Regularization and Feedback Passivation in Cooperative Control of Passivity-Short Systems: A Network Optimization Perspective

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***Abstract*—In this letter, we establish a connection between cooperative control of *passivity-short* systems, and the regularization of a pair of dual network optimization problems. We build upon existing works that established, under a passivity requirement, an equivalence between the steady-state behavior of diffusively coupled network system and the solutions to a pair of convex-dual optimization problems. We show that when the agents are passivity-short, the resulting optimization problems are no-longer convex. By introducing a regularization term to the problem, we then establish that this corresponds to a feedback passivation of each system via an appropriately chosen linear output-feedback gain. We also obtain conditions on the regularization term such that the resultant closed system possess the so-called maximally equilibrium-independent passivity property and exhibits a solution to their network-level interactions. Finally, we illustrate theoretical results with two case studies.**

***Index Terms*—Multi-agent systems, cooperative systems, nonlinear control, optimization, passivity.**

I. Introduction

PASSIVITY has been instrumental in recent years to study distributed control and optimization of large-scale networks comprising multiple dynamical systems interacting with each other with widespread applications [1]–[4]. Several variations and extensions of passivity have been explored in the literature in a variety of contexts. One important variant particularly useful for the analysis of multi-agent systems, is equilibrium-independent passivity (EIP), which encompasses nontrivial equilibria, often desirable in interconnected systems [5], [6]. Unlike the classical definition of passivity [7], EIP requires a passivity inequality to hold between any system trajectory and any forced equilibrium point [6]. Because of this property, EIP is also known as shifted passivity of a control system and is widely used to study Port-Hamiltonian systems [8]. As an extension to EIP, the concept of maximally equilibrium-independent passivity (MEIP) was introduced in [9], which includes a general class of systems that widely occurs in applications.

A general analysis result about cooperative control of diffusively-coupled MEIP systems, presented in [9], relates the steady-state input-output signals of the closed loop system to two canonical dual network optimization problems [10]. Based on these result, a control synthesis technique is discussed in [11] to achieve desired coordination goals in networked systems. It is to be noted that these analysis [9] and synthesis [11] results were based on an assumption that the underlying dynamical systems already satisfy the MEIP properties.

However, this letter extends the work in [9] by relaxing the passivity requirement on the systems, and assumes them to be characterized by their *shortage of passivity*, which is more common in practice [12], [13]. This consideration, unlike in [9] and [11], leads to a different perspective of the problem. To tackle such systems, we propose a regularization-based network optimization framework, which relies on finding an appropriate regularizing term so as to take control of the passivity-shortage and to assure a solution to the closed loop network model. One of the main results of this letter shows that the regularization of the network flow problems is equivalent to a *feedback passivation* of each heterogeneous passive-short (PS) system by an output-feedback.

Feedback passivation[1] is widely studied in the literature in the context of multi-agent systems, for instance [15]–[17], in particular, for the concept of exponential feedback passivity. However, unlike these works, in this letter, we present a network optimization perspective for the diffusively coupled networked dynamical systems, by considering the "equilibrium-independent" aspect of the passivity.

In the literature, passivity indices are used to quantify the excess or shortage of passivity in a system and are often compensated for using feedback or feedforward controllers [18], [19]. One of the classical results about feedback passivation is that a control system is feedback equivalent to a passive system if and only if it has relative degree one (in a vector sense) and is weakly minimum phase [20]. For the case of output feedback, an additional condition of minimum

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[1] Feedback passivation is also referred to as *feedback passification* [14].

phase is required to be satisfied by the Jacobian linearization about the equilibrium point [7]. To support these results, we consider in this letter the SISO dynamical systems that satisfy so called "passivity-short" inequality for a given supply rate and a storage function. The main contributions of this letter are therefore:

1) We show that passivity-short systems have input-output relations that are not monotone. This leads to the associated network optimization problems being non-convex, precluding the analysis results of [9].
2) We show that introducing a Tikhonov-type regularization to the non-convex optimization problem corresponds to a feedback passivation of each system via an appropriately chosen linear output-feedback gain.
3) Furthermore, we provide an analysis showing the regularized optimization problem corresponds to the system recovering its MEIP properties if the control gain of individual output-feedback is strictly greater than the absolute value of the passivity index.

Additionally, we provide two case studies to illustrate the theoretical findings. This letter unfolds as follows: Section II reviews the necessary results from [9]. Section III motivates and formulates the problem. The connection between regularization and feedback passivation is established in Section IV. We also obtain the required condition on the regularization term in Section IV. Theoretical results are illustrated by two case studies in Section V. Finally, we conclude this letter in Section VI.

