Bearing-only Cyclic Pursuit in 2-D for Capture of Moving Target

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The present paper studies the stability of formations around a moving target using bearing-only measurements for agents in cyclic pursuit. A control law is proposed for every agent using bearing information of its leader and the target and also the target's velocity. This control law is proved to be locally asymptotically stable with respect to a desired arbitrary feasible formation around the target in the two-dimensional space. A detailed stability analysis of the equilibrium formations is also carried out. Simulations back up the theoretical results.

I. Introduction

Capturing a moving target co-operatively is a challenging task for any multi-agent system. The problem has been looked at from various perspectives, such as locating non-adversarial targets in a search space [1], assignment of targets to agents or vice versa [2], tracking a target, using a potential function, while avoiding obstacles at the same time [3], dynamic coalition formation for co-operative tracking of trajectories [4], and several other paradigms [5–10]. This paper casts the problem of capturing a moving target as one of obtaining a desired formation around the target, while the agents are in cyclic pursuit. This is in the same spirit as some earlier work [11–14]. Moreover, it is assumed that only information about the bearing and target velocity is available to each agent.

Cyclic pursuit is a well known strategy for multi-agent systems, with n agents, where each agent, indexed i, receives information about its leader, agent $i+1$ (modulo n), and chooses its control law based on this information. In graph theoretic terms, this implies that cyclic pursuit can be represented by a directed cycle graph, whose nodes are the agents and the directed edges depict the information flow, as shown in Fig. 1. In

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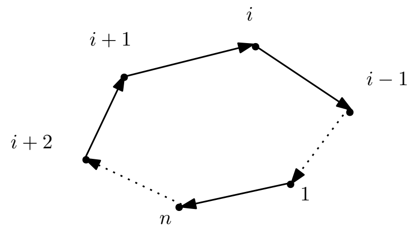


Figure 1: Information flow in cyclic pursuit.

studies pertaining to the consensus problem, cyclic pursuit has been well studied over a considerable period [15–20]. This has further resulted in several applications of cyclic pursuit, such as target capture [11, 12, 14, 21, 22], boundary tracking [23] and vehicular formations [18, 24, 25] emerging, with promising results. Inspired by these findings, this paper looks into co-operative capture of a target by agents in cyclic pursuit. As mentioned earlier, here the problem is addressed by designing control laws that result in a desired formation of vehicles around the moving target. Thus, the problem is essentially one of formation control.

Formation control is a widely investigated problem in the domain of multi-agent systems ([26–30] and the references therein). Proceeding along similar lines, this paper considers a bearing-only formation control around a target, while the agents are in cyclic pursuit. In recent years, bearing-only formation control has received significant attention from researchers [31–37]. Most of them consider an undirected communication topology, while using bearing-only information. Ref. [34] considered a directed graph for exchanging bearing information and sufficient conditions for the stability of formations were derived. However, information about relative distances was also used therein. Hence, stable bearing-only formation control over agents communicating via general digraphs is still an open problem. To address this problem partially, this paper considers bearing-only formation control over a particular directed topology− the cycle digraph.

The main focus in this work is to study the possibility of achieving *any* desired formation shape around a target, under the bearing-only cyclic pursuit paradigm. Thus, the present paper adds to the body of existing literature along two specific directions. Firstly, a particular directed information exchange topology (directed cycle) is used to study the stability of desired formations in $\mathbb{R}^2$ and secondly, the cyclic pursuit problem is cast with bearing-only measurements and tailored for capturing a moving target. It may be pointed out that formations of planar bearing-only cyclic pursuit have been recently studied in [38], and the problem of capturing a stationary target was considered in [39].

This paper is organized in the following manner. Section II presents some preliminary results on bearing-only formation and leads to the formulation of the problem. In Section III the main stability results, pertaining to capture of a moving target using bearing-only formation control for agents in cyclic pursuit, are provided. Simulations vindicate the theoretical results in Section IV. Section V concludes the paper by suggesting avenues for future work.

II. Problem Formulation

Consider a group of n agents, modeled by single integrator dynamics, given by

$$\dot{\mathbf{p}}_i = \mathbf{u}_i, \tag{1}$$

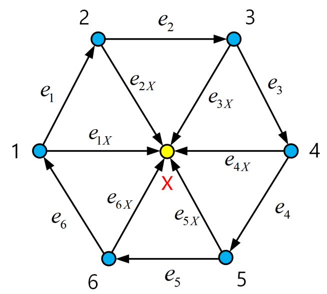


Figure 2: A group consisting of six agents and a target $\mathbf{X}$.

where, $\mathbf{p}_i, \mathbf{u}_i \in \mathbb{R}^2$ are the position and the control input of agent i, $i = 1, \ldots, n$, respectively. The agent indices are in modulo n throughout this paper. Suppose there is a target $\mathbf{X}$, whose position is given by $\mathbf{p}_X \in \mathbb{R}^2$ and it has a velocity given by $\dot{\mathbf{p}}_X = \mathbf{v}_T$. Fig. 2 shows the pursuit graph for a cyclic pursuit system with six agents and a target $\mathbf{X}$, at some instant in time. Let $\mathbf{p} = [\mathbf{p}_1^T, \ldots, \mathbf{p}_n^T]^T \in \mathbb{R}^{2n}$ represent the vector containing the positions of the agents in $\mathbb{R}^2$. Define the displacement vectors $\mathbf{z}_i = \mathbf{p}_{i+1} - \mathbf{p}_i$ and $\mathbf{z}_{iX} = \mathbf{p}_X - \mathbf{p}_i$, $i = 1, \ldots, n$. Denote $d_i = \|\mathbf{z}_i\|$ and $d_{iX} = \|\mathbf{z}_{iX}\|$ as the distance from agent i to agent $i+1$ and that between agent i and the target $\mathbf{X}$, respectively. Other inter-agent distances are given by $d_{ij} = ||\mathbf{p}_j - \mathbf{p}_i||$, $j \neq i+1, i-1$.

