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$\mathcal{H}_2$ Performance of Relative Sensing Networks: Analysis and Synthesis

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This work provides a general framework for the analysis and synthesis of a class of relative sensing networks (RSN) in the context of its $\mathcal{H}_2$ performance. In an RSN, the underlying connection topology couples each agent at their outputs. A distinction is made between RSN with homogeneous agent dynamics and RSN with heterogeneous RSN. In both cases, an expression for the system $\mathcal{H}_2$ norm is developed that explicitly shows the dependance of the connection topology on that property. In the homogeneous setting, the norm expression reduces to the Frobenius norm of the underlying connection topology incidence matrix, $E(\mathcal{G})$, scaled by the $\mathcal{H}_2$ norm of the agents comprising the RSN. In the heterogeneous case, the $\mathcal{H}_2$ norm becomes the weighted Frobenius norm of the incidence matrix, where the weights appear on the nodes of the graph, and correspond to the $\mathcal{H}_2$ norm of each agent in the RSN. The $\mathcal{H}_2$ norm characterization is then used to synthesize RSN with certain $\mathcal{H}_2$ performance. Specifically, a semi-definite programming solution is presented to design a local controller for each agent when the underlying topology is fixed. A solution using Kruskal's algorithm for finding a minimum weight spanning tree is used to design the optimal RSN topology given fixed agent dynamics.

Nomenclature

(A, B, C, D)	State-space realization for a linear system
$x_i(t),\ \mathbf{x}(t)$	State vector
$u_i(t),\ \mathbf{u}(t)$	Control vector
$w_i(t),\ \mathbf{w}(t)$	Exogenous input vector
$y_i(t),\ \mathbf{y}(t)$	Measured output vector for an individual agent and all agents
$z_i(t),\ \mathbf{z}(t)$	Controlled variable
$\mathbf{y}_{\mathcal{G}}(t)$	Global RSN output
$Y_o,\ X_c$	Observability and controllability grammian
$\mathcal{G},\ \mathcal{V},\ \mathcal{E}$	A graph and its vertex and edge sets
$E(\mathcal{G}),\ L(\mathcal{G}),\ \Delta(\mathcal{G}),\ A(\mathcal{G})$	Incidence matrix, graph Laplacian, degree matrix, and adjacency matrix
$\Sigma_{hom}(\mathcal{G}),\ \Sigma_{het}(\mathcal{G})$	Homogeneous and heterogeneous RSN
$T_i^{w\mapsto y},\ T_i^{w\mapsto z}$	Closed-loop map from $w_i(t)$ to $y_i(t)$ and $z_i(t)$
$T_{hom}^{w\mapsto\mathcal{G}},\ T_{het}^{w\mapsto\mathcal{G}}$	Homogeneous and heterogeneous map from $w(t)$ to RSN output
$A < B$	Equivalent to $(A - B)$ a symmetric negative-definite matrix
$\mathbf{1}$	Vector with all entries equal to one
$\mathbb{R}^n$	Real n-dimensional Euclidean space
$\mathbf{tr}[\bullet]$	Trace operator
$\lvert\bullet\rvert$	Absolute value of argument

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I. Introduction

The National Aeronautics and Space Administration (NASA) and the European Space Agency (ESA) have both identified formation flying of spacecraft as an essential technology for future missions. The use of multiple coordinated spacecraft have many advantages over monolithic spacecraft, both in terms of cost, sensitivity to failure, and mission capabilities. Examples of such missions include spacecraft constellations for studying the structure of the heliopause, stereographic imaging and tomography for space physics, and space borne optical interferometry for probing the origins of the cosmos and identifying Earth-like planets (e.g., TPF, MAXIM).[1–4] The success and feasibility of these missions depend on both the scalability and the development of a sound theory for the analysis and synthesis of such systems.

In addition to the aforementioned space applications, there is a large research effort focusing on sensor fusion applications. Distributed sensor networks span a range of applications from environmental surveillance, modeling, localization, and collaborative information processing.[5–8] The challenges for these applications also relate to the development of scalable and distributed algorithms.

Fundamental to all these systems is the implicit presence of a network. The exchange of information between each agent, whether sensed or transmitted, describes an underlying connection topology. The role of the topology can have profound implications in the analysis and synthesis of these systems. Studying system theoretic notions from the perspective of the underlying topology can lead to interpretations that explicitly characterize the effects of the network on the behavior of the system.

For linear and time-invariant systems, all the essential systems theoretic properties can be derived from the quadruple system matrices (A, B, C, D). When considering mutli-agent systems, the underlying connection topology, $\mathcal{G}$, can typically be embedded into the system matrices. It becomes enlightening to consider how certain properties of the system depend on that topology. Therefore, when studying linear multi-agent systems, one should consider the quintuple $(A, B, C, D, \mathcal{G})$ and describe the dependance of the underlying topology on the system properties. Recent examples of such graph-centric analysis include relating closed-loop stability properties of multi-agent systems to the spectral properties of the graph Laplacian,[13] relating controllability in consensus seeking systems to graph symmetry,[14] and graph-centric observability properties of relative sensing networks.[15]

In this work we focus on systems that rely on relative sensing to achieve their mission objectives. We refer to this class of systems as Relative Sensing Networks (RSN). In RSN's the underlying connection topology couples the agents at their outputs. Such systems are prevalent in formation flying applications where relative sensing is used to measure inter-agent distances.[10,16] More fundamentally, these types of networks are relevant for any application involving distributed sensing for purposes of estimation and control.

The main contribution of this paper is a graph-centric characterization of the system $\mathcal{H}_2$ norm for both analysis and synthesis purposes. A distinction is made between RSN with homogeneous agent dynamics and RSN with heterogeneous agent dynamics. Although homogenous RSNs can be considered a subset of heterogeneous RSNs, it is more illuminating to consider these cases separately due to the algebraic simplicity of the former case.

For the synthesis portion of this paper we consider two general design scenarios. In the first, we focus on the design of a local $\mathcal{H}_2$ controller for each agent when the underlying connection topology is given and fixed. In addition to satisfying local performance objectives (such as those typically found in $\mathcal{H}_2$ synthesis), the proposed synthesis procedure also satisfies a global RSN objective related to the underlying connection topology. A semi-definite program is derived as a solution method for this problem.