Preliminaries: An undirected graph is a pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, which consist a finite set of vertices $\mathcal{V}$ and undirected edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. The incidence matrix $E \in \mathbb{R}^{|\mathcal{V}| \times |\mathcal{E}|}$ of graph $\mathcal{G}$ with an arbitrary orientation is defined such that for edge $k = (i, j) \in \mathcal{E}$ (where $i \in \mathcal{V}$ is the head and $j \in \mathcal{E}$ is the tail of edge k), $[E]_{ik} = +1$, $[E]_{jk} = -1$, and $[E]_{lk} = 0$ for $l \neq i, j$. This definition implies that $\mathbf{1}^T E = 0$, where $\mathbf{1} \in \mathbb{R}^{|\mathcal{V}|}$ is the vector of all ones. A function $\phi : \mathbb{R}^q \mapsto \mathbb{R}$ is said to be a convex function on a convex set $\mathcal{D}$ if for any two points $\eta, \xi \in \mathcal{D}$ and for all $\mu \in [0, 1]$, $\phi(\mu\eta + (1 - \mu)\xi) \leq \mu\phi(\eta) + (1 - \mu)\phi(\xi)$. The convex conjugate or a dual function of convex function ϕ is defined as $\phi^\star(\xi) = \sup_{\eta \in \mathcal{D}} \{\eta^T \xi - \phi(\eta)\}$ [21]. We follow convention that italic letters denote dynamic variables and letters in normal font denote constant signals.

II. Background

Consider a collection of agents interacting over a network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Assign to each node $i \in \mathcal{V}$ the dynamical system Σ_i (the agents), and to each edge $e \in \mathcal{E}$ the dynamical systems Π_e (the controllers),

$$\Sigma_i : \begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = h_i(x_i) \end{cases}, \ \Pi_e : \begin{cases} \dot{\eta}_e = \phi_e(\eta_e, \zeta_e) \\ \mu_e = \psi_e(\eta_e) \end{cases}, \tag{1}$$

with state $x_i \in \mathcal{X}_i \subseteq \mathbb{R}^{p_i}$, control input $u_i \in \mathcal{U}_i \subseteq \mathbb{R}$ and output $y_i \in \mathcal{Y}_i \subseteq \mathbb{R}$. We define the state, input, and output spaces respectively as $\mathcal{X}_i, \mathcal{U}_i, \mathcal{Y}_i$. Furthermore, η_e, ζ_e and μ_e are the state, input and output of the controllers Π_e. We adopt the following vector notation $x(t) = [x_1(t), \ldots, x_{|\mathcal{V}|}(t)]^T$, $u(t) = [u_1(t), \ldots, u_{|\mathcal{V}|}(t)]^T$ and $y(t) = [y_1(t), \ldots, y_{|\mathcal{V}|}(t)]^T$. Similarly, we can define stack vectors ζ and μ for the controller input and output. The functions $f_i(\cdot, \cdot)$, $h_i(\cdot)$ and $\phi_e(\cdot, \cdot)$, $\psi_e(\cdot)$ represent system's state/output and controller's state/output dynamics, respectively, and are of appropriate dimensions.

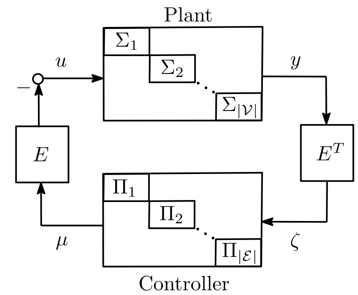


Fig. 1. Cooperative control structure of (1).

We assume that there exists a nonempty set $\overline{\mathcal{U}}_i \subseteq \mathcal{U}_i$ such that for every constant $\mathrm{u}_i \in \overline{\mathcal{U}}_i$, there exists a constant $\mathrm{y}_i \in \mathcal{Y}_i$. Define k_i as the input-output map which contains the set of all the steady-state input-output pairs $(\mathrm{u}_i, \mathrm{y}_i)$. For EIP systems, it is shown in [5] that k_i is continuous and monotonically increasing. However, EIP systems exclude some important system classes, such as integrators. As an extension, [9] introduced a refined version of EIP, called MEIP, which does not require k_i to be functions, but instead allows them to be set-valued maps (or curves in $\mathbb{R}^2$), i.e., $k_i = \{(\mathrm{u}_i, \mathrm{y}_i) : \mathrm{u}_i \in \overline{\mathcal{U}}_i\}$. In an abuse of notation and to improve clarity, we will often denote k_i by $k_i(u_i)$. To facilitate further analysis, we follow notations in Rockafellar [21] and define the set of all steady-state inputs associated to the output y_i by $k_i^{-1} = \{(\mathrm{y}_i, \mathrm{u}_i) : (\mathrm{u}_i, \mathrm{y}_i) \in k_i(\mathrm{u}_i)\}$. We similarly define the steady-state input-output map associated to each controller at edge $e \in \mathcal{E}$ by $\gamma_e \triangleq \gamma_e(\zeta_e)$. Furthermore, the relation k_i is said to be *maximally monotone* if $(\mathrm{u}_i', \mathrm{y}_i'), (\mathrm{u}_i'', \mathrm{y}_i'') \in k_i$ then either $(\mathrm{u}_i' \leq \mathrm{u}_i''$ and $\mathrm{y}_i' \leq \mathrm{y}_i'')$, or $(\mathrm{u}_i' \geq \mathrm{u}_i''$ and $\mathrm{y}_i' \geq \mathrm{y}_i'')$, and k_i is not contained in any larger monotone relation [10]. It is important to note that the relation k_i is monotone if and only if k_i^{-1} is maximally monotone [10].

