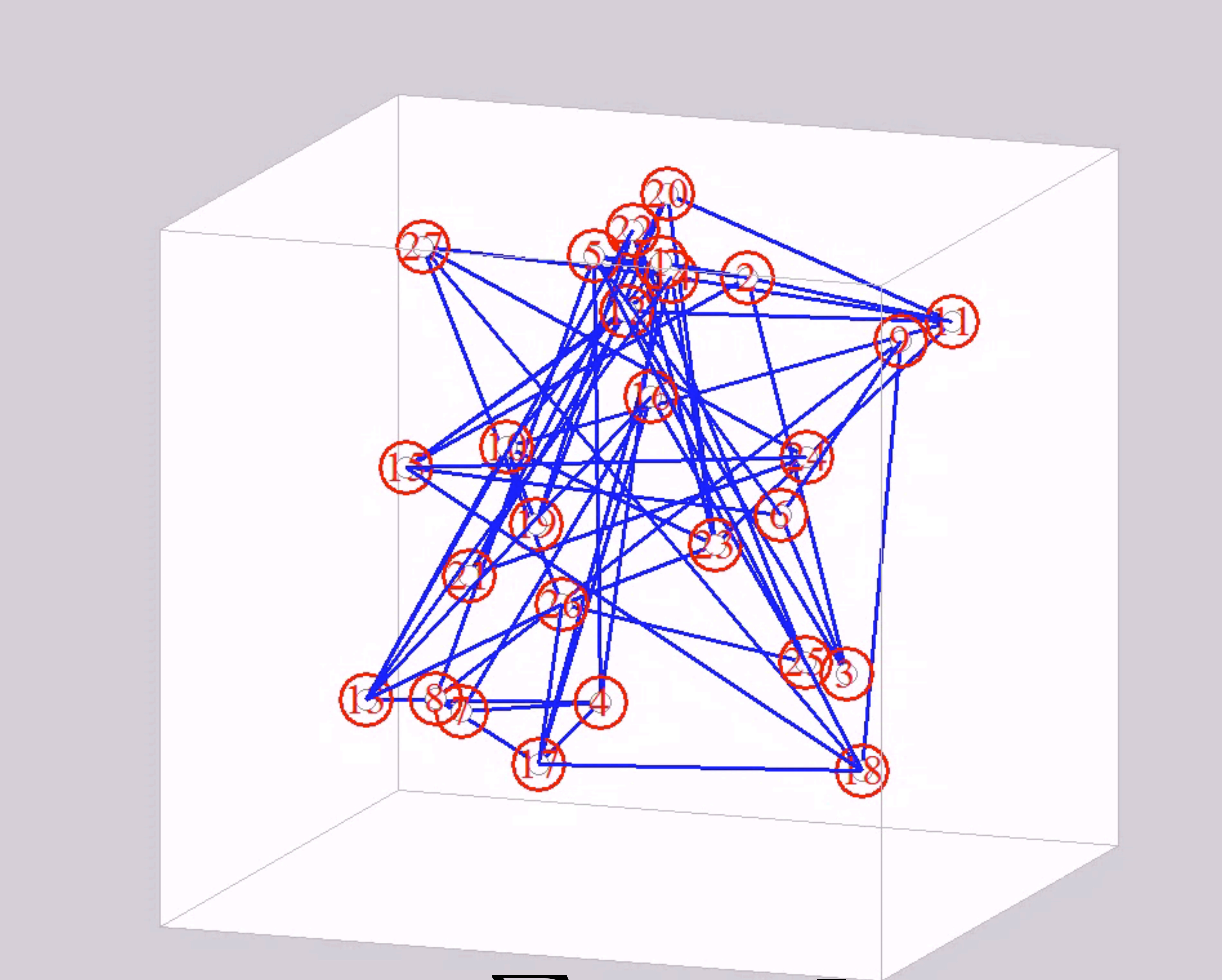
almost global stability
1 undesirable equilibriums
[Zhao, Z 2016]