The second synthesis objective focuses on the design of the connection topology that optimizes the $\mathcal{H}_2$ performance of the RSN. Topology design can be considered a problem in combinatorial optimization, which can be a prohibitively hard problem to solve when the number of agents is large. The results of this paper shows that the problem can be solved using Kruskal's minimum spanning tree algorithm.

The paper is organized as follows. Section §II gives a brief overview of notions from algebraic graph theory and properties of the Kronecker product for matrices. In section §III, general models for homogeneous and heterogeneous RSNs are developed. Section §IV derives expressions for the $\mathcal{H}_2$ norm of homogeneous and heterogeneous RSNs, with an emphasis given to the role of the underlying topology. Section §V presents synthesis procedures for RSNs, and a few numerical examples are given in §VI.

II. Preliminaries and Notations

We provide here some notations and preliminaries that will be used throughout the remainder of the paper.

A. Graphs and their Algebraic Representation

An undirected (simple) graph $\mathcal{G}$ is specified by a vertex set $\mathcal{V}$ and an edge set $\mathcal{E}$ whose elements characterize the incidence relation between distinct pairs of $\mathcal{V}$. Two vertices i and j are called *adjacent* (or neighbors) when $\{i,j\} \in \mathcal{E}$; we denote this by writing $i \sim j$. The cardinalities of the vertex and edge sets of $\mathcal{G}$ will be denoted by $|\mathcal{V}|$ and $|\mathcal{E}|$, respectively.

An *orientation* of an undirected graph $\mathcal{G}$ is the assignment of directions to its edges, i.e., an edge e_k is an ordered pair (i,j) such that i and j are, respectively, the initial and the terminal nodes of e_k.

In this work we make extensive use of the $|\mathcal{V}| \times |\mathcal{E}|$ incidence matrix, $E(\mathcal{G})$, for a graph with arbitrary orientation. The incidence matrix is a $\{0, \pm 1\}$-matrix with rows and columns indexed by the vertices and edges of $\mathcal{G}$ such that

$$[E(\mathcal{G})]_{ik} = \begin{cases} +1 & \text{if } i \text{ is initial node of edge } e_k \\ -1 & \text{if } i \text{ is terminal node of edge } e_k \\ \;\;0 & \text{otherwise.} \end{cases} \tag{1}$$

The degree of vertex i, d_i, is the cardinality of the set of vertices adjacent to it. The diagonal matrix $\Delta(\mathcal{G})$ contains the degree of each vertex on its diagonal. A graph is complete if all possible pairs of vertices are adjacent, or equivalently, if the degree of all vertices is $|\mathcal{V}|-1$. A sequence of $r+1$ distinct and consecutively adjacent vertices, starting from vertex i and ending at vertex j, is called a path of length r (form i to j); when $i=j$, we call this path a *cycle*. We call a graph *connected* if there exists a path between any pair of vertices. A connected graph without cycles is referred to as a *tree*.

The (graph) Laplacian of $\mathcal{G}$,

$$L(\mathcal{G}) := E(\mathcal{G})E(\mathcal{G})^T, \tag{2}$$

is a rank deficient positive semi-definite matrix. An alternative definition of the graph Laplacian can be written in terms of the degree matrix, $\Delta(\mathcal{G})$, and the graph adjacency matrix $A(\mathcal{G})$,

$$L(\mathcal{G}) := \Delta(\mathcal{G}) - A(\mathcal{G}). \tag{3}$$

The adjacency matrix is defined as

$$[\,A(\mathcal{G})\,]_{ij} := \begin{cases} 1 & \text{if} \quad \{i,j\} \in \mathcal{E} \\ 0 & \text{otherwise.} \end{cases}$$

B. Matrix Kronecker Products

Some important results on the Kronecker product are given here. The Kronecker product of two matrices A and B is written as $A \otimes B$.

Theorem II.1 ([19]) *Let $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ each have a singular value decomposition of $A = U_A\Sigma_A V_A^T$ and $B = U_B\Sigma_B V_B^T$. The singular value decomposition of the Kronecker product of A and B is*

$$A \otimes B = (U_A \otimes U_B)(\Sigma_A \otimes \Sigma_B)(V_A^T \otimes V_B^T). \tag{4}$$

An immediate consequence of Theorem II.1 is the following result on the matrix 2-norm,

$$\|A \otimes B\|_2 \quad = \quad \|A\|_2 \, \|B\|_2. \tag{5}$$

We also make extensive use of the following Kronecker product matrix multiplication property,

$$(A \otimes B)(C \otimes D) \quad = \quad (AC \otimes BD), \tag{6}$$

where the matrices are all of commensurate dimension.

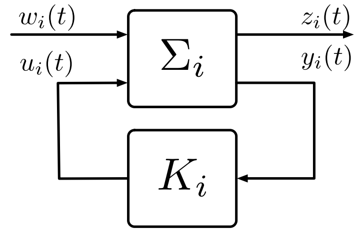


Figure 1. Local agent layer with control

III. Homogeneous and Heterogeneous RSN

An RSN consists of two system layers. The first can be considered a local layer corresponding to the dynamics of the individual agents in the ensemble. The second layer is a global RSN layer that represents the complete interconnected system. This section develops a general linear model for RSN that includes both the local and global layers.

We identify two classes of RSN in this paper: 1) homogeneous RSN, and 2) heterogeneous RSN. For both cases, we will work with linear and time-invariant systems,

$$\Sigma_i : \begin{cases} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + \Gamma_i w_i(t) \\ z_i(t) &= C_i^z x_i(t) + D_i^z u_i(t) \\ y_i(t) &= C_i^y x_i(t), \end{cases} \tag{7}$$

where each agent is indexed by the sub-script i. Here, $x_i(t)$ represents the state, $u_i(t)$ the control, $w_i(t)$ an exogenous input (e.g., disturbances), $z_i(t)$ the controlled variable, and $y_i(t)$ the measured output.

In the homogeneous case, it is assumed that each dynamic agent in the RSN is described by the same set of linear state-space dynamics (e.g., $(A_i, B_i, \Gamma_i, C_i^z, D_i^z, C_i^y) = (A_j, B_j, \Gamma_j, C_j^z, D_j^z, C_j^y)$ for all i, j). When working with homogeneous RSN, we drop the sub-script for all state-space and operator representations of the system.