Definition 1 (Maximal Equilibrium Independent Passivity [9]): A dynamical SISO system Σ_i is *maximal equilibrium independent passive (MEIP)* if there exists a maximal monotone relation $k_i \subset \mathbb{R}^2$ such that for all $(\mathrm{u}_i, \mathrm{y}_i) \in k_i$, there exists a once-differentiable and positive semi-definite storage function $S_i(x_i) : \mathcal{X}_i \to \mathbb{R}^+$ such that

$$\dot{S}_i \leq (y_i - \mathrm{y}_i)(u_i - \mathrm{u}_i). \tag{2}$$

Furthermore, it is *output-strictly MEIP* if additionally there exists a constant $\sigma_i > 0$ such that

$$\dot{S}_i \leq -\sigma_i(y_i - \mathrm{y}_i)^2 + (y_i - \mathrm{y}_i)(u_i - \mathrm{u}_i). \tag{3}$$

We now consider a diffusively-coupled network model $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ such that $u = -E\mu$, $\zeta = E^T y$. This structure is illustrated in Fig. 1, which has been of considerable interest in passivity-based cooperative control [1], [3], [9]. We denote the complete closed-loop system by the triple $(\mathcal{G}, \Sigma, \Pi)$.

We emphasize that the steady-state input-output relations of the MEIP systems are monotone [5], [9]. An interesting result about monotone relations states that the subdifferential for the closed proper convex functions on $\mathbb{R}$ are the maximal monotone relations from $\mathbb{R}$ to $\mathbb{R}$ [21]. Thus, one can associate to each steady-state monotone relation $k_i(\mathrm{u}_i)$ and $\gamma_e(\zeta_e)$ the convex functions[2] $K_i(\mathrm{u}_i)$ and $\Gamma_e(\zeta_e)$, respectively, such that $\partial K_i(\mathrm{u}_i) = k_i(\mathrm{u}_i)$ and $\partial\Gamma_e(\zeta_e) = \gamma_e(\zeta_e)$. For convenience, we consider the stack relations $k(\mathrm{u})$ and $\gamma(\zeta)$ by assimilating the $k_i(\mathrm{u}_i)$'s and $\gamma_e(\zeta_k)$'s, respectively. We also define the composite convex functions $K(\mathrm{u}) = \sum_{i=1}^{|\mathcal{V}|} K_i(\mathrm{u}_i)$ and $\Gamma(\zeta) = \sum_{e=1}^{|\mathcal{E}|}\Gamma_e(\zeta_e)$, which implies $\partial K(\mathrm{u}) = k(\mathrm{u})$ and $\partial\Gamma(\zeta) = \gamma(\zeta)$. Analogously, the convex dual functions $K^\star(\mathrm{y})$ and $\Gamma^\star(\mu)$ possess the property that $\partial K^\star(\mathrm{y}) = k^{-1}(\mathrm{y})$ and $\partial\Gamma^\star(\mu) = \gamma^{-1}(\mu)$, respectively [11].

According to [9, Th. 4.10], the dynamical network, comprising output-strictly MEIP SISO dynamical systems on the nodes, and the MEIP controllers on the edges (1), asymptotically converge to a steady-state. That is, $\lim_{t\to\infty} u \to \mathrm{u}$, $\lim_{t\to\infty} y \to \mathrm{y}$, $\lim_{t\to\infty}\zeta \to \zeta$ and $\lim_{t\to\infty}\mu \to \mu$. Furthermore, the steady-state values $\mathrm{u}, \mathrm{y}, \zeta$ and μ are the (primal-dual) solutions of the following pair of convex optimization problems:

Optimal Potential Problem		**Optimal Flow Problem**	
$\min_{\mathrm{y},\zeta}$	$K^\star(\mathrm{y}) + \Gamma(\zeta)$	$\min_{\mathrm{u},\mu}$	$K(\mathrm{u}) + \Gamma^\star(\mu)$
$s.t.$	$E^T\mathrm{y} = \zeta$	$s.t.$	$\mathrm{u} = -E\mu.$

We explore these problems in the context of PS systems in the subsequent section.

III. Motivation and Problem Formulation

In this section, we characterize passivity-short systems and formulate the main problem of this letter. Note that σ_i in (3) is referred to as the *passivity index* [7]. If $\sigma_i = 0$, the system Σ_i in (1) is MEIP. If $\sigma_i > 0$, Σ_i is said to have an *excess of passivity* or is output strictly passive. Moreover, if $\sigma_i < 0$, the system has a *shortage of passivity*, which is the main focus of this letter. It is clear from (3) that MEIP systems are a special case of PS systems. Thus, PS systems include a general class of systems. For the remainder of this letter, we make the following assumption on the dynamics in (1).

Assumption 1: Each SISO dynamical system Σ_i is passivity-short, i.e., there exists a $\sigma_i < 0$ and storage function S_i such that (3) is satisfied for each $i \in \mathcal{V}$. Moreover, we assume that the inverse steady-state relation k_i^{-1} is a function defined over $\mathbb{R}$. In this case, one can choose the integral functions for k_i^{-1} by defining $K_i^\star(\mathrm{y}) = \int_{\mathrm{y}_0}^{\mathrm{y}} k_i^{-1}(\tilde{\mathrm{y}})d\tilde{\mathrm{y}}$, so that $K_i^\star$ are differentiable and $\nabla K_i^\star = k_i^{-1}$.