The bearing vectors $\mathbf{g}_i$ and $\mathbf{g}_{iX}$ are defined as the unit vectors directed from agent i to agent $i+1$, and from agent i to the target $\mathbf{X}$, given by:

$$\mathbf{g}_i = \frac{\mathbf{p}_{i+1} - \mathbf{p}_i}{\|\mathbf{p}_{i+1} - \mathbf{p}_i\|} = \frac{\mathbf{z}_i}{\|\mathbf{z}_i\|}, \ \mathbf{g}_{iX} = \frac{\mathbf{p}_X - \mathbf{p}_i}{\|\mathbf{p}_X - \mathbf{p}_i\|} = \frac{\mathbf{z}_{iX}}{\|\mathbf{z}_{iX}\|}. \tag{2}$$

Next, it is assumed that the system of $n-$agents and the target satisfies the following conditions:

Assumption 1. *Every agent has access to a global reference frame in $\mathbb{R}^2$. The positions of the agents, $\mathbf{p}_i \in \mathbb{R}^2$, are initially non-collocated, i.e., $\mathbf{p}_i(0) \neq \mathbf{p}_j(0)$, for all $1 \leq i \neq j \leq n$.*

Assumption 2. *Each agent i senses the bearing vectors with respect to agent $i+1$ and the target $\boldsymbol{X}$. Thus the sensing topology of the agents is a directed cycle graph with n nodes and an additional node whose information is sensed by all other nodes. Additionally, the target's velocity is available to every agent in the group.*

Assumption 2 implies that the overall graph may be represented as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, \ldots, v_n, v_X\}$, $|\mathcal{V}| = n+1$, $\mathcal{E} = \{(v_i, v_{i+1}), (v_i, v_X) | i \in \mathcal{V}\}$ and $|\mathcal{E}| = 2n$. Based on the assumptions stated above, a feasible bearing vector set can now be defined.

Definition 1. *The set $\mathcal{B}_n = \{\mathbf{g}_i^*, \mathbf{g}_{iX}^*\}_{i=1,\ldots,n}$ is called a feasible bearing vector set if and only if the following conditions hold $\forall i$:*

(a) $\mathbf{g}_i^* \neq \pm\mathbf{g}_{i+1}^*$, $\mathbf{g}_i^* \neq \pm\mathbf{g}_{iX}^*$, $\mathbf{g}_{i-1}^* \neq \pm\mathbf{g}_{iX}^*$, and there exist positive scalars d_i^* such that $\sum_{i=1}^n d_i^* \mathbf{g}_i^* = \mathbf{0}$, and

(b) positive scalars d_{iX}^* exist such that $d_i^* \mathbf{g}_i^* - d_{iX}^* \mathbf{g}_{1X}^* + d_{i+1,X}^* \mathbf{g}_{i+1,X}^* = \mathbf{0}$.

The condition $\sum_{i=1}^{n} d_i^* \mathbf{g}_i^* = \mathbf{0}$ guarantees that the desired formation shape is a closed polygon. This is because the vector $d_i^* \mathbf{g}_i^*$ is an edge of the desired polygon joining the positions of agent i and $i+1$. Also, the desired formation is such that every agent's position forms the vertex of a polygon. Hence, a necessary condition for this to hold is that no three successive agents $i-1$ through $i+1$ are collinear. Neither is the desired formation such that any two successive agents i and $i+1$ are collinear with the target $\mathbf{X}$. Hence, $\mathbf{g}_{i+1,X}^* \neq \pm \mathbf{g}_{iX}^*$. Part (b) of Definition 1 ensures that every agent i, its leader $i+1$ and the target $\mathbf{X}$ form a triangle. For brevity, all bearing vectors are stacked in a column vector as $\mathbf{g} = [\mathbf{g}_1^T, \dots, \mathbf{g}_n^T, \mathbf{g}_{1X}^T, \dots, \mathbf{g}_{nX}^T]^T \in \mathbb{R}^{4n}$ and all the desired bearing vectors are stacked in a column vector as $\mathbf{g}^* = [\mathbf{g}_1^{*T}, \dots, \mathbf{g}_n^{*T}, \mathbf{g}_{1X}^{*T}, \dots, \mathbf{g}_{nX}^{*T}]^T$.

The definitions of bearing equivalency and bearing congruency, as stated by [35], aid in establishing the relationship between bearing congruency and bearing equivalence for the problem addressed in this paper. Hence, they will be revisited here for the sake of completeness. However, to understand these definitions, first a closer look needs to be taken at the orthogonal projection matrix $\mathbf{P}_{\mathbf{w}} = I_2 - \frac{\mathbf{w}\mathbf{w}^{\mathbf{T}}}{\mathbf{w}^T\mathbf{w}} \in \mathbb{R}^{2\times 2}$, for a given vector $\mathbf{w} \in \mathbb{R}^2$. The projection matrix $\mathbf{P}_{\mathbf{w}}$ satisfies the following properties: it is symmetric, positive semidefinite, and idempotent, that is, $\mathbf{P}_{\mathbf{w}}^2 = \mathbf{P}_{\mathbf{w}}$. Moreover, the projection matrix is singular and $\mathbf{P}_{\mathbf{w}}\mathbf{w} = \mathbf{0}$. Further, if $\mathbf{w}^\perp$ is a unit vector orthogonal to $\mathbf{w}$, then it follows that $\mathbf{P}_{\mathbf{w}}\mathbf{w}^\perp = \mathbf{w}^\perp$, implying that 0 and 1 are the two eigenvalues of $\mathbf{P}_{\mathbf{w}} \in \mathbb{R}^{2\times 2}$ with $\mathbf{w}$ and $\mathbf{w}^\perp$ being the corresponding eigenvectors. Thus, the nullity of the projection matrix $\mathbf{P}_{\mathbf{w}}$ is unity. Also, both the right and the left null spaces are spanned by $\mathbf{w}$.