As we are focusing on the $\mathcal{H}_2$ properties of this system, we assume no feedforward term of the control $u_i(t)$ and no noises in the measurements (e.g., strictly proper system). Additionally, we assume a minimal realization for each agent with the outputs of each agent being compatible (e.g., system outputs correspond to the same physical quantity). It should be noted that in a heterogeneous system, the dimension of each agent need not be the same. However, without loss of generality we assume each agent to have the same dimension.

A two-port block diagram for a local agent in a feedback configuration is shown Figure 1. We denote the open-loop map from $w_i(t)$ to $y_i(t)$ as $T_i^{w\mapsto y}$, and the closed-loop map from $w_i(t)$ to $z_i(t)$ as $T_i^{w\mapsto z}$. The $\mathcal{H}_2$ synthesis problem for a local agent is to design a feedback controller of the form $u_i(t) = K_i y_i(t)$ that minimizes the closed-loop system norm, $\|T_i^{w\mapsto z}\|_2$.

The parallel interconnection of all the agents is described with the following state-space description:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(\mathbf{t}) + \mathbf{B}\mathbf{u}(\mathbf{t}) + \mathbf{\Gamma}\mathbf{w}(\mathbf{t}) \\ \mathbf{z}(t) &= \mathbf{C}^{\mathbf{z}}\mathbf{x}(\mathbf{t}) + \mathbf{D}^{\mathbf{z}}\mathbf{u}(\mathbf{t}) \\ \mathbf{y}(t) &= \mathbf{C}^{\mathbf{y}}\mathbf{x}(\mathbf{t}), \end{aligned} \tag{8}$$

with $\mathbf{x}(t)$, $\mathbf{u}(t)$, $\mathbf{w}(t)$, $\mathbf{z}(t)$, and $\mathbf{y}(t)$ denoting respectively, the concatenated state vector, control vector, exogenous input vector, controlled vector, and output vector of all the agents in the RSN. The matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{\Gamma}$, $\mathbf{C}^z$, $\mathbf{D}^z$, and $\mathbf{C}^y$ are the block diagonal aggregation of each agent's state-space matrices.

The global RSN layer we examine for the duration of this paper is motivated by the relative sensing problem. The sensed output of the RSN is the vector $\mathbf{y}_{\mathcal{G}}(t)$ containing the relative state information of each agent and its neighbors. For example, the output sensed across an edge $e = (i, i')$ would be of the form $y_i(t) - y_{i'}(t)$. This can be compactly written as

$$\mathbf{y}_{\mathcal{G}}(t) = (E(\mathcal{G})^T \otimes I)\mathbf{y}(t). \tag{9}$$

The global layer is visualized in the block diagram in Figure 2.

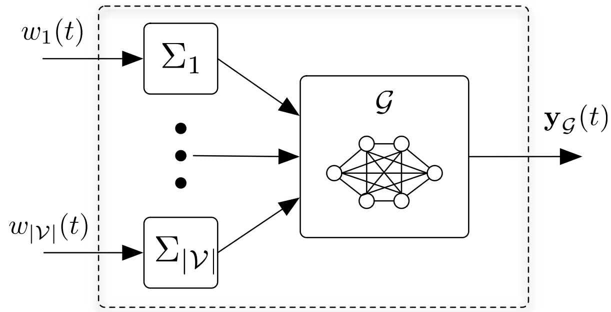


Figure 2. Global RSN layer block diagram

When considering the analysis of the global layer, we are interested in studying the map from the agent's exogeneous inputs to the RSN sensed output, which we denote by the operator $T_{hom}^{w\mapsto\mathcal{G}}$ for homogeneous RSN, and $T_{het}^{w\mapsto\mathcal{G}}$ for heterogeneous RSN. Using the above notations and (6), we can express the homogeneous and heterogeneous RSN in a compact form,

$$\Sigma_{hom}(\mathcal{G}) \begin{cases} \dot{\mathbf{x}}(t) &= (I_{|\mathcal{V}|}\otimes A)\mathbf{x}(t) + (I_{|\mathcal{V}|}\otimes B)\mathbf{u}(t) + (I_{|\mathcal{V}|}\otimes \Gamma)\mathbf{w}(t) \\ \mathbf{z}(t) &= (I_{|\mathcal{V}|}\otimes C^z)\mathbf{x}(t) + (I_N\otimes D^z)\mathbf{u}(t) \\ \mathbf{y}(t) &= (I_{|\mathcal{V}|}\otimes C^y)\mathbf{x}(t) \\ \mathbf{y}_{\mathcal{G}}(t) &= (E(\mathcal{G})^T\otimes C^y)\mathbf{x}(t) \end{cases}, \tag{10}$$

$$\Sigma_{het}(\mathcal{G}) \begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{\Gamma}\mathbf{w}(t) \\ \mathbf{z}(t) &= \mathbf{C}^z\mathbf{x}(t) + \mathbf{D}^z\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}^y\mathbf{x}(t) \\ \mathbf{y}_{\mathcal{G}}(t) &= (E(\mathcal{G})^T\otimes I)\mathbf{C}^y\mathbf{x}(t) \end{cases}. \tag{11}$$

IV. $\mathcal{H}_2$ System Norm of RSN

The $\mathcal{H}_2$ norm of a system is an important performance metric in the analysis and design of feedback systems. This section aims to explicitly characterize the affect of the network on the $\mathcal{H}_2$ norm of the system.