The following result is a direct consequence of (3).

Proposition 1: The SISO dynamical system Σ_i, satisfying (3), is passivity-short for all steady-state pairs $(\mathrm{u}_i, \mathrm{y}_i) \in k_i$, if and only if there exists a constant $\sigma_i < 0$ such that it is passive with respect to the augmented input $\tilde{\mathrm{u}}_i = \mathrm{u}_i - \sigma_i\mathrm{y}_i$. That is, Σ_i is passive with respect to the pair $(\tilde{\mathrm{u}}_i, \mathrm{y}_i)$.

Proof: Rearranging terms in (3), we obtain $\dot{S}_i \le (y_i - \mathrm{y}_i)(\tilde{u}_i - \tilde{\mathrm{u}}_i)$, where $\tilde{u}_i = u_i - \sigma_i y_i$ and $\tilde{\mathrm{u}}_i = \mathrm{u}_i - \sigma_i\mathrm{y}_i$. ■

Proposition 1 means there exists a $\sigma_i > 0$ and a storage function S_i, $\forall i \in \mathcal{V}$, such that $\dot{S}_i = \nabla S_i^T \cdot f_i(x_i, \tilde{u}_i) \le (y_i - \mathrm{y}_i)(\tilde{u}_i - \tilde{\mathrm{u}}_i)$, with respect to the input transformation $\tilde{u}_i = u_i - \sigma_i y_i$. For the linear systems, this leads to the celebrated KYP property for passive systems. We do not focus on these aspects in this letter. A detailed description can be found in [14], [20], and [22].

One can notice that, unlike EIP systems in [5] and [9], the steady-state input-output relations of the PS systems are not necessarily monotone or co-coercive, which can be readily seen by the following example.

Example 1: Consider a SISO dynamical system $\dot{x} = -x + \sqrt[3]{x} + u;\ y = \sqrt[3]{x}$. Let (u, y) be some equilibrium input-output pair and note that x must converge to y^3, where $\mathrm{u} = \mathrm{y}^3 - \mathrm{y}$. Consider the storage function $S(x) = \frac{3}{4}x^{\frac{4}{3}} - \mathrm{y}x + \frac{1}{4}\mathrm{y}^4$.[3] We have:

$$\begin{aligned}\dot{S} &= \frac{dS}{dx}\dot{x} = (\sqrt[3]{x} - \mathrm{y})(-x + \sqrt[3]{x} + u)\\ &= (y - \mathrm{y})(u - \mathrm{u}) + (y - \mathrm{y})^2 - (y - \mathrm{y})(y^3 - \mathrm{y}^3)\\ &\le (y - \mathrm{y})(u - \mathrm{u}) + (y - \mathrm{y})^2,\end{aligned}$$

where we use the fact that the function $\sqrt[3]{\cdot}$ is monotone. The inequality implies that this system is passivity-short with passivity index $\sigma = -1$. Moreover, the steady-state input-output relation $\mathrm{u} = k^{-1}(\mathrm{y}) = \mathrm{y}^3 - \mathrm{y}$ is not monotone, and therefore k is not monotone. The integral function associated with this SISO dynamical system is $K^\star(\mathrm{y}) = \frac{1}{4}\mathrm{y}^4 - \frac{1}{2}\mathrm{y}^2$, which is non-convex.

We now continue this example to provide the main motivation for this letter. In this direction, we note that the integral function $K^\star(\mathrm{y}) = \frac{1}{4}\mathrm{y}^4 - \frac{1}{2}\mathrm{y}^2$ can be *convexified* by adding the regularization term $\frac{1}{2}\mathrm{y}^2$. That is, the function $\tilde{K}^\star(\mathrm{y}) = K^\star(\mathrm{y}) + \frac{1}{2}\mathrm{y}^2$ is strictly convex. One may ask, therefore, if there exist some loop-transformation for the original system that leads to an input-output relation corresponding the function $\tilde{K}^\star(\mathrm{y})$.

Example 1 (Continued): Consider now a feedback linearization of the system defined earlier, with $u = v - \sqrt[3]{x}$, where v is the new input to the system. The closed-loop system is now $\dot{x} = -x + v,\ y = \sqrt[3]{x}$ and the input-output relation is $\mathrm{v} = \mathrm{y}^3$. The corresponding integral function is precisely $\tilde{K}^\star(\mathrm{y})$. The feedback linearized system can be shown to be MEIP with the same storage function defined earlier.

This example presents a unique phenomenon. In general, the integral functions associated to PS systems are non-convex, which is problematic for proving convergence of the closed-loop network using the framework presented in Section II. However, in the case of Example 1, convexifying the integral function corresponds to a feedback that passified the system. In the sequel, we formally examine the implications of regularizing the network optimization problems associated with the system $(\mathcal{G}, \Sigma, \Pi)$ in the case the agents Σ_i are passivity-short. Specifically, we look into the following problems in this letter.

Problem 1: What are the conditions such that the regularized network optmization problems are convex and there exists a steady-state solution to the network in Fig. 1.

Problem 2: What is the equivalence, if one exists, between the regularization term and a passifying loop transformation for each PS system Σ_i.

[2]We also refer to these as the *integral functions*. Note that these integral functions define a family of convex functions, and are not unique [9], [11].