Definition 2 ([35])**.** *(Bearing Equivalency) Frameworks $\mathcal{G}(\mathbf{p})$ and $\mathcal{G}(\mathbf{p}')$ are* bearing equivalent *if $\mathbf{P}_{(\mathbf{p}_i - \mathbf{p}_j)}(\mathbf{p}_i' - \mathbf{p}_j') = \mathbf{0}$ for all $(i,j) \in \mathcal{E}$.*

Definition 3 ([35])**.** *(Bearing Congruency) Frameworks $\mathcal{G}(\mathbf{p})$ and $\mathcal{G}(\mathbf{p}')$ are* bearing congruent *if $\mathbf{P}_{(\mathbf{p}_i - \mathbf{p}_j)}(\mathbf{p}_i' - \mathbf{p}_j') = \mathbf{0}$ for all $i,\ j \in \mathcal{V}$.*

Lemma 1. *Under the assumptions **A1**-**A2**, given two formations $\mathcal{G}(\mathbf{p})$ and $\mathcal{G}(\mathbf{p}')$ with the graph as described in **A2**, if $\mathcal{G}(\mathbf{p})$ and $\mathcal{G}(\mathbf{p}')$ are bearing equivalent, they are also bearing congruent. Moreover, $d_{ij}/d_{ij}' = \eta \in \mathbb{R}$, for all $i, j \in \mathcal{V}$, $i \neq j$.*

Proof. Let $\mathcal{G}(\mathbf{p})$ and $\mathcal{G}(\mathbf{p}')$ be bearing equivalent. Then $\mathbf{g}_i^* = \mathbf{g}_i'^*$ and $\mathbf{g}_{iX}^* = \mathbf{g}_{iX}'^*$ for all $i = 1, \dots, n$. Note that

$$d_i \mathbf{g}_i^* - d_{iX}\mathbf{g}_{iX}^* + d_{i+1,X}\mathbf{g}_{i+1,X}^* = \mathbf{0}, \tag{3}$$

$$d_i' \mathbf{g}_i'^* - d_{iX}'\mathbf{g}_{iX}'^* + d_{i+1,X}'\mathbf{g}_{i+1,X}'^* = \mathbf{0}. \tag{4}$$

Further, since $\mathbf{g}_i^* = \mathbf{g}_i'^*$, $\mathbf{g}_{iX}^* = \mathbf{g}_{iX}'^*$, $\mathbf{g}_{i+1,X}^* = \mathbf{g}_{i+1,X}'^*$ and $\mathbf{g}_i^* \neq \pm\mathbf{g}_{iX}^*$, $\mathbf{g}_i^* \neq \pm\mathbf{g}_{i+1,X}^*$, it is apparent that:

$$\frac{d_i'}{d_i} = \frac{d_{iX}'}{d_{iX}} = \frac{d_{i+1,X}'}{d_{i+1,X}} = \eta, \quad \forall i = 1, \dots, n. \tag{5}$$

The relation (5) follows from applying to (3)-(4) the fact that in $\mathbb{R}^2$, any vector can be uniquely represented as a linear combination of two linearly independent vectors. Now, for any bearing unit vector $\mathbf{g}_{ij}$, $j \neq i+1, i-1$, directed from agent i to agent j, the following holds:

$$\mathbf{g}_{ij}^* = \frac{1}{d_{ij}}(d_{iX}\mathbf{g}_{iX}^* - d_{jX}\mathbf{g}_{jX}^*), \tag{6}$$

$$\text{and for } \mathcal{G}(\mathbf{p}'),\ \mathbf{g}_{ij}'^* = \frac{1}{d_{ij}'}(d_{iX}'\mathbf{g}_{iX}'^* - d_{jX}'\mathbf{g}_{jX}'^*). \tag{7}$$

Since $\mathbf{g}_{iX}^* = \mathbf{g}_{iX}'^*$, $d_{iX}' = \eta d_{iX}$, from (7) it immediately follows that

$$\mathbf{g}_{ij}'^* = \frac{1}{d_{ij}'}(\eta d_{iX}\mathbf{g}_{iX}^* - \eta d_{jX}\mathbf{g}_{jX}^*) = \eta\frac{d_{ij}}{d_{ij}'}\mathbf{g}_{ij}^*. \tag{8}$$

Thus, $\mathbf{g}_{ij}^* = \mathbf{g}_{ij}'^*$ and $d_{ij}' = \eta d_{ij}$. Hence, the two frameworks $\mathcal{G}(\mathbf{p})$ and $\mathcal{G}(\mathbf{p}')$ are bearing equivalent and their shapes differ by a scale factor η. □

The main problem may now be stated.

Problem: Given a group of n-agents in cyclic pursuit and a moving target, satisfying Assumptions 1-2, design control laws for the agents using bearing-only measurements and target velocity information such that the agents asymptotically attain a desired formation shape, described by a feasible bearing vector set as in Definition 1, around the moving target.

III. Main Results

A. Proposed Control Law

The proposed bearing-only control law for any agent (say agent i) is given by

$$\mathbf{u}_i = -\mathbf{P}_{\mathbf{g}_i}\mathbf{g}_i^* - \mathbf{P}_{\mathbf{g}_{iX}}\mathbf{g}_{iX}^* + \mathbf{v}_T, \tag{9}$$

where $\mathbf{P}_{\mathbf{g}_i} = \mathbf{I}_d - \mathbf{g}_i\mathbf{g}_i^T$, $\mathbf{P}_{\mathbf{g}_{iX}} = \mathbf{I}_d - \mathbf{g}_{iX}\mathbf{g}_{iX}^T$ are orthogonal projection matrices as described in Section II. Furthermore, in $\mathbb{R}^2$, it is evident that $\mathbf{P}_{\mathbf{g}_i} = \mathbf{g}_i^\perp(\mathbf{g}_i)^{\perp T}$, where

$$\mathbf{g}_i^\perp = \mathbf{J}\mathbf{g}_i = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\mathbf{g}_i$$

is a unit vector orthogonal to $\mathbf{g}_i$. Let $\mathbf{z}_i = \mathbf{p}_{i+1} - \mathbf{p}_i$, $\mathbf{z}_{iX} = \mathbf{p}_X - \mathbf{p}_i$, $d_i = \|\mathbf{z}_i\|$, $d_{iX} = \|\mathbf{z}_{iX}\|$, for $i = 1, \ldots, n$ as before. Further, $\mathbf{P}_{\mathbf{g}_i}\mathbf{z}_i = d_i\mathbf{P}_{\mathbf{g}_i}\mathbf{g}_i = \mathbf{0}$. Similar results are also true of $\mathbf{P}_{\mathbf{g}_{iX}} = \mathbf{g}_{iX}^\perp(\mathbf{g}_{iX})^{\perp T}$.

The following lemma is about the equilibrium of the cyclic pursuit system described by (9).