BEARING RIGIDITY THEORY



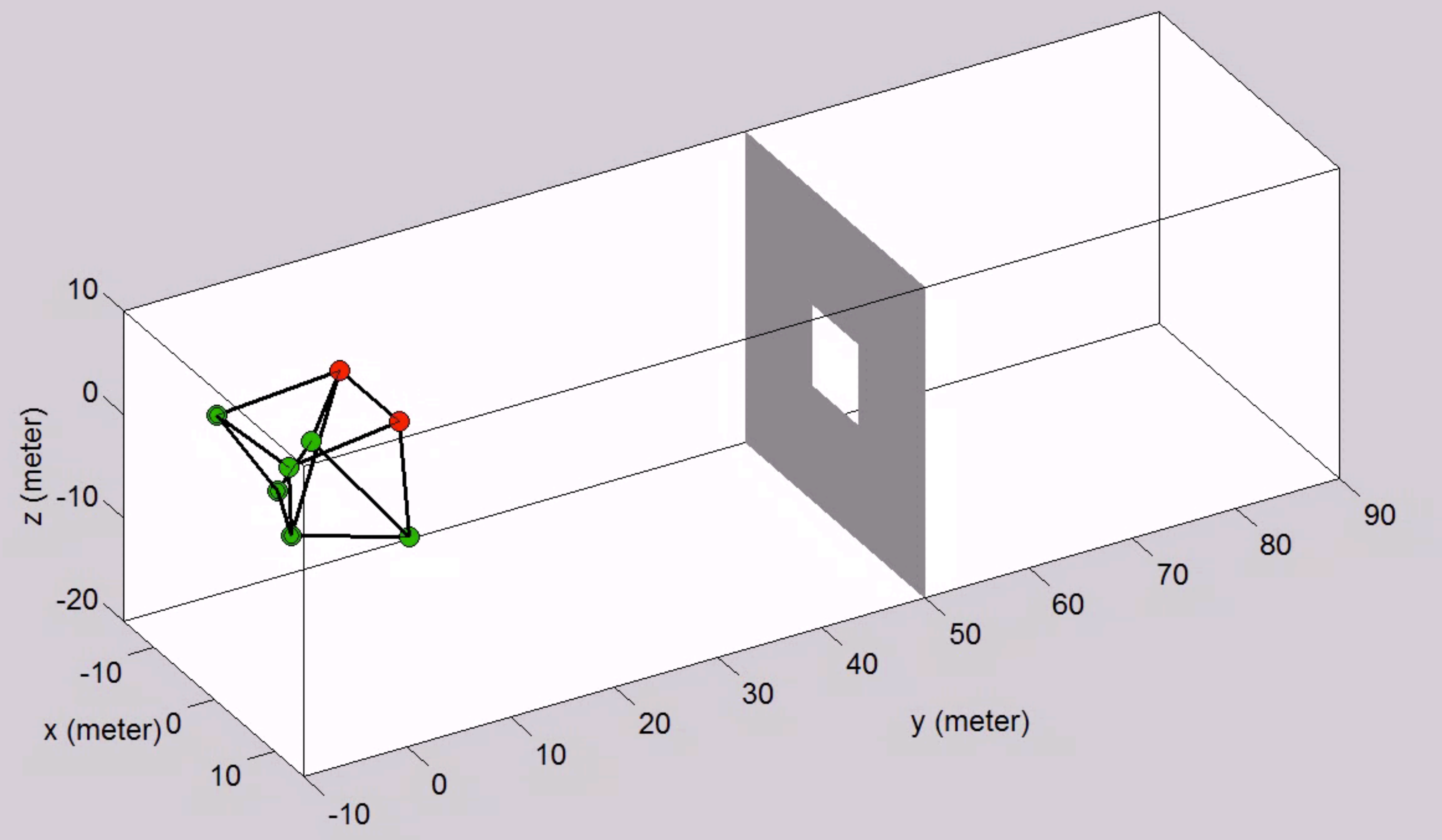
$$u_i = -\sum_{i \sim j} (I - g_{ij} g_{ij}^T) g_{ij}^*$$