The $\mathcal{H}_2$ norm of a system can be calculated in a variety of ways. One description involves the observability grammian of the system. The observability grammian for an individual agent based on the dynamics in (7) is defined as

$$Y_o^{(i)} = \int_0^\infty e^{A_i^T t}(C_i^y)^T C_i^y e^{A_i t} dt\,. \tag{12}$$

The observability grammian can be calculated by solving a system of linear equations, described by the Lyapunov equation

$$A_i^T Y_o^{(i)} + Y_o^{(i)} A_i + (C_i^y)^T C_i^y = 0. \tag{13}$$

Another description involves the controllability grammian of the system. The controllability grammian for an individual agent (from the exogenous input channel) based on the dynamics in (7) is defined as

$$X_c^{(i)} = \int_0^\infty e^{A_i t}\Gamma_i \Gamma_i^T e^{A_i^T t} dt\,. \tag{14}$$

The controllability grammian can be calculated by solving the corresponding Lyapunov equation,

$$A_i X_c^{(i)} + X_c^{(i)} A_i^T + \Gamma_i\Gamma_i^T = 0. \tag{15}$$

The $\mathcal{H}_2$ norm of each agent from the exogenous input channel to the measured output can be expressed in terms of the grammians as

$$\|T_i^{w\mapsto y}\|_2 = \sqrt{\mathbf{tr}(\Gamma_i^T Y_o^{(i)} \Gamma_i)} \tag{16}$$

$$= \sqrt{\mathbf{tr}(C_i X_c^{(i)} C_i^T)}. \tag{17}$$

Using the above description we can begin to understand how the underlying network topology influences the system norm. We separate our analysis into the homogeneous and heterogeneous cases.

A. Homogeneous RSN $\mathcal{H}_2$ Norm

The $\mathcal{H}_2$ norm of the homogeneous RSN described in (10) can be written in terms of the observability grammian. As mentioned in §III, when examining the global RSN layer, we consider the map $T_{hom}^{w\mapsto\mathcal{G}}$. Therefore, the expression for the observability grammian of the global RSN layer in (10) is

$$\begin{aligned}\mathbf{Y_o} &= \int_0^\infty e^{(I_N\otimes A)^T t}(E(\mathcal{G})^T\otimes C^y)^T(E(\mathcal{G})^T\otimes C^y)e^{(I_N\otimes A)t}dt \\ &= L(\mathcal{G})\otimes Y_o,\end{aligned} \tag{18}$$

where Y_o represents the observability grammian of a single agent in the network (described in (12)).

Using (18), we have the following characterization of the $\mathcal{H}_2$ norm,

$$\begin{aligned}\left\|T_{hom}^{w\mapsto\mathcal{G}}\right\|_2 &= \sqrt{\mathbf{tr}((I_N\otimes\Gamma)^T(L(\mathcal{G})\otimes Y_o)(I_N\otimes\Gamma))} \\ &= \|E(\mathcal{G})\|_F\|T^{w\mapsto y}\|_2,\end{aligned} \tag{19}$$

where $\|M\|_F$ denotes the Frobenius norm of the matrix M.

The expression in (19) gives an explicit characterization of how the network affects the overall gain of the RSN. In the homogeneous case, we can focus our attention on how the Frobenius norm of the incidence matrix changes with the addition or removal of an edge. To elaborate on this, we can write the Frobenius norm of a matrix in terms of the 2-norm of the matrix columns:

$$\|M\|_F = \left(\sum_{i=1}^n \|m_i\|_2^2\right)^{1/2},$$

where m_i is the ith column of the matrix M.

In the case of the incidence matrix, each column, representing a single edge of the graph, always has the same structure, as described in (1). Therefore, the Frobenius norm of the incidence matrix can be expressed in terms of the number of edges in the graph, $|\mathcal{E}|$, as

$$\|E(\mathcal{G})\|_F = (2\,|\mathcal{E}|)^{1/2}. \tag{20}$$

One immediate consequence of this description is that the RSN $\mathcal{H}_2$ norm is only dependent on the number of edges in the graph rather than the actual structure of the topology. This makes intuitive sense, as more edges would correspond to additional amplification of the disturbances entering the system.

If we consider only connected graphs, then we have immediate lower and upper bounds on the $\mathcal{H}_2$ norm of the system,

$$\left\|T_{hom}^{w\mapsto\mathcal{G}}\right\|_2^2 \geq 2\,\|T^{w\mapsto y}\|_2^2\,(|\mathcal{V}|-1)\,. \tag{21}$$

The lower bound is attained with equality whenever the underlying graph is a spanning tree. It is clear from the definition of the Frobenius norm that the choice of tree is irrelevant (e.g., a star or a path).

If we assume that all graphs are simple, that is they do not have multiple edges between a single pair of nodes, then the upper bound for the system norm is achieved by the complete graph,

$$\left\|T_{hom}^{w\mapsto\mathcal{G}}\right\|_2^2 \leq 2\,\|T^{w\mapsto y}\|_2^2\,|\mathcal{V}|\,(|\mathcal{V}|-1)\,. \tag{22}$$

B. Heterogeneous RSN $\mathcal{H}_2$ Norm

In the heterogeneous case, the RSN $\mathcal{H}_2$ norm can be derived by using (17) as,

$$\left\|T_{het}^{w\mapsto\mathcal{G}}\right\|_2 = \left(\mathbf{tr}\left\{(E(\mathcal{G})^T\otimes I)\mathbf{C}^y\mathbf{X}_c(\mathbf{C}^y)^T(E(\mathcal{G})\otimes I)\right\}\right)^{1/2}, \tag{23}$$

where $\mathbf{X}_c$ denotes the block diagonal aggregation of each agent's controllability grammian, as definied in (14). First, we make the following observation,

$$\mathbf{tr}\left\{\mathbf{C}^y\mathbf{X}_c(\mathbf{C}^y)^T\right\} = \sum_{i=1}^{|\mathcal{V}|}\|T_i^{w\mapsto y}\|_2^2.$$

Using the cycle property of the trace operator and exploiting the block diagonal structure of the argument leads to the following identity simplification,

$$\begin{aligned}\mathbf{tr}\left\{\mathbf{C}^y\mathbf{X}_c(\mathbf{C}^y)^T((\Delta(\mathcal{G})-A(\mathcal{G}))\otimes I)\right\} &= \sum_i \mathbf{tr}\left\{C_i^y X_c^{(i)}(C_i^y)^T(d_i\otimes I)\right\}\\ &= \sum_i d_i\,\|T_i^{w\mapsto y}\|_2^2\,,\end{aligned} \tag{24}$$

where d_i is the degree of the ith agent in the graph.