Lemma 2. *The cyclic pursuit system with control law* (9) *has two types of equilibria which are symmetric about the target's position: the desired equilibrium* $\mathbf{p}^*$ *corresponding to* $\mathbf{g} = \mathbf{g}^*$ *and the undesired equilibrium corresponding to* $\mathbf{g} = -\mathbf{g}^*$.

Proof. Combining (1) and (9), and noting that $\dot{\mathbf{p}}_X = \mathbf{v}_T$, it may be observed that by defining $\tilde{\mathbf{p}}_i = \mathbf{p}_i - \mathbf{p}_X$, the overall system dynamics transform to

$$\dot{\tilde{\mathbf{p}}}_i = -\mathbf{P}_{\mathbf{g}_i}\mathbf{g}_i^* - \mathbf{P}_{\mathbf{g}_{iX}}\mathbf{g}_{iX}^* \tag{10}$$

From Definition 1, there exist non-zero scalars $d_i^*, d_{i,X}^*$ and $d_{i+1,X}^*$ such that

$$d_i^*\mathbf{g}_i^* - d_{i,X}^*\mathbf{g}_{i,X}^* + d_{i+1,X}^*\mathbf{g}_{i+1,X}^* = \mathbf{0}. \tag{11}$$

Clearly, at steady state it is desired that the agents attain a desired formation around moving target and continue to follow the target with the same velocity as that of the target. So the agents do not come to rest but must each attain a velocity $\mathbf{v}_T$. Hence, the equilibria of the system driven by (9) is attained when $\dot{\tilde{\mathbf{p}}}_i = 0 \ \forall i$. These equilibria therefore satisfy

$$-\mathbf{P}_{\mathbf{g}_i}\mathbf{g}_i^* - \mathbf{P}_{\mathbf{g}_{iX}}\mathbf{g}_{iX}^* = \mathbf{0}, \tag{12}$$

for all $i = 1, \dots, n$. Premultiplication by $\mathbf{g}_i^T$ on both side of equation (12) yields

$$\mathbf{g}_i^T\mathbf{P}_{\mathbf{g}_{iX}}\mathbf{g}_{iX}^* = 0. \tag{13}$$

Equation (13) is satisfied if and only if

$$\text{either} \quad \mathbf{g}_i = \pm\mathbf{g}_{iX}, \tag{14}$$

$$\text{or} \quad \mathbf{g}_{iX} = \pm\mathbf{g}_{iX}^*, \tag{15}$$

for all $i = 1, \dots, n$. The following exhaustive possibilities are now considered:

Case 1: Suppose some agents satisfy (14) while others satisfy (15). Then there is some $i \in \{1, \dots, n\}$ such that (14) holds for agent i while (15) holds for agent $i+1$, i.e., $\mathbf{g}_i = \pm\mathbf{g}_{iX}$ and $\mathbf{g}_{i+1,X} = \pm\mathbf{g}_{i+1,X}^*$.

Now, $\mathbf{g}_i = \pm\mathbf{g}_{iX}$ means that agent i, agent $i+1$ and the target $\mathbf{X}$ are collinear. Hence, $\mathbf{P}_{\mathbf{g}_i} = \mathbf{P}_{\mathbf{g}_{iX}}$. Using this in (12), one obtains

$$\mathbf{P}_{\mathbf{g}_i}(\mathbf{g}_i^* + \mathbf{g}_{iX}^*) = \mathbf{0}, \tag{16}$$

which holds if and only if $\mathbf{g}_{iX}^* + \mathbf{g}_i^* = k\mathbf{g}_i$ since the one dimensional null space of $\mathbf{P}_{\mathbf{g}_i}$ is spanned by $\mathbf{g}_i$. Equivalently,

$$\mathbf{g}_{iX}^* = k\mathbf{g}_i - \mathbf{g}_i^* \tag{17}$$

where $k \in \mathbb{R} \setminus \{0\}$. Further, since agents i, $i+1$ and the target are collinear, it follows that

$$\mathbf{g}_i = \pm\mathbf{g}_{i+1,X} = \pm\mathbf{g}_{i+1,X}^*. \tag{18}$$

Substituting $\mathbf{g}_{i,X}^*$ from (17) and $\mathbf{g}_{i+1,X}^*$ from (18) into (11) the following expression, in terms of $\mathbf{g}_i^*$ and $\mathbf{g}_i$, is obtained:

$$\begin{aligned} d_i^*\mathbf{g}_i^* - d_{iX}^*(k\mathbf{g}_i - \mathbf{g}_i^*) \pm d_{i+1,X}^*\mathbf{g}_i &= \mathbf{0} \\ \implies (d_i^* + d_{iX}^*)\mathbf{g}_i^* + (-k_i d_{iX}^* \pm d_{i+1,X}^*)\mathbf{g}_i &= \mathbf{0}. \end{aligned} \tag{19}$$

Combining (19) and (17) one may conclude that the vectors $\mathbf{g}_i^*$ and $\mathbf{g}_i$ are both aligned with $\mathbf{g}_{iX}^*$. But Definition 1, prevents $\mathbf{g}_i^*$ and $\mathbf{g}_{iX}^*$ from being aligned. Therefore, there is a contradiction.

Case 2: Consider the condition $\mathbf{g}_i = \pm\mathbf{g}_{iX}$ to hold for all $i = 1, \dots, n$. Thus, the positions of all agents and that of the target are collinear, and

$$\mathbf{g}_1 = \dots = \pm\mathbf{g}_n = \pm\mathbf{g}_{1X} = \dots = \pm\mathbf{g}_{n,X}. \tag{20}$$

This further leads to the following:

$$\mathbf{P}_{\mathbf{g}_1} = \dots = \mathbf{P}_{\mathbf{g}_n} = \mathbf{P}_{\mathbf{g}_{1X}} = \dots = \mathbf{P}_{\mathbf{g}_{nX}}. \tag{21}$$

Using relation (21) in (12) results in

$$\mathbf{g}_{iX}^* + \mathbf{g}_i^* = k_i \mathbf{g}_i, \quad i = 1, \ldots, n, \tag{22}$$

or equivalently, in conjunction with (20), this leads to

$$\mathbf{g}_1 = \ldots = \pm\mathbf{g}_n = \frac{\mathbf{g}_{1X}^* + \mathbf{g}_1^*}{k_1} = \ldots = \frac{\mathbf{g}_{nX}^* + \mathbf{g}_n^*}{k_n}. \tag{23}$$

where $k_i \in \mathbb{R} \setminus \{0\}$.