BEARING RIGIDITY THEORY



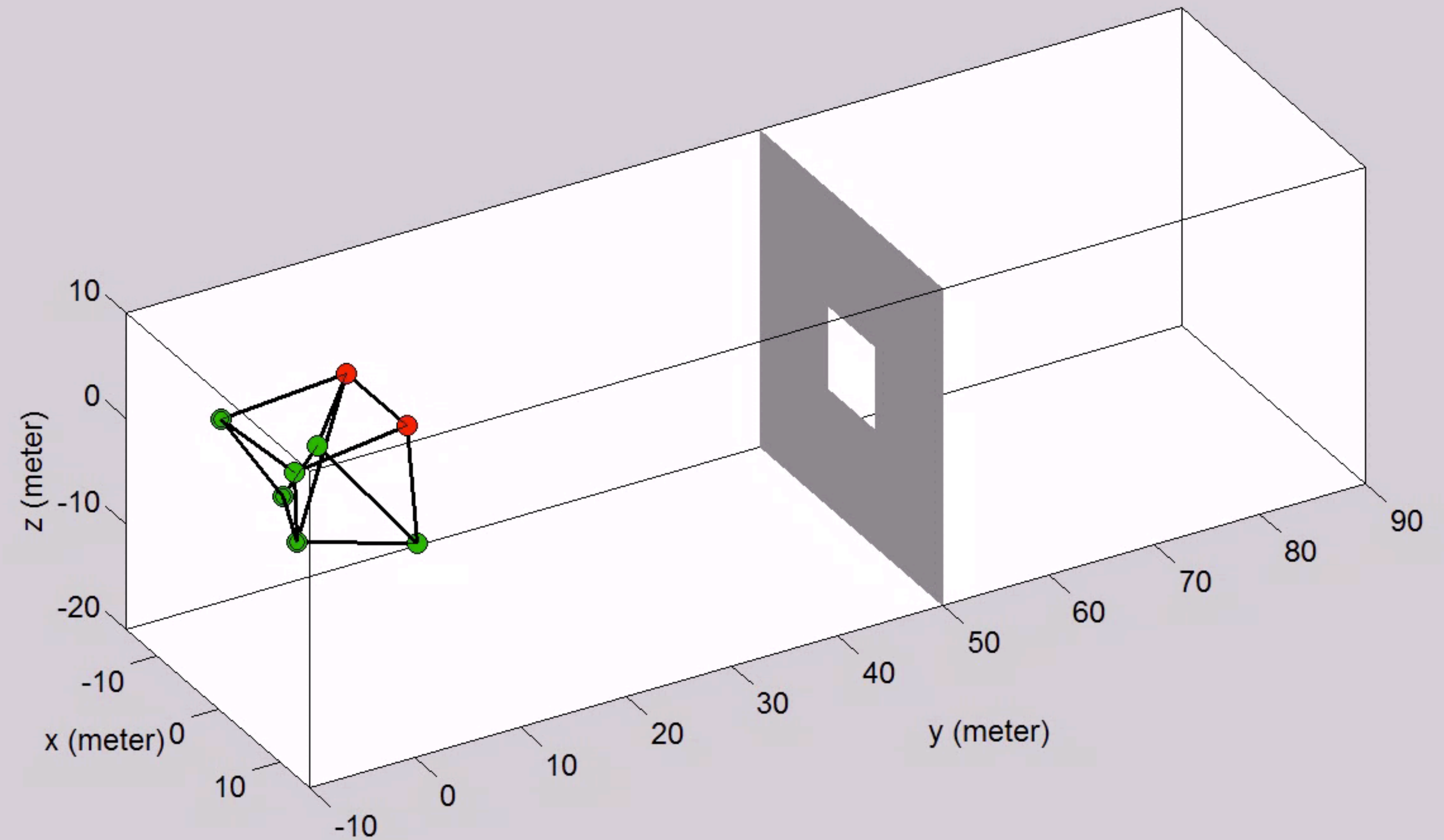
$$u_i = -\sum_{i \sim j} (I - g_{ij} g_{ij}^T) g_{ij}^*$$

BEARING RIGIDITY THEORY



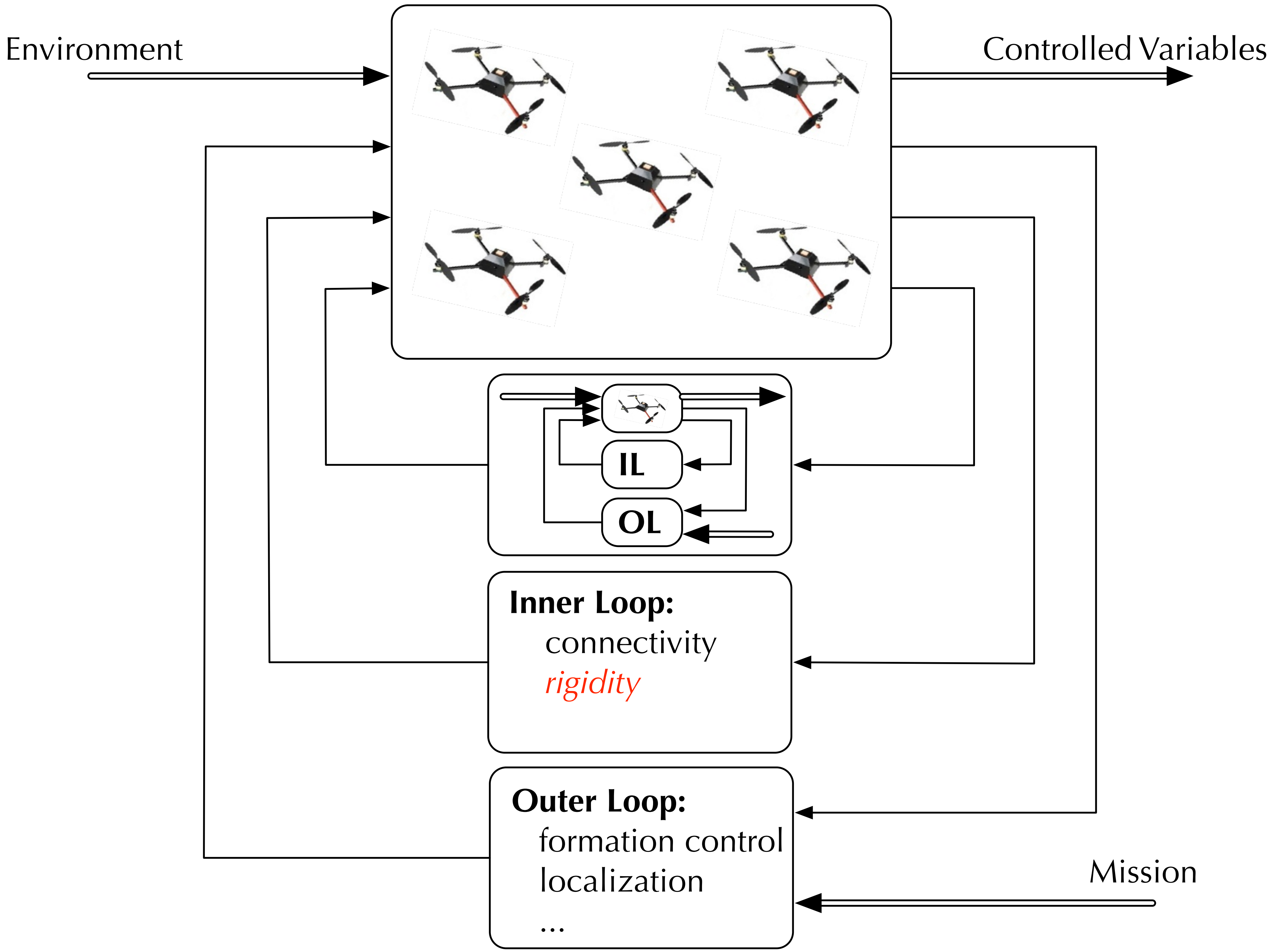
[Zhao,Zelazo2017]

