2017 ASIAN CONTROL CONFERENCE WORKSHOP

ADVANCES IN DISTRIBUTED CONTROL AND FORMATION CONTROL SYSTEMS

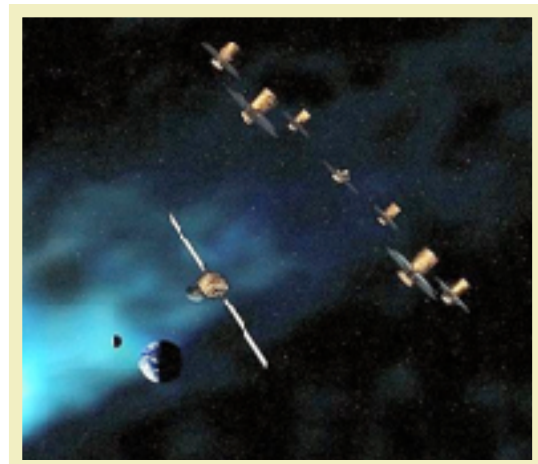
FORMATIONS OVER DIRECTED GRAPHS AND LOCAL COORDINATE FRAMES

Daniel Zelazo
Faculty of Aerospace Engineering

CoNeCt | Cooperative Networks and Controls Lab

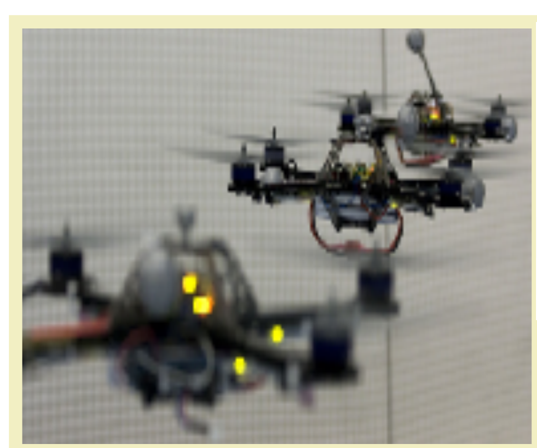
TECHNION Israel Institute of Technology

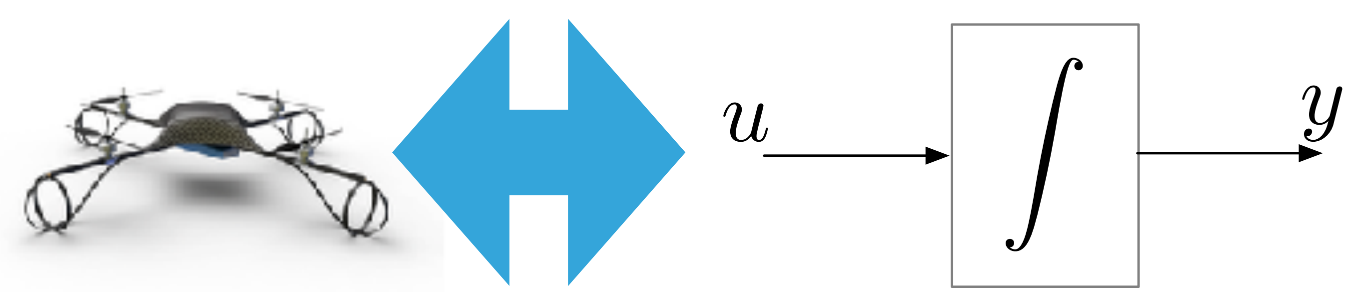
SIMPLIFY, SIMPLIFY, SIMPLIFY!



THE SYSTEMS WE WISH TO DESIGN, ANALYZE, AND CONTROL ARE COMPLEX!

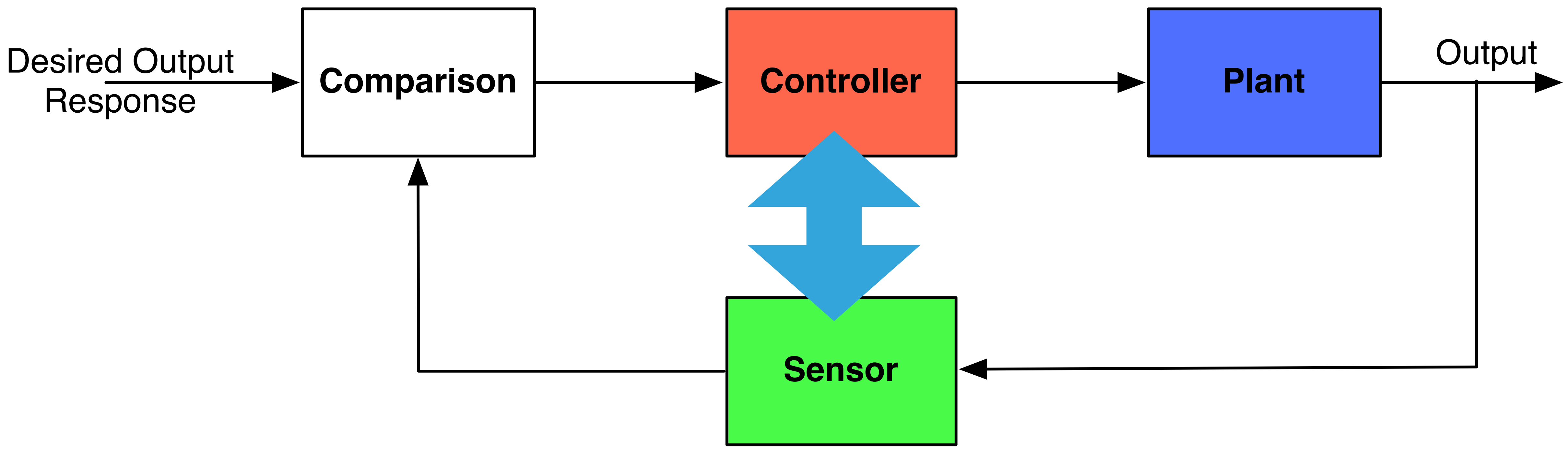
CONTROL THEORY PROVIDES US WITH AN ANALYTICAL JUSTIFICATION FOR USING SIMPLE MODELS!



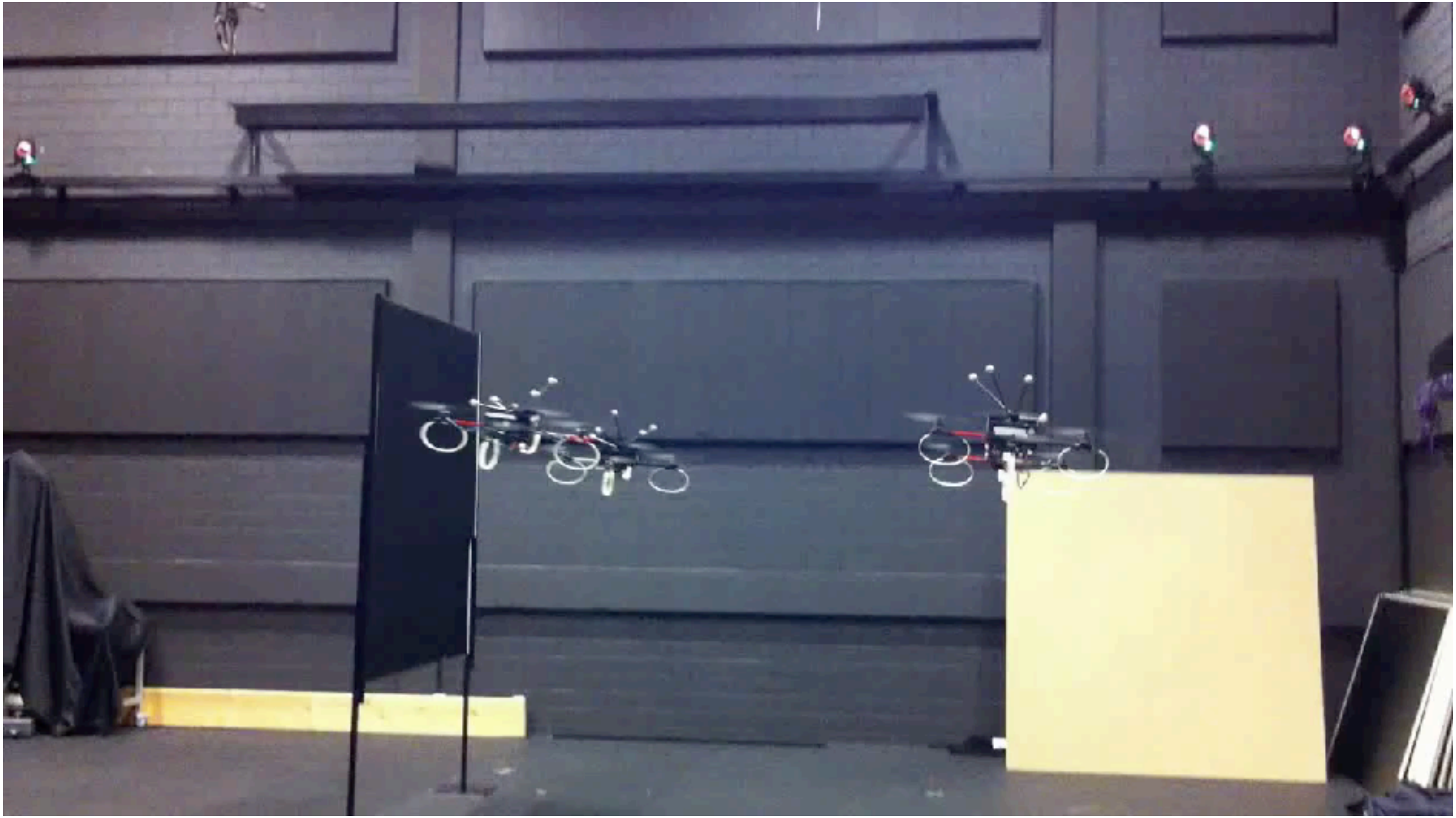


LET'S MAKE EVERYTHING AN INTEGRATOR!

WHAT ABOUT SENSING?

