Symmetry-Constrained Formation Maneuvering

Zamir Martinez[1], Daniel Zelazo[2]
[1]Technion Israel Institute of Technology
Email: z.m@campus.technion.ac.il
[2]Technion Israel Institute of Technology
Email: dzelazo@technion.ac.il

Abstract

The demand for advanced coordination schemes in multi-agent systems (MAS) has significantly increased in recent years, driven by applications such as vehicle platooning in autonomous driving [1], drone swarm coordination for surveillance [2], and coordination of satellite constellations for efficient communication relays [3]. While traditional centralized coordination schemes have been widely used for such tasks, they are prone to vulnerability and scalability issues, motivating the exploration of decentralized approaches relying on local sensing and interactions among agents.

This work addresses distance-constrained formation control, a decentralized strategy that enforces desired formations by maintaining specified relative positions and distances between agents. A key challenge in implementing this strategy is achieving a balance between communication requirements and performance constraints. Rigidity theory provides a robust framework for understanding and addressing this trade-off by modeling the decentralized interactions and geometric constraints. Distance constrained rigid frameworks ensure the desired formations by maintaining specific inter-agent distances. Among these, minimal infinitesimal rigid (MIR) frameworks are known to be the minimal architectural requirement that ensures the ensemble converges to the correct shape [4]. In the Euclidean space $\mathbb{R}^2$, MIR translates to having at least $(2n-3)$ communication constraints, where n is the number of agents.

Recent advancements in the theory of symmetry-forced rigidity provided a novel approach that demonstrates the potential of symmetry constrained frameworks as a promising alternative for MIR frameworks [5]. By exploiting specified symmetric constraints found in the formation, this approach notably reduces the required interaction constrains to $(1+1/|\Gamma|)n$, where n is the number of agents and Γ is the underlying symmetry group of the formation. Since symmetry forced formations by definition have point-group symmetries defined with respect to some fixed inertial point, preserving symmetric relationships during motion requires additional considerations.

A key contribution of this work is a modification of the augmented control strategy presented in [5]. This modification introduces a virtual state for each agent to ensure convergence with respect to any arbitrary time-varying centroid, enabling the MAS to achieve and maintain the required spatial pattern during maneuvering. Specifically, consider a network of n agents, described by the gradient-based dynamics:

$$\begin{bmatrix} \dot{p}_0(t) \\ \dot{p}_f(t) \end{bmatrix} = \begin{bmatrix} -\mathcal{O}^T(\mathcal{G}_0, c_0(t))\Big(\mathcal{O}(\mathcal{G}_0, c_0(t))c_0(t) - \mathbf{d}_0^2\Big) \\ 0 \end{bmatrix} - PQP^T \begin{bmatrix} c_0(t) \\ c_f(t) \end{bmatrix}, \tag{1}$$

where $\mathcal{O}$ is the orbit rigidity matrix describing the symmetric infinitesimal rigidity properties of the formation, Q is the symmetry-forcing term, and $\bar{c}(t) = \begin{bmatrix} c_0^T(t) & c_f^T(t) \end{bmatrix}^T = P(p(t) - r(t))$ is the shifted state of the formation according to the virtual state $r(t)$ (see [5]).

Manuscript 56 submitted to 64th Israel Annual Conference on
Aerospace Sciences (IACAS). Received January 8, 2025.

By introducing a reference velocity command $v_{ref}(t)$ to the cascade system (1), the system achieves the required formation, rendering the error dynamics exponentially stable while the agents move cohesively according to $v_{ref}(t)$. Fig. 1 illustrates an example of the resulting trajectories when running the implemented control law for $n = 6$ agents. The formation achieves zero steady-state error, as shown in Fig. 2.

Additional results show that by implementing a PI controller in a consensus version of the virtual state, a single leader agent with access to the reference velocity command is sufficient to distributively guide the entire formation while preserving its integrity. Numerical simulations are shown to illustrate the main results.

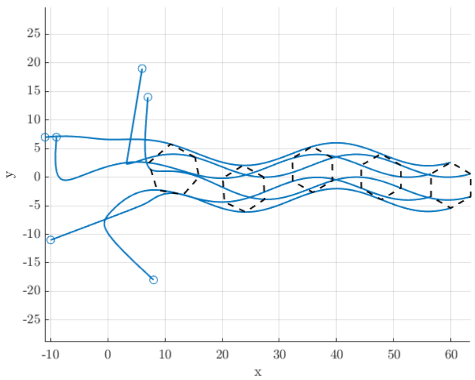

Fig. 1. Maneuvering of a hex-shaped forced symmetric formation according to a reference velocity command $v_{ref}(t) = [1, \pi/4 \cdot \sin(\pi/8 \cdot t)]^T$.

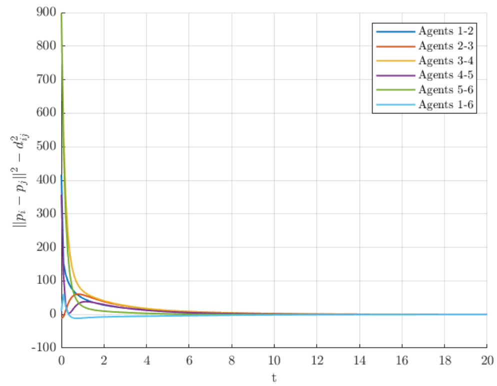

Fig. 2. Error signals $\|p_i - p_j\|^2 - d_{ij}^2$ where d_{ij} is the set of desired inter-agent distances.

References

[1] S.-L. Dai, S. He, H. Lin, and C. Wang, “Platoon formation control with prescribed performance guarantees for usvs,” IEEE Transactions on Industrial Electronics, vol. 65, no. 5, pp. 4237–4246, 2018.

[2] S. Martinez and F. Bullo, “Optimal sensor placement and motion coordination for target tracking,” Automatica, vol. 42, no. 3, pp. 661–668, 2006.

[3] L. Pedroso and P. Batista, “Distributed decentralized receding horizon control for very large-scale networks with application to satellite mega-constellations,” arXiv, 2023.

[4] L. Krick, M. E. Broucke, and B. A. Francis, “Stabilisation of infinitesimally rigid formations of multi-robot networks,” International Journal of Control, vol. 82, no. 3, pp. 423–439, 2009.

[5] D. Zelazo, S. ichi Tanigawa, and B. Schulze, “Forced symmetric formation control,” arXiv, 2024.