[3]This storage function is derived using the *Bregman divergence* $S(x) = V(x) - V(\mathrm{x}) - \nabla V^T(\mathrm{x})(x - \mathrm{x})$ with $V(x) = \frac{3}{4}x^{\frac{4}{3}}$, which is convex $\forall x$.

IV. Regularization and Feedback Passivation

As seen in Example 1, the integral functions associated to a general PS system are not necessarily convex and hence a steady-state solution of their network-level interaction may not exist. In this section, we show that introducing a regularzation term to the network optimization problems is equivalent to passifying the individual agents and recovering the MEIP property, thus ensuring converngence of the closed-loop system.

In this direction, consider the optimal potential problem (OPP) of Section II with a Tikhonov type regularization term $\frac{1}{2}\|y\|_{\beta}^2 = \frac{1}{2}\sum_{i=1}^{|\mathcal{V}|}\beta_i y_i^2$ [23],

$$\begin{aligned}\min_{\mathrm{y},\zeta}\ & K^\star(\mathrm{y}) + \Gamma(\zeta) + \frac{1}{2}\mathrm{y}^T\mathrm{diag}(\beta)\mathrm{y}\\ s.t.\ & E^T\mathrm{y} = \zeta,\end{aligned} \tag{ROPP}$$

where $\beta = [\beta_1, \dots, \beta_{|\mathcal{V}|}]^T$ is a design parameter and can be appropriately chosen such that the Regularized Optimal Potential Problem (ROPP) is convex. Later, we provide condition on β_i in terms of the system parameters to ensure this requirement. In this letter, we restrict ourselves to PS systems on the nodes and assume that the controllers are already MEIP. Let us denote

$$\Lambda^\star(\mathrm{y}) = K^\star(\mathrm{y}) + \frac{1}{2}\mathrm{y}^T\mathrm{diag}(\beta)\mathrm{y} \tag{4}$$

as the augmented integral function of the (ROPP). We now show in the following theorem that the regularization of this type is equivalent to feedback passivation of the PS systems by an individual output-feedback, as seen in Example 1.

Theorem 1: Consider the SISO dynamical system Σ_i and suppose that Assumption 1 is satisfied. Let $\Lambda^\star(\mathrm{y})$, given by (4), be the augmented integral function of the (ROPP). Then the output-feedback control of the form

$$u_i = v_i - \beta_i y_i, \tag{5}$$

with new exogenous input v_i for each $i \in \mathcal{V}$, gives rise to the integral function $\Lambda^\star(\mathrm{y})$, in the sense of Assumption 1.

Proof: Note that $\Lambda^\star(\mathrm{y})$ is a differentiable function. Thus we can define an input-output relation λ_i such that, for each $i \in \mathcal{V}$, $\partial\Lambda_i^\star(\mathrm{y}_i) = \lambda_i^{-1}(\mathrm{v}_i)$ for any v_i, where λ_i^{-1} is the inverse relation of λ_i. Consequently, it follows from (4) that

$$\lambda_i^{-1}(\mathrm{y}_i) = k_i^{-1}(\mathrm{y}_i) + \beta_i \mathrm{y}_i, \tag{6}$$

where k_i^{-1} is given according to Assumption 1. If $(\mathrm{u_i}, \mathrm{y_i})$ is a steady-state input-output pair of Σ_i, and $\mathrm{v}_i = \lambda^{-1}(\mathrm{y}_i)$, then we get from (6) that in steady-state,

$$\mathrm{v}_i = \mathrm{u}_i + \beta_i \mathrm{y}_i, \tag{7}$$

holds for each node i, meaning that $(\mathrm{v_i}, \mathrm{y_i})$ is a steady-state input-output pair of the system with output-feedback control as in (5). This completes the proof. ∎

We now consider the new system obtained by implementing the feedback (5). Under control (5), each dynamical system Σ_i becomes individually compensated to

$$\hat{\Sigma}_i : \begin{cases} \dot{x}_i = f_i(x_i, v_i - \beta_i y_i) \triangleq f_i^c(x_i, v_i),\\ y_i = h_i(x_i), \quad i \in \mathcal{V},\end{cases} \tag{8}$$

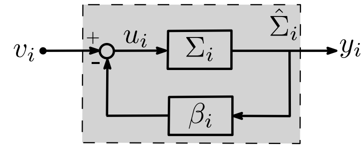


Fig. 2. The i^{th} closed loop dynamical system.

which has λ_i as the steady-state input-output relation. This configuration is illustrated in Fig. 2. Note that β_i is the feedback gain and v_i is the new network-level control input.

It remains to realize the passivity properties of the individual closed loop system $\hat{\Sigma}_i$ with respect to new set of steady-state input-output signals $(\mathrm{v}_i, \mathrm{y}_i)$. We now show in the following theorem that the closed loop system $\hat{\Sigma}_i$ is MEIP if $\beta_i > |\sigma_i|$. Consequently, the regularization term $\frac{1}{2}\mathrm{y}^T\beta\mathrm{y}$ convexifies the integral function (4).