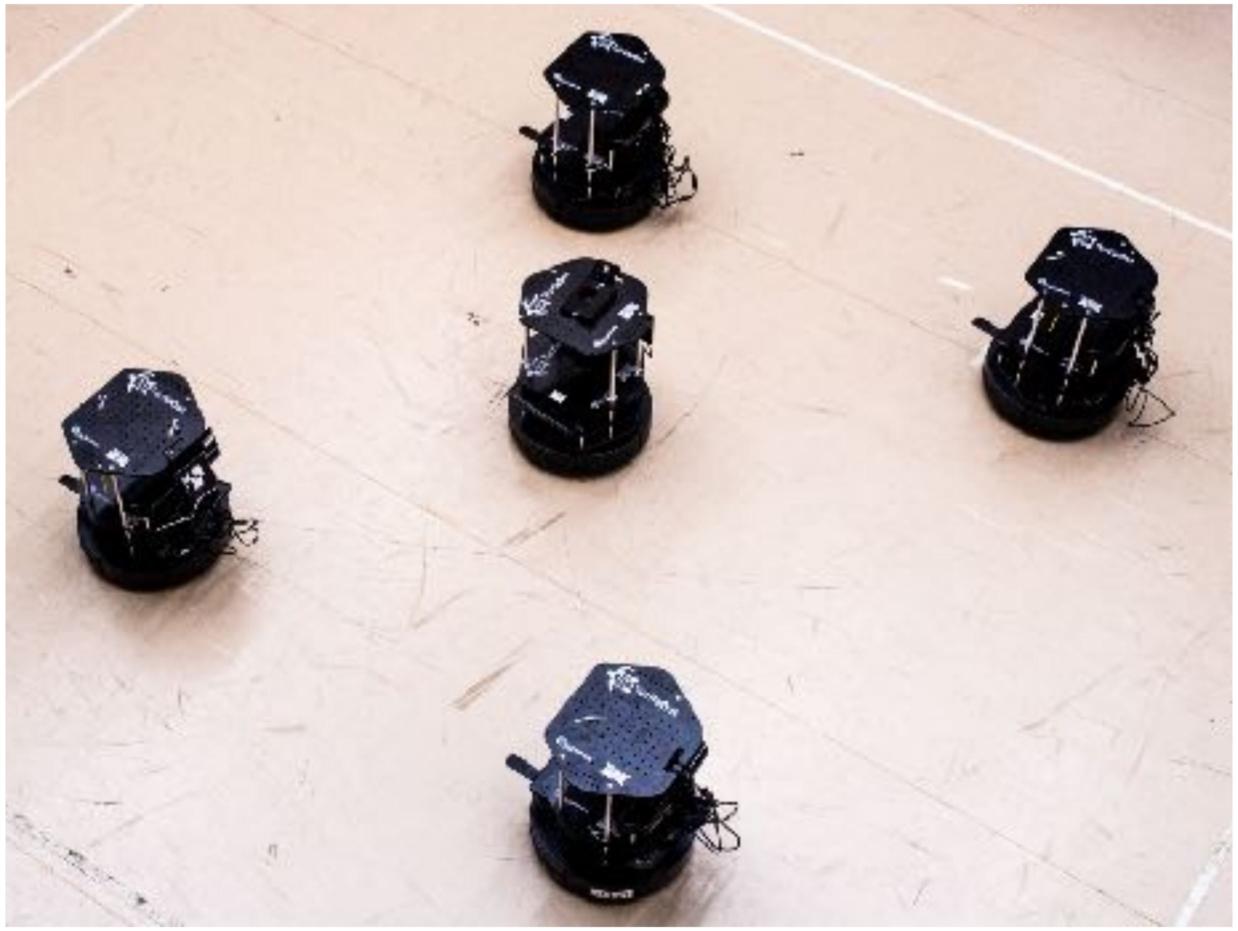


THE “DYNAMICS” OF THE SENSOR IN A CONTROL SYSTEM IS LESS IMPORTANT THAN THE QUANTITY IT IS MEASURING



Courtesy of P. Robuffo Giordano and A. Franchi

Solutions to coordination problems in multi-robot systems are highly dependent on the sensing and communication mediums available!

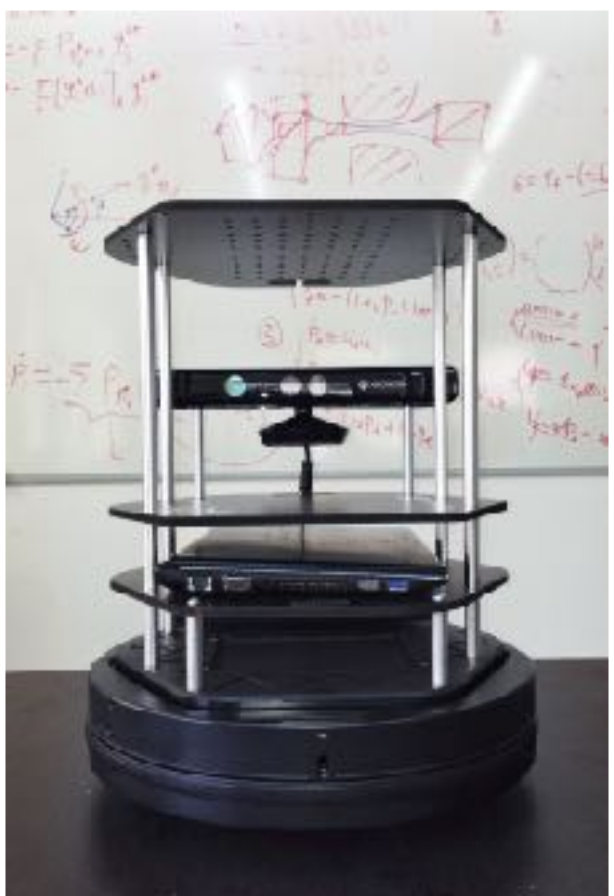


Sensing

- GPS
- Relative Position Sensing
- Range Sensing
- Bearing Sensing

Communication

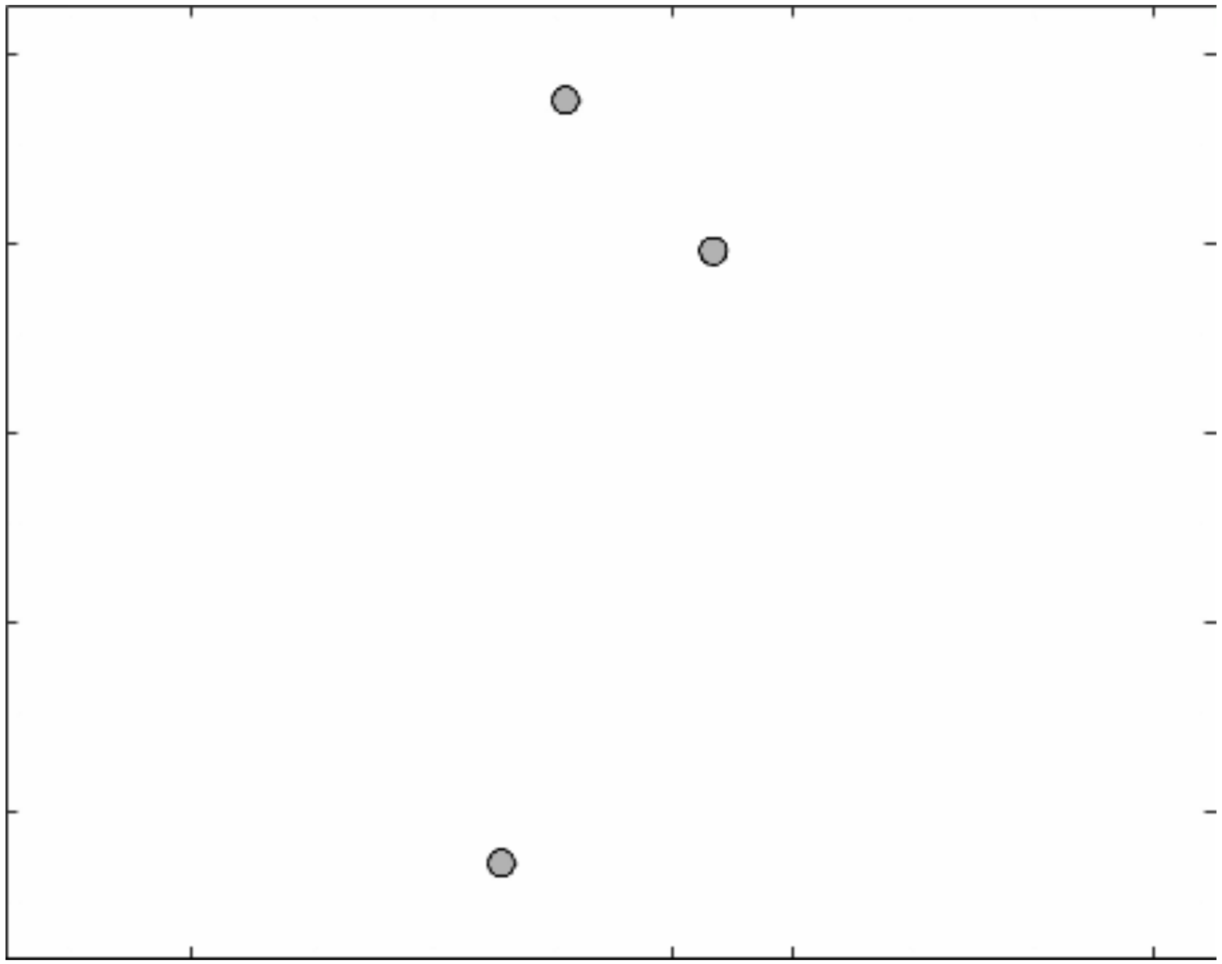
- Internet
- Radio
- Sonar
- MANet



TurtleBot II

"robots" – modeled as kinematic point mass

$$\dot{x}_i = u_i$$



Assumptions

- GLOBAL COORDINATE FRAME
- RELATIVE POSITION MEASUREMENTS
- DISTANCE MEASUREMENTS
- NO SENSING CONSTRAINTS (360°)
- SENSING

Formation

- SPECIFIED BY DISTANCES BETWEEN PAIRS OF ROBOTS

$$d_{ij} \in \mathbb{R}$$

Control

$$u_i = \sum_{i \sim j} (\|x_i - x_j\|^2 - d_{ij}^2)(x_j - x_i)$$

[Krick2009]