BEARING RIGIDITY THEORY

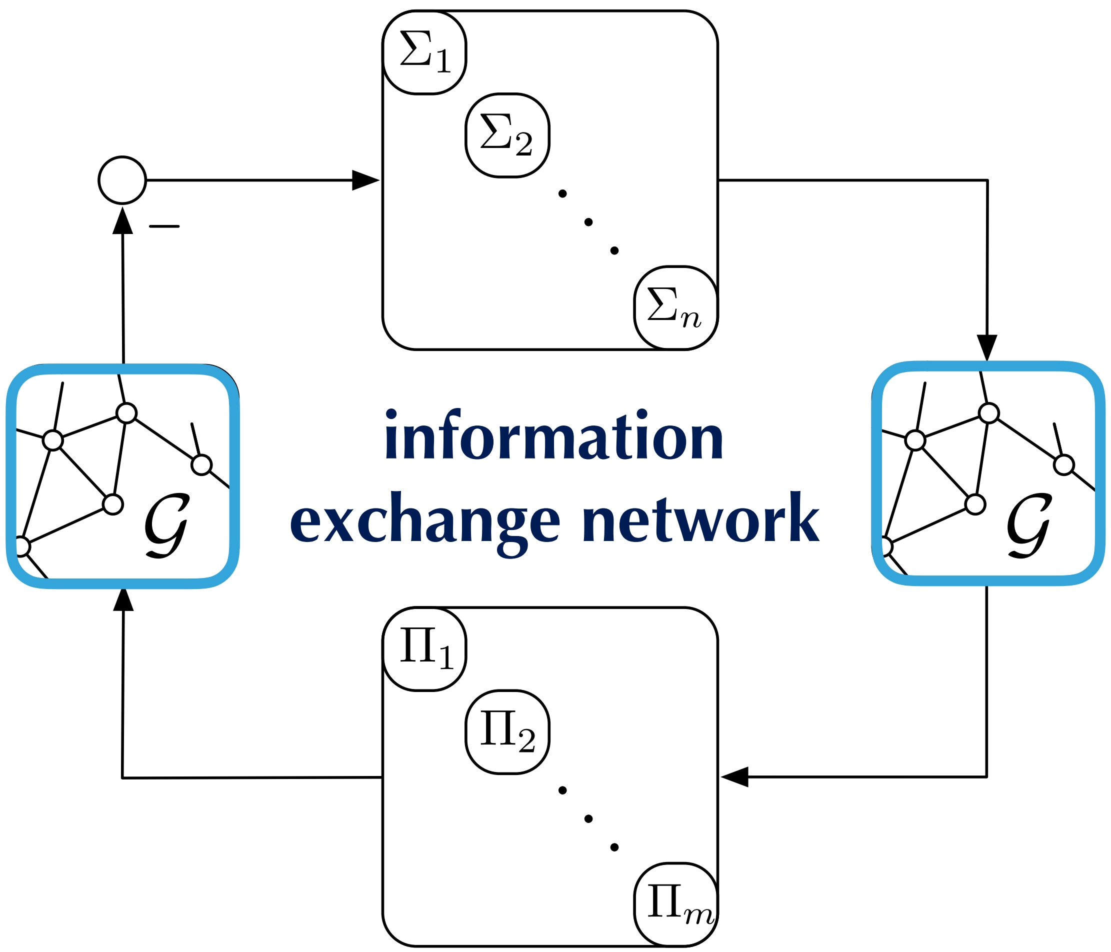


[Zhao,Zelazo2017]