Theorem 2: Consider the SISO dynamical system Σ_i with steady-state input-output relation k_i and suppose that Assumption 1 is satisfied. Then for any $\beta_i > |\sigma_i|$, the feedback (5) defines a new dynamical system $\hat{\Sigma}_i$ with steady-state input-output relation λ_i, which is output-strictly maximally monotone equilibrium-independent passive.

Proof: Substituting (5) and (7) in (3), yields

$$\dot{S}_i \leq -\rho_i(y_i - \lambda_i(\mathrm{v}_i))^2 + (y_i - \lambda_i(\mathrm{v}_i))(v_i - \mathrm{v}_i), \tag{9}$$

where $\rho_i = \beta_i + \sigma_i > 0$ since $\beta_i > |\sigma_i|$. Thus, the new dynamical system $\hat{\Sigma}_i$ is output-strictly equilibrium-independent passive with respect to any steady-state input-output pair it possesses, so it is output-strictly maximally monotone equilibrium-independent passive if and only if its input-output relation, λ_i, is maximally monotone, which we prove next.

In particular, for v_i equal to another arbitrary constant $\hat{\mathrm{v}}_i$ and $y_i = \hat{\mathrm{y}}_i = \lambda_i(\hat{\mathrm{v}}_i)$, it follows from (9) that λ_i is a monotone relation, so we only need to consider its maximality. Rockafellar's Theorem [24] implies that λ_i is contained in the subdifferential of some convex function P_i, meaning that λ_i^{-1} is contained in the subdifferential of the convex function $P_i^\star$. But $\lambda_i^{-1} = \nabla\Lambda_i^\star = \partial\Lambda_i^\star$, so uniqueness of the subgradient implies that $\Lambda_i^\star = P_i^\star$ up to an additive constant. In that case, $\Lambda_i^\star$ must be also convex, so $\lambda_i^{-1} = \partial\Lambda_i^\star$ implies that λ_i^{-1}, and hence λ_i, is actually *maximally* monotone. This concludes the proof. ∎

According to [9] and also mentioned previously, the network comprising output-strictly MEIP agents at nodes and MEIP controllers on the edges asymptotically converges, and the steady-state signals relate to dual "convex" network optimization problems. In the spirit of this result, one can conclude from the above discussion that Tikhonov-type regularization term $\frac{1}{2}\mathrm{y}^T\beta\mathrm{y}$ convexifies the (OPP) (leading to (ROPP)) and assures a solution of the closed-loop model if $\beta_i > |\sigma_i|, \forall i \in \mathcal{V}$, that is, the control gain β_i is strictly greater than the absolute value $|\sigma_i|$ of the passivity index σ_i.

Based on the above results, we now obtain in the following corollary the dual Regularized Optimal Flow Problem (ROFP).

Corollary 1: Consider SISO dynamical system Σ_i and suppose that Assumption 1 is satisfied. Let us assume that there exists a steady-state output-feedback control (7) according to

Theorem 1 such that $\beta_i > |\sigma_i|, \forall i \in \mathcal{V}$. Then the (ROFP) associated to networks of passivity-short systems is given by

$$\begin{aligned} \min_{\mathrm{u},\mu} \quad & K(\mathrm{u}) + \Gamma^\star(\mu) + F(\mathrm{u},\mathrm{y}) \\ s.t. \quad & \mathrm{v} = -E\mu, \end{aligned} \tag{ROFP}$$

where $F(\mathrm{u},\mathrm{y}) = \Lambda(\mathrm{u}+\mathrm{diag}(\beta)\mathrm{y}) - K(\mathrm{u}) - \mathrm{y}^{\mathrm{T}}\mathrm{diag}(\beta)\mathrm{y}$ is the regularization term.

Proof: Since the (ROPP) and (ROFP) are dual optimization problems associated to systems $\hat{\Sigma}_i$, the objective function of the (ROFP) is given by $\Lambda(\mathrm{v}) + \Gamma^\star(\mu)$. Moreover, since the steady-state equilibrium trajectory y is an optimal multiplier for the dual optimization problem [9], the Lagrangian of the (ROFP) with optimal Lagrange multiplier y is given by

$$\mathcal{L}(\mathrm{v},\mu,\mathrm{y}) = \Lambda(\mathrm{v}) + \Gamma^\star(\mu) + \mathrm{y}^T(-\mathrm{v} - E\mu). \tag{10}$$

According to Theorem 1, substituting (7) in terms of vector notation as $\mathrm{v} = \mathrm{u} + \mathrm{diag}(\beta)\mathrm{y}$, we have

$$\begin{aligned} \mathcal{L}(\mathrm{u},\mu,\mathrm{y}) = \; & \Lambda(\mathrm{u}+\mathrm{diag}(\beta)\mathrm{y}) + \Gamma^\star(\mu) \\ & - \mathrm{y}^T(\mathrm{u} + \mathrm{diag}(\beta)\mathrm{y} + E\mu). \end{aligned}$$