THE "DISTANCE CONSTRAINED" FORMATION CONTROL PROBLEM

DISTANCE CONSTRAINED

Formation

- SPECIFIED BY DISTANCES BETWEEN PAIRS OF ROBOTS

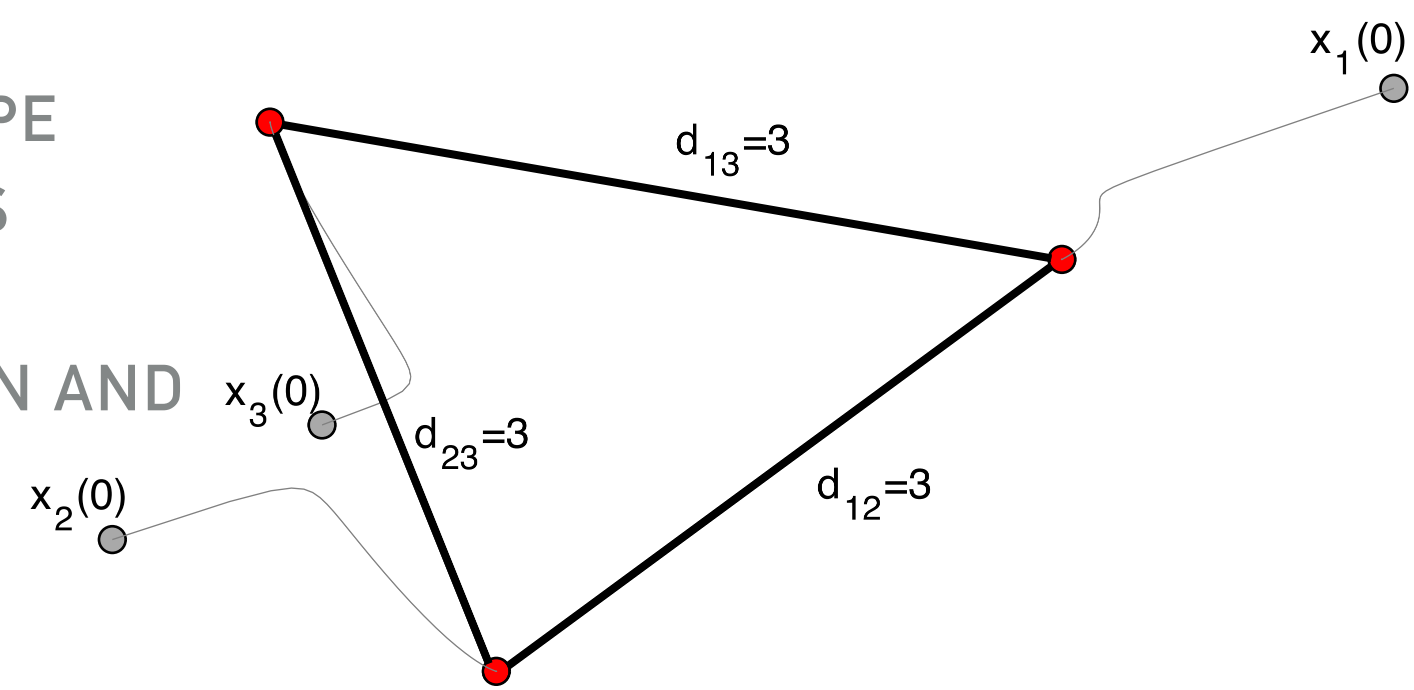
$$d_{ij} \in \mathbb{R}$$

Control

$$u_i = \sum_{i \sim j} (\|x_i - x_j\|^2 - d_{ij}^2)(x_j - x_i)$$

- FINAL FORMATION WILL BE A TRANSLATION OR ROTATION OF SHAPE SATISFYING DISTANCE CONSTRAINTS

- AGENTS REQUIRE RELATIVE POSITION AND DISTANCES



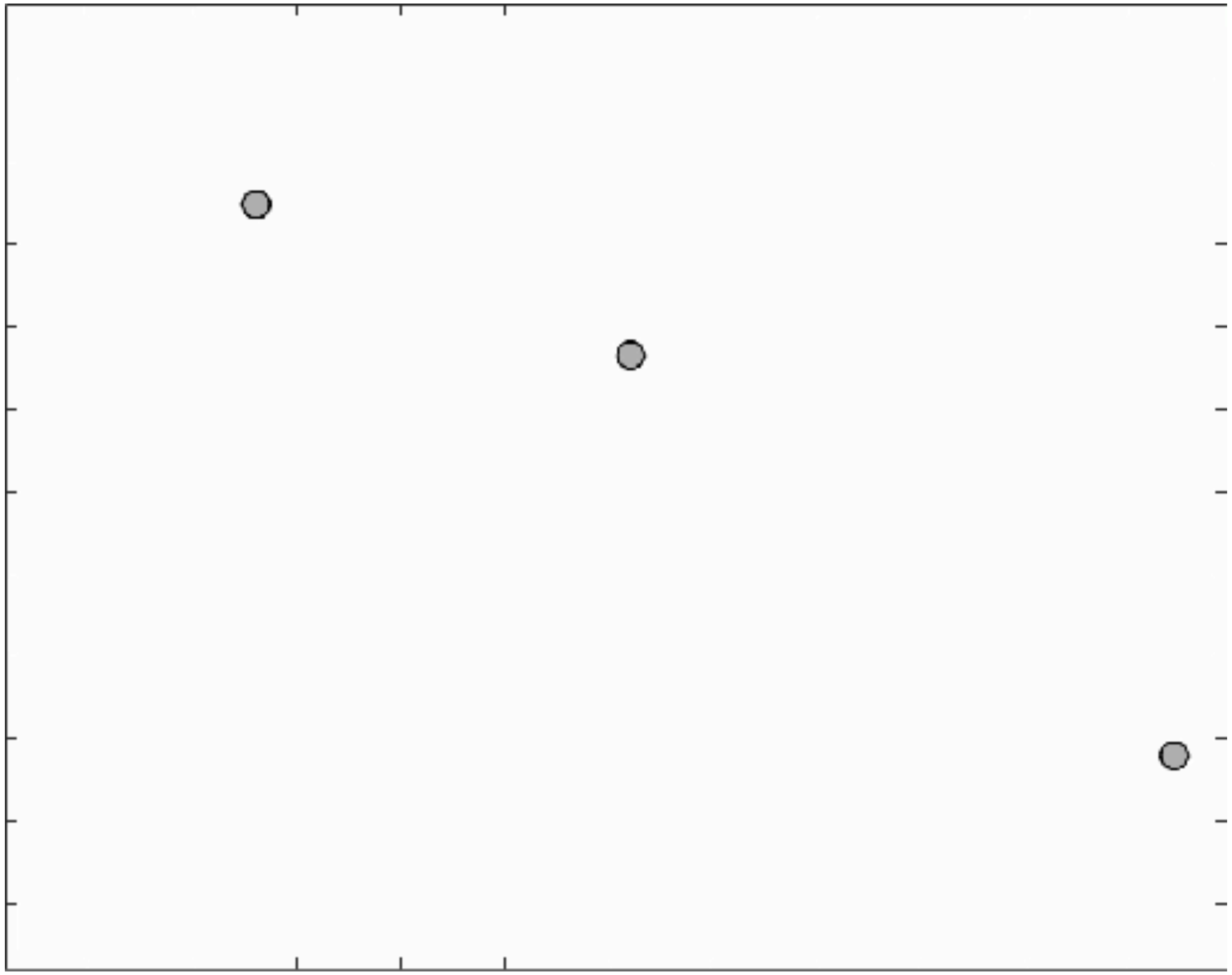
NEGLECTS RANGE CONSTRAINT OF RELATIVE POSITION SENSORS

"robots" – modeled as kinematic point mass

$$\dot{x}_i = u_i$$

Assumptions

- GLOBAL COORDINATE FRAME
- BEARING MEASUREMENTS
- NO SENSING CONSTRAINTS (360°)
- SENSING



Formation

- SPECIFIED BY BEARING VECTORS

$$g_{ij}^* \in \mathbb{R}^2, \; \|g_{ij}^*\| = 1$$

Control

$$u_i = -\sum_{i \sim j} (I - g_{ij} g_{ij}^T) g_{ij}^*$$

THE "BEARING ONLY" FORMATION CONTROL PROBLEM

[Zhao,Zelazo2016]

BEARING ONLY

Formation

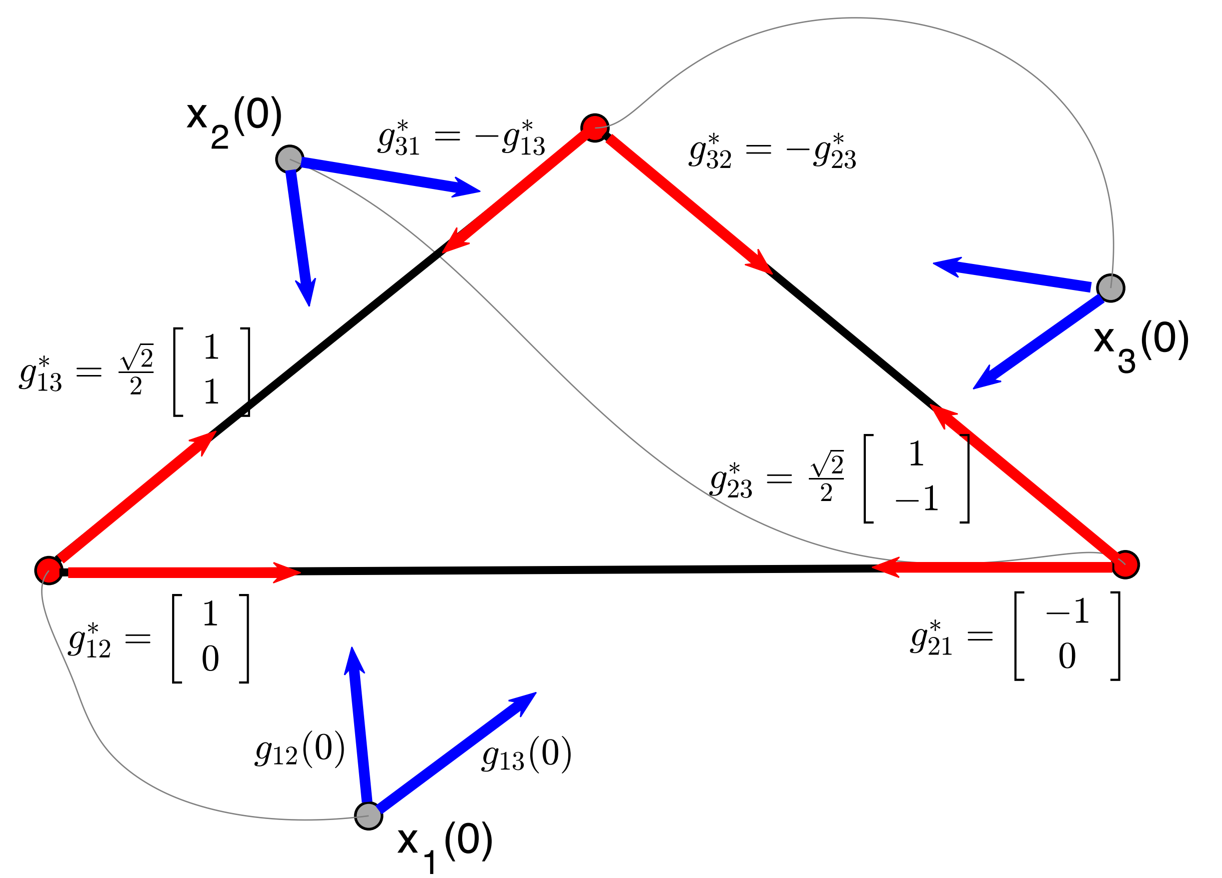
- SPECIFIED BY BEARING VECTORS

$$g_{ij}^* \in \mathbb{R}^2, \;\; \|g_{ij}^*\| = 1$$

- FINAL FORMATION WILL BE A TRANSLATION OR SCALING OF SHAPE SATISFYING BEARING CONSTRAINTS
- AGENTS REQUIRE BEARING MEASUREMENTS

Control

$$u_i = -\sum (I - g_{ij} g_{ij}^T) g_{ij}^*$$

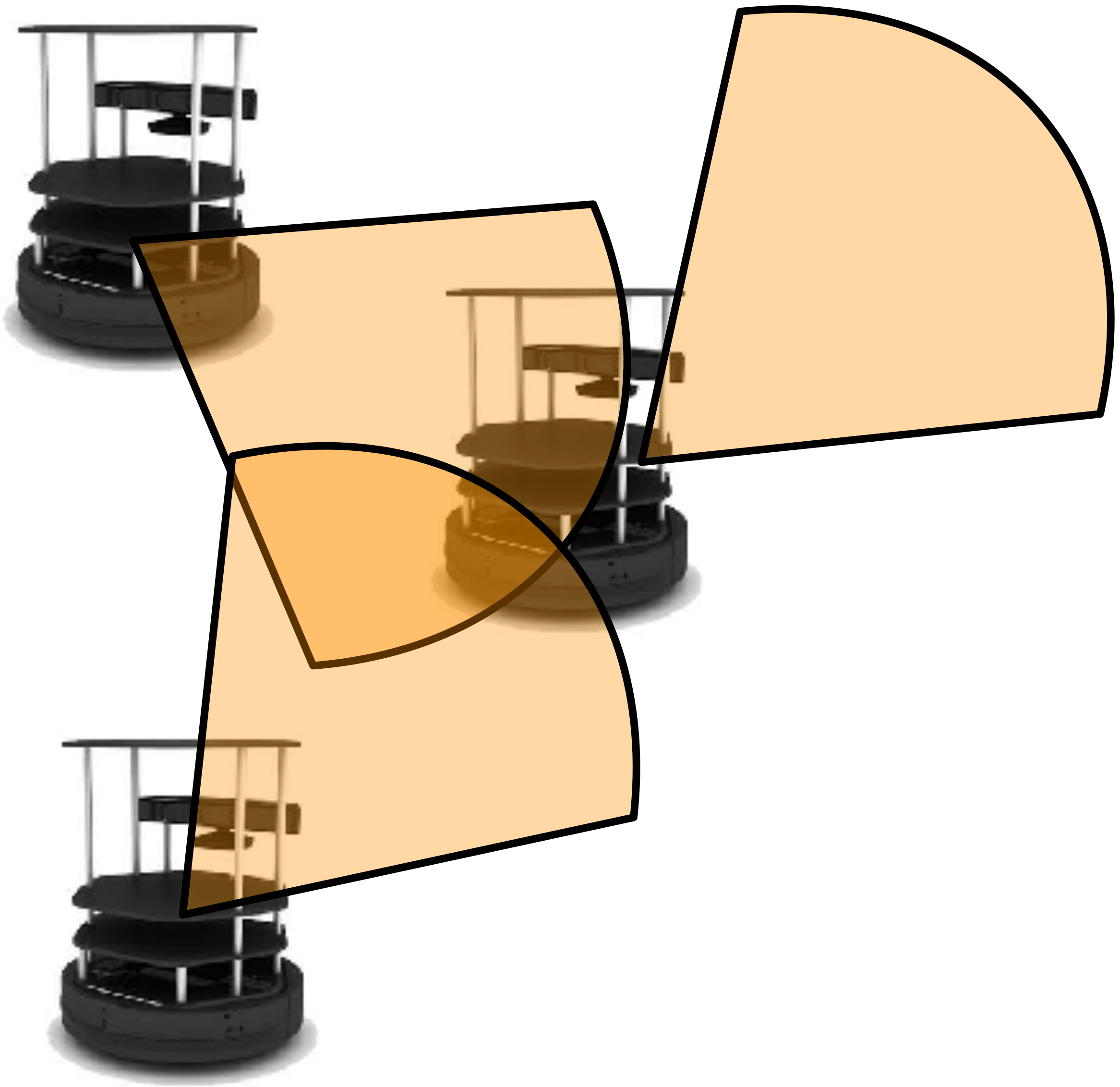


NEGLECTS FIELD-OF-VIEWS CONSTRAINT OF RELATIVE BEARING SENSORS

GRASP Lab

Motion capture systems allow us to "simulate" ideal sensors and test our control strategies