This can now be used to obtain the following expression for the $\mathcal{H}_2$ norm of the system,

$$\left\|T_{het}^{w\mapsto\mathcal{G}}\right\|_2 = \left(\sum_i d_i\,\|T_i^{w\mapsto y}\|_2^2\right)^{1/2}. \tag{25}$$

An even further examination of the above term reveals that it can be written as the Frobenius norm of a node-weighted incidence matrix,

$$\left\|T_{het}^{w\mapsto\mathcal{G}}\right\|_2 = \left\|\begin{bmatrix}\|T_1^{w\mapsto y}\|_2 & & \\ & \ddots & \\ & & \|T_{|\mathcal{V}|}^{w\mapsto y}\|_2\end{bmatrix}E(\mathcal{G})\right\|_F. \tag{26}$$

When each agent has the same dynamics, (26) reduces to the expression in (19). This characterization paints a very clear picture of how the placement of an agent within a certain topology affects the overall system gain. In order to minimize the gain, it is beneficial to keep systems with high norm in locations with minimum degree.

V. Synthesis of RSN

The results of §IV can be used to develop a performance metric for the synthesis of RSN. The objective is to design a local controller K_i for each agent in the ensemble that minimizes some local performance objective, $\|T_i^{w\mapsto z}\|_2$ while additionally minimizing the global RSN objective, $\left\|T_{het}^{w\mapsto\mathcal{G}}\right\|_2$. This is visualized in the block diagram in Figure 3. It should be noted that this problem does not consider the design of feedback controllers to achieve higher level objectives for the network, such as formation control.

In this setting, we propose two scenarios for the synthesis of RSN. In the first case, we consider designing the local controller for each agent when the underlying topology and the placement of agents within that topology is given and fixed. A semi-definite program is derived to solve this problem.

The second case examines how to design the optimal topology and placement of agents within the topology, assuming that each agent already has a local controller designed. We cite a result from combinatorial optimization, Kruskal's algorithm, and describe how it can be applied to this problem.

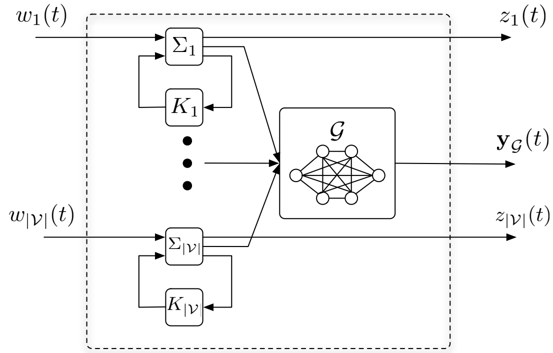


Figure 3. $\mathcal{H}_2$ Synthesis of RSN

A. Local Agent Control for Fixed Topology

For this problem we will consider a heterogeneous RSN with a given and fixed topology, $E(\mathcal{G})$. Each agent, Σ_i, is also assigned a fixed location within the network. From a synthesis point of view, each agent behaves independently and does not use information from the RSN for its control.

To simplify this discussion, we will assume that each agent has full-state feedback available for its controller ($C_i^y = I$). For this example, we also assume that the global RSN output corresponds to a relative position measurement. Therefore, the RSN output $\mathbf{y}_{\mathcal{G}}(t)$ will be described as

$$\begin{aligned} \mathbf{y}_{\mathcal{G}}(t) &= E(\mathcal{G})^T \otimes \begin{bmatrix} \mathbf{1}^T & 0 & \cdots & 0 \end{bmatrix} \\ &= E(\mathcal{G})^T \otimes C_p ; \end{aligned} \tag{27}$$

here we have assumed the states corresponding to the position of each agent are the first p states of $x_i(t)$.

The state-feedback optimal $\mathcal{H}_2$ control problem for a single agent without considering the global RSN layer can be formulated as an SDP.[20]

$$\min_{W_i, X_i, Z_i} \quad \mathbf{tr}[W_i] \tag{28}$$

s.t.

$$\begin{bmatrix} A_i & B_i \end{bmatrix} \begin{bmatrix} X_i \\ Z_i \end{bmatrix} + \begin{bmatrix} X_i & Z_i^T \end{bmatrix} \begin{bmatrix} A_i^T \\ B_i^T \end{bmatrix} + \Gamma_i \Gamma_i^T < 0$$

$$\begin{bmatrix} X_i & (C_i^z X_i + D_i^z Z_i)^T \\ (C_i^z X_i + D_i^z Z_i) & W_i \end{bmatrix} > 0 \,;$$

the control can be reconstructed as $K_i = Z_i X_i^{-1}$.

From the above SDP, we have that $\|T_i^{w \mapsto z}\|_2^2 = \mathbf{tr}(W_i)$. Here, we note that X_i corresponds to the controllability grammian of the closed-loop system for agent i. That is, it is the controllability grammian for a realization of the system $T_i^{w \mapsto z}$.

The SDP in (28), however, does not incorporate the global RSN performance objective into the problem. While each agent can generate a solution to (28) independently of each other, the addition of the global RSN layer couples the design of each agent's controller. To illustrate this, we should examine the map $T_{het}^{w \mapsto \mathcal{G}}$ in the context of Figure 3. This is easily accomplished by considering the system in (11). We will treat the RSN output $\mathbf{y}_{\mathcal{G}}(t)$ as an additional performance variable, and rewrite the system as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{\Gamma}\mathbf{w}(t) \\ \begin{bmatrix} \mathbf{z}(t) \\ \mathbf{y}_{\mathcal{G}}(t) \end{bmatrix} &= \begin{bmatrix} \mathbf{C}^z \\ E(\mathcal{G})^T \otimes C_p \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} \mathbf{D}^z \\ 0 \end{bmatrix} \mathbf{u}(t) \\ \mathbf{y}(t) &= I\mathbf{x}(t). \end{aligned} \tag{29}$$

Using the augmented state-space description in (29) we have the following result for the synthesis of controllers for each agent while incorporating the global RSN objecitve.

