PASSIVITY, MONOTONICITY, AND NETWORK OPTIMIZATION: NEW PERSPECTIVES FOR NETWORK SYSTEMS ANALYSIS

Daniel Zelazo

Current Trends in Systems and Control Theory
a celebratory colloquium

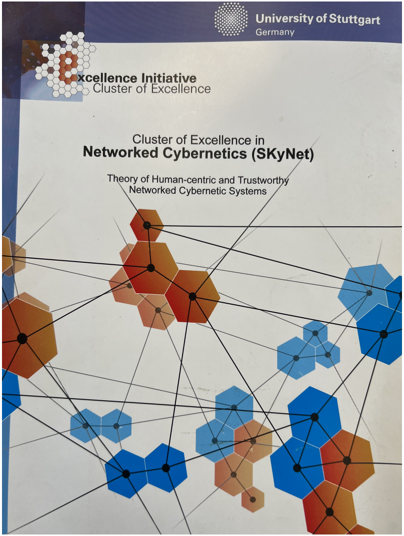
June 7, 2022

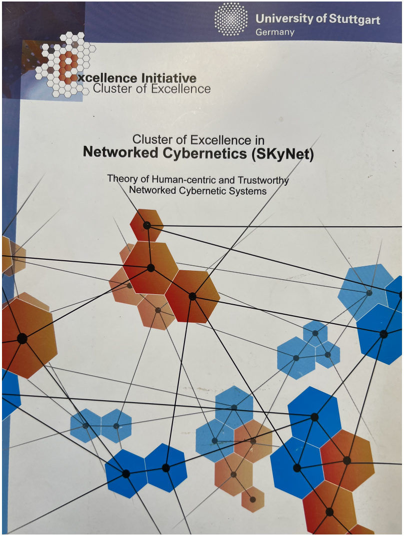
DEPARTMENT OF AEROSPACE ENGINEERING
TECHNION Israel Institute of Technology

CoNeCt Cooperative Networks and Controls Lab

Philadelphia Flight Control Laboratory

IST TRAJECTORIES





Collaborators


Dr. Mathias Bürger

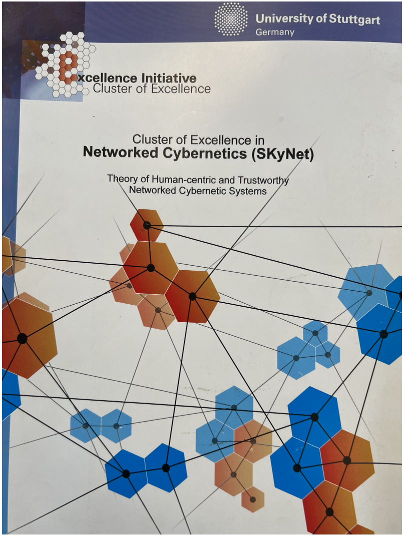

Dr. Max Montenbruck


Dr. Simone Schuler


Dr. Anne (Koch) Rommer


Daniel Frank

IST TRAJECTORIES



Collaborators


Dr. Mathias Bürger

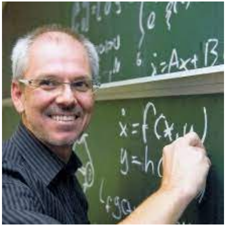

Dr. Max Montenbruck


Dr. Simone Schuler


Dr. Anne (Koch) Rommer

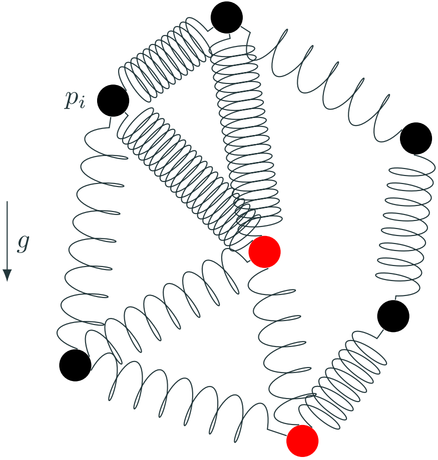

Daniel Frank

Happy Birthday!



A PHYSICS WARM-UP

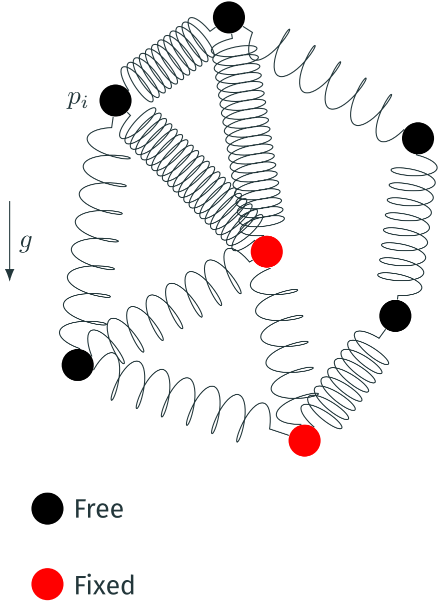
A MASS-SPRING NETWORK



Free

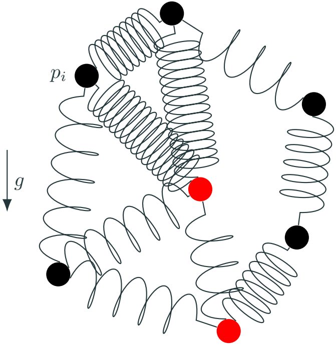
Fixed

- A fixed network of (linear) springs
- springs connected to masses with position $p_i \in \mathbb{R}^2$ and mass m_i
- r masses have a fixed position (anchors)



- A fixed network of (linear) springs
- springs connected to masses with position $p_i \in \mathbb{R}^2$ and mass m_i
- r masses have a fixed position (anchors)

Determine the positions of the free masses that minimize the total potential energy of the mass-spring network.



- Potential Energy due to gravity

$$m_i g^T p_i$$

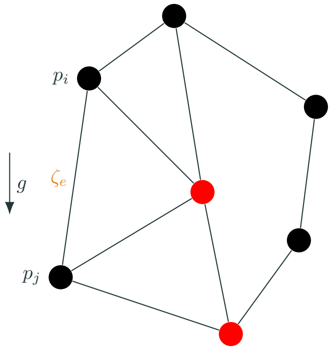
- Elastic Potential Energy of springs

$$\frac{1}{2} k_{ij} (\|p_i - p_j\| - r_{ij})^2$$

an optimization problem (take 1)

$$\min_{p_i} \quad \sum_i m_i g^T p_i + \sum_{i \sim j} \frac{1}{2} k_{ij} (\|p_i - p_j\| - r_{ij})^2$$

$$\texttt{s.t.}\, p_i = \mathbf{p}_i^*,\ i = 1, \ldots, r \ \texttt{(fixed nodes)}$$



- Potential Energy due to gravity (nodes)

$$m_i g^T p_i,\ i = 1, \ldots, n$$

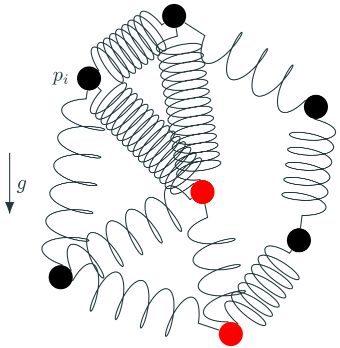
- Elastic Potential Energy of springs (edges)

$$\frac{1}{2} k_e (\| \underbrace{p_i - p_j}_{\zeta_e} \| - r_e)^2,\ e = 1, \ldots, m$$

an optimization problem (take 2)

$$\min_{p_i, \zeta_e} \quad \sum_{i=1}^{r} (m_i g^T p_i + \mathbb{I}_{\mathbf{p_i^*}}(p_i)) + \sum_{i=r+1}^{n} m_i g^T p_i + \sum_{e} \frac{1}{2} k_e (\|\zeta_e\| - r_e)^2$$

$$\texttt{s.t.}\, p_i - p_j = \zeta_e,\ \forall e = (i, j)$$



A Convex Program!

an optimization problem (take 2)

$$\min_{p_i,\zeta_e} \quad \sum_{i}^{r}(m_i g^T p_i + \mathbb{I}_{\mathbf{p_i^*}}(p_i)) + \sum_{i=r+1}^{n} m_i g^T p_i + \sum_{e} \frac{1}{2} k_{ij} (\|\zeta_e\| - r_e)^2$$

$$\texttt{s.t.}\, p_i - p_j = \zeta_e, \; \forall e = (i,j)$$

A MASS-SPRING NETWORK - THE DYNAMICS

- dynamic model for the masses

$$\Sigma_i : \begin{cases} \begin{bmatrix} \dot{p}_i \\ \ddot{p}_i \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_i \\ \dot{p}_i \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u_i + m_i g \\ y_i = \begin{cases} \begin{bmatrix} p_i \\ 0 \end{bmatrix}, & i = 1, \ldots, r \ \texttt{(anchors)} \\ \begin{bmatrix} p_i \\ \dot{p}_i \end{bmatrix}, & i = r+1, \ldots, n \end{cases} \end{cases}$$

- springs couple masses together

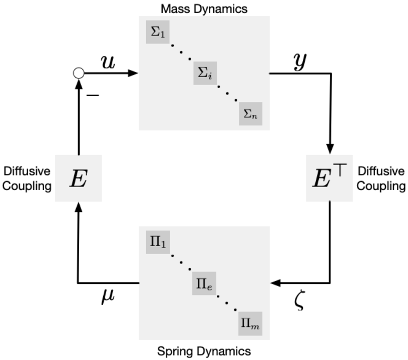
$$\Pi_e : \begin{cases} u_i & = \sum_{i \sim j} k_{ij}(\|p_i - p_j\| - r_{ij}) \frac{p_j - p_i}{\|p_j - p_i\|} + \\ & \quad b_{ij}(\dot{p}_j - \dot{p}_i) \\ & = \sum_{i \sim j} \kappa_{ij}(y_i - y_j) \end{cases}$$