REAL SENSORS, REAL CHALLENGES



- sensing is typically *physically attached to the body frame* of the robot
- sensing is inherently directed
- knowledge of common inertial frame is *not* a realistic assumption
- sensing is inherently limited

FIELD-OF-VIEW CONSTRAINTS

$$\dot{p}_i = -\sum_{j \sim i} \left(I - \frac{(p_j - p_i)(p_j - p_i)^T}{\|p_j - p_i\|^2} \right) g_{ij}^*$$

- bearing measurement only available when neighbor is in field-of-view of camera

Bearing only control law with limited view constraint

1) agents faces in direction of motion

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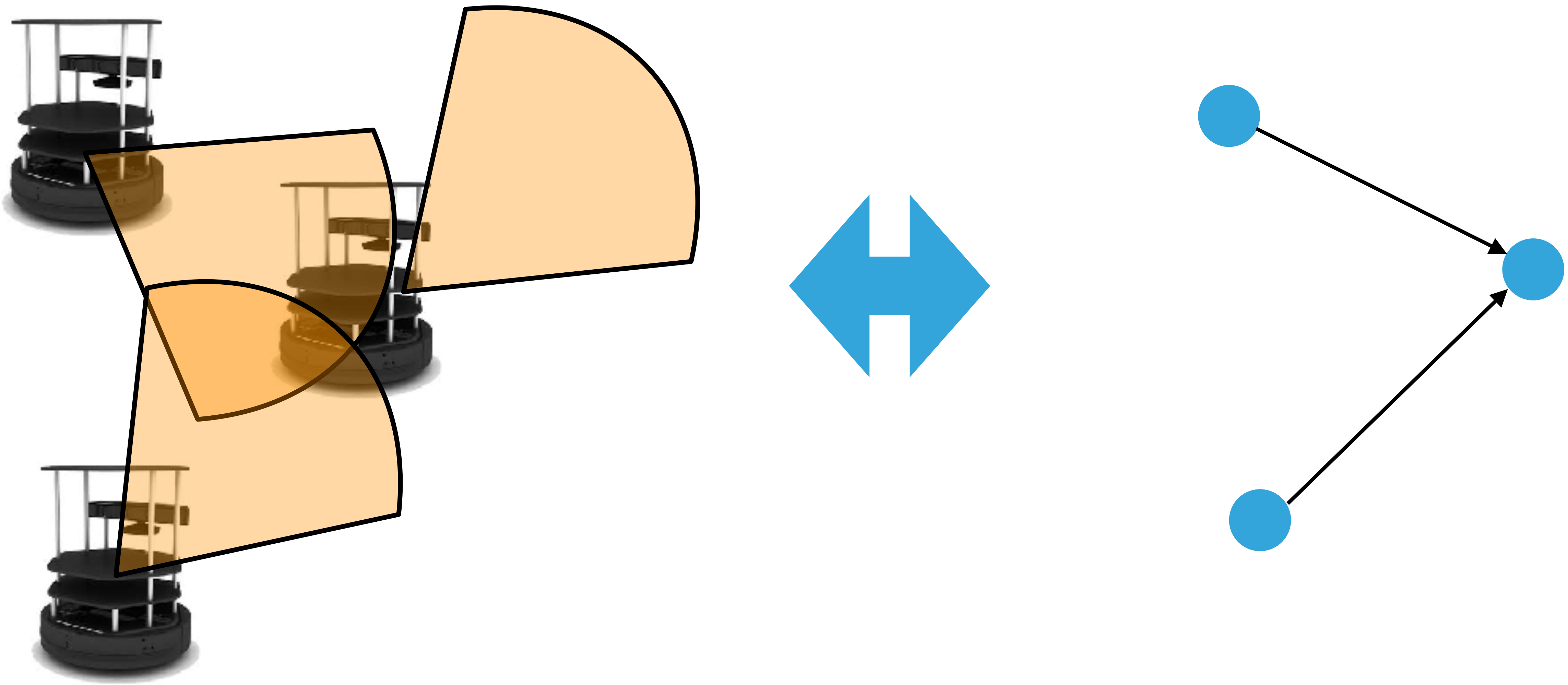
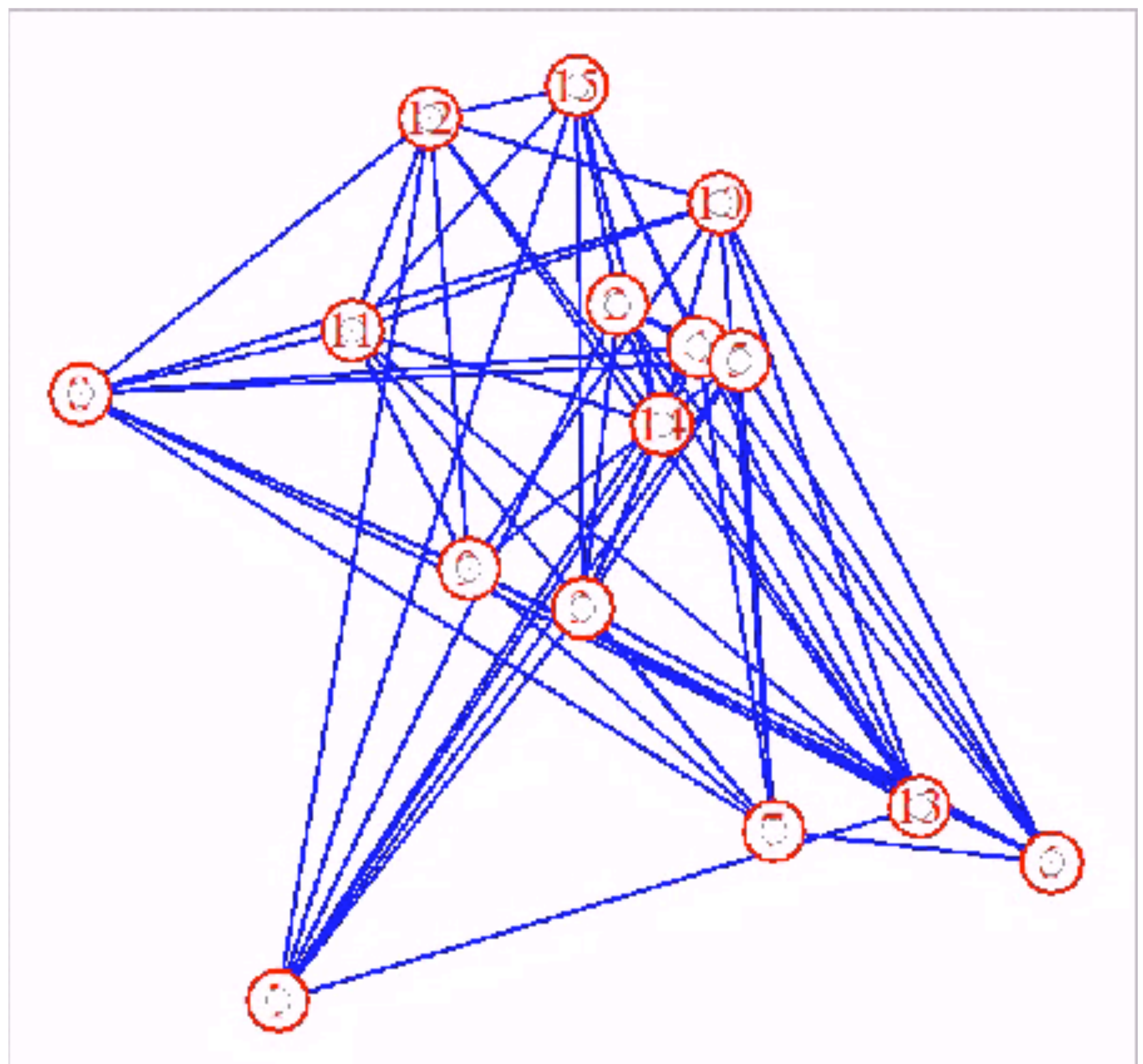
REAL SENSING MEANS DIRECTED INFORMATION



HOW DO WE ADAPT OUR EXISTING THEORY TO HANDLE REAL SENSING?

FORMATION CONTROL

Given a team of robots endowed with the ability to sense/communicate with neighboring robots, design a control for each robot using only *local information* that moves the team into a desired formation shape.



Rigidity Theory

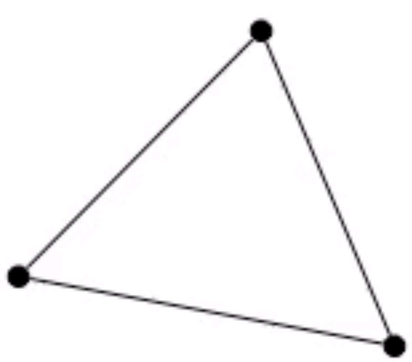
Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