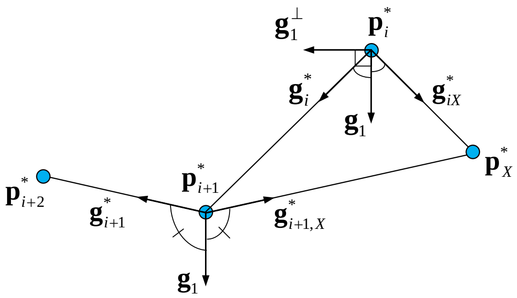


Figure 3: Proof for Case 2.

The unit vector $\mathbf{g}_1^\perp$ which is perpendicular to $\mathbf{g}_1$ is given by $\mathbf{g}_1^\perp = \mathbf{J}\mathbf{g}_1$, where $\mathbf{J} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Thus, from (11) and (23), it follows that

$$\begin{aligned} & d_i^* \mathbf{g}_i^* + d_{iX}^* \mathbf{g}_i^* = d_{iX}^* \mathbf{g}_{iX}^* - d_{i+1,X}^* \mathbf{g}_{i+1,X}^* + d_{iX}^* \mathbf{g}_i^* \\ \implies & (d_i^* + d_{iX}^*)\mathbf{g}_i^* = d_{iX}^*(\mathbf{g}_{iX}^* + \mathbf{g}_i^*) - d_{i+1,X}^*(k_{i+1}\mathbf{g}_1 - \mathbf{g}_{i+1}^*) \\ \implies & (d_i^* + d_{iX}^*)\mathbf{g}_i^* = (k_i d_{iX}^* - k_{i+1} d_{i+1,X}^*)\mathbf{g}_1 + d_{i+1,X}^* \mathbf{g}_{i+1}^* \end{aligned} \tag{24}$$

Premultiplication of both sides of (24) by $(\mathbf{g}_1^\perp)^T$ leads to

$$(d_i^* + d_{iX}^*)(\mathbf{g}_1^\perp)^T \mathbf{g}_i^* = d_{i+1,X}^* (\mathbf{g}_1^\perp)^T \mathbf{g}_{i+1}^*, \tag{25}$$

since $(\mathbf{g}_1^\perp)^T \mathbf{g}_1 = 0$. This implies that

$$\operatorname{sgn}((\mathbf{g}_1^\perp)^T \mathbf{g}_i^*) = \operatorname{sgn}((\mathbf{g}_1^\perp)^T \mathbf{g}_{i+1}^*). \tag{26}$$

Since it has been assumed that $\mathbf{g}_i^* \neq \pm\mathbf{g}_{i+1}^*$, hence $(\mathbf{g}_1^\perp)^T \mathbf{g}_i^* \neq 0 \ \forall i$. From (26), the principle of mathematical induction leads to

$$\operatorname{sgn}((\mathbf{g}_1^\perp)^T \mathbf{g}_1^*) = \ldots = \operatorname{sgn}((\mathbf{g}_n^\perp)^T \mathbf{g}_n^*) \neq 0. \tag{27}$$

But it is known that $\sum_{i=1}^n \mathbf{z}_i^* = 0$. Hence, one may conclude that

$$\begin{aligned} 0 = (\mathbf{g}_1^\perp)^T \mathbf{0} = (\mathbf{g}_1^\perp)^T \sum_{i=1}^n \mathbf{z}_i^* & \\ = (\mathbf{g}_1^\perp)^T \sum_{i=1}^n d_i^* \mathbf{g}_i^* & \\ = \sum_{i=1}^n d_i^* (\mathbf{g}_1^\perp)^T \mathbf{g}_i^*. & \end{aligned} \tag{28}$$

From equation (27) it may be deduced that either $(\mathbf{g}_1^{\perp})^T \mathbf{g}_i^* > 0$ or $(\mathbf{g}_1^{\perp})^T \mathbf{g}_i^* < 0$ holds for all i. In either case, the right hand side of (28) is not zero. This is a contradiction, which rules out Case 2.

Case 3: Here some agents are assumed to satisfy $\mathbf{g}_{iX} = \mathbf{g}_{iX}^*$ while others satisfy $\mathbf{g}_{iX} = -\mathbf{g}_{iX}^*$. As before, it may be concluded that there is some index j for which $\mathbf{g}_{jX} = \mathbf{g}_{jX}^*$ and $\mathbf{g}_{j+1,X} = -\mathbf{g}_{j+,1X}^*$. Note that a positive solution (in terms of the distances) exists for the following equation:

$$\mathbf{g}_j^* = \frac{d_{jX}^*}{d_j^*}\mathbf{g}_{jX}^* - \frac{d_{j+1,X}^*}{d_j^*}\mathbf{g}_{j+1,X}^* \tag{29}$$

up to a scaling factor. This is because for a feasible formation shape, the agents j and $j+1$ must form a triangle with the target $\mathbf{X}$ at steady state. Also, $d_j\mathbf{g}_j - d_{jX}\mathbf{g}_{jX} + d_{j+1,X}\mathbf{g}_{j+1,X} = \mathbf{0}$ always holds as this is the triangle criteria between agents j, $j+1$ and the target $\mathbf{X}$ at any instant. Using this triangle criteria at the equilibrium (i.e. $\mathbf{g}_{jX} = \mathbf{g}_{jX}^*$ and $\mathbf{g}_{j+1,X} = -\mathbf{g}_{j+,1X}^*$), it turns out that there must be positive solutions for the equation

$$\mathbf{g}_j^* = \pm\left[\frac{d_{jX}'^*}{d_j'^*}\mathbf{g}_{jX}^* + \frac{d_{j+1,X}'^*}{d_j'^*}\mathbf{g}_{j+1,X}^*\right]. \tag{30}$$

Now, $\mathbf{g}_{jX}^*$ and $\mathbf{g}_{j+1,X}^*$ form the basis set for $\mathbb{R}^2$ as they are linearly independent due to Definition 1. Also the representation of $\mathbf{g}_j^*$ in terms of this basis set must be unique. But, clearly a contradiction arises between (29) and (30). This is because the coefficients of $\mathbf{g}_{jX}^*$ and $\mathbf{g}_{j+1,X}^*$ in the representation of $\mathbf{g}_j^*$ have opposite signs in (29) while in (30) they are of the same sign.