RIGIDITY AS AN ARCHITECTURAL REQUIREMENT

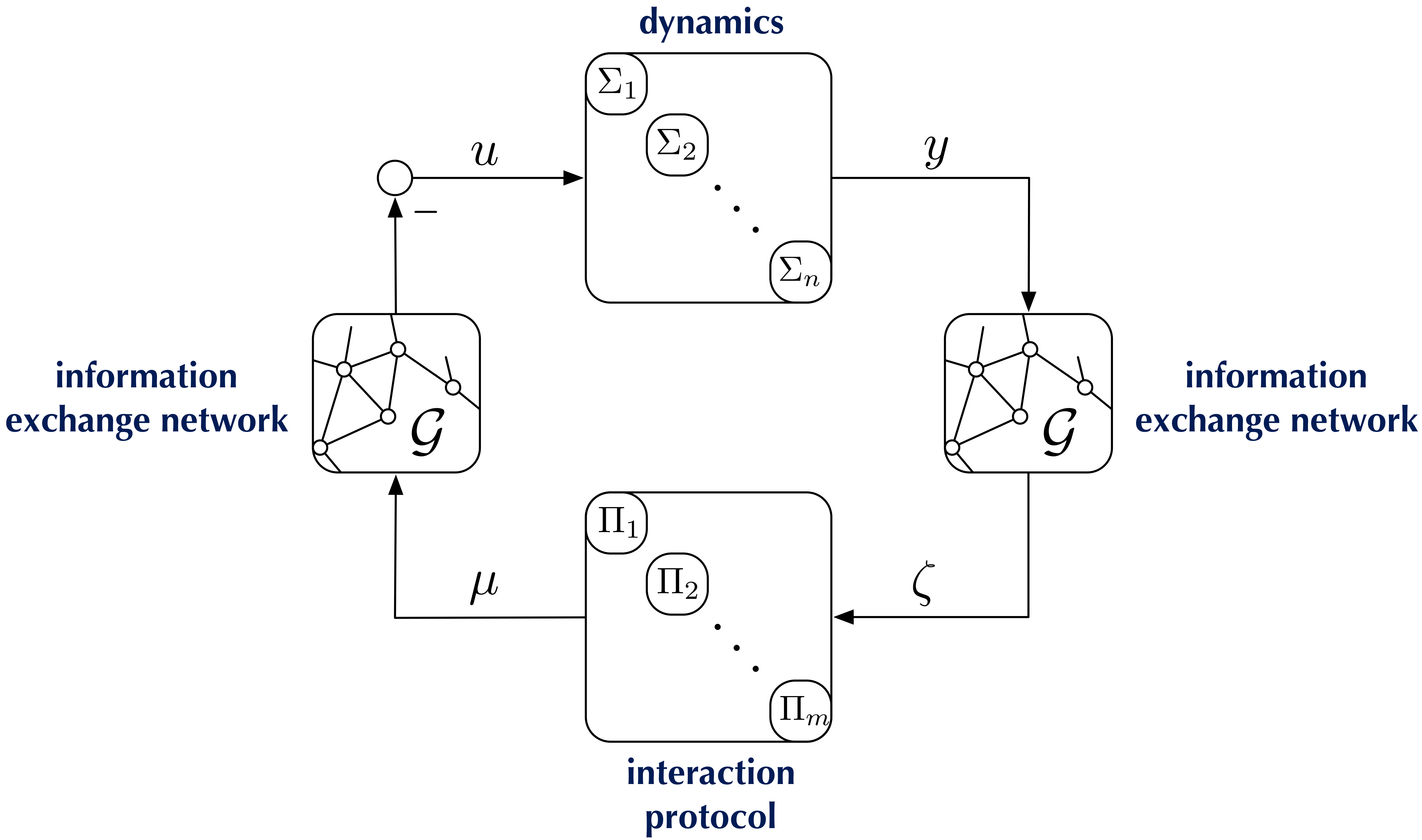


RIGIDITY THEORY FOR MULTI-ROBOT COORDINATION



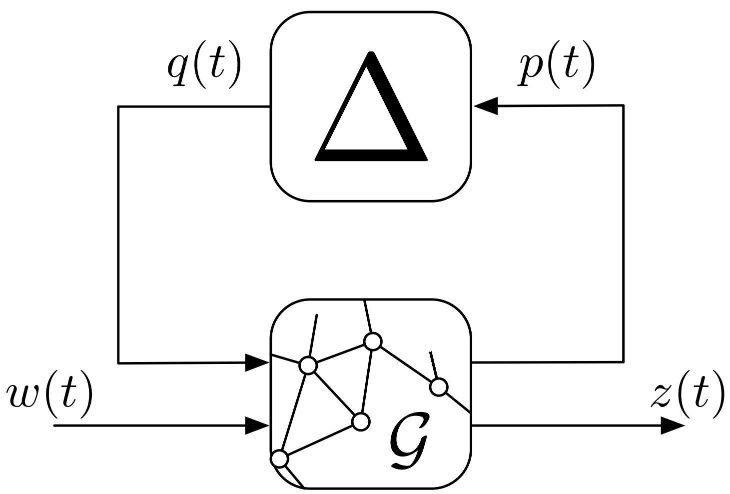
- **bearing rigidity theory for formation control and localization**
  [Automatica '16, TAC '16, TCNS '17, CSM '18]
- **multi-robot coordination for state-dependent and directed sensing**
  [IJRR '14, ECC '14, CDC '15, IJRNC '18, TAC '18]
- **implementation on robotic testbed**
  [IJRR '14, IROS '17, IFAC '18 (to be submitted)]