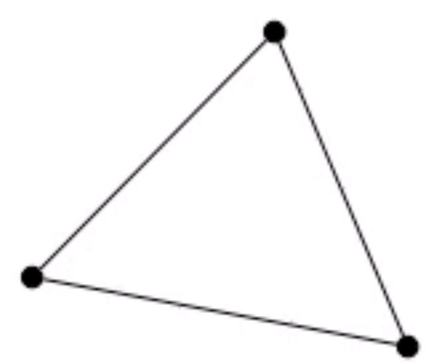
Distance Rigidity

- maintain distance pairs
- rigid body rotations and translations

Bearing Rigidity

- maintain angles (shape)
- rigid body translations and dilations



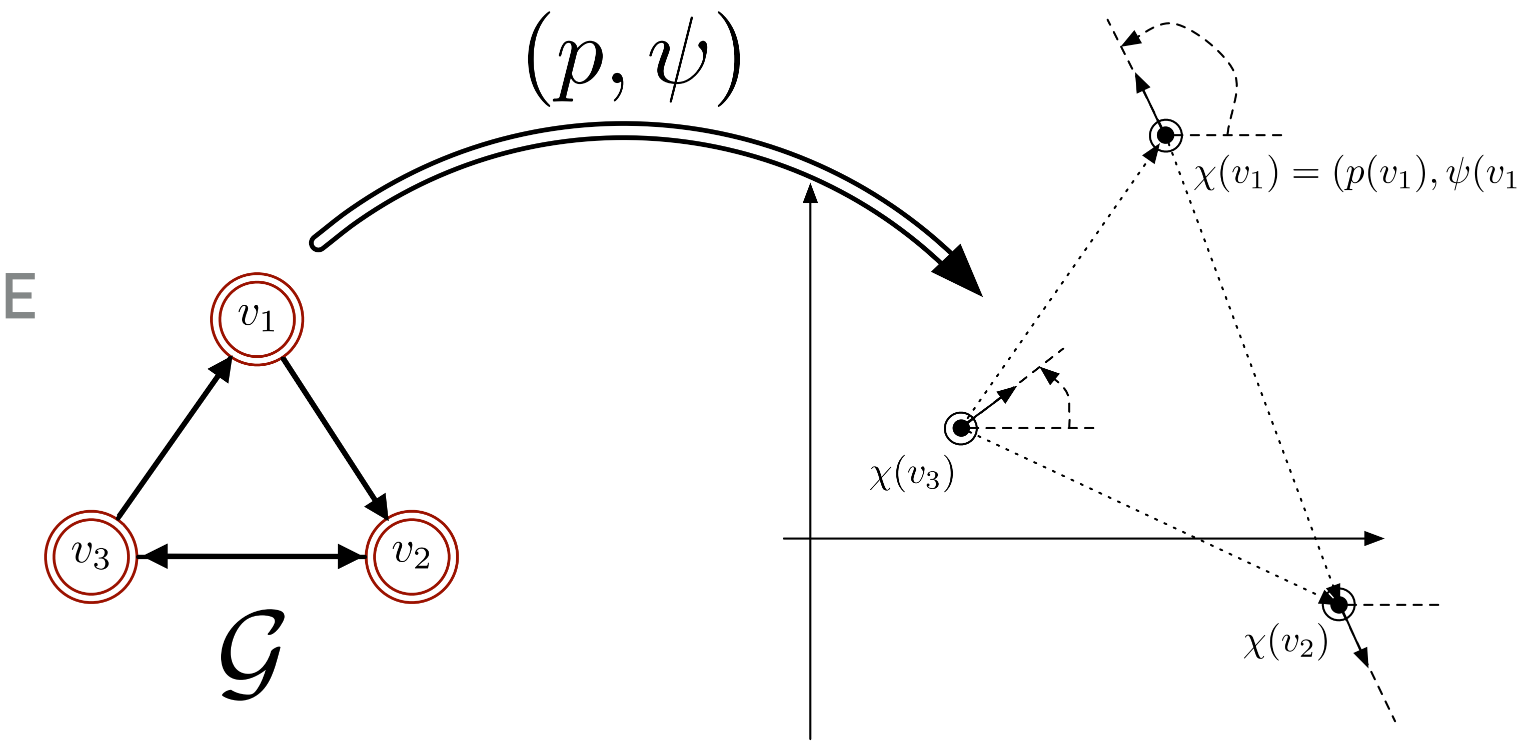


SE(2) RIGIDITY THEORY

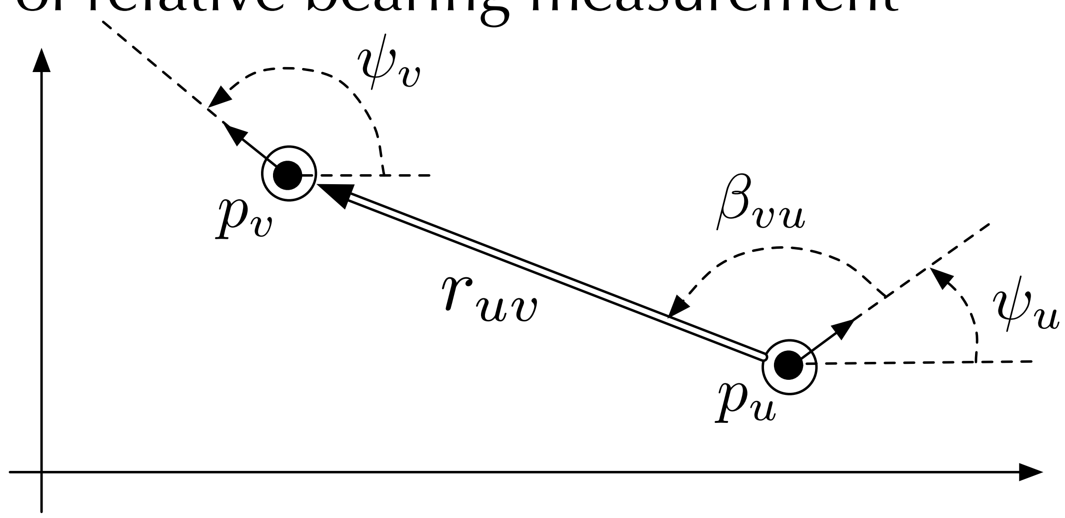
A *framework*

- A **DIRECTED** GRAPH
- A MAPPING TO A METRIC SPACE

$$\begin{aligned} \mathcal{G} &= (\mathcal{V}, \mathcal{E}) \\ p &: \mathcal{V} \rightarrow \mathbb{R}^2 \\ \psi &: \mathcal{V} \rightarrow \mathcal{S}^1 \end{aligned}$$



a directed edge indicates availability of relative bearing measurement

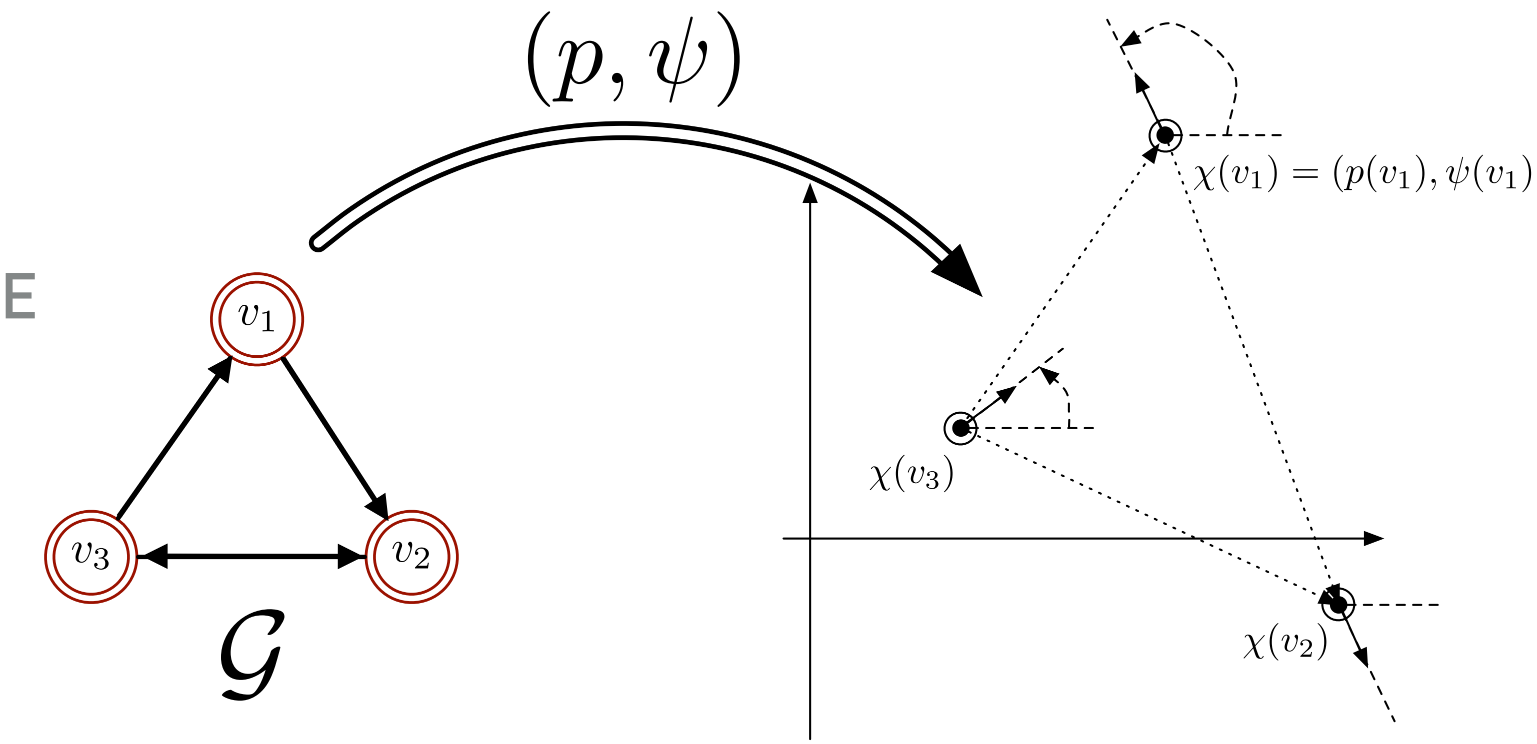


SE(2) RIGIDITY THEORY

A *framework*

- A **DIRECTED** GRAPH
- A MAPPING TO A METRIC SPACE

$$
\begin{aligned}
\mathcal{G} &= (\mathcal{V}, \mathcal{E}) \\
p &: \mathcal{V} \rightarrow \mathbb{R}^2 \\
\psi &: \mathcal{V} \rightarrow \mathcal{S}^1
\end{aligned}
$$



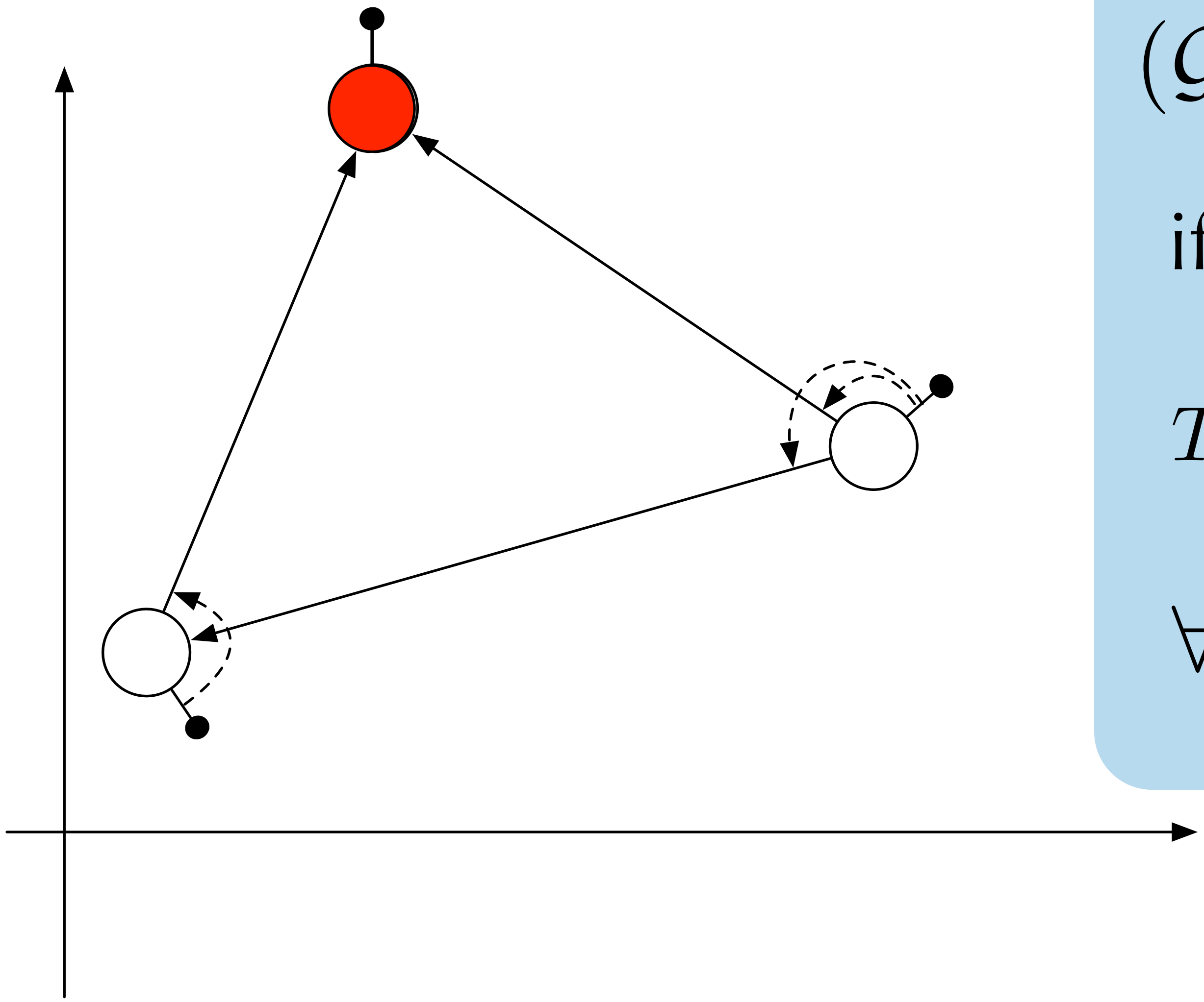
$$
r_{uv} = \underbrace{\begin{bmatrix} \cos(\psi_u) & \sin(\psi_u) \\ -\sin(\psi_u) & \cos(\psi_u) \end{bmatrix}}_{T(\psi_u)^T} \frac{p_v - p_u}{\|p_v - p_u\|}
$$