- dynamic model for the masses

$$\Sigma_i : \begin{cases} \begin{bmatrix} \dot{p}_i \\ \ddot{p}_i \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_i \\ \dot{p}_i \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u_i + m_i g \\ y_i = \begin{cases} \begin{bmatrix} p_i \\ 0 \end{bmatrix}, & i = 1, \ldots, r \quad \texttt{(anchors)} \\ \begin{bmatrix} p_i \\ \dot{p}_i \end{bmatrix}, & i = r+1, \ldots, n \end{cases} \end{cases}$$

- springs couple masses together

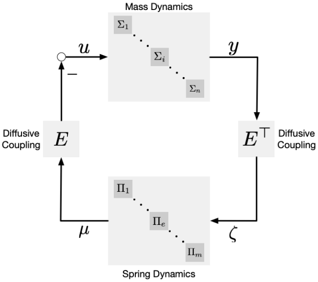
$$\Pi_e : \begin{cases} u_i & = \sum_{i \sim j} k_{ij}(\|p_i - p_j\| - r_{ij}) \frac{p_j - p_i}{\|p_j - p_i\|} + \\ & \quad b_{ij}(\dot{p}_j - \dot{p}_i) \\ & = \sum_{i \sim j} \kappa_{ij}(y_i - y_j) \end{cases}$$



An example of a diffusively coupled network!

- System Equilibrium

$$\begin{cases} 0 &= \dot{p}_i \\ 0 &= m_i g + \sum_{i \sim j} k_{ij}(\|p_i - p_j\| - r_{ij}) \frac{p_j - p_i}{\|p_j - p_i\|} \end{cases}$$

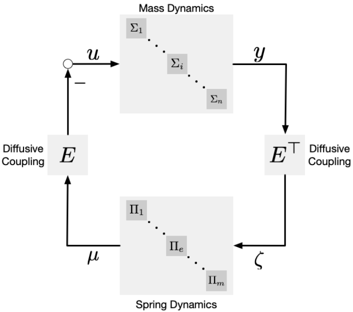


Minimum Total Potential Energy Principle (MTPE)

Equilibrium configurations extremize the total potential energy. Stable equilibriums correspond to minimizers of the total potential energy.

Dynamics

- Diffusively Coupled Network

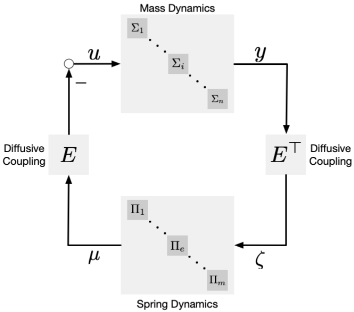


- Dissipasivity Theory

$$V(x) = \frac{1}{2}\sum_i \|\dot{p}_i\|^2 + \frac{1}{2}\sum_{i\sim j} k_{ij}\|p_i - p_j\|_2^2$$

Dynamics

- Diffusively Coupled Network



- Dissipasivity Theory

$$V(x) = \frac{1}{2}\sum_i \|\dot{p}_i\|^2 + \frac{1}{2}\sum_{i\sim j} k_{ij}\|p_i - p_j\|_2^2$$

Optimization

- Convex Optimization

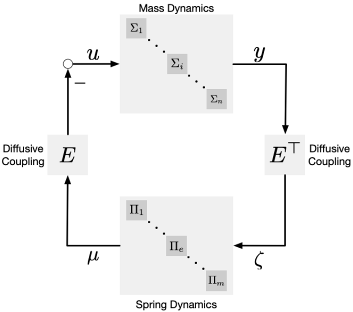
$$\min_{p_i,\zeta_e} \quad J(p,\zeta)$$
$$\texttt{s.t.}\, p_i - p_j = \zeta_e, \forall\, e = (i,j)$$

- Optimality Conditions

$$0 \in \partial J(p,\zeta)$$

Dynamics

- Diffusively Coupled Network



- Dissipasivity Theory

$$V(x) = \frac{1}{2}\sum_i \|\dot{p}_i\|^2 + \frac{1}{2}\sum_{i\sim j} k_{ij}\|p_i - p_j\|_2^2$$

Optimization

- Convex Optimization

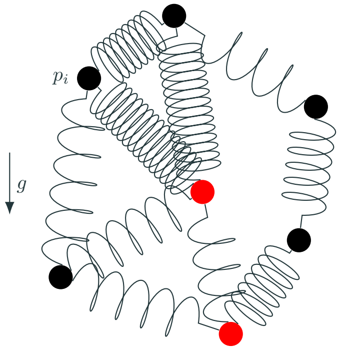
$$\min_{p_i,\zeta_e} \quad J(p,\zeta)$$

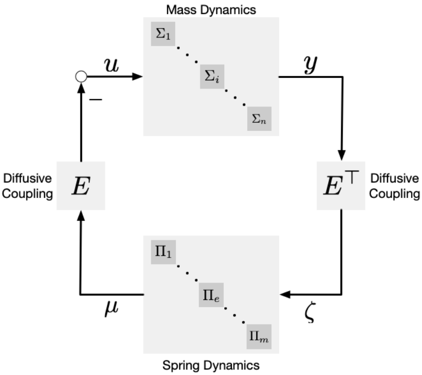
$$\texttt{s.t.}\, p_i - p_j = \zeta_e, \forall\, e = (i,j)$$

- Optimality Conditions

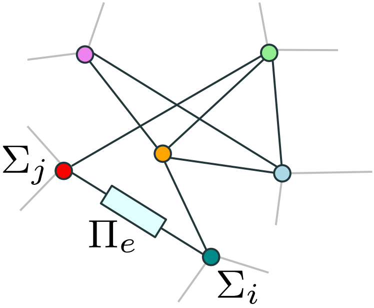
MTPE Principle ensures that the dynamics of the diffusively coupled network solve the optimization problem, and vice versa.

- What class of systems can be “solved” by examining a related optimization problem?
- What class of optimization problems can be be “solved” by a dynamical system?



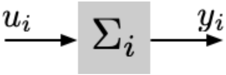


DIFFUSIVELY COUPLED NETWORKS



A network system is comprised of dynamical systems that interact with eachother over an information exchange network (a graph).

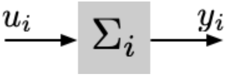
Agent dynamics:



$$\Sigma_i : \begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = h_i(x_i, u_i) \end{cases}$$

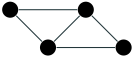
A NETWORK MODEL

Agent dynamics:



$$\Sigma_i : \begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = h_i(x_i, u_i) \end{cases}$$

Information Exchange Network:



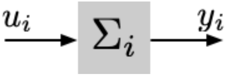
$$\mathcal{G} = (\mathbb{V}, \mathbb{E})$$

$$E = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

$$[E]_{ij} = \begin{cases} \pm 1 & (i, j) \in \mathbb{E} \\ 0 & \texttt{o.w.} \end{cases}$$

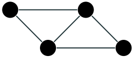
$$E^\top \mathbf{1} = 0$$

Agent dynamics:



$$\Sigma_i : \begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = h_i(x_i, u_i) \end{cases}$$

Information Exchange Network:



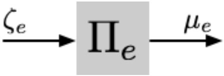
$$\mathcal{G} = (\mathbb{V}, \mathbb{E})$$

$$E = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

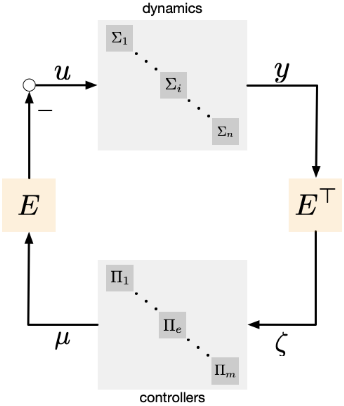
$$[E]_{ij} = \begin{cases} \pm 1 & (i,j) \in \mathbb{E} \\ 0 & \texttt{o.w.} \end{cases}$$

$$E^\top \mathbf{1} = 0$$

Controller dynamics:



$$\Pi_e : \begin{cases} \dot{\eta}_e = \phi_e(\eta_e, \zeta_e) \\ \mu_e = \psi_e(\eta_e, \zeta_e) \end{cases}$$



$$(\Sigma, \Pi, \mathcal{G})$$

- Consensus Dynamics

$$\dot{x}_i = -\sum_{i\sim j} w_{ij}(x_j - x_i)$$

- Kumamoto Model

$$\dot{\theta}_i = -k\sum_{i\sim j} \sin(\theta_i - \theta_j)$$

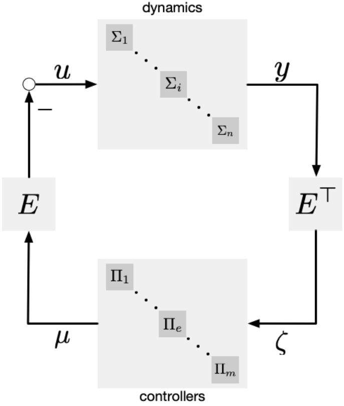
- Traffic Dynamics

$$\dot{v}_i = \kappa_i\left(V_i^0 - v_i + V_i^1 \sum_{i\sim j} \tanh(p_j - p_i)\right)$$

- Neural Network

$$C\dot{V}_i = f(V_i, h_i) + \sum_{i\sim j} g_{ij}(V_j - V_i)$$

$$\dot{h}_i = g(V_i, h_i)$$



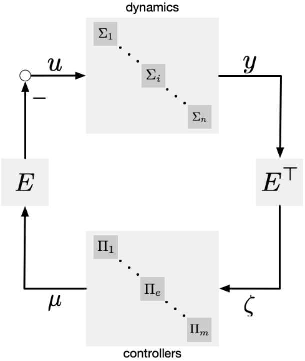
What properties must the systems Σ_i and Π_e possess such that $(\Sigma, \Pi, \mathcal{G})$ admits and converges to a steady-state solution?

$$u(t) \to \mathrm{u}$$
$$y(t) \to \mathrm{y}$$
$$\zeta(t) \to \zeta$$
$$\mu(t) \to \mu$$

- Consensus: $\mathrm{y} = \alpha \mathbf{1}$ ($\zeta = 0$)
- Formation: $\zeta \neq 0$ constant

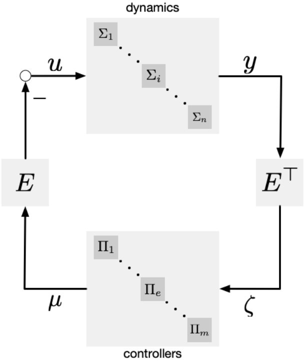
All signals converge to a constant steady-state

NETWORK OPTIMIZATION MEETS PASSIVITY THEORY



Assumption 1

Each agent Σ_i and controller Π_e admit forced steady-state solutions.