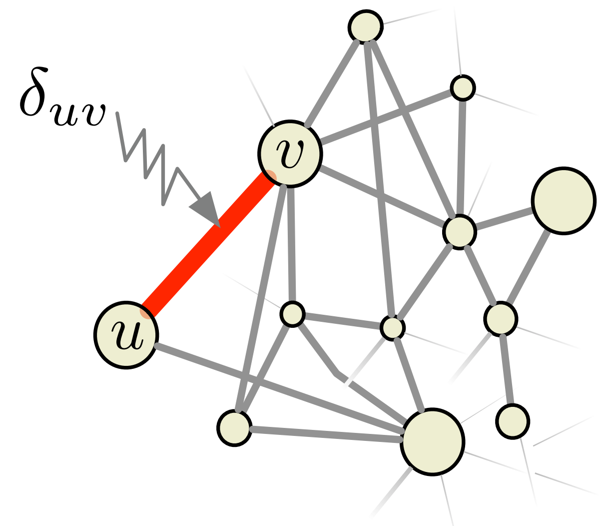
NETWORKED DYNAMIC SYSTEMS

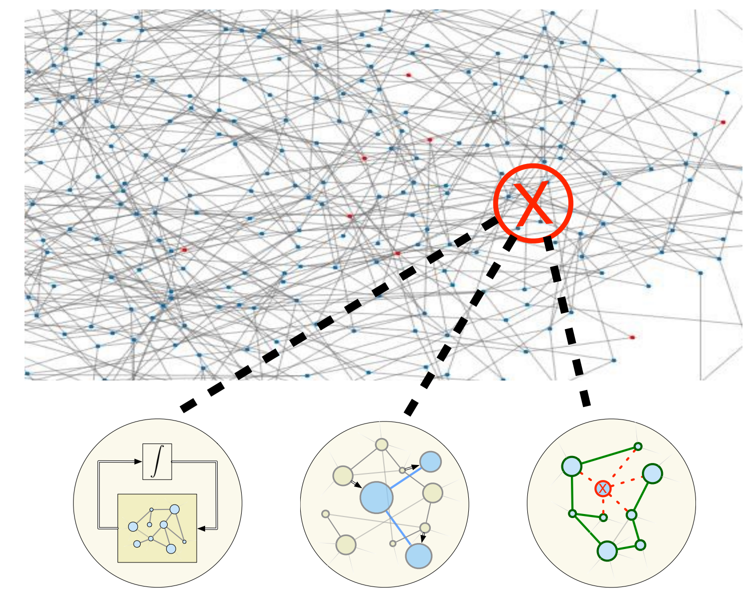


RESEARCH HORIZONS

Security, Robustness, and Fault Detection



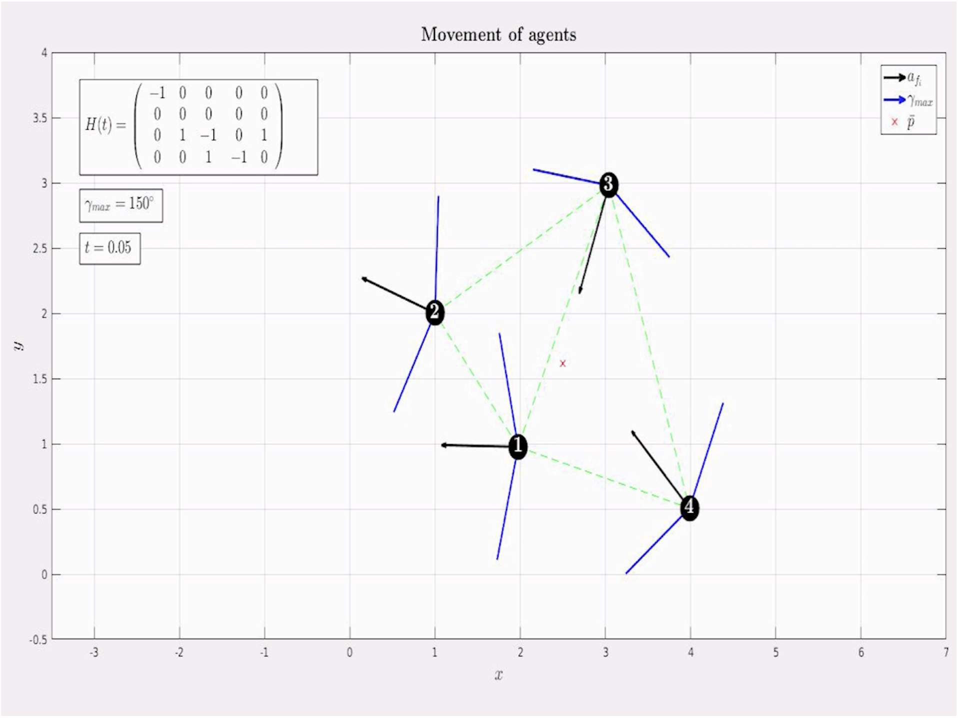