It may thus be concluded that (13) holds if and only if $\mathbf{g}_{iX} = \mathbf{g}_{iX}^*\ \forall i$ or $\mathbf{g}_{iX} = -\mathbf{g}_{iX}^*\ \forall i$. Using this assertion in (12), it follows that, the system also satisfies $\mathbf{g}_i = \mathbf{g}_i^*\ \forall i$ or $\mathbf{g}_i = -\mathbf{g}_i^*$ $\forall i$ respectively, at the corresponding equilibria. Using Definition 1, it is apparent that there exists an equilibrium of the system at the desired formation $\mathbf{p}^*$ which satisfies $\mathbf{g} = \mathbf{g}^*$. Moreover, there is another formation $\mathbf{p}'$ and this is symmetric with $\mathbf{p}^*$ about the target $\mathbf{X}$. Further, this formation $\mathbf{p}'$ is an undesired one corresponding to $\mathbf{g} = -\mathbf{g}^*$. □

B. Stability Analysis

The following sets are defined to aid in the stability analysis of the desired formation shape:

$$\begin{aligned}
\mathcal{Q} &:= \{\mathbf{p} \in \mathbb{R}^{2n} | \mathbf{g}_i = \pm\mathbf{g}_i^* \text{ and } \mathbf{g}_{iX} = \pm\mathbf{g}_{iX}^*,\ i = 1, \dots, n\},\\
\mathcal{D} &:= \{\mathbf{p} \in \mathbb{R}^{2n} | \mathbf{g}_i = \mathbf{g}_i^* \text{ and } \mathbf{g}_{iX} = \mathbf{g}_{iX}^*,\ i = 1, \dots, n\},\\
\mathcal{U} &:= \{\mathbf{p} \in \mathbb{R}^{2n} | \mathbf{g}_i = -\mathbf{g}_i^* \text{ and } \mathbf{g}_{iX} = -\mathbf{g}_{iX}^*,\ i = 1, \dots, n\}.
\end{aligned}$$

The set $\mathcal{Q}$ contains all the equilibria of the system described by the control law (9). This set $\mathcal{Q}$ is partitioned into two sets, $\mathcal{D}-$ the set of desired equilibria, and $\mathcal{U}-$ the set of undesired equilibria. These equilibria are as described in Lemma 2.

Consider a directed cycle in $\mathbb{R}^2$. Let α_i be the magnitude of the angle between $\mathbf{g}_i$, the actual bearing and $\mathbf{g}_i^*$, the desired bearing of agent i with respect to its leader, agent $i+1$, such that $0 \leq \alpha_i \leq \pi$. Similarly, define β_i, ϕ_i and γ_i to be the magnitudes of the angles between $\mathbf{g}_{iX}$ and $\mathbf{g}_{iX}^*$, $\mathbf{g}_i$ and $\mathbf{g}_{iX}$, and $\mathbf{g}_i$ and $\mathbf{g}_{i+1}$, respectively, as illustrated in Fig. 4.

Since this paper investigates local stability, the behavior of the system in the vicinity of the equilibrium will be considered for stability analysis. Each equilibrium $\mathbf{p}^* \in \mathcal{D}$ corresponds to $\alpha_i = \beta_i = 0, i = 1, 2, \dots, n$. Similarly, for each equilibrium $\mathbf{p}^* \in \mathcal{U}$, it follows that $\alpha_i = \beta_i = \pi, \forall\ i$. As defined earlier, $d_i = \|\mathbf{p}_{i+1} - \mathbf{p}_i\|$ and $d_{iX} = \|\mathbf{p}_{iX} - \mathbf{p}_i\|$.

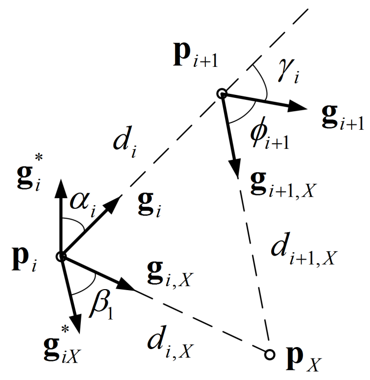


Figure 4: Illustration for proof of local stability

Consider

$$\cos\beta_i = (\mathbf{g}_{iX}^*)^T \mathbf{g}_{iX}, \tag{31}$$

which, upon differentiating both sides with respect to time, yields

$$\begin{aligned} \sin\beta_i \dot{\beta}_i &= -(\mathbf{g}_{iX}^*)^T \dot{\mathbf{g}}_{iX} \\ &= -(\mathbf{g}_{iX}^*)^T \frac{\mathbf{P}_{\mathbf{g}_{iX}}}{d_{iX}} (\dot{\mathbf{p}}_X - \dot{\mathbf{p}}_i) \\ &= -(\mathbf{g}_{iX}^*)^T \frac{\mathbf{P}_{\mathbf{g}_{iX}}}{d_{iX}} (\mathbf{v}_T - \dot{\mathbf{p}}_i) \\ &= -(\mathbf{g}_{iX}^*)^T \frac{\mathbf{P}_{\mathbf{g}_{iX}}}{d_{iX}} (-\dot{\tilde{\mathbf{p}}}_i) \end{aligned} \tag{32}$$

where $\dot{\tilde{\mathbf{p}}}_i$ is given by (10). Using the relation $\mathbf{P}_{\mathbf{g}_{iX}} = \mathbf{g}_{iX}^{\perp}(\mathbf{g}_{iX}^{\perp})^T$ in (32) one arrives at

$$\begin{aligned} d_{iX} \sin\beta_i \dot{\beta}_i =& - (\mathbf{g}_{iX}^*)^T \mathbf{g}_{iX}^{\perp} (\mathbf{g}_{iX}^{\perp})^T \mathbf{g}_{iX}^{\perp} (\mathbf{g}_{iX}^{\perp})^T \mathbf{g}_{iX}^* \\ &- (\mathbf{g}_{iX}^*)^T \mathbf{g}_{iX}^{\perp} (\mathbf{g}_{iX}^{\perp})^T \mathbf{g}_i^{\perp} (\mathbf{g}_i^{\perp})^T \mathbf{g}_i^* \\ =& - \sin^2\beta_i + (\pm\sin\beta_i)(\cos\phi_i)(\pm\sin\alpha_i). \end{aligned}$$