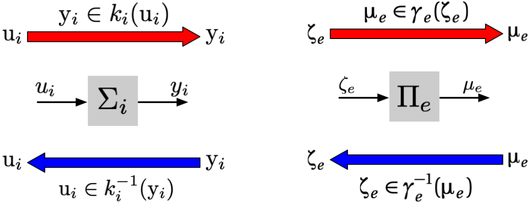


Assumption 1

Each agent Σ_i and controller Π_e admit forced steady-state solutions.

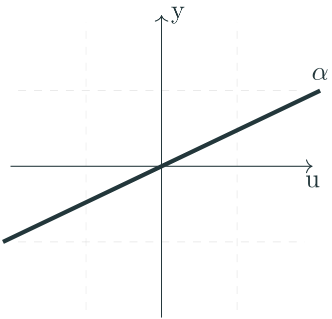
Input-Output Maps

The *steady-state input-output map* $k : \mathcal{U} \to \mathcal{Y}$ associated with Σ is the set consisting of all steady-state input-output pairs (u, y) of the system.



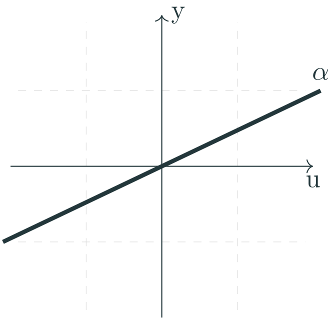
INPUT-OUTPUT RELATIONS

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$\Rightarrow k(\mathrm{u}) = \{\mathrm{y} \mid \underbrace{(-CA^{-1}B + D)}_{\alpha}\,\mathrm{u}\}$$



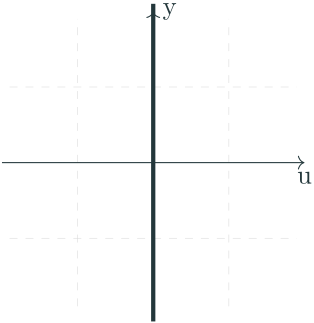
SISO and stable linear system

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$\Rightarrow k(\mathrm{u}) = \{\mathrm{y} \mid \underbrace{(-CA^{-1}B + D)}_{\alpha}\,\mathrm{u}\}$$



SISO and stable linear system

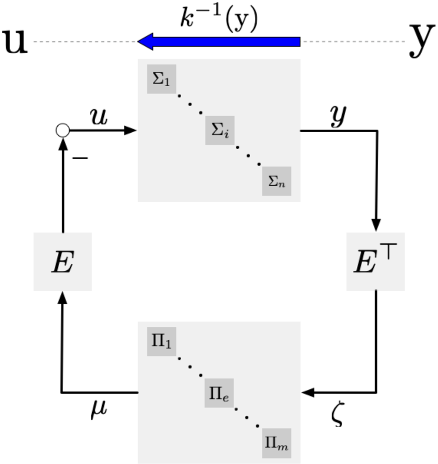
$$\dot{x} = u$$
$$y = x$$
$$\Rightarrow k = \{(0, \mathrm{y}),\ \mathrm{y} \in \mathbb{R}\}$$



simple integrator

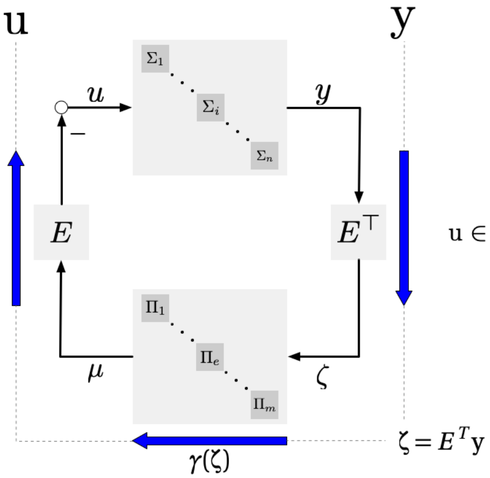
The network interconnection imposes constraints on allowable steady-states

The network interconnection imposes constraints on allowable steady-states



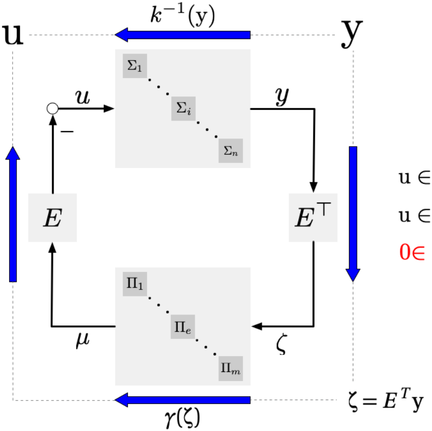
$\mathrm{u} \in k^{-1}(\mathrm{y})$

The network interconnection imposes constraints on allowable steady-states



$$u \in -E\gamma(E^T y)$$

The network interconnection imposes constraints on allowable steady-states

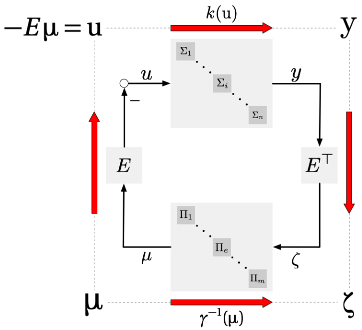


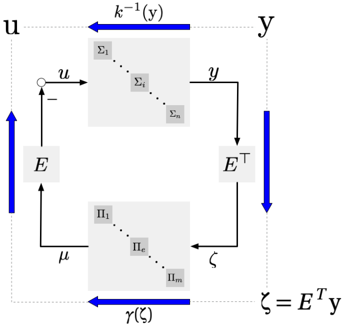
$\mathrm{u} \in k^{-1}(\mathrm{y})$

$\mathrm{u} \in -E\gamma(E^T \mathrm{y})$

$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T \mathrm{y})$

The network interconnection imposes constraints on allowable steady-states





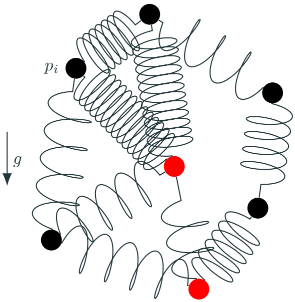
$\zeta \in \gamma^{-1}(\mu)$
$\zeta \in E^T k(-E\mu)$
$0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$

$u \in k^{-1}(y)$
$u \in -E\gamma(E^T y)$
$0 \in k^{-1}(y) + E\gamma(E^T y)$

The network system $(\Sigma, \Pi, \mathcal{G})$ admits a steady-state if and only if there exists a solution to the system of non-linear inclusions

$$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T \mathrm{y})$$
$$0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$$

- When do solutions exist?
- How do we find them?

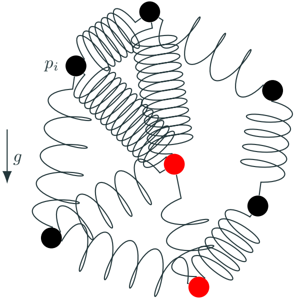


A Convex Program!

Minimum Total Potential Energy Problem

$$\min_{p_i,\zeta_e} \quad \sum_i^r (m_i g^T p_i + \mathbb{I}_{\mathbf{p_i^*}}(p_i)) + \sum_{i=r+1}^n m_i g^T p_i + \sum_e \frac{1}{2} k_{ij} (\|\zeta_e\| - r_e)^2$$

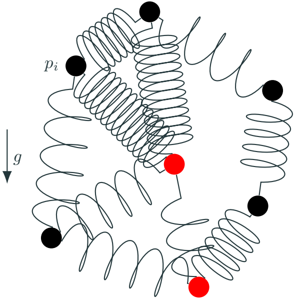
$$\texttt{s.t.}\, p_i - p_j = \zeta_e, \forall e = (i,j)$$



A Convex Program!

Minimum Total Potential Energy Problem

$$\min_{p_i, \zeta_e} \quad \sum_i J_i(p_i) + \sum_e \Gamma_e(\zeta_e)$$
$$\texttt{s.t.}\, E^T p = \zeta$$



A Convex Program!