- bearings are expressed in the **body frame** of a point

directed bearing rigidity function

$$
F_{SE(2)}(p, \psi) = \frac{1}{2} \begin{bmatrix} r_{e_1}^T & \cdots & r_{e_{|\mathcal{E}|}}^T \end{bmatrix}^T
$$

EQUIVALENCE AND CONGRUENCE



$(\mathcal{G}, p, \psi)$ **is equivalent to**

if

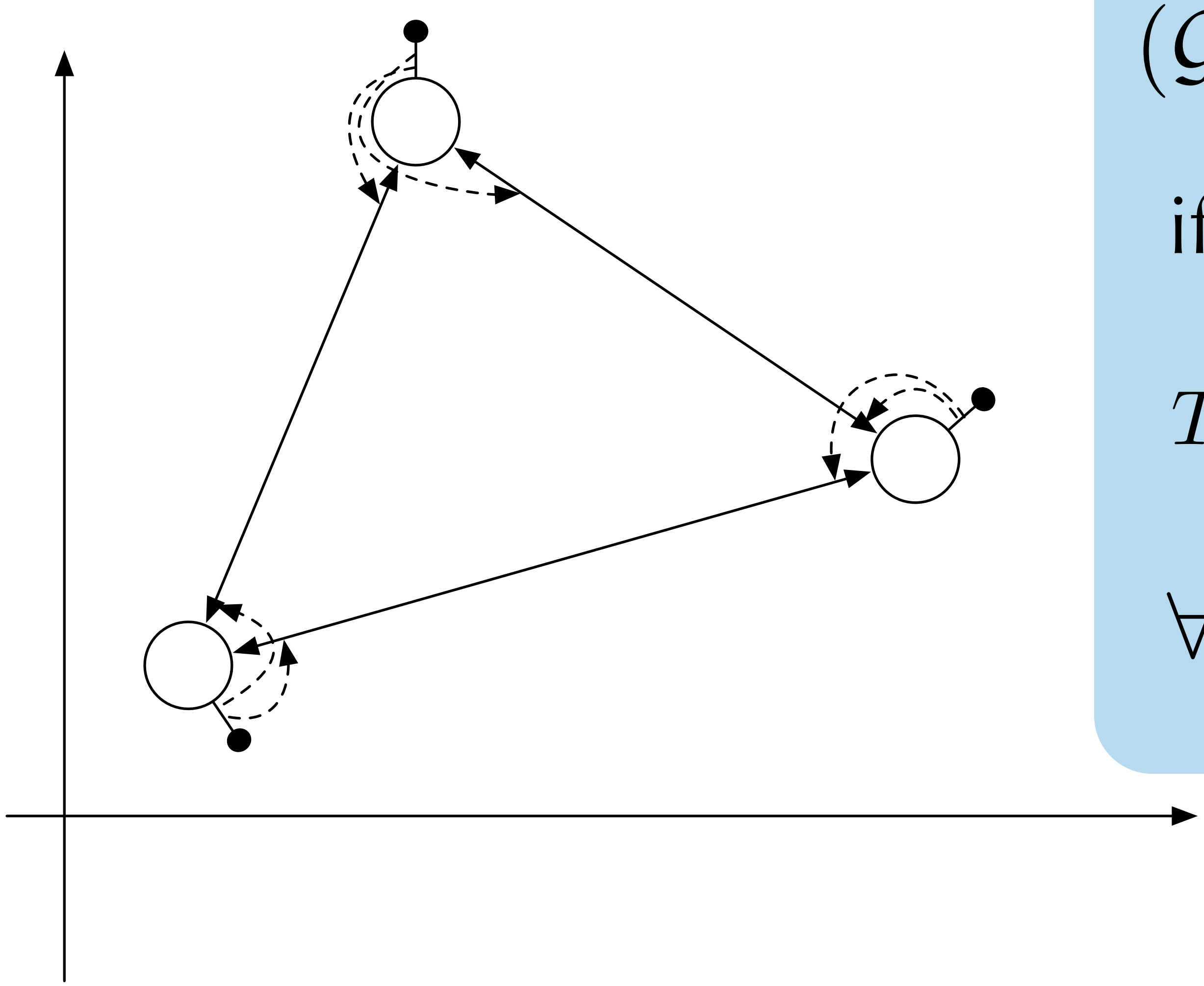
$$T(\psi_u)^T \frac{p_v - p_u}{\|p_u - p_v\|} = T(\phi_u)^T \frac{q_v - q_u}{\|q_u - q_v\|}$$

$$\forall \ (u, v) \in \mathcal{E}$$

$|\mathcal{E}| = 3$

- (local) bearings determined by the edge-set should be the same

EQUIVALENCE AND CONGRUENCE



$(\mathcal{G}, p, \psi)$ **is congruent to** $(\mathcal{G}, q, \phi)$

if

$$T(\psi_u)^T \frac{p_v - p_u}{\|p_u - p_v\|} = T(\phi_u)^T \frac{q_v - q_u}{\|q_u - q_v\|}$$

$\forall\ u, v \in \mathcal{V}$

$|\mathcal{E}| = 6$

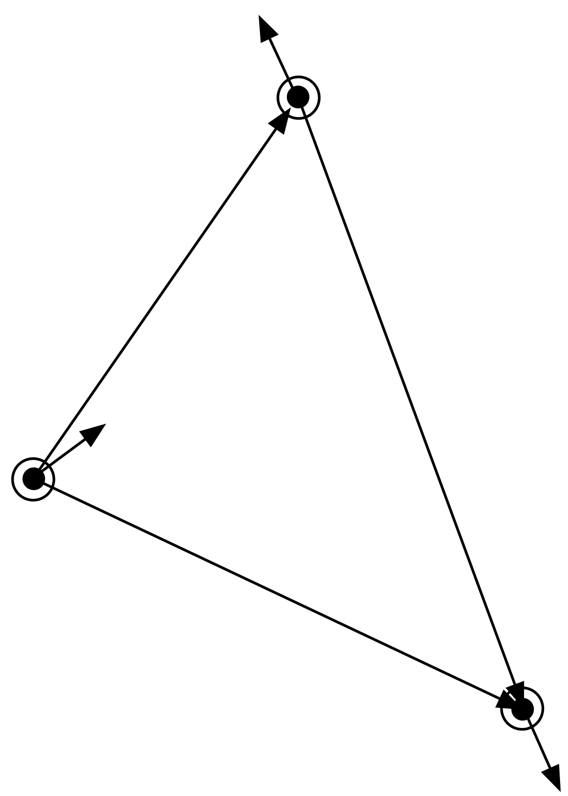
- all (local) bearings pairs should be the same

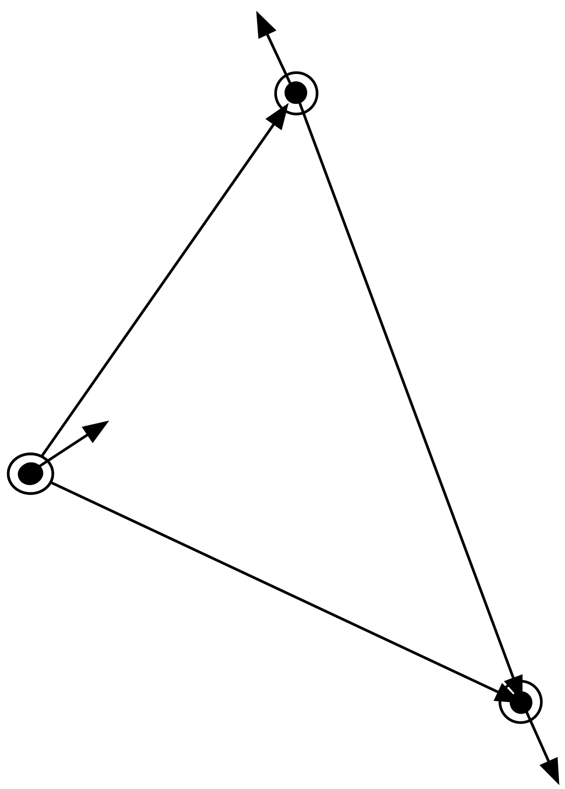
INFINITESIMAL MOTIONS IN SE(2)

Infinitesimal motions are bearing preserving (in local frame) motions of the framework.

SE(2) Rigidity

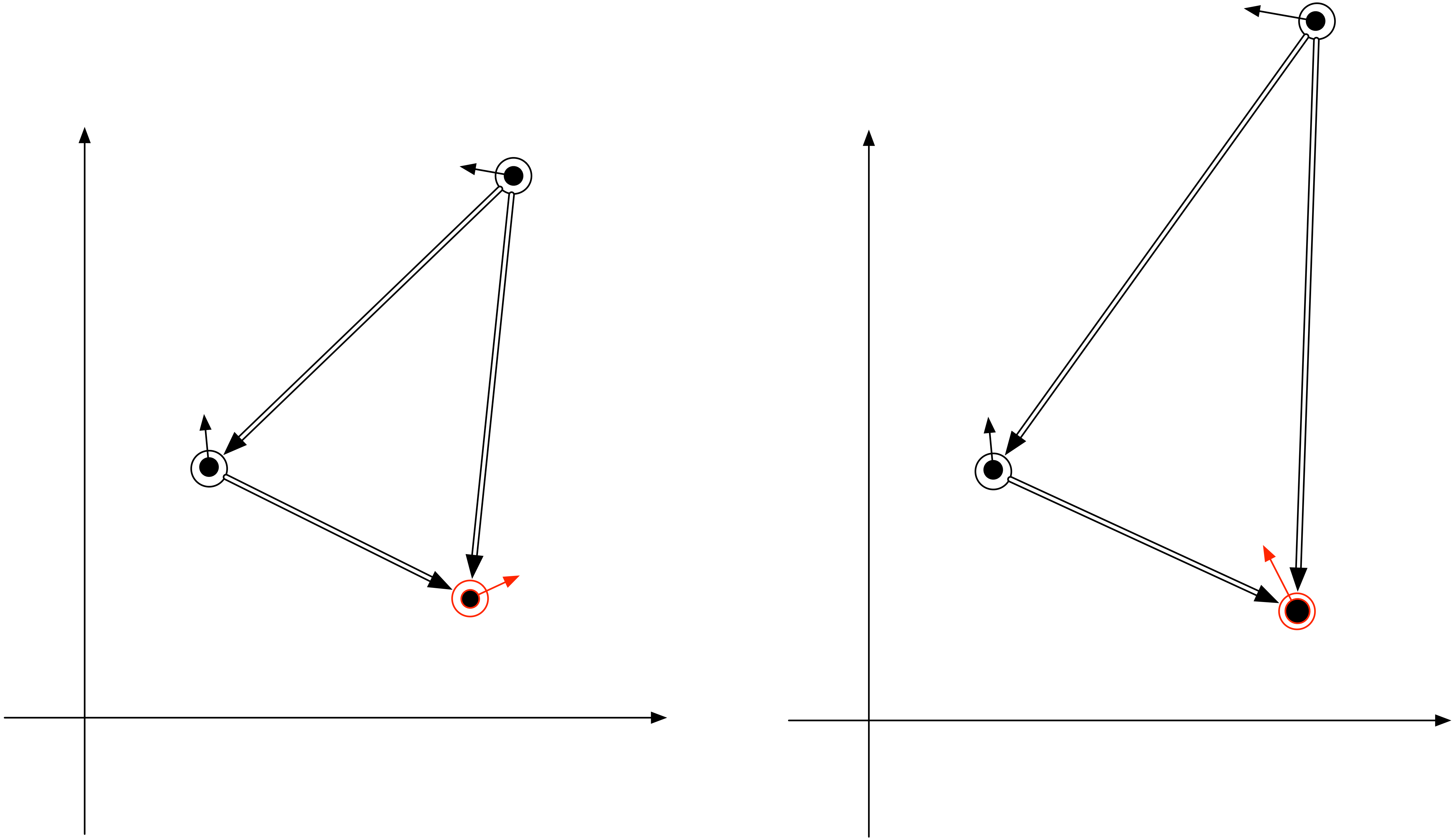
- maintain bearings in local frame
- rigid body rotations and scaling + coordinated rotations