Hence, the dynamics in terms of the angle β_i is given by

$$\dot{\beta}_i = -\frac{\sin\beta_i}{d_{iX}} \pm \frac{\sin\alpha_i \cos\phi_i}{d_{iX}}. \tag{33}$$

Similarly, the following relation is obtained:

$$\cos\alpha_i = (\mathbf{g}_i^*)^T \mathbf{g}_i. \tag{34}$$

Upon differentiating both sides of (34) with respect to time and performing some algebraic manipulations similar to the ones used to obtain (33), it follows that

$$\dot{\alpha}_i = -\frac{\sin\alpha_i}{d_i} \pm \frac{\sin\alpha_{i+1}\cos\gamma_i}{d_i} \pm \frac{\sin\beta_{i+1}\cos(\gamma_i \pm \phi_i)}{d_i} \pm \frac{\sin\beta_i \cos\phi_i}{d_i}. \tag{35}$$

Define $\mathbf{\Theta} = [\alpha_1 \ldots \alpha_{n-1}\ \beta_1 \ldots \beta_{n-1}\beta_n] \in \mathbb{R}^{2n-1}$ as the vector where the error angles are stacked together. All the $2n$ angles are not considered because in order to define the formation shape, with n agents and a target, only $2n-1$ angles suffice. From the earlier discussion in Section III.B, the desired equilibria in $\mathcal{D}$, correspond to $\mathbf{\Theta} = \mathbf{0}_{2n-1}$ and the undesired ones in $\mathcal{U}$ correspond to $\mathbf{\Theta} = \pi\mathbf{1}_{2n-1}$. The following theorem states results on the local stability of these two types of equilibria.

Theorem 1. *In $\mathbb{R}^2$, the equilibria corresponding to $\mathcal{D}$ are locally asymptotically stable, while those corresponding to $\mathcal{U}$ are unstable.*

Proof. Linearization of equations (33) and (35) near the desired equilibria, leads to the following perturbed system close to a desired equilibria in $\mathcal{D}$:

$$\Delta\dot{\boldsymbol{\theta}} = \mathbf{M}\Delta\boldsymbol{\theta} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \Delta\boldsymbol{\theta}, \tag{36}$$

where

$$\mathbf{A} = \begin{bmatrix} -\frac{1}{d_1^*} & \pm\frac{\cos\gamma_1^*}{d_1^*} & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & -\frac{1}{d_{n-2}^*} & \pm\frac{\cos\gamma_{n-2}^*}{d_{n-2}^*} \\ 0 & \cdots & 0 & -\frac{1}{d_{n-1}^*} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} -\frac{\cos\phi_1^*}{d_1^*} & \pm\frac{\cos\left(\gamma_1^*-\phi_1^*\right)}{d_1^*} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & -\frac{\cos\phi_{n-1}^*}{d_{n-1}^*} & \pm\frac{\cos\left(\gamma_{n-1}^*-\phi_{n-1}^*\right)}{d_{n-1}^*} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_1 & \begin{matrix} \mathbf{0}_{n-2} \\ \pm\frac{\cos\left(\gamma_{n-1}^*\pm\phi_{n-1}^*\right)}{d_{n-1}^*} \end{matrix} \end{bmatrix}.$$

$$\mathbf{C} = \begin{bmatrix} -\frac{1}{d_{1,X}^*} & \cdots & 0 \\ 0 & \ddots & \vdots \\ \vdots & \ddots & -\frac{1}{d_{n-1,X}^*} \\ 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{0}_{n-1}^T \end{bmatrix}, \mathbf{D} = \mathrm{diag}\left(-\frac{1}{d_{1X}^*}, \ldots, -\frac{1}{d_{n-1,X}^*}\right),$$

The eigenvalues of $\mathbf{M}$ are the roots of the polynomial equation in λ given by

$$\begin{aligned} det(\lambda\mathbf{I}_{2n-1} - \mathbf{M}) = &\left(\lambda + \frac{1}{d_{n,X}^*}\right) det\left(\begin{bmatrix} \lambda\mathbf{I}_{n-1} - \mathbf{A} & -\mathbf{B}_1 \\ -\mathbf{C}_1 & \lambda\mathbf{I}_{n-1} - \mathbf{D}_1 \end{bmatrix}\right) \\ &+ (-1)^{(2n-1)+(n-1)} \left(\pm\frac{\cos\left(\gamma_{n-1}^* \pm \phi_{n-1}^*\right)}{d_{n-1}^*}\right) \times 0. \end{aligned} \tag{37}$$

Note that we have used the Laplace expansion to get (37), whose the second term is zero since it is the determinant of a matrix with a zero row (the $(2n-1)$-th row). Thus, we have

$$\begin{aligned} det(\lambda\mathbf{I}_{2n-1} - \mathbf{M}) &= \left(\lambda + \frac{1}{d_{n,X}^*}\right) \prod_{i=1}^{n-1} \left(\left(\lambda + \frac{1}{d_i^*}\right)\left(\lambda + \frac{1}{d_{iX}^*}\right) \pm \frac{\left(\cos\phi_1^*\right)^2}{d_1^* d_{1X}^*}\right) \\ &= \left(\lambda + \frac{1}{d_{n,X}^*}\right) \prod_{i=1}^{n-1} \left(\lambda^2 + \left(\frac{1}{d_1^*} + \frac{1}{d_{iX}^*}\right)\lambda + \frac{1 \pm \left(\cos\phi_1^*\right)^2}{d_i^* d_{iX}^*}\right). \end{aligned}$$