Minimum Total Potential Energy Problem

$$\min_p \quad J(p) + \Gamma(E^T p)$$

First-order Optimality Condition:

$$0 \in \partial J(p) + E\partial\Gamma(E^T p)$$

SOLUTION OF NETWORK EQUATIONS

The network system $(\Sigma, \Pi, \mathcal{G})$ admits a steady-state if and only if there exists a solution to the system of non-linear inclusions

$$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T\mathrm{y})$$
$$0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$$

RECALL First-order Optimality Condition:

$$0 \in \partial J(p) + E\partial\Gamma(E^T p)$$

Network equations are the first-order optimality conditions of a corresponding optimization problem!

The network system $(\Sigma, \Pi, \mathcal{G})$ admits a steady-state if and only if there exists a solution to the system of non-linear inclusions

$$0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T\mathrm{y})$$
$$0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$$

RECALL First-order Optimality Condition:

$$0 \in \partial J(p) + E\partial\Gamma(E^T p)$$

Network equations are the first-order optimality conditions of a corresponding optimization problem!

What is it?

Definition

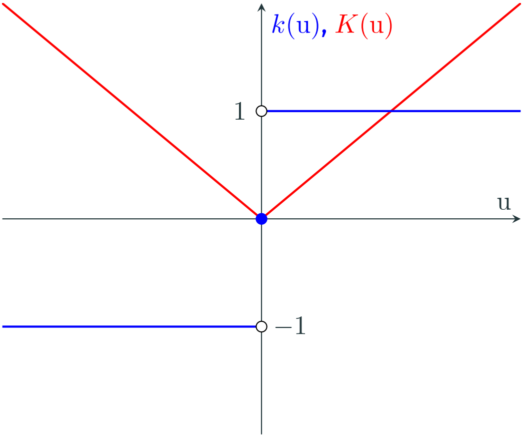
Let k_i be the input-output relation for system Σ_i. Define the function $K_i : \mathbb{R} \to \mathbb{R}$ such that $\partial K_i(\mathrm{u}_i) = k_i(\mathrm{u}_i)$ and $K = \sum_i K_i$. The function K is called the *cost function* associated with the system Σ_i.

Similarly,

$$\partial K_i^\star(\mathrm{y}_i) = k_i^{-1}(\mathrm{y}_i),\; K^\star = \sum_i K_i^\star$$

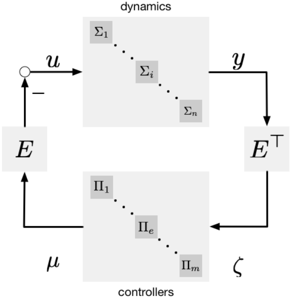
$$\partial \Gamma_e(\zeta_e) = \gamma_e(\zeta_e),\, \Gamma = \sum_e \Gamma_e$$

$$\partial \Gamma_e^\star(\mu_e) = \gamma_e^{-1}(\mu_e)\, \Gamma^\star = \sum_e \Gamma_e^\star$$



$K(u) = |u|$

$y = k(u) = \mathrm{sgn}(u)$

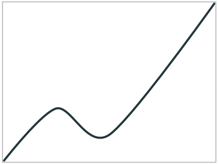


Steady-state values $\mathrm{u}, \mathrm{y}, \zeta$ and μ are the solutions of the following pair of optimization problems[1]:

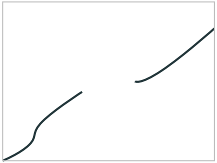
$\min_{\mathrm{y},\zeta} \quad \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e)$ $s.t. \quad E^T\mathrm{y} = \zeta.$	$\min_{\mathrm{u},\mu} \quad \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e)$ $s.t. \quad \mathrm{u} = -E\mu.$
First-order Optimality Condition $0 \in k^{-1}(\mathrm{y}) + E\gamma(E^T\mathrm{y})$	First-order Optimality Condition $0 \in \gamma^{-1}(\mu) - E^T k(-E\mu)$

[1] [Bürger, Z, Allgower, 2014]

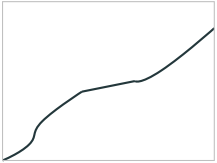
MONOTONE MAPS AND CONVEXITY



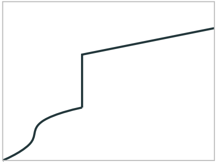
Not Monotone



Monotone but not maximal



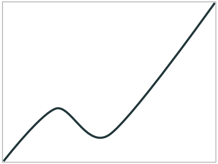
Maximal monotone function



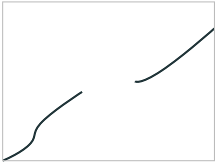
Maximal monotone relation

A relation on $\mathbb{R}$ is monotone
if they are non-decreasing curves in $\mathbb{R}^2$

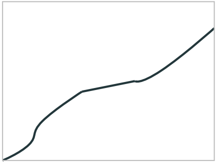
MONOTONE MAPS AND CONVEXITY



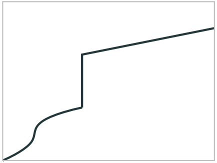
Not Monotone



Monotone but not maximal



Maximal monotone function

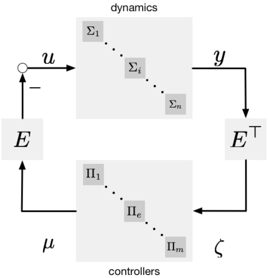


Maximal monotone relation

Theorem

The subdifferentials of convex functions on $\mathbb{R}$ are maximally monotone relations from $\mathbb{R}$ to $\mathbb{R}$.[a]

[a] [R. T. Rockafellar, Convex Analysis. Princeton University Press, 1997]

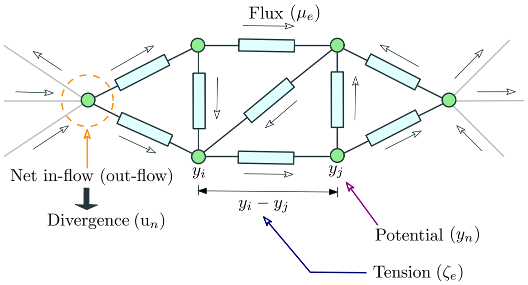


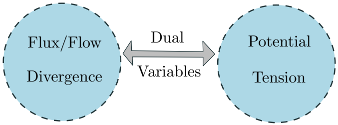
Theorem[1]

If the input-output maps k_i and γ_e are maximally monotone, then the steady-state values $\mathrm{u}, \mathrm{y}, \zeta$ and μ are the solutions of the following pair of convex dual optimization problems:

Optimal Flow Problem (OFP)	Optimal Potential Problem (OPP)
$\min_{\mathrm{y},\zeta} \quad \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e)$	$\min_{\mathrm{u},\mu} \quad \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e)$
$s.t. \quad E^T\mathrm{y} = \zeta.$	$s.t. \quad \mathrm{u} = -E\mu.$

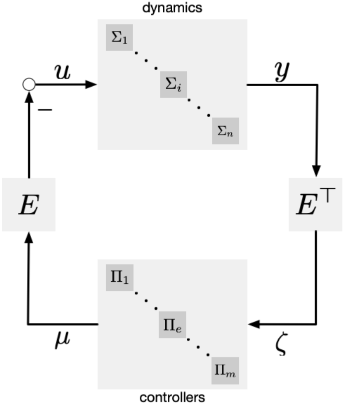
[1] [Bürger, Z, Allgower, 2014]





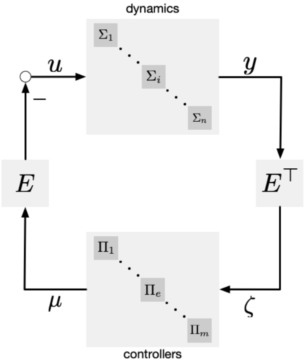
Optimal Flow Problem[1]	**Optimal Potential Problem**[1]
$\min_{u,\mu} \sum_{n=1}^{\lvert\mathcal{V}\rvert} C_n^{\text{div}}(u_n) + \sum_{e=1}^{\lvert\mathcal{E}\rvert} C_e^{\text{flux}}(\mu_e)$	$\min_{y,\zeta} \sum_{n=1}^{\lvert\mathcal{V}\rvert} C_n^{\text{pot}}(y_n) + \sum_{e=1}^{\lvert\mathcal{E}\rvert} C_e^{\text{ten}}(\zeta_e)$
$s.t. \quad u + E\mu = 0.$	$s.t. \quad E^T y = \zeta.$

[1] [R. T. Rockafellar, Network Flows and Monotropic Optmizations. John Wiley and Sons, Inc., 1984]



Diffusively coupled dynamic networks can be associated to static network optimization problems!

Monotone steady-state maps ⇔ Network Duality



Assumption 1

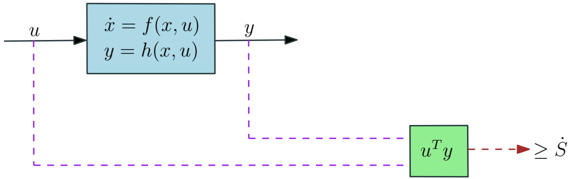
Each agent Σ_i and controller Π_e admit forced steady-state solutions.

Assumption 2

The input-output maps of each agent, k_i, and controller, γ_e, are maximally monotone.

Under what conditions does the network actually *converge* to these steady states?