Theorem V.1 *Given the RSN system described in (29), a local state-feedback controller of the form* $u_i(t) = K_i x_i(t)$ *that minimizes local performance objectives in addition to the global RSN performance objective can be found by solving*

$$\min_{W_i,X_i,Z_i,V_i} \sum_i^{|\mathcal{V}|} \mathbf{tr}[W_i] + \mathbf{tr}[V_i] \tag{30}$$

s.t.

$$\begin{bmatrix} A_i & B_i \end{bmatrix} \begin{bmatrix} X_i \\ Z_i \end{bmatrix} + \begin{bmatrix} X_i & Z_i^T \end{bmatrix} \begin{bmatrix} A_i^T \\ B_i^T \end{bmatrix} + \Gamma_i \Gamma_i^T \leq 0 \tag{31}$$

$$\begin{bmatrix} X_i & (C_i^z X_i + D_i^z Z_i)^T \\ (C_i^z X_i + D_i^z Z_i) & W_i \end{bmatrix} > 0 \tag{32}$$

$$\begin{bmatrix} X_i & (C_p X_i)^T \\ C_p X_i & \frac{1}{d_i} V_i \end{bmatrix} > 0 \tag{33}$$

where

$$K_i = Z_i X_i^{-1}.$$

Proof Consider the system in (29) with a control $\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t)$ implemented, where $\mathbf{K} = \mathbf{diag}(K_1, \ldots, K_{|\mathcal{V}|})$. The closed-loop system becomes

$$\Sigma_{cl} \left\{ \begin{array}{rcl} \dot{\mathbf{x}}(t) & = & (\mathbf{A} + \mathbf{B}\mathbf{K})\mathbf{x}(t) + \mathbf{\Gamma}\mathbf{w}(t) \\ \begin{bmatrix} \mathbf{z}(t) \\ \mathbf{y}_{\mathcal{G}}(t) \end{bmatrix} & = & \begin{bmatrix} \mathbf{C}^z + \mathbf{D}^z\mathbf{K} \\ E(\mathcal{G})^T \otimes C_p \end{bmatrix} \mathbf{x}(t). \end{array} \right. \tag{34}$$

To guarantee the stability of the closed loop system, we require that $(\mathbf{A}+\mathbf{B}\mathbf{K})$ be Hurwitz. This is guaranteed by the LMI given in (31) by noting the block diagonal structure of the matrix, and defining $Z_i = K_i X_i$. In fact, when the constraint (31) is satisfied at equality, we note that X_i is the controllability grammian for the system in (34).

The $\mathcal{H}_2$ norm of (34) can be calculated as

$$\begin{aligned} \|\Sigma_{cl}\|_2^2 &= \mathbf{tr}\left\{ \begin{bmatrix} \mathbf{C}^z + \mathbf{D}^z\mathbf{K} \\ E(\mathcal{G})^T \otimes C_p \end{bmatrix} \mathbf{X} \begin{bmatrix} \mathbf{C}^z + \mathbf{D}^z\mathbf{K} \\ E(\mathcal{G})^T \otimes C_p \end{bmatrix}^T \right\} \\ &= \mathbf{tr}\{(\mathbf{C}^z + \mathbf{D}^z\mathbf{K})\mathbf{X}(\mathbf{C}^z + \mathbf{D}^z\mathbf{K})^T\} + \mathbf{tr}\{(E(\mathcal{G})^T \otimes C_p)\mathbf{X}(E(\mathcal{G})^T \otimes C_p)^T\}, \end{aligned} \tag{35}$$

where $\mathbf{X} = \mathbf{diag}(X_1, \ldots, X_{|\mathcal{V}|})$. The first term on the right hand side corresponds precisely to the $\mathcal{H}_2$ norm of the system in (8) with the feedback law $\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t)$ implemented. The second term is the $\mathcal{H}_2$ norm of $T_{het}^{w \mapsto \mathcal{G}}$. Using the results from §IV we can express the performance as

$$\begin{aligned} \left\| T_{het}^{w \mapsto \mathcal{G}} \right\|_2^2 &= \mathbf{tr}\{(E(\mathcal{G})^T \otimes C_p)\mathbf{X}(E(\mathcal{G}) \otimes C_p)\} \\ &= \sum_i^{|\mathcal{V}|} d_i \mathbf{tr}\{C_p X_i C_p^T\}. \end{aligned} \tag{36}$$

The objective is to minimize $\|\Sigma_{cl}\|_2$, which can be accomplished by minimizing both terms in the right-hand side of (35). Using the matrix Schur-complement,[18] we note that

$$d_i C_p X_i C_p^T \quad < \quad V_i \tag{37}$$

is equivalent to

$$\begin{bmatrix} X_i & (C_p X_i)^T \\ C_p X_i & \frac{1}{d_i} V_i \end{bmatrix} \quad > \quad 0. \tag{38}$$

We now note that if $d_iC_pX_iC_p^T < V_i$, then $d_i\mathbf{tr}\{C_pX_iC_p^T\} < \mathbf{tr}\{V_i\}$.

A similar derivation is used to arrive at the LMI in (32). ■

Remark V.2 *The full-state feedback assumption can be relaxed without loss of generality using an LMI formulation for the more general output-feedback problem (such as LQG).*[21] *The LMI (32) will consequently be modified, but the LMI corresponding to the global RSN performance (33) remains the same.*

A striking feature of the SDP (30)-(33) is its structure. Although the global RSN layer couples each agent, we see that the coupling can be removed via the formulation of the $\mathcal{H}_2$ norm. The SDP is therefore separable across each of the agents which has implications for the parallelization of the computation and decision-making process.

B. Topology Design and Agent Placement

In this section we consider how to design the underlying connection topology and where to place agents within that topology. Recall from §IV that in terms of the $\mathcal{H}_2$ norm objective, an optimal topology should always correspond to a spanning tree. The design problem, therefore, is to determine which spanning tree will achieve the smallest $\mathcal{H}_2$ norm for the RSN.

We assume in this case that each agent has already adopted a feedback controller for its operation. Using the same relative position sensing model, the RSN state-space description can be written as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{\Gamma}\mathbf{w}(t) \\ \mathbf{y}_{\mathcal{G}}(t) &= (E(\mathcal{G})^T \otimes C_p)\mathbf{x}(t). \end{aligned} \tag{39}$$

The design of the topology reduces to the design of the incidence matrix, $E(\mathcal{G})$. This problem is combinatorial in nature, as there are only a finite number of graphs that can be constructed from a set of N nodes. As the number of agents in the RSN becomes large, solving this problem becomes prohibitively hard. However, we find that with an appropriate modification of the problem statement, results from combinatorial optimization can be used, leading to a polynomial-time algorithm.