Let us write $\Lambda(\mathrm{u}+\mathrm{diag}(\beta)\mathrm{y}) = K(\mathrm{u}) + (\Lambda(\mathrm{u}+\mathrm{diag}(\beta)\mathrm{y}) - K(\mathrm{u}))$, we have

$$\mathcal{L}(\mathrm{u},\mu,\mathrm{y}) = K(\mathrm{u}) + \Gamma^\star(\mu) + \mathrm{y}^T(-\mathrm{u} - E\mu) + F(\mathrm{u},\mathrm{y}), \tag{11}$$

where $F(\mathrm{u},\mathrm{y}) = \Lambda(\mathrm{u}+\mathrm{diag}(\beta)\mathrm{y}) - K(\mathrm{u}) - \mathrm{y}^{\mathrm{T}}\mathrm{diag}(\beta)\mathrm{y}$ is the regularization term since the first three terms on the RHS of (11) relates to the PS systems with structural property $u + E\mu = 0$. This complies the proof. ■

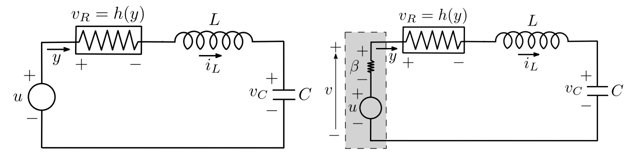
Next, we present a network-level perspective of the above results. One can obtain the regularized network, as in Fig. 1, by replacing each Σ_i by $\hat{\Sigma}_i$ and u by v. The steady-states $(\mathrm{v},\mathrm{y},\zeta,\mu)$ solve the (ROPP) and (ROFP) for some $\beta_i > |\sigma_i|, \forall i \in \mathcal{V}$. We conclude by stating this in the following theorem.

Theorem 3: Consider the network system $(\mathcal{G},\Sigma,\Pi)$, suppose that Assumption 1 holds, and that the controllers are MEIP with integral functions Γ_e. Then for any $\beta_i > |\sigma_i|$, the augmented systems $\hat{\Sigma}_i$ achieved by the output-feedback controller (5) are MEIP. Moreover, the closed loop system with agents $\hat{\Sigma}_i$ and controllers Π_e converges, and the steady-state limit $(\mathrm{v},\mathrm{y},\zeta,\mu)$ is the minimizer of the following network optimization problems:

ROPP		**ROFP**	
$\min_{\mathrm{y},\zeta}$	$\Lambda^\star(\mathrm{y}) + \Gamma(\zeta)$	$\min_{\mathrm{v},\mu}$	$\Lambda(\mathrm{v}) + \Gamma^\star(\mu)$
$s.t.$	$E^T\mathrm{y} = \zeta$	$s.t.$	$\mathrm{v} = -E\mu$

where the function Λ is defined as $\sum_i \Lambda_i(y_i)$ and $\Lambda_i^\star(\mathrm{y}) = K_i^\star(\mathrm{y}) + \frac{\beta_i}{2}\mathrm{y}_i^2$.

Remark 1: Note that, for the special case where $\beta_i = |\sigma_i|$, new systems $\hat{\Sigma}_i$ recover EIP properties and the mapping λ_i is monotonically increasing as in [5].



(a) (b)

Fig. 3. RLC circuit. (a) Passivity-short circuit (b) MEIP circuit.

V. Case Studies

We now illustrate the results with two motivating examples. We first consider an RLC circuit with a sector-bounded nonlinear resistor, and then study a traffic dynamic network model.

A. An RLC Circuit With Nonlinear Resistor

Consider an RLC circuit as shown in Fig. 3(a), where a voltage source u is connected with a linear inductor L, a linear capacitor C, and a nonlinear resistor, represented by its i-v characteristic $v_R = h(i_L)$, where the nonlinearity $h(i_L)$ belongs to the sector $[\sigma,\infty]$ with $\sigma < 0$. By taking the current $x_1 = i_L$ and the voltage $x_2 = v_c$ as the state variables, we can write the state model as

$$L\dot{x}_1 = -x_2 - h(x_1) + u; \quad C\dot{x}_2 = x_1; \quad y = x_1.$$

The equilibrium states of this system are $(\mathrm{x}_1,\mathrm{x}_2) = (0,\mathrm{u}), \mathrm{u} \in \mathbb{R}$, and the steady-state input-output relation is given by $k_y = \{(\mathrm{u},0) : \mathrm{u} \in \mathbb{R}\}$, which, in contrast to [5], is not a single valued function. Moreover, the system is equilibrium independent PS with the storage function $S(x) = (1/2)L(x_1 - \mathrm{x}_1)^2 + (1/2)C(x_2 - \mathrm{x}_2)^2$. This is verified by noting that $\dot{S} \le (u - \mathrm{u})(y - \mathrm{y}) - \sigma(y - \mathrm{y})^2$ since $\mathrm{y} = 0$ and $y(h(y) - \sigma y) \ge 0$ due to $h \in [\sigma,\infty]$ [25]. The corresponding integral function of the (OPP) is given by $K^\star(\mathrm{y}) = \mathrm{uy}, \mathrm{u} \in \mathbb{R}$, which is the power input to the circuit. As discussed above, let us regularize this function by $\beta\mathrm{y}^2$ and denote the regularized objective function $\Lambda^\star(y) = \mathrm{uy} + \beta\mathrm{y}^2$, which is convex in y. Physically, it implies that an extra energy dissipation by an amount of βy^2 is required to achieve the convexification of the (ROPP). This suggests the feedback control $u = v - \beta y \implies v = u + \beta y$, and with $\beta > |\sigma|$, the circuit possess MEIP properties. In other words, a linear resistive element $\beta > |\sigma|$ is required to passify the system as shown in Fig. 3(b). Obviously, the equilibrium input-output relation of the MEIP system in Fig. 3(b) is given by $\lambda_y = \{(\mathrm{v},0) : \mathrm{v} \in \mathbb{R}\}$.