PASSIVITY FOR DYNAMICAL SYSTEMS



$$0 = f(0,0)$$
$$0 = h(0,0)$$

Definition [Khalil 2002]

A system is passive if there exists a C^1 storage function $S(x)$ such that

$$u^T y \geq \dot{S} = \frac{\partial S}{\partial x} f(x,u), \quad \forall (x,u) \in \mathbb{R}^n \times \mathbb{R}^p$$

Moreover, it is said to be

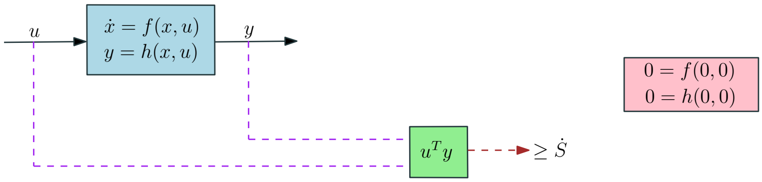
- Input-strictly passive if $\dot{S} \leq u^T y - u^T \phi(u)$ and $u^T \phi(u) > 0, \forall u \neq 0$
- Output-strictly passive if $\dot{S} \leq u^T y - y^T \rho(y)$ and $y^T \rho(y) > 0, \forall y \neq 0$

Definition

Let Σ be a SISO system with a constant input-output steady-state pair (u, y). The system is said to be *input-output* (ρ, ν)*-passive* wrt (u, y) if there exists a storage function $S(x)$ and numbers $\rho, \nu \in \mathbb{R}$, such that $\rho\nu < 1/4$ and

$$\dot{S} = \frac{\partial S}{\partial x} f(x, u) \leq (y - \mathrm{y})(u - \mathrm{u}) - \rho(y - \mathrm{y})^2 - \nu(u - \mathrm{u})^2,$$

for any trajectory u, y.



Definition

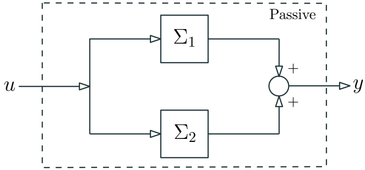
Let Σ be a SISO system with a constant input-output steady-state pair (u, y). The system is said to be *input-output* (ρ, ν)*-passive* wrt (u, y) if there exists a storage function $S(x)$ and numbers $\rho, \nu \in \mathbb{R}$, such that $\rho\nu < 1/4$ and

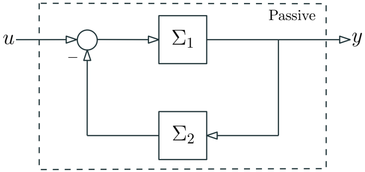
$$\dot{S} = \frac{\partial S}{\partial x} f(x, u) \leq (y - \mathrm{y})(u - \mathrm{u}) - \rho(y - \mathrm{y})^2 - \nu(u - \mathrm{u})^2,$$

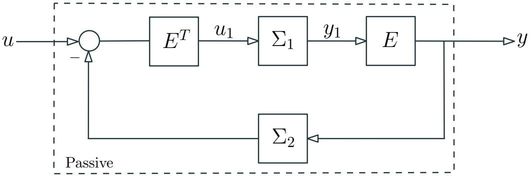
for any trajectory u, y.

- $\rho = \nu = 0 \Rightarrow$ **passivity**
- $\rho, \nu > 0 \Rightarrow$ **strict input/output passivity**
- $\rho, \nu < 0 \Rightarrow$ **passive short**

- Parallel Interconnection
- Negative Feedback Interconnection
- Symmetric Interconnection







Theorem[1]

Consider the network system $(\Sigma, \Pi, \mathcal{G})$ comprised of SISO agents and controllers. Suppose that there are vectors $\mathrm{u_i}, \mathrm{y_i}, \zeta_e$ and μ_e such that

i) the systems Σ_i are output strictly-passive with respect to $\mathrm{u_i}$ and $\mathrm{y_i}$;
ii) the systems Π_e are passive with respect to ζ_e and μ_e;
iii) the vectors $\mathrm{u}, \mathrm{y}, \zeta$ and μ satisfy $\mathrm{u} = -\mathcal{E}\mu$ and $\zeta = \mathcal{E}^T \mathrm{y}$.

Then the output vector $y(t)$ converges to y as $t \to \infty$.

[1] [Arcak, 2007], [Bürger, Z, Allgower, 2014]

A CONVERGENCE RESULT

Theorem[1]

Consider the network system $(\Sigma, \Pi, \mathcal{G})$ comprised of SISO agents and controllers. Suppose that there are vectors $\mathrm{u_i}, \mathrm{y_i}, \zeta_e$ and μ_e such that

i) the systems Σ_i are output strictly-passive with respect to $\mathrm{u_i}$ and $\mathrm{y_i}$;
ii) the systems Π_e are passive with respect to ζ_e and μ_e;
iii) the vectors $\mathrm{u}, \mathrm{y}, \zeta$ and μ satisfy $\mathrm{u} = -\mathcal{E}\mu$ and $\zeta = \mathcal{E}^T \mathrm{y}$.

Then the output vector $y(t)$ converges to y as $t \to \infty$.

- requires passivity w.r.t. to specific equilibrium configuration

[1] [Arcak, 2007], [Bürger, Z, Allgower, 2014]

EIP[1]

A SISO system $\Sigma : u \mapsto y$ is said to be *equilibrium-independent input-output* (ρ, ν)*-passive* if it is input-output (ρ, ν)-passive with respect to any equilibrium $(\mathrm{u}, k(\mathrm{u}))$.

EIP systems ($\rho, \nu \geq 0$) have monotone steady-state input-output maps!

$$\dot{S} \leq (y - \mathrm{y})^T (u - \mathrm{u}) \implies k \text{ monotonically increasing function}$$

[1] [G.H. Hines et al., 2011], [M. Sharf, A. Jain, Z., 2020]

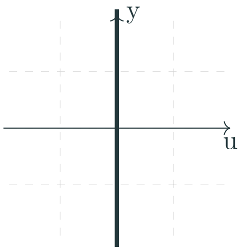
EQUILIBRIUM-INDEPENDENT PASSIVITY (EIP)

EIP[1]

A SISO system $\Sigma : u \mapsto y$ is said to be *equilibrium-independent input-output* (ρ, ν)*-passive* if it is input-output (ρ, ν)-passive with respect to any equilibrium $(\mathrm{u}, k(\mathrm{u}))$.

EIP systems ($\rho, \nu \geq 0$) have monotone steady-state input-output maps!

$$\dot{S} \leq (y - \mathrm{y})^T (u - \mathrm{u}) \implies k \text{ monotonically increasing function}$$



$$\dot{x}(t) = u(t), y(t) = x(t)$$

- Passive with respect to $\mathcal{U} = \{0\}$ and any output value $\mathrm{y} \in \mathbb{R}$ with storage function $S(x) = \frac{1}{2}(x - \mathrm{y})^2$.
- The equilibrium input-output map $k = \{(0, \mathrm{y}) : \mathrm{y} \in \mathbb{R}\}$ is not a single valued function and hence the integrator is **NOT** *EIP*.

[1] [G.H. Hines et al., 2011], [M. Sharf, A. Jain, Z., 2020]

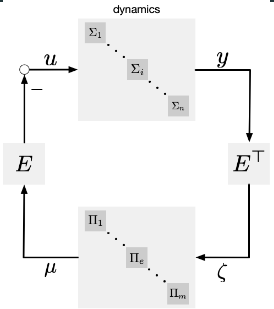
MAXIMALLY EQUILIBRIUM-INDEPENDENT PASSIVITY (MEIP)

MEIP[1]

A dynamical SISO system Σ is *maximal equilibrium independent* passive if the following conditions hold:

- The system Σ is passive with respect to any steady-state $(\mathrm{u}, \mathrm{y}) \in k$.
- The relation k is maximally monotone.

[1] [M. Bürger et al., 2014]



Assumption 1

Each agent Σ_i and controller Π_e admit forced steady-state solutions.

Assumption 2

The agent dynamics Σ_i are output-strictly MEIP and the controllers are MEIP.

Theorem[1]

Assume Assumptions 1 and 2 hold. Then the signals $u(t), y(t), \zeta(t), \mu(t)$ converge to the solutions of the following pair of convex dual optimization problems:

Optimal Flow Problem (OFP)	Optimal Potential Problem (OPP)
$\min_{\mathrm{y},\zeta} \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e)$	$\min_{\mathrm{u},\mu} \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e)$
$s.t. \quad E^T\mathrm{y} = \zeta.$	$s.t. \quad \mathrm{u} = -E\mu.$

[1] [Bürger, Z, Allgower, 2014]

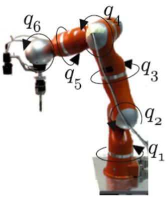
NEW PERSPECTIVES ON PASSIVATION

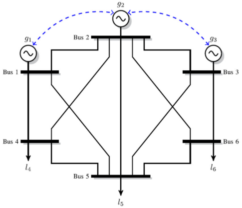
What else can we say about MEIP systems?

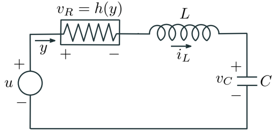
PASSIVITY-SHORT SYSTEMS

In practice, systems are usually passivity-short (or non-passive)!

- Generator (always generates energy) [R. Harvey , 2016]
- Oscillating systems with small or nonexistent damping [R. Harvey, 2017]
- Dynamics of robot system from torque to position [D. Babu, 2018]
- Power-system network (turbine-governor dynamics) [S. Trip, 2018]
- Electrical circuits with nonlinear components
- More general as include non-minimum phase systems and systems with relative degree greater than 1 [Z. Qu, 2014]



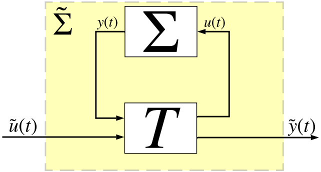




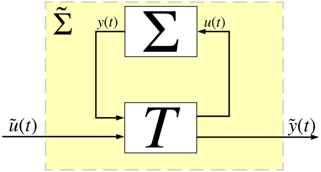
Passive short systems can destroy
the developed network optimization framework!