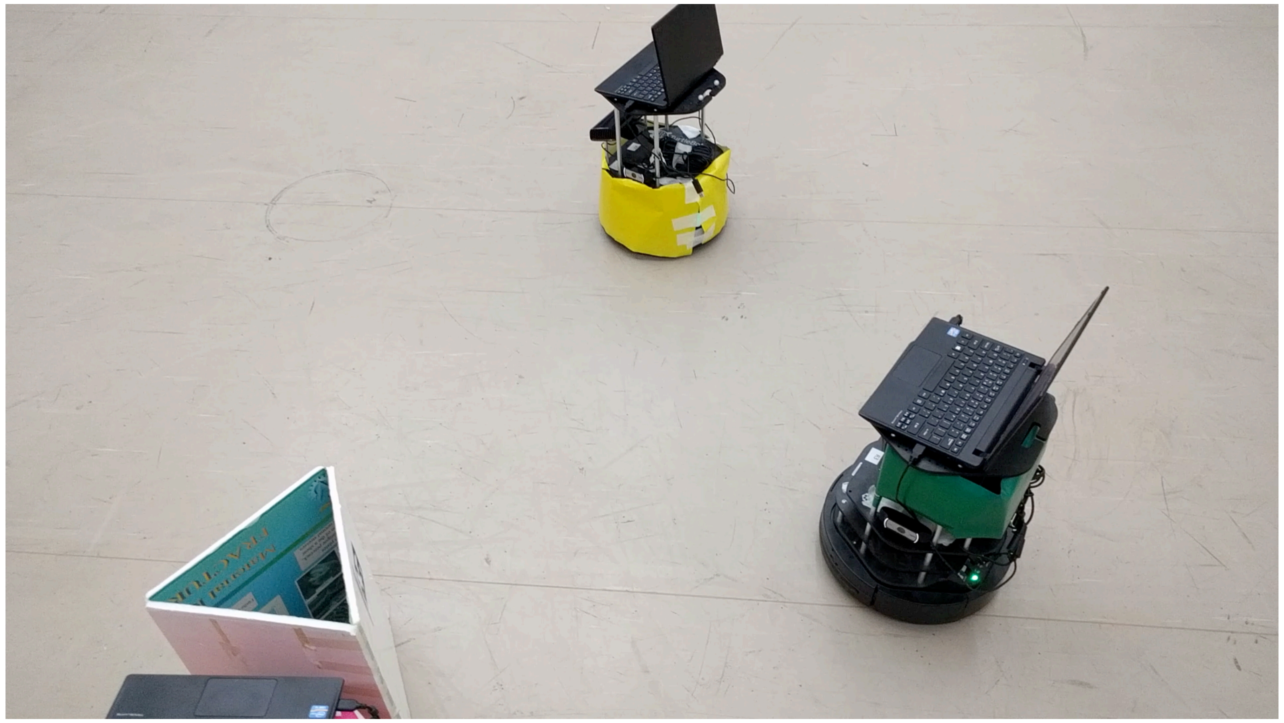


- what is the right way to study and design *secure* networked systems?
- how can understand *robustness* and *uncertainty* for networked systems?
- how can we *detect* and *isolate faults* in a large network?