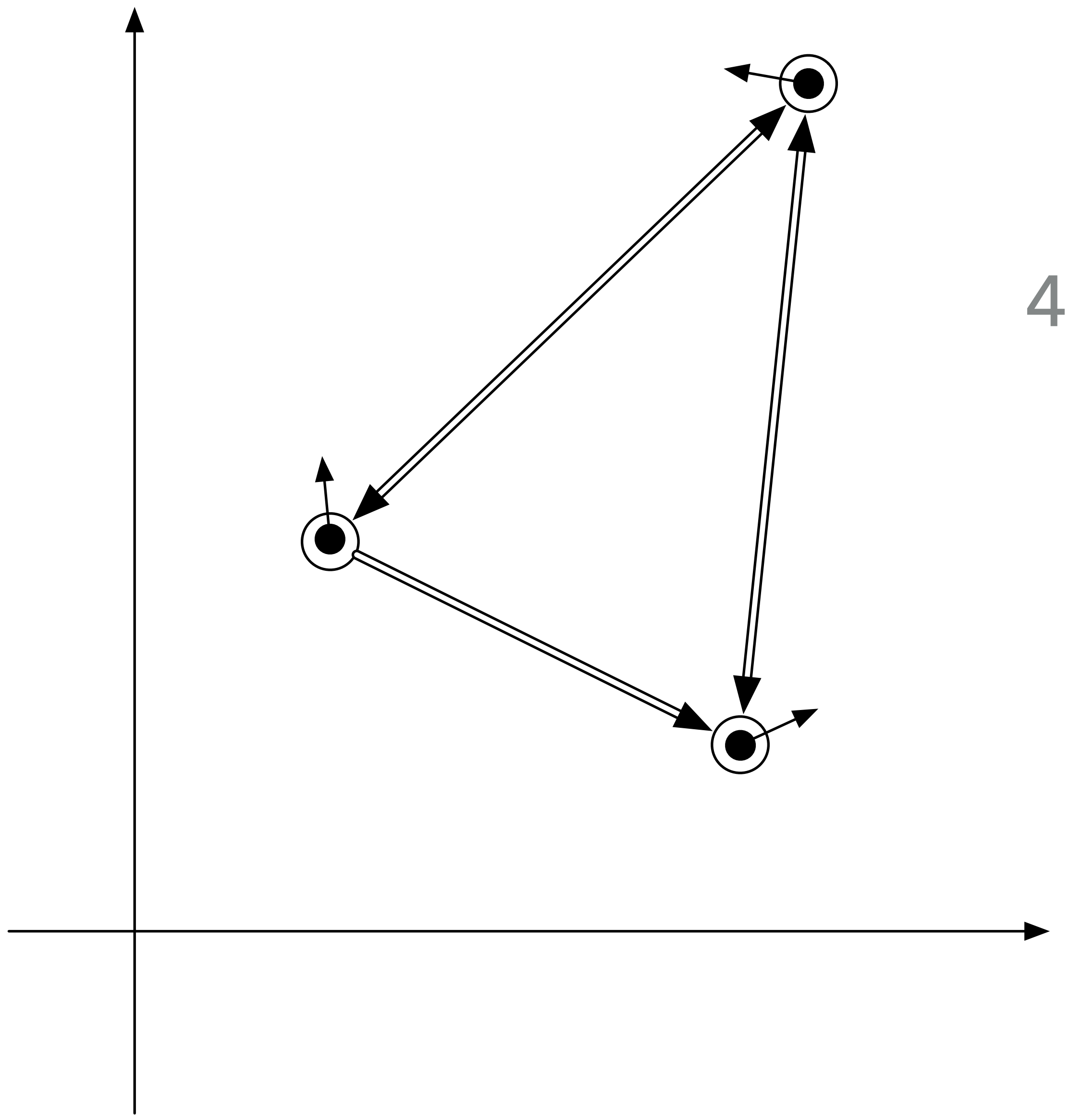
[Zelazo et al. ECC2014]
[Schiano et al. ICRA2016]

AN EXAMPLE: THE TRIANGLE



equivalent but not congruent

AN EXAMPLE: THE TRIANGLE



4 SE(2) preserving infinitesimal motions

we need 5 edges for triangle!

INFINITESIMAL RIGIDITY

A framework is ***infinitesimally rigid*** if every infinitesimal motion is *trivial*

Distance Function

$$F_D(p) = \frac{1}{2}\begin{bmatrix} \vdots \\ \|p(v_i) - p(v_j)\|^2 \\ \vdots \end{bmatrix}$$

Bearing Function

$$F_B(p) = \begin{bmatrix} \vdots \\ \frac{p(v_j)-p(v_i)}{\|p(v_i)-p(v_j)\|} \\ \vdots \end{bmatrix}$$

Distance Rigidity Matrix

$$R_D(p) = \frac{\partial F_D(p)}{\partial p}$$

Bearing Rigidity Matrix

$$R_B(p) = \frac{\partial F_B(p)}{\partial p}$$

infinitesimal motions are precisely the motions that satisfy

$$R(p)\delta_p = \frac{\partial F(p)}{\partial p}\delta_p = 0$$

INFINITESIMAL RIGIDITY

Distance Function

$$F_D(p) = \frac{1}{2} \begin{bmatrix} \vdots \\ \|p(v_i) - p(v_j)\|^2 \\ \vdots \end{bmatrix}$$

Bearing Function

$$F_B(p) = \begin{bmatrix} \vdots \\ \frac{p(v_j) - p(v_i)}{\|p(v_i) - p(v_j)\|} \\ \vdots \end{bmatrix}$$

Distance Rigidity Matrix

$$R_D(p) = \frac{\partial F_D(p)}{\partial p}$$

Bearing Rigidity Matrix

$$R_B(p) = \frac{\partial F_B(p)}{\partial p}$$

THEOREM

A framework is infinitesimally (distance, bearing) rigid if and only if the rank of the rigidity matrix is 2n-3.

3 trivial motions in the plane

INFINITESIMAL RIGIDITY

Directed Bearing Function

$$F_{SE(2)}(p,\psi) = \frac{1}{2}\begin{bmatrix} \vdots \\ T(\psi_u)g_{uv} \\ \vdots \end{bmatrix}$$

SE(2) Bearing Rigidity Matrix

$$R_{SE(2)}(p,\psi) = \frac{\partial F_{SE(2)}(p,\psi)}{\partial(p,\psi)}$$

THEOREM

A framework is infinitesimally SE(2) rigid if and only if the rank of the rigidity matrix is 3n-4.

SE(2) FORMATION CONTROL

The **SE(2) bearing-based formation control** problem is to design a (distributed) control law that drives the agents to a desired spatial configuration determined by interagent bearings measured in the local body frame of each agent.

A gradient controller

$$\Phi(p,\psi) = \frac{1}{2}\sum_{(i,j)\in\mathcal{E}} \|r_{ij} - r_{ij}^*\|^2$$

$$\begin{bmatrix} \dot{p} \\ \dot{\psi} \end{bmatrix} = -\nabla_{(p,\psi)}\Phi(p,\psi) = R_{SE(2)}(p,\psi)^T b_{\mathcal{G}}^*$$

$u_i = T(\psi_i)^T \dot{p}_i$ control expressed in local frame

SE(2) FORMATION CONTROL

The **SE(2) bearing-based formation control** problem is to design a (distributed) control law that drives the agents to a desired spatial configuration determined by interagent bearings measured in the local body frame of each agent.

A gradient controller

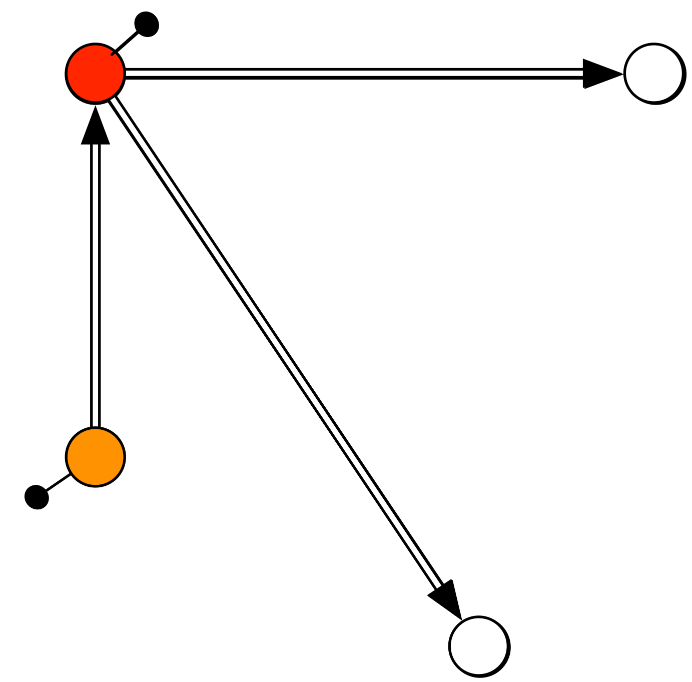
$$\dot{p}_i = \sum_{(i,j)\in\mathcal{E}} \frac{P_{r_{ij}}}{\|p_j - p_i\|} r_{ij}^d + \sum_{(j,i)\in\mathcal{E}} T(\psi_j - \psi_i) \frac{P_{r_{ji}}}{\|p_i - p_j\|} r_{ji}^d$$

$$\dot{\psi}_i = - \sum_{(i,j)\in\mathcal{E}} (r_{ij}^{\perp})^T r_{ij}^d$$

x requires distances

x requires communication

x requires relative orientation



SE(2) FORMATION CONTROL

a scale-free SE(2) formation control

$$T(\psi_i)^T \dot{p}_i = -\sum_{(i,j)\in\mathcal{E}} P_{r_{ij}} r_{ij}^d + \sum_{(j,i)\in\mathcal{E}} T(\psi_i - \psi_j)^T P_{r_{ji}} r_{ji}^d$$