System Type	Relations	Integral Function
MEIP	k, k^{-1} max. monotone	$K(\mathrm{u}), K^\star(\mathrm{y})$ are convex
Input PS	k is not monotone	$K(\mathrm{u})$ is non-convex
Output PS	k^{-1} is not monotone	$K^\star(\mathrm{y})$ is non-convex
Input-output PS	k, k^{-1} are not monotone	May not exist

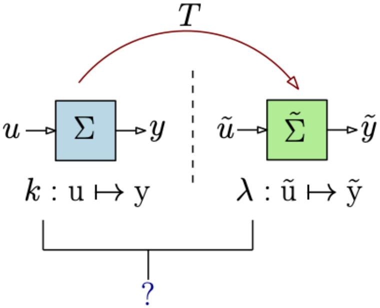
Optimal Flow Problem (OFP)	Optimal Potential Problem (OPP)
$\min\limits_{\mathrm{y},\zeta} \quad \sum_i K_i^\star(\mathrm{y}_i) + \sum_e \Gamma_e(\zeta_e)$	$\min\limits_{\mathrm{u},\mu} \quad \sum_i K_i(\mathrm{u}_i) + \sum_e \Gamma_e^\star(\mu_e)$
$s.t. \quad E^T\mathrm{y} = \zeta.$	$s.t. \quad \mathrm{u} = -E\mu.$



For a passive-short system $\Sigma : u \mapsto y$, we aim to find a map T such that the closed-loop system $\tilde{\Sigma} : \tilde{u} \mapsto \tilde{y}$ is passive. This is known as feedback passivation.



For a passive-short system $\Sigma : u \mapsto y$, we aim to find a map T such that the closed-loop system $\tilde{\Sigma} : \tilde{u} \mapsto \tilde{y}$ is passive. This is known as feedback passivation.



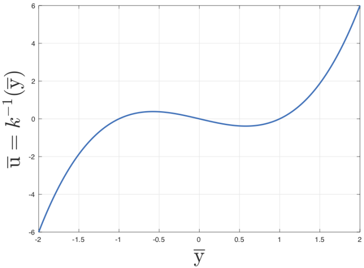
how does feedback passivation affect the steady-state input/output maps?

an example

$$\dot{x} = -x + \sqrt[3]{x} + u$$
$$y = \sqrt[3]{x}$$
$$\overline{\mathrm{u}} = k^{-1}(\overline{\mathrm{y}}) = \overline{\mathrm{y}}^3 - \overline{\mathrm{y}}$$

not a monotone input-output relation!

equilibrium input-output map



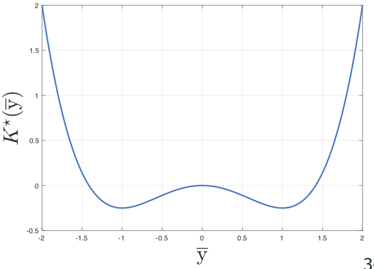
System is output passivity-short

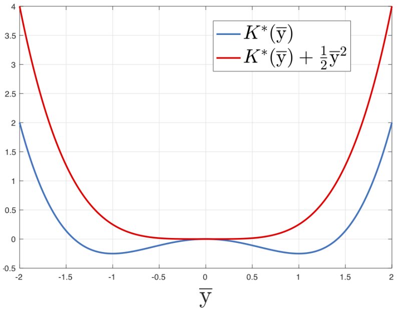
$$S(x) = \frac{3}{4}x^{4/3} - \overline{\mathrm{y}}x + \frac{1}{4}\overline{\mathrm{y}}$$

$$\dot{S} \leq (y - \bar{y})(u - \bar{u}) + (y - \bar{y})^2$$

(passivity index $\rho = -1$)

integral function





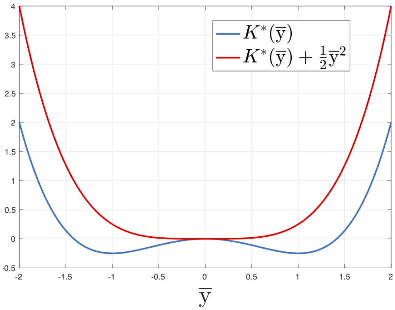
what is the system interpretation of a "convexified" integral function?

$$K^\star(\overline{y}) = \frac{1}{4}\overline{y}^4 - \frac{1}{2}\overline{y}^2$$

$$\tilde{K}^\star(\overline{y}) = K^\star(\overline{y}) + \frac{1}{2}\overline{y}^2$$

(Tikhonov regularization term)

AN EXAMPLE



what is the system interpretation of a "convexified" integral function?

$$K^\star(\overline{y}) = \frac{1}{4}\overline{y}^4 - \frac{1}{2}\overline{y}^2$$

$$\tilde{K}^\star(\overline{y}) = K^\star(\overline{y}) + \frac{1}{2}\overline{y}^2$$

(Tikhonov regularization term)

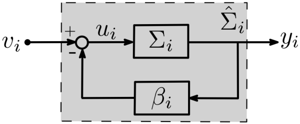
$$\partial\tilde{K}^\star(\overline{y}) = \partial K^\star(\overline{y}) + \overline{y}$$
$$\tilde{k}^{-1}(\overline{y}) = k^{-1}(\overline{y}) + \overline{y}$$
$$= \overline{y}^3 - \overline{y} + \overline{y} = \overline{y}^3$$

a monotone function!

what system yields this steady-state I/O map?

$$\dot{x} = -x + \sqrt[3]{x} - \underbrace{\sqrt[3]{y}}_{u} + v = -x + v$$

$$y = \sqrt[3]{x}$$



regularization is realized by output feedback!

$$u = v - y$$
$$\Rightarrow \dot{x} = -x + v$$
$$\Rightarrow \overline{\mathrm{v}} = \tilde{k}^{-1}(\overline{\mathrm{y}}) = \overline{\mathrm{y}}^3$$

(maximally monotone!)

Theorem[1]

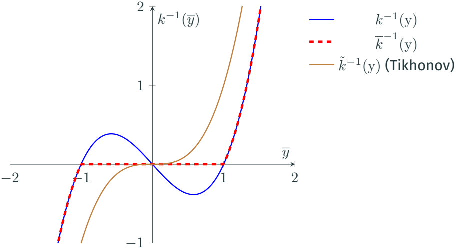
Consider the passive-short SISO dynamical system $\Sigma : u \mapsto y$ with I/O steady-state map k and output passivity index $\rho < 0$. Then for any $\beta > |\rho|$, the feedback

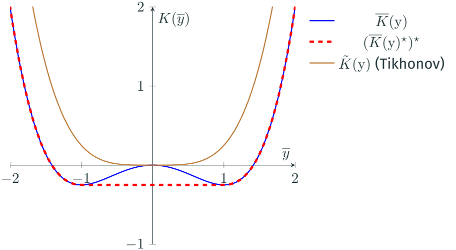
$$u = v - \beta y$$

renders the system $\tilde{\Sigma} : v \mapsto y$ output-strictly maximally monotone EIP with steady-state input map $\tilde{k}$ satisfying

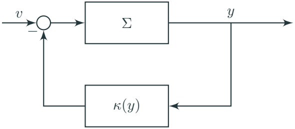
$$\tilde{k}^{-1}(\overline{\mathrm{y}}) = k^{-1}(\overline{\mathrm{y}}) + \beta\overline{\mathrm{y}}.$$

[1] [Jain, Sharf, Z, 2018]





A "better" convexification leads to different feedback passivation!



the feedback

$$\kappa(y) = \begin{cases} 0, & |x| = |y^3| > 1 \\ y^3 - y, & |x| = |y^3| \leq 1 \end{cases}$$

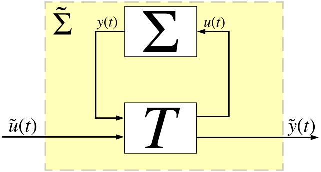
the closed-loop

$$\dot{x} = \begin{cases} -x + \sqrt[3]{x} + v, & |x| > 1 \\ v, & |x| \leq 1 \end{cases}$$

$$y = \sqrt[3]{x}.$$

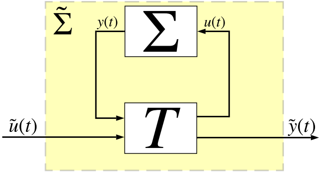
Is it possible to find a linear transformation $T : (\mathrm{u}, \mathrm{y}) \mapsto (\tilde{\mathrm{u}}, \tilde{\mathrm{y}})$ for a non-monotone I/O map $k : \mathrm{u} \mapsto \mathrm{y}$ such that $\tilde{k} : \tilde{\mathrm{u}} \mapsto \tilde{\mathrm{y}}$ is monotone?

Is it possible to find a linear transformation $T : (\mathrm{u}, \mathrm{y}) \mapsto (\tilde{\mathrm{u}}, \tilde{\mathrm{y}})$ for a non-monotone I/O map $k : \mathrm{u} \mapsto \mathrm{y}$ such that $\tilde{k} : \tilde{\mathrm{u}} \mapsto \tilde{\mathrm{y}}$ is monotone?



For a passive-short system $\Sigma : u \mapsto y$, we aim to find a map T such that the closed-loop system $\tilde{\Sigma} : \tilde{u} \mapsto \tilde{y}$ is passive. This is known as feedback passivation.

Is it possible to find a linear transformation $T : (\mathrm{u}, \mathrm{y}) \mapsto (\tilde{\mathrm{u}}, \tilde{\mathrm{y}})$ for a non-monotone I/O map $k : \mathrm{u} \mapsto \mathrm{y}$ such that $\tilde{k} : \tilde{\mathrm{u}} \mapsto \tilde{\mathrm{y}}$ is monotone?



For a passive-short system $\Sigma : u \mapsto y$, we aim to find a map T such that the closed-loop system $\tilde{\Sigma} : \tilde{u} \mapsto \tilde{y}$ is passive. This is known as feedback passivation.

Are these T maps the same?