Specifically, the *minimum spanning tree* (MST) problem solves this problem. The MST can be efficiently solved using Kruskal's algorithm in $\mathcal{O}(|\mathcal{E}|\log(|\mathcal{V}|))$ time. The algorithm is given below and a proof of its correctness can be found in.[22]

Algorithm 1: Kruskal's Algorithm

Data: A connected undirected graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ and weights $w: \mathcal{E} \mapsto \mathbb{R}$.
Result: A spanning tree $\mathcal{G}_t$ of minimum weight.
begin
  Sort the edges such that $w(e_1) \le w(e_2) \le \cdots \le w(e_{|\mathcal{E}|})$, where $e_i \in \mathcal{E}$
  Set $\mathcal{G}_t := \mathcal{G}_t(\mathcal{V},\emptyset)$
  for $i := 1$ **to** $|\mathcal{E}|$ **do**
    if $\mathcal{G}_t + e_i$ *contains no cycle* **then**
      set $\mathcal{G}_t := \mathcal{G}_t + e_i$
end

In order to apply the MST to the $\mathcal{H}_2$ synthesis problem we must reformulate the original problem statement. To begin, we first write the expression for the $\mathcal{H}_2$ norm of the system in (39).

$$\begin{aligned} \left\| T_{het}^{w\mapsto\mathcal{G}} \right\|_2^2 &= \sum_i^{|\mathcal{V}|} d_i\mathbf{tr}\{C_pX_iC_p^T\} \\ &= \sum_i^{|\mathcal{V}|} d_i\|T_i^{w\mapsto p}\|_2^2 \end{aligned} \tag{40}$$

We reiterate here that the RSN norm description is related to the degree of each node in the network. Using the weighted incidence graph interpretation of the norm, as in (26), we see that the gain of each agent, $\|T_i^{w\mapsto p}\|_2^2$, acts as a weight on the nodes.

As each agent is assumed to have fixed dynamics, the problem of minimizing the RSN $\mathcal{H}_2$ norm reduces to finding the degree of each agent while ensuring the resulting topology is a spanning tree. This objective is related to properties of the nodes of the graph. To use the MST results, we must convert the objective from weights on the nodes to weights on the edges.

To develop this transformation, consider the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with fixed weights w_i on each node $(i = 1, \ldots, |\mathcal{V}|)$. The node-weighted Frobenius norm of the incidence matrix is

$$\|WE(\mathcal{G})\|_F^2 \quad = \quad \sum_i d_i w_i^2 \;, \tag{41}$$

where $W = \mathbf{diag}(w_1, \ldots, w_{|\mathcal{V}|})$.

Next, consider the effect of adding an edge $\hat{e} = (i,j)$ to $\mathcal{E}$ in terms of the Frobenius norm of the augmented incidence matrix,

$$\left\| W \begin{bmatrix} E(\mathcal{G}) & \hat{e} \end{bmatrix} \right\|_F^2 = \left(\sum_k d_k w_k^2 \right) + w_i^2 + w_j^2 \;, \tag{42}$$

where d_k represents the degree of node k *before* adding the new edge $\hat{e}$. This shows that each edge $\hat{e} = (i,j)$ contributes $(w_i^2 + w_j^2)$ to the overall norm. Therefore, weights on the edges can be constructed by adding the node weights corresponding to the nodes adjacent to each edge as

$$\mathbf{w}_e = |E(\mathcal{G})^T| \mathbf{w}_n^2 \;. \tag{43}$$

This result can be used to generate an equivalent norm characterization to the one presented in (40)

$$\left\| T_{het}^{w \mapsto \mathcal{G}} \right\|_2^2 \quad = \quad \left\| \, |E(\mathcal{G})^T| \begin{bmatrix} \|T_1^{w\mapsto p}\|_2^2 \\ \vdots \\ \|T_{|\mathcal{V}|}^{w\mapsto p}\|_2^2 \end{bmatrix} \right\|_1 , \tag{44}$$

where $\|x\|_1 = \sum_i |x_i|$.

Using the above transformation from node weights to edge weights, we arrive at the following result.

Theorem V.3 *The connection topology that minimzes the $\mathcal{H}_2$ norm of (39), can be found using Kruskal's MST algorithm with input data $\mathcal{G}$, and weights*

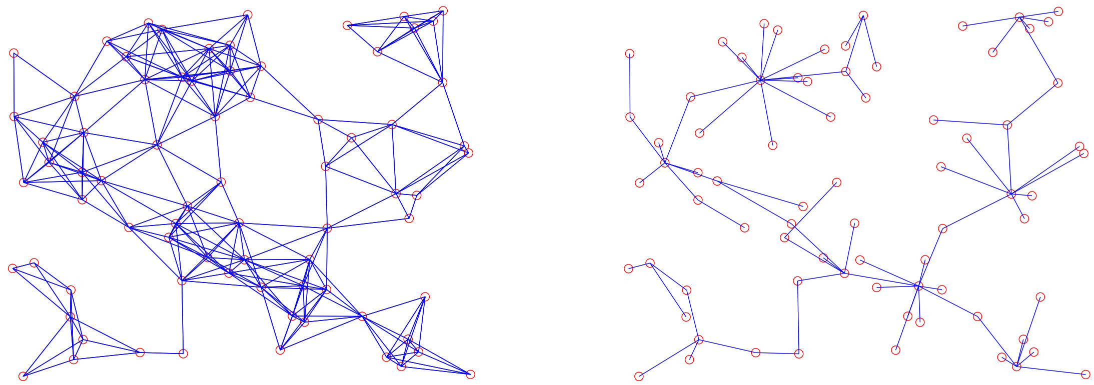
$$w \quad = \quad |E(\mathcal{G})|^T \begin{bmatrix} \|T_1^{w\mapsto p}\|_2^2 \\ \vdots \\ \|T_{|\mathcal{V}|}^{w\mapsto p}\|_2^2 \end{bmatrix} . \tag{45}$$

Proof The proof follows from (40) and the transformation from node weights to edge weights described in (41)-(43). ■

Remark V.4 *The choice of the input graph $\mathcal{G}$ may be application specific, and can capture certain communication or sensing constraints between agents. For example, one may consider a scenario where agents are initially randomly distributed (a geometric random graph) upon deployment and can only sense neighboring agents within a specified range. The results of Theorem V.3 can be used to determine the optimal spanning tree for that initial configuration.*

Remark V.5 *There are many distributed algorithms that solve the MST problem.[23, 24] These could be used in place of the centralized version when the optimal spanning tree topology needs to be reconfigured. This scenario can arise due to the initialization problem discussed in Remark V.4, or in situations when certain agents are disabled, lost, or reallocated for different mission purposes.*

If there are no initial constraints on the input graph for Theorem V.3, then we arrive at the following result.