$$\dot{\psi}_i = -\sum_{(i,j)\in\mathcal{E}} (r_{ij}^{\perp})^T r_{ij}^d,$$

proof

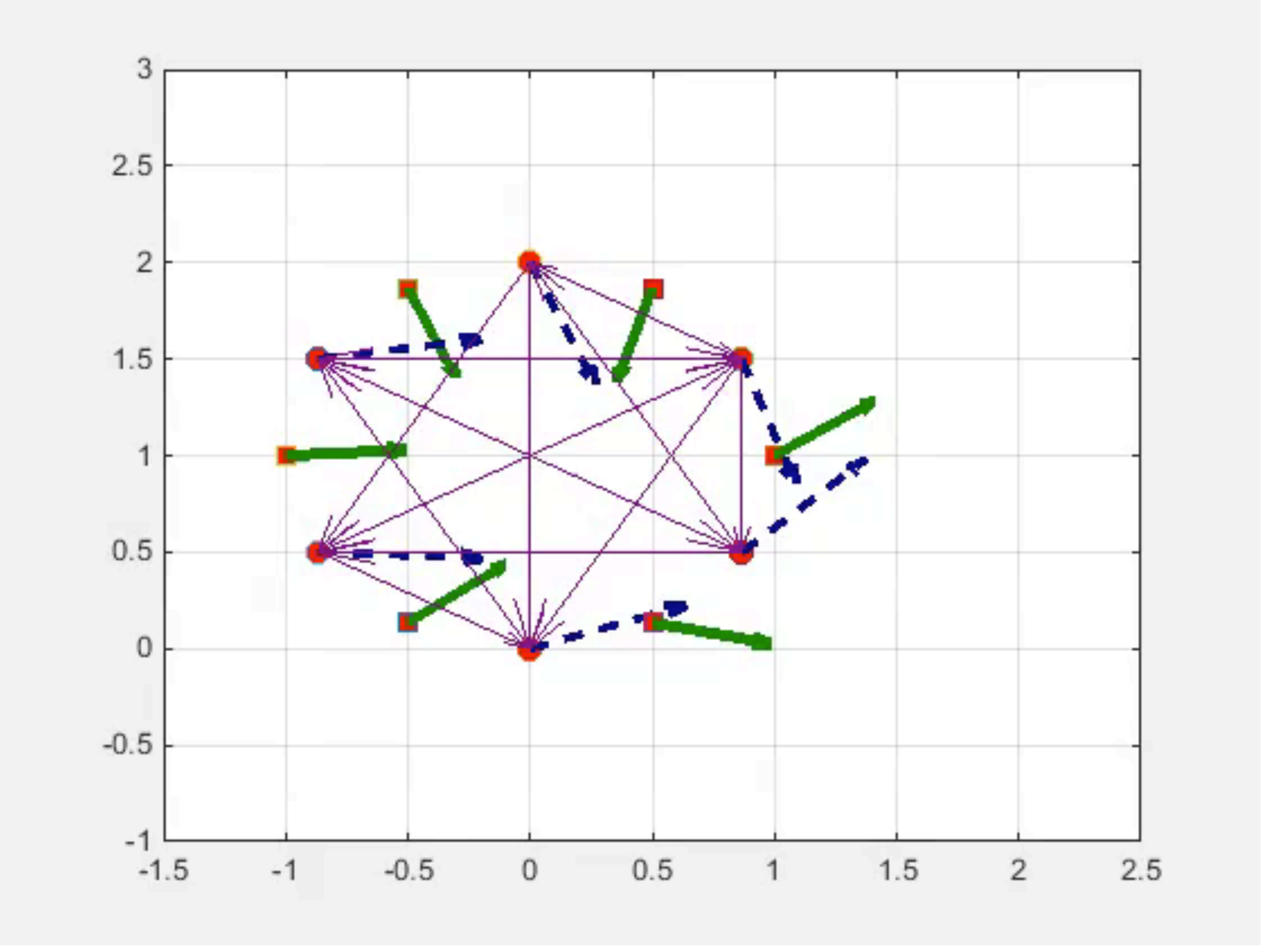
$$\delta = F_{SE(2)}(p, \psi) - b_{\mathcal{G}}^*$$

$$\dot{\delta} = -R_{SE(2)}^T \underbrace{\hat{R}_{SE(2)}}_{\texttt{scale-free rigidity}} \delta$$

D-semistability, Lyapunov, LaSalle

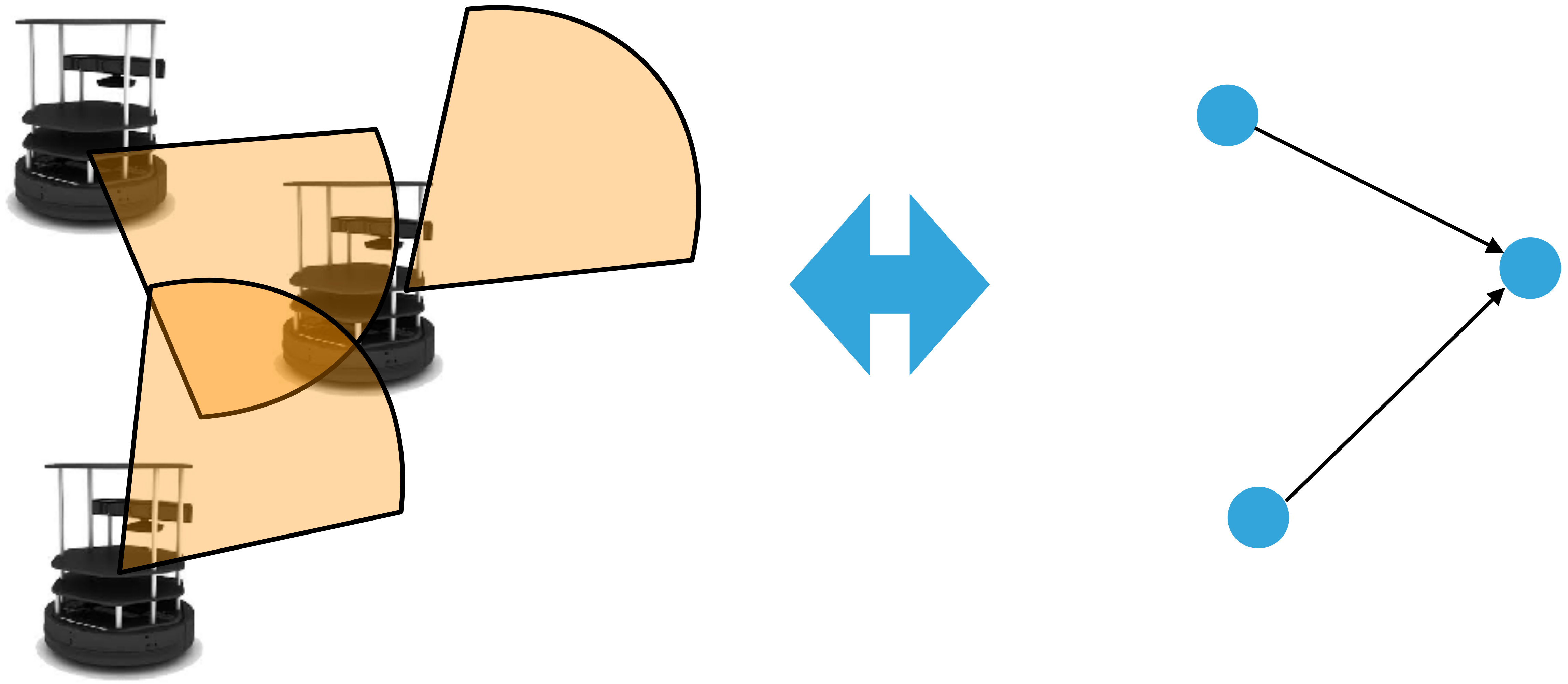
stability analysis depends on the **SE(2) bearing rigidity** of the formation!

SE(2) FORMATION CONTROL



[Zelazo, Franchi, Giordano2015]

REAL SENSING MEANS DIRECTED INFORMATION



HOW DO WE ADAPT OUR EXISTING THEORY TO HANDLE REAL SENSING?

SE(2) FORMATION CONTROL

[Schiano, Franchi, Zelazo, Giordano2016]

FIELD-OF-VIEW CONSTRAINTS

<u>**Bearing only control law with limited view constraint**</u>

1) agents faces in direction of motion

2) agent faces the middle of its neighbours

$$w_i = \frac{1}{|\mathcal{N}_i(p(t))|} \sum_{j \in \mathcal{N}_i(p(t))} \gamma_{ij}(p(t))$$

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FIELD-OF-VIEW CONSTRAINTS

"robots" – modeled as kinematic point mass with heading

$$\begin{bmatrix} \dot{p} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} u \\ \omega \end{bmatrix}$$

Assumptions

- GLOBAL COORDINATE FRAME
- BEARING MEASUREMENTS
- FIELD OF VIEW CONSTRAINTS
- SENSING

Formation

- SPECIFIED BY BEARING VECTORS

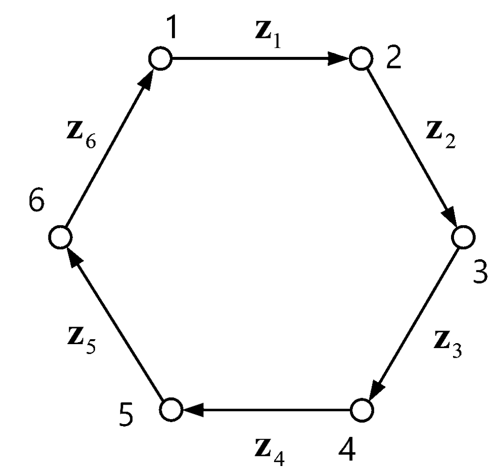
$$g_{ij}^* \in \mathbb{R}^2, \ \|g_{ij}^*\| = 1$$

Control

$$u_i = -\sum_{i \sim j} (I - g_{ij} g_{ij}^T) g_{ij}^*$$

$$\omega_i = \frac{1}{|\mathcal{N}_i(p(t))|} \sum_{j \in \mathcal{N}_i(p(t))} \gamma_{ij}(p(t))$$

- ALWAYS FACE IN THE MIDDLE OF THE NEIGHBORS YOU ARE SENSING



STABILITY?
CONVERGENCE?

Formations on directed cycles with bearing-only measurements

Minh Hoang Trinh[1] | Dwaipayan Mukherjee[2] | Daniel Zelazo[2] | Hyo-Sung Ahn[1]

IJRNC2017

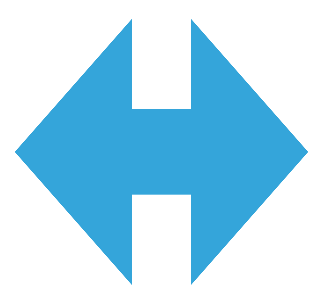
OUTLOOKS

Do we need to develop rigidity theory extensions for every kind of sensor?

G. Stacey and R. Mahony, "*The Role of Symmetry in Rigidity Analysis: A Tool for Network Localisation and Formation Control*," in *IEEE Transactions on Automatic Control*, vol. PP, no. 99, pp. 1-1.

Extensions for directed sensing network

control and estimation algorithms

THEORY  APPLICATION

ACKNOWLEDGEMENTS

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lagadic

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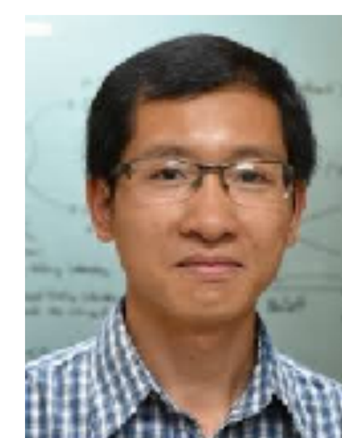
Dr. Antonio Franchi

Prof. Hyo-Sung Ahn

Daniel Frank

Fabrizio Schiano



Minh Hoang Trinh

The 2nd International Symposium on Formation Control and Multi-Agent Systems

Date: June 9-10, 2018
Venue: University of Sheffield, UK

General Chair:
Hyo-Sung Ahn, Korea
Daniel Zelazo, Israel
Shiyu Zhao, UK

Advisory Committee:
Ben M. Chen, Singapore
Zhengtao Ding, UK
Magnus Egerstedt, USA
Karl H. Johansson, Sweden
Zhiyun Lin, China
Toshiharu Sugie, Japan
Lihua Xie, Singapore

The 2nd International Symposium on Formation Control and Multi-Agent Systems will be held at the University of Sheffield, UK on June 8-9, 2018. This symposium aims to create a forum for scientists and engineers throughout the world to present their latest research findings and encourage discussions on formation control and multi-agent systems.

Topic of the Year:
20 Years of Multi-Agent Formation Control —— The Future of Formation Control

http://formationcontrol.group.shef.ac.uk