For an EI-IOP(ρ, ν) system, for any two points $(\mathrm{u}_1, \mathrm{y}_1), (\mathrm{u}_2, \mathrm{y}_2) \in k$, the following inequality holds:

$$0 \leq -\rho(\mathrm{y}_1 - \mathrm{y}_2)^2 + (\mathrm{u}_1 - \mathrm{u}_2)(\mathrm{y}_1 - \mathrm{y}_2) - \nu(\mathrm{u}_1 - \mathrm{u}_2)^2.$$

Projective Quadratic Inequalities and EI-IOP

A *projective quadratic inequality (PQI)* is an inequality with variables $\xi, \chi \in \mathbb{R}$ of the form

$$0 \leq a\xi^2 + b\xi\chi + c\chi^2 = F(\xi, \chi),$$

for some numbers a, b, c, not all zero. The inequality is called *non-trivial* if $b^2 - 4ac > 0$. The associated solution set $\mathcal{A}$ of the PQI is the set of all points $(\xi, \chi) \in \mathbb{R}^2$ satisfying the inequality.

- passivity inequality is a PQI: $\xi = \mathrm{u}_1 - \mathrm{u}_2$, $\chi = \mathrm{y}_1 - \mathrm{y}_2$
- monotonicity is a PQI: $0 \leq (\mathrm{u}_1 - \mathrm{u}_2)(\mathrm{y}_1 - \mathrm{y}_2)$ with $a = c = 0$ and $b = 1$

$$0 \leq a\xi^2 + b\xi\chi + c\chi^2 = F(\xi, \chi)$$

A Recap:

- $F(\mathrm{u}_1 - \mathrm{u}_2, \mathrm{y}_1 - \mathrm{y}_2) \geq 0$ is a PQI for a EI-IOP(ρ, ν) system

$$0 \leq a\xi^2 + b\xi\chi + c\chi^2 = F(\xi, \chi)$$

A Recap:

- $F(\mathrm{u}_1 - \mathrm{u}_2, \mathrm{y}_1 - \mathrm{y}_2) \geq 0$ is a PQI for a EI-IOP(ρ, ν) system
- For the linear map $T : (\mathrm{u}, \mathrm{y}) \mapsto (\tilde{\mathrm{u}}, \tilde{\mathrm{y}})$,

  $$F(\tilde{\mathrm{u}}_1 - \tilde{\mathrm{u}}_2, \tilde{\mathrm{y}}_1 - \tilde{\mathrm{y}}_2) \geq 0$$

  is also a PQI for a EI-IOP($\tilde{\rho}, \tilde{\nu}$) system

$$0 \le a\xi^2 + b\xi\chi + c\chi^2 = F(\xi, \chi)$$

A Recap:

- $F(\mathrm{u}_1 - \mathrm{u}_2, \mathrm{y}_1 - \mathrm{y}_2) \ge 0$ is a PQI for a EI-IOP(ρ, ν) system
- For the linear map $T : (\mathrm{u}, \mathrm{y}) \mapsto (\tilde{\mathrm{u}}, \tilde{\mathrm{y}})$,

$$F(\tilde{\mathrm{u}}_1 - \tilde{\mathrm{u}}_2, \tilde{\mathrm{y}}_1 - \tilde{\mathrm{y}}_2) \ge 0$$

  is also a PQI for a EI-IOP($\tilde{\rho}, \tilde{\nu}$) system
- $F(\tilde{\mathrm{u}}_1 - \tilde{\mathrm{u}}_2, \tilde{\mathrm{y}}_1 - \tilde{\mathrm{y}}_2) = (\tilde{\mathrm{u}}_1 - \tilde{\mathrm{u}}_2)(\tilde{\mathrm{y}}_1 - \tilde{\mathrm{y}}_2)$ corresponds to monotonicity

$$0 \leq a\xi^2 + b\xi\chi + c\chi^2 = F(\xi, \chi)$$

A Recap:

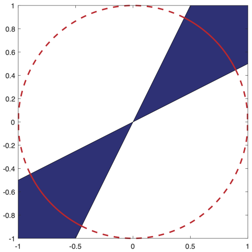
- $F(\mathrm{u}_1 - \mathrm{u}_2, \mathrm{y}_1 - \mathrm{y}_2) \geq 0$ is a PQI for a EI-IOP(ρ, ν) system
- For the linear map $T : (\mathrm{u}, \mathrm{y}) \mapsto (\tilde{\mathrm{u}}, \tilde{\mathrm{y}})$,

$$F(\tilde{\mathrm{u}}_1 - \tilde{\mathrm{u}}_2, \tilde{\mathrm{y}}_1 - \tilde{\mathrm{y}}_2) \geq 0$$

  is also a PQI for a EI-IOP($\tilde{\rho}, \tilde{\nu}$) system
- $F(\tilde{\mathrm{u}}_1 - \tilde{\mathrm{u}}_2, \tilde{\mathrm{y}}_1 - \tilde{\mathrm{y}}_2) = (\tilde{\mathrm{u}}_1 - \tilde{\mathrm{u}}_2)(\tilde{\mathrm{y}}_1 - \tilde{\mathrm{y}}_2)$ corresponds to monotonicity

Study the effect of the map T on the solution sets of the PQIs, $T(\mathcal{A})$

The solution set of any non-trivial PQI is a symmetric double-cone. Moreover, any symmetric double-cone is the solution set of some non-trivial PQI.



Theorem[1]

Let (ξ_1, χ_1), (ξ_2, χ_2) be non-colinear solutions of $a_1\xi^2 + \xi\chi + c_1\chi^2 = 0$, and $(\tilde{\xi}_1, \tilde{\chi}_1)$,$(\tilde{\xi}_2, \tilde{\chi}_2)$ be non-colinear solutions of $a_2\xi^2 + \xi\chi + c_2\chi^2 = 0$. Define

$$T_1 = \begin{bmatrix} \tilde{\xi}_1 & \tilde{\xi}_2 \\ \tilde{\chi}_1 & \tilde{\chi}_2 \end{bmatrix} \begin{bmatrix} \xi_1 & \xi_2 \\ \chi_1 & \chi_2 \end{bmatrix}^{-1}, T_2 = \begin{bmatrix} \tilde{\xi}_1 & -\tilde{\xi}_2 \\ \tilde{\chi}_1 & -\tilde{\chi}_2 \end{bmatrix} \begin{bmatrix} \xi_1 & \xi_2 \\ \chi_1 & \chi_2 \end{bmatrix}^{-1}.$$

Then one of T_1, T_2 transforms the PQI $a_1\xi^2 + \xi\chi + c_1\chi^2 \geq 0$ to the PQI $\tau a_2\xi^2 + \tau\xi\chi + \tau c_2\chi^2 \geq 0$ for some $\tau > 0$.

[1] [Sharf, Jain, Z, 2021]

EXAMPLE

Consider the system

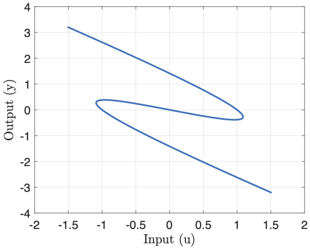
$$\Sigma : \dot{x} = -\sqrt[3]{x} + .5x + .5u,\ y = .5x - .5u$$

Using $S(x) = \frac{1}{6}(x - \mathrm{x})^2$ we have

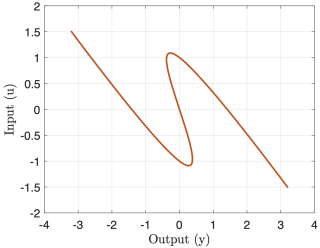
$$\dot{S}(x) \leq (u - \mathrm{u})(y - \mathrm{y}) + \frac{1}{3}(u - \mathrm{u})^2 + \frac{2}{3}(y - \mathrm{y})^2$$

System is EI-IOP(ρ, ν) with
$\rho = -2/3,\ \nu = -1/3$

> Passive-short system with non-monotone input-output relations (not even a function!)



(a) k.



(b) k^{-1}

Consider the system

$$\Sigma : \dot{x} = -\sqrt[3]{x} + .5x + .5u,\ y = .5x - .5u$$

Using $S(x) = \frac{1}{6}(x - \mathrm{x})^2$ we have

$$\dot{S}(x) \leq (u - \mathrm{u})(y - \mathrm{y}) + \frac{1}{3}(u - \mathrm{u})^2 + \frac{2}{3}(y - \mathrm{y})^2$$

System is EI-IOP(ρ, ν) with $\rho = -2/3,\ \nu = -1/3$

Corresponding PQI:

$$0 \leq \frac{1}{3}\xi^2 + \xi\chi + \frac{2}{3}\chi^2$$

Find a linear map T that monotonizes the input-output relations, i.e., leads to the PQI

$$\tilde{\xi}\tilde{\chi} \geq 0$$

non-colinear solutions to PQI

$$\tilde{\xi}\tilde{\chi} = 0$$

$$\begin{bmatrix} \tilde{\xi}_1 \\ \tilde{\chi}_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \tilde{\xi}_2 \\ \tilde{\chi}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

non-colinear solutions to original PQI

$$0 = \frac{1}{3}\xi^2 + \xi\chi + \frac{2}{3}\chi^2$$

$$\begin{bmatrix} \xi_1 \\ \chi_1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} \xi_2 \\ \chi_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The map

$$T_1 = \begin{bmatrix} \tilde{\xi}_1 & \tilde{\xi}_2 \\ \tilde{\chi}_1 & \tilde{\chi}_2 \end{bmatrix} \begin{bmatrix} \xi_1 & \xi_2 \\ \chi_1 & \chi_2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