(a) Random Geometric Graph with $r = 0.20$ (b) Optimal spanning tree

Figure 4. Application example of Theorem V.3

Lemma V.6 *When the input graph in Theorem V.3 is the complete graph, then the star graph with center node corresponding to the agent with minimum norm is the (non-unique) optimal topology.*

Proof The degree of the center node in a star graph is $N-1$, and all other nodes have degree one. Assume the node weights are sorted as $w_1 \leq \cdots \leq w_N$, then the $\mathcal{H}_2$ norm of the NDS is $\|T_{het}^{w\mapsto\mathcal{G}}\|_2^2 = (N-1)w_1 + \sum_{i=2}^{N} w_i$. Any other tree can be obtained by removing and adding a single edge, while ensuring connectivity. With each such operation, the cost is non-decreasing, as any new edge will increase the degree of node $i > 1$ and by assumption $w_1 \leq w_i$. ■

Lemma V.6 shows that if there are no restrictions on the initial configuration, the optimal topology can be obtained without the MST algorithm. The computational effort required is only to determine the agent with smallest norm. The non-uniqueness of the star graph can occur if certain agents have identical norm, resulting in other possible configuration with an equivalent cost.

VI. Simulation Example

In this section we consider an application of our results to a mission scenario related to the *Autonomous NanoTechnology Swarm* project, or ANTS, currently under investigation by NASA.[27] One component of the ANTS mission involves the deployment of 1,000 pico-satellites to the asteroid belt for observational study. The spacecraft are deployed en-route to the asteroid belt, and after deployment must organize into smaller teams which will coordinate to search for various resources and materials. We consider here two aspects of this mission.

When the pico-satellites are initially deployed they must be configured into teams. One scenario is to consider forming a team with a topology that minimizes the $\mathcal{H}_2$ performance of the team, corresponding to the results developed in §VB. For this example, we will consider a system comprised of 75 heterogeneous pico-satellites. Each agent's state-space was generated randomly using MATLAB, with a single input and a single output (corresponding to the position, as in C_p defined in (27)). It is worth mentioning for this mission there may be certain pico-satellites that contain different sensors depending on their mission objectives. This variation would introduce heterogeneity, but for ease of presentation we use random models. Each of the agents are randomly distributed and the initial topology is determined by assigning an edge between two agents if their Euclidean distance is less than $r = 0.20$. This could correspond to the relative sensing capabilities available on each spacecraft. The initial connection graph is given in Figure 4(a), and the resulting MST is given in Figure 4(b). A key point in this example is to highlight the non-triviality of the resulting topology. When designing a topology based on heuristics, this result most likely would not be found, especially when dealing with large networks.

Another component of the mission involves collecting data from an asteroid. To accomplish this the pico-satellite team must rendezvous with an asteroid. For this scenario, we consider a rendezvous problem

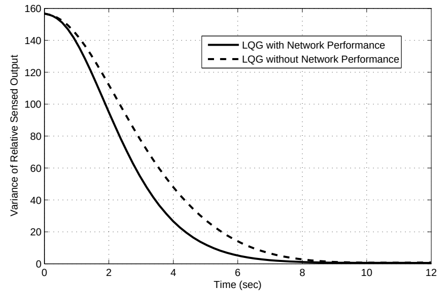


Figure 5. Variance of $y_{\mathcal{G}}(t)$ for a system with additional network performance constraints (solid) and without these constraints (dashed)

for each pico-satellite individually. Each satellite is assumed to have continuous actuation on each axis. We also introduce disturbances in the form of process noise for the actuators and measurement noise for the sensors. The noises are assumed to be white Gaussian with $\sigma_w^2 = 0.1$ for the process and $\sigma_v^2 = 0.01$ for the sensors. Contrary to the previous example, we will assume homogeneous agent dynamics generated by the Hill's equations.[28] The target asteroid is assumed to be in a circular orbit around the Sun with radius $r_o = 3 \times 10^9$km. A random spanning tree graph is generated and the results of Theorem V.1 are applied to generate a control for each pico-satellite to drive them to the asteroid. We also address the issue in Remark V.2 regarding the full-state information. For this example we employ LQG for estimation and control while including the additional performance constraint for the network. Figure 5 shows the variance of the RSN output $y_{\mathcal{G}}(t)$ for the system using the network performance constraint and the system without the constraint. This shows that the inclusion of the network performance constraint will tend to keep the agents closer together even in the presence of noise.

VII. Concluding Remarks

This paper focused on the analysis and synthesis of a class of relative sensing networks (RSN) that have a wide range of applications in new aerospace technologies. The results of this paper highlight an important connection between certain graph-theoretic concepts and systems-theoretic properties. The analysis of RSNs pointed to the importance of spanning trees and the node degree of each agent in the context of the overall $\mathcal{H}_2$ performance. When considering the synthesis of RSN with $\mathcal{H}_2$ performance, a semi-definite program was derived that uses the node degree information of each agent in the network as an additional constraint to address the global network performance. Perhaps the most salient feature of this work pertains to the application of the celebrated MST algorithm from combinatorial optimization for designing the interconnection topology for overall optimal $\mathcal{H}_2$ performance.

A natural extension of these results is to examine the $\mathcal{H}_\infty$ system norm for RSNs. In the $\mathcal{H}_\infty$ setting the central issue relates to the largest eigenvalue of the graph Laplacian.[26] This points to a more structure dependent result, rather than the results in this paper that relates to the number of edges in the graph.

This work also suggests that the relationship between systems-theoretic properties and graph properties in RSNs can be examined further in the systems and aerospace communities. In fact, we believe that developing efficient solution methods for the design of such systems will involve connecting and interpreting results from graph theory and combinatorial optimization in a systems-theoretic context.

Acknowledgements

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