Since $\cos \phi_i^* < 1$ at the desired equilibrium, each quadratic equation

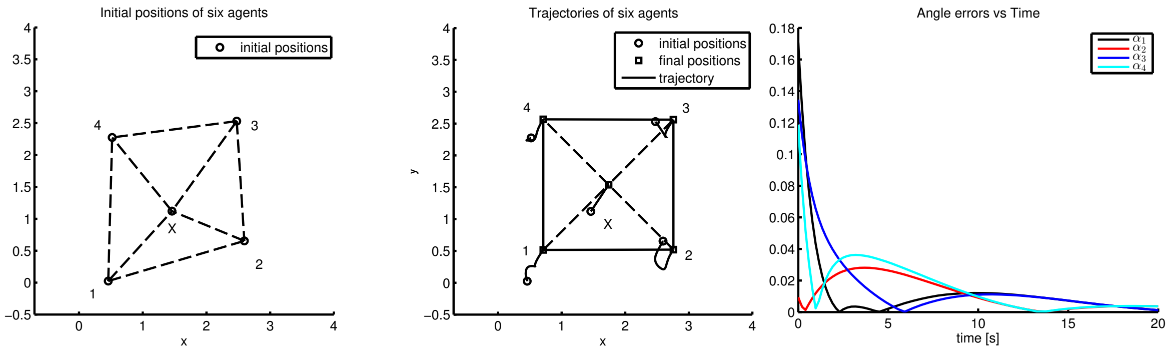
$$\lambda^2 + \left(\frac{1}{d_1^*} + \frac{1}{d_{iX}^*}\right)\lambda + \frac{1 \pm (\cos \phi_i^*)^2}{d_i^* d_{iX}^*} = 0$$

has two roots in the open left half plane. Thus, the matrix $\mathbf{M}$ is Hurwitz and local asymptotic stability of the system about any point in $\mathcal{D}$ is guaranteed.

Using similar reasoning as above, the equilibrium corresponding to a point in $\mathcal{U}$ is an unstable one. This completes the proof. □

IV. Simulations

For all the three examples in this section, a four-agent system is considered. The desired bearing vectors are chosen such that the agents form a square around the moving target in each case. The initial configurations are shown for the three cases in the Figs. 5a, 6a and 7a, respectively. The three scenarios consider the target to move along a straight line, a parabola and a circular trajectory, respectively. In each of the figures, 5b, 6b, and 7b it may be observed that the agents successfully achieve a square formation around the moving target and continue to track it. The evolution of the errors in the angles between the desired bearing and actual bearing of the agents are shown in Figs. 5c, 6c, and 7c for the three cases. These decay to zero asymptotically.



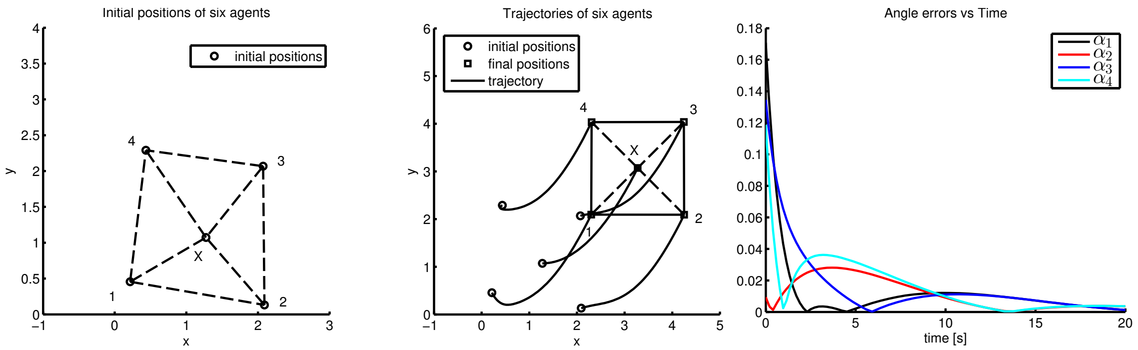
(a) The initial formation. (b) Trajectories and the final formation. (c) Angle errors vs. time.

Figure 5: Capturing a target moving along a line.

V. Conclusions

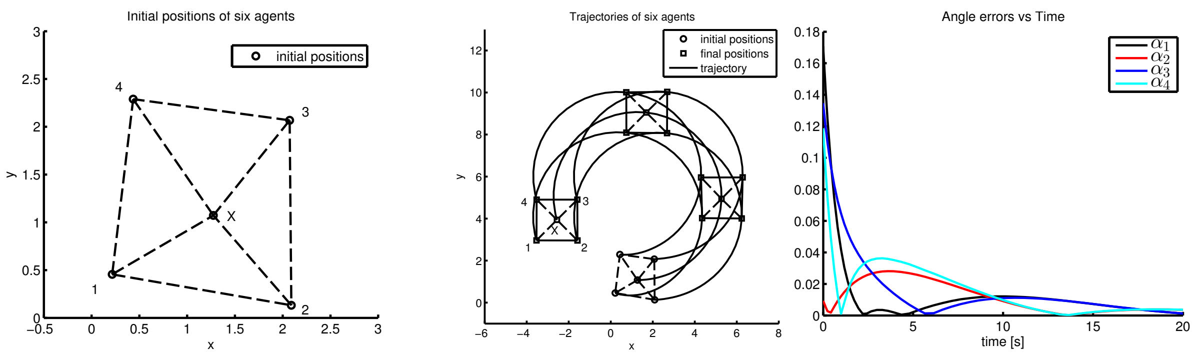
This paper proposed a bearing-only cyclic pursuit strategy for capturing a moving target, when the velocity of the target is known to each agent. It was first shown that when all desired bearing vectors are satisfied, a target formation shape will be achieved up to a scaling factor. Further, it was proved that the desired formation is locally asymptotically stable, and the undesired formation is unstable.

The present work opens up several possibilities for further investigations. Firstly, although the formation asymptotically reaches a desired formation for the digraph considered here, the size of the target formation, given by the scale factor η, is yet to be obtained. Extension of the present set-up to three dimensions may also be an interesting problem. Even in two-dimensions, an estimate of the region of attraction, in terms of the



(a) The initial formation.
(b) Trajectories and the final formation.
(c) Angle errors vs. time.

Figure 6: Capturing a target moving along a parabola.



(a) The initial formation.
(b) Trajectories and the final formation.
(c) Angle errors vs. time.

Figure 7: Capturing a target moving along a circle.

error in bearings may provide further insights. Finally, the bearing-only formation control problem with imprecise target velocity information is another challenging problem, open for future investigations.

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