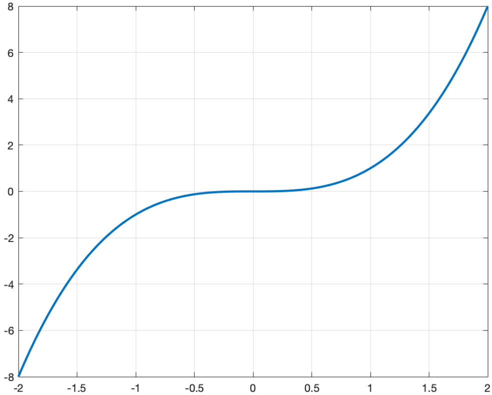
can be used to monotonize the relation! Indeed, for $(\xi, \chi) = T^{-1}(\tilde{\xi}, \tilde{\chi})$

$$\begin{aligned} 0 \leq& \frac{1}{3}\xi^2 + \xi\chi + \frac{2}{3}\chi^2 \\ =& \frac{1}{3}(2\tilde{\xi} - \tilde{\chi})^2 + (2\tilde{\xi} - \tilde{\chi})(-\tilde{\xi} + \tilde{\chi}) + \frac{2}{3}(-\tilde{\xi} + \tilde{\chi})^2 = \frac{1}{3}\tilde{\xi}\tilde{\chi}, \end{aligned}$$

EXAMPLE

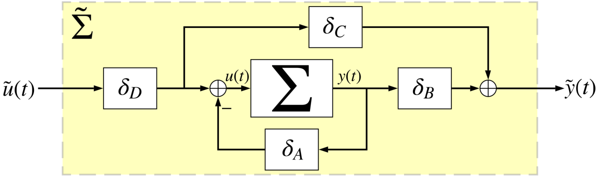
Steady-state input-output maps under T_1,

$$\begin{bmatrix} \tilde{u} \\ \tilde{y} \end{bmatrix} = T_1 \begin{bmatrix} u \\ y \end{bmatrix}$$



Theorem[1]

Let Σ be EI-IOP(ρ, ν). If the map T monotizes the input-output relation k, then it passivizes the system Σ.

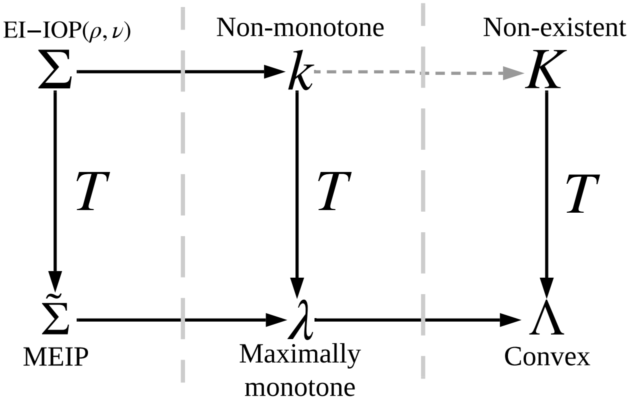


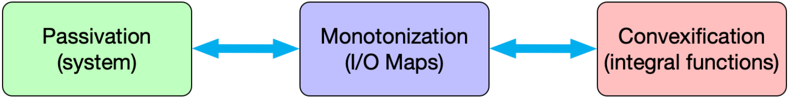
$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \underbrace{\begin{bmatrix} \delta_D & 0 \\ 0 & 1 \end{bmatrix}}_{L_D} \underbrace{\begin{bmatrix} 1 & 0 \\ \delta_C & 1 \end{bmatrix}}_{L_C} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \delta_B \end{bmatrix}}_{L_B} \underbrace{\begin{bmatrix} 1 & \delta_A \\ 0 & 1 \end{bmatrix}}_{L_A},$$

[1] [Sharf, Jain, Z, 2020]

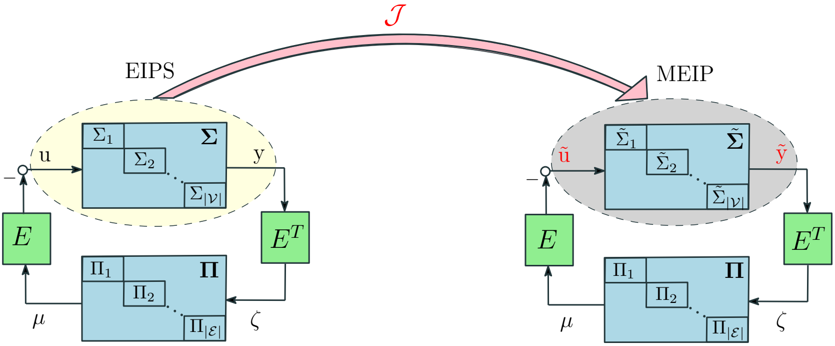
MONOTIZATION AND PASSIVATION

Elementary Transformation	Relation between I/O of Σ and $\tilde{\Sigma}$	Effect on Steady-State Relations	Realization	Effect on Integral Functions
$L_A = \begin{bmatrix} 1 & \delta_A \\ 0 & 1 \end{bmatrix}$	$\tilde{u} = u + \delta_A y$ $\tilde{y} = y$	$\lambda_A^{-1}(\tilde{y}) = k^{-1}(\tilde{y}) + \delta_A \tilde{y}$	output-feedback	$\Lambda^\star(y) = K^\star(y) + \frac{1}{2}\delta_A y^2$
$L_B = \begin{bmatrix} 1 & 0 \\ 0 & \delta_B \end{bmatrix}$	$\tilde{u} = u$ $\tilde{y} = \delta_B y$	$\lambda_B(u) = \delta_B k(u)$ or $\lambda_B^{-1}(\tilde{y}) = k^{-1}(\frac{1}{\delta_B}\tilde{y})$	post-gain	$\Lambda^\star(y) = \frac{1}{\delta_B} K^\star(\frac{1}{\delta_B} y)$ or $\Lambda(u) = \delta_B K(u)$
$L_C = \begin{bmatrix} 1 & 0 \\ \delta_C & 1 \end{bmatrix}$	$\tilde{u} = u$ $\tilde{y} = y + \delta_C u$	$\lambda_C(\tilde{u}) = k(\tilde{u}) + \delta_C \tilde{u}$	input-feedthrough	$\Lambda(u) = K(u) + \frac{1}{2}\delta_C u^2$
$L_D = \begin{bmatrix} \delta_D & 0 \\ 0 & 1 \end{bmatrix}$	$\tilde{u} = \delta_D u$ $\tilde{y} = y$	$\lambda_D^{-1}(y) = \delta_D k^{-1}(y)$ or $\lambda_D(\tilde{u}) = k(\frac{1}{\delta_D}\tilde{u})$	pre-gain	$\Lambda^\star(y) = \delta_D K^\star(y)$ or $\Lambda(u) = \frac{1}{\delta_D} K(\frac{1}{\delta_D} u)$


EI−IOP(ρ, ν)
Non-monotone
Non-existent
Σ
k
K
T
T
T
Σ̃
λ
Λ
MEIP
Maximally monotone
Convex


Passivation (system)
Monotonization (I/O Maps)
Convexification (integral functions)

PASSIVATION OF DIFFUSIVELY-COUPLED NETWORKS OF EIPS SYSTEMS



- Without loss of generality assume that the systems at nodes are EIPS (applicable if some of the systems are EIPS)
- Loop Transformation results in a pair of regularized network optimization problems

$$\mathcal{J} = \mathrm{diag}(T_i)$$

CONCLUDING REMARKS

Systems (Passivity) ⟷ **Monotonicity (I/O Relations)** ⟷ **Optimization (Convexity)**

New perspectives on networks and passivity

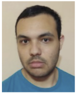
- networks of EIP agents can be understood through solutions of a pair of static dual optimization problems
- passivity and monotonicity of input-output maps are essential
- passivation means monotonization - monotonization means convexification

ACKNOWLEDGEMENTS

CoNeCt
Cooperative Networks and Controls Lab

GIF German-Israeli Foundation for Scientific Research and Development

Dr. Anoop Jain
IIT-Jodhpur



Dr. Miel Sharf

BOSCH

Dr. Mathias Bürger

ist

Prof. Dr.-Ing.
Frank Allgöwer

QUESTIONS?

SELECTED PUBLICATIONS

- M. Bürger, D. Zelazo and F. Allgower, “Duality and network theory in passivity-based cooperative control,”, *Automatica*, 50(8): 2051-2061, 2014.
- M. Sharf and D. Zelazo, “Analysis and Synthesis of MIMO Multi-Agent Systems Using Network Optimization,” *IEEE Transactions on Automatic Control*, 64(11):1558-2523, 2019.
- M. Sharf and D. Zelazo, “A Network Optimization Approach to Cooperative Control Synthesis,” *IEEE Control Systems Letters*, 1(1):86-91, 2017.
- A. Jain, M. Sharf and D. Zelazo, “Regularization and Feedback Passivation in Cooperative Control of Passivity-Short Systems : A Network Optimization Perspective”, *IEEE Control Systems Letters*, (2):4:731-736, 2018.
- M. Sharf, A. Jain and D. Zelazo, “A Geometric Method for Passivation and Cooperative Control of Equilibrium-Independent Passivity-Short Systems”, *IEEE Transactions on Automatic Control*, 66(12):5877-5892, 2021.
- M. Sharf and D. Zelazo, ”Passivity-Based Network Identification Algorithm with Minimal Time Complexity,” arXiv preprint, 2019.
- M. Sharf and D. Zelazo, “A Characterization of All Passivizing Input-Output Transformations of a Passive-Short System,” arXiv preprint, 2020.

Temporary page!

LaTeX was unable to guess the total number of pages correctly. As t
some unprocessed data that should have been added to the fina
this extra page has been added to receive it.

If you rerun the document (without altering it) this surplus page w
away, because LaTeX now knows how many pages to expect for this
document.