RESEARCH HORIZONS

Multi-Robot Coordination

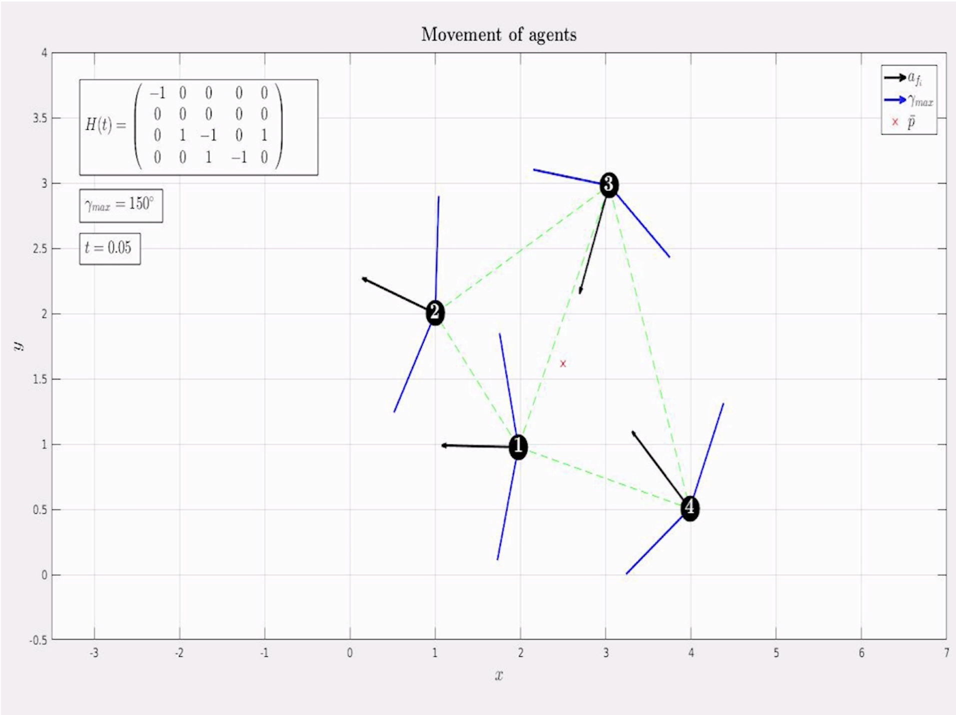


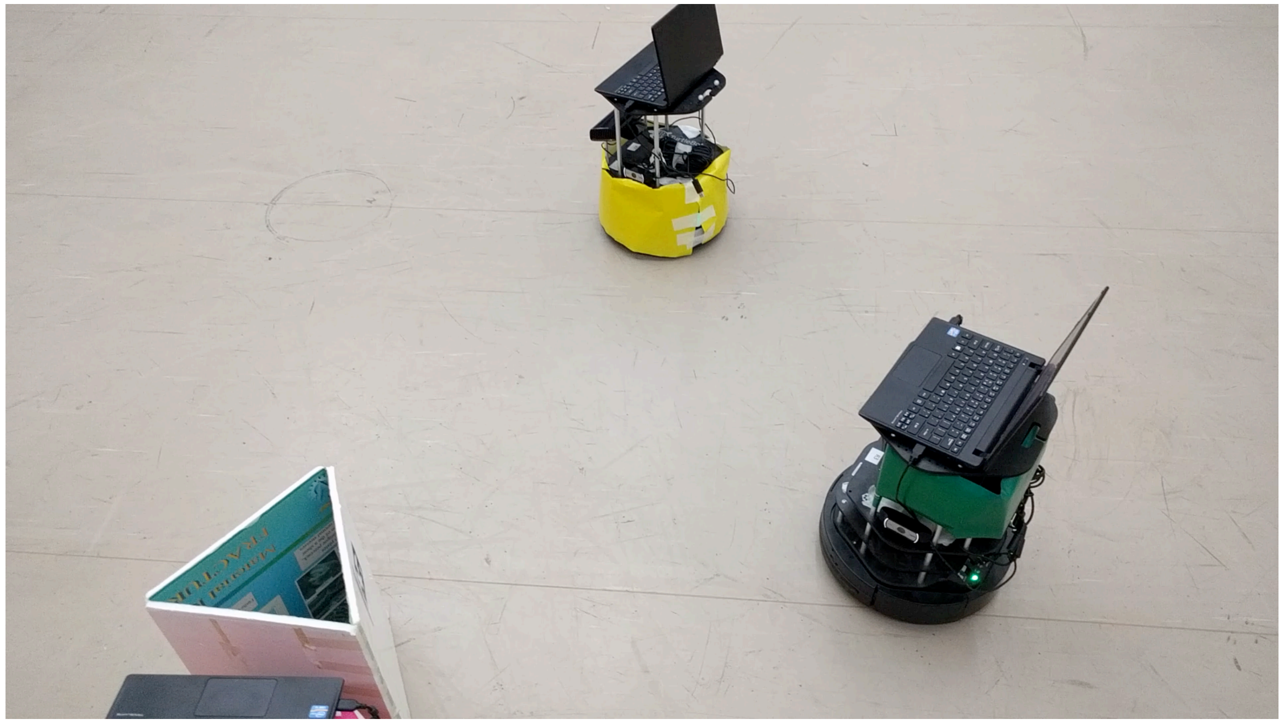


- how to bridge theory to implementation - coordination using *cheap sensing*
- higher level coordination tasks - constrained deployment, finite-time multi-objective coordination

RESEARCH HORIZONS

Multi-Robot Coordination





- how to bridge theory to implementation - coordination using *cheap sensing*
- higher level coordination tasks - constrained deployment, finite-time multi-objective coordination

ACKNOWLEDGEMENTS

CoNeCt
Cooperative Networks and Controls Lab

GIF German-Israeli Foundation for Scientific Research and Development

קרן לאומית למדע
ISRAEL SCIENCE FOUNDATION

BOSCH

ist

The University Of Sheffield.

Dr. Mathias Bürger

Prof. Dr.-Ing. Frank Allgöwer

Dr. Shiyu Zhao

lagadic

LAAS-CNRS

GIST
Gwangju Institute of Science and Technology