B. Traffic Dynamics Network

We consider an optimal velocity model of the traffic dynamics of vehicles as proposed in [26], according to which, each vehicle adjust its velocity x_i as:

$$\dot{x}_i = \kappa_i[V_i(\Delta p) - x_i], \tag{12}$$

where, κ_i is a non-zero constant representing the driver's sensitivities and the adjustment $V_i(\Delta p)$ depends on the relative position $\Delta p = p_j - p_i$, as

$$V_i(\Delta p) = V_i^0 + V_i^1 \sum_{j\in\mathcal{N}(i)} \tanh(p_j - p_i), \tag{13}$$

where, the constants V_i^0 are "preferred velocities" and $V_i^1 = 1/\kappa_i$ are "sensitivity coefficients". In the form of (1), the agent dynamics can be identified as

$$\dot{x}_i = \kappa_i[-x_i + V_i^0 + V_i^1 u_i], \quad y_i = x_i, \tag{14}$$

where, u_i is the control input. Note that the sensitivities κ_i are heterogeneous and could be positive or negative depending on *attentiveness* of individual driver. The case of positive sensitivities, i.e., $\kappa_i > 0$, was studied in [9] and it has been shown that the system (14) possess passivity properties and hence stable and exhibit a clustering behavior. However, we consider the case that the sensitivities κ_i are negative for some of the drivers (one of the reasons for this could be *drowsy driving*). In this situation, the system (14) is PS with the storage function $S_i(x_i) \triangleq \frac{1}{2}(x_i - \mathrm{x}_i)^2$ (alternatively, one can also utilize Proposition 1 to check this fact) for some drivers, and hence it is not possible to ensure a stable traffic flow network. Regularization suggests the existence of the feedback control $u_i = v_i - \beta_i y_i$, where v_i is the new control input. The modified dynamics of each node is given by $\dot{x}_i = \kappa_i[-(1+\beta_i V_i^1)x_i + V_i^0 + V_i^1 v_i]$, which is MEIP with the earlier storage function provided $\beta_i > |\kappa_i|$. The input-output relation of the new dynamics is given by $\lambda_i(\mathrm{v}_i) = V_i^0 + V_i^1 \mathrm{v}_i$. The associated integral functions are

$$\Lambda_i(\mathrm{v}_i) = \frac{V_i^1}{2}\mathrm{v}_i^2 + V_i^0 \mathrm{v}_i \quad \text{and} \quad \Lambda_i^\star(\mathrm{y}_i) = \frac{1}{2V_i^1}(\mathrm{y}_i - V_i^0)^2.$$

Substituting v_i in terms of the original input u_i, the objective function $\Lambda_i(\mathrm{v}_i)$ becomes

$$\Lambda_i(\mathrm{v}_i) = \frac{V_i^1}{2}(\mathrm{u}_i + \beta_i \mathrm{y}_i)^2 + V_i^0(\mathrm{u}_i + \beta_i \mathrm{y}_i),$$

which can be re-written as $\Lambda_i(\mathrm{v}_i) \triangleq K_i(\mathrm{u}_i) + \beta_i V_i^1 \mathrm{u}_i \mathrm{y}_i + \frac{V_i^1}{2}\beta_i^2 \mathrm{y}_i^2 + \beta_i V_i^0 \mathrm{y}_i$, where $K_i(\mathrm{u}_i) = \frac{V_i^1}{2}\mathrm{u}_i^2 + V_i^0 \mathrm{u}_i$. Following Corollary 1, in this case, we get the regularizing function as

$$F(\mathrm{u}, \mathrm{y}) = \left(\frac{V_i^1}{2}\beta_i^2 - \beta_i\right)\mathrm{y}_i^2 + \beta_i V_i^0 \mathrm{y}_i + \beta_i V_i^1 \mathrm{u}_i \mathrm{y}_i,$$

for the (ROFP). Now, the regularized systems are ready for network operation as in Fig. 1, where $v_i = \sum_{j \in \mathcal{N}(i)} \tanh(p_j - p_i)$, and the relative velocities of the neighbouring agents are $\zeta(t) = E^T y$. Since $\dot{p}_i = x_i$, we define relative positions of the neighbouring vehicles as $\eta_e(t) = p_j - p_i$, where e connects node i and j. Thus, the controller dynamics can be represented, in vector notation, as $\dot{\eta} = \zeta, \quad \mu = \tanh(\eta)$ and $u = -E\mu$, where $\tanh(\cdot)$ is vector valued function [9].

VI. Conclusion

We investigated the networked duality properties for the passivity-short cooperative control systems. We proposed Tikhonov-based regularized network flow problems to account for the shortage of passivity. It was proved that the proposed regularization is equivalent to the feedback passivation of each heterogeneous PS system by a self output-feedback control. Furthermore, we have shown that the regularized network optimization problems are convex and the closed-loop network possess MEIP properties if the control gain of individual output-feedback is strictly greater than the absolute value of the passivity index.

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