Distributed Identification of Leader Agents in Semi-Autonomous Networks

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The growing ubiquity of semi-autonomous network (SAN) has raised, where the multi-agents cooperate with each other in a consensus protocol. In a consensus algorithm, where some agents receive an external mission command then the network becomes leaders-followers network. This work focusses on computational strategy of the network structure to identify the leader agents.

I Introduction

One of the cornerstone algorithms for cooperative multi-agent coordination is the consensus protocol. The consensus protocol is a distributed algorithm were a team of dynamic systems exchange information with each other in order to reach an agreement on a common state or trajectory. Consensus serves as a basic layer in many higher-level coordination tasks, such as formation control and localization problems.

An appeal of consensus algorithms is their ability to operate autonomously over simple trusting agents. This has the added benefit that external (control) agents, perceived as simple agents, can seamlessly attach to the network. These additional agents, ignoring consensus rules, can influence the system dynamics compared to the unforced networked system resulting in scenarios such as leader-follower and drift correction. We refer to this class of systems as *semi-autonomous networks* (SANs).

In other words, SANs are *multi-agent systems where a subset of agents, referred to as leaders or informed agents, are selected to receive external control signals so as to steer the entire network towards a desired state.* Fig. 1 illustrates an example of SAN.

II Network formulation and SANs

In this section, the adopted model for SANs is described and basic facts about linear consensus in such systems are reported. In particular, it is then discussed how the behavior of SANs can be understood through the so-called "Following the slower neighbor" networks and a couple of novel theoretical results from DNS are yielded.

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Figure 1: Networks of agents in which part of them are leaders and the remaining part are followers.

II.A Graph-based network model for SANs

The basic network model is classic. An n-agent system can be modeled through a weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$ so that each element in the *vertex set* $\mathcal{V} = \{1, \dots, n\}$ is related to an agent in the group, while the *edge set* $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ characterizes the agents' interactions in terms of both sensing and communication capabilities. Also, $\mathcal{W} = \{w_k\}_{k=1}^{m}$, with $m = |\mathcal{E}|$, represents the set of weights assigned to each edge. Throughout the report, bidirectional interactions among agents are supposed, hence $\mathcal{G}$ is assumed to be *undirected*. The set $\mathcal{N}_i = \{j \in \mathcal{V} \setminus \{i\} \mid (i,j) \in \mathcal{E}\}$ identifies the *neighborhood* of the vertex i, i.e., the set of agents interacting with the i-th one and the cardinality $d_i = |\mathcal{N}_i|$ of neighborhood $\mathcal{N}_i$ defines the degree of node i. Furthermore, we denote the *incidence matrix* as $E \in \mathbb{R}^{n \times m}$, in which each column $k \in \{1, \dots, m\}$ is defined through the k-th (ordered) edge $(i,j) \in \mathcal{E}$, where $i < j$ is adopted w.l.o.g., and for edge k corresponding to (i,j) one has $[E]_{lk} = -1$, if $l = i$; $[E]_{lk} = 1$, if $l = j$; $[E]_{lk} = 0$, otherwise. For all $k = 1, \dots, m$, the weight $w_k = w_{ij} = w_{ji} \in \mathbb{R}$ is associated to k-th edge (i,j), and $W = \mathrm{diag}_{k=1}^{m}(w_k)$ is the diagonal matrix of edge weights. Also, the *Laplacian matrix* containing the topological information about $\mathcal{G}$ is addressed as $L(\mathcal{G}) = EWE^\top$. Henceforward, we also assume that graph $\mathcal{G}$ is *connected* and $L(\mathcal{G}) \succeq 0$, having eigenvalues λ_i^L and eigenvectors ν_i^L, for $i = 1, \dots, n$, such that $0 = \lambda_1^L < \lambda_2^L \leq \cdots \leq \lambda_n^L$. A sufficient condition to satisfy the latter requirement, which is adopted throughout the paper, is setting $w_{ij} > 0$ for all (i,j).

However, distinct from fully-autonomous networks, in SANs, a subset of agents, referred to as leaders or informed agents, are selected to receive external control signals so as to steer the entire network towards a desired state. In this direction, we assume that the vertex set $\mathcal{V}$ is split into two complementary subsets $\mathcal{V}_L \neq \varnothing$ and $\mathcal{V}_F \neq \varnothing$, such that $\mathcal{V}_L \cap \mathcal{V}_F = \varnothing$ and $\mathcal{V}_L \cup \mathcal{V}_F = \mathcal{V}$. Subsets $\mathcal{V}_L$ and $\mathcal{V}_F$ represents the group of *leader* and *follower* agents (see, e.g., partition in Fig. 2), respectively, assigning $n_L = |\mathcal{V}_L|$ and $n_F = |\mathcal{V}_F|$. Each node l in $\mathcal{V}_L$ are then subject to an *external input* $u_l \in \mathbb{R}^d$, which is assumed to be constant in this specific framework. In this setup, it is finally assumed that each leader is at most influenced by one external input.

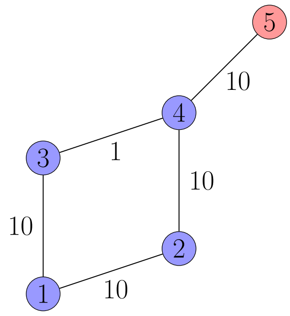


Figure 2: Partition of a graph in leaders (red) $\mathcal{V}_L = \{5\}$ and followers (blue) $\mathcal{V}_F = \{1, 2, 3, 4\}$.

II.B Linear consensus in SANs

Denoting with $x_i(t) \in \mathbb{R}^d$ the state of the i-th agent, the interaction protocol for each element of a SAN can be written as

$$\dot{x}_i(t) = -\sum_{i=1}^{n} w_{ij}(x_i(t) - x_j(t)) - \sum_{l=1}^{n_L} b_{il}(x_i(t) - u_l), \quad \forall i \in \mathcal{V} \tag{1}$$

where $b_{il} = 1$ if and only if $i \in \mathcal{V}_L$ and $b_{il} = 0$ otherwise. Subsequently, the collective behavior of SAN (1) can be characterized by

$$\dot{\mathbf{x}} = -(L_B(\mathcal{G}) \otimes I_d)\mathbf{x} + (B \otimes I_d)\mathbf{u}, \tag{2}$$

where $\mathbf{x} = \begin{bmatrix} x_1^\top(t) & \cdots & x_n^\top(t) \end{bmatrix} \in \mathbb{R}^{nd}$, $B = [b_{il}] \in \mathbb{R}^{n \times n_L}$, $\mathbf{u} = \begin{bmatrix} u_1^\top & \cdots & u_n^\top \end{bmatrix} \in \mathbb{R}^{n_L d}$ and

$$L_B(\mathcal{G}) = L(\mathcal{G}) + \mathrm{diag}(B\mathbb{1}_{n_L}). \tag{3}$$

Finally, a SAN is said to achieve consensus if

$$\lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0 \tag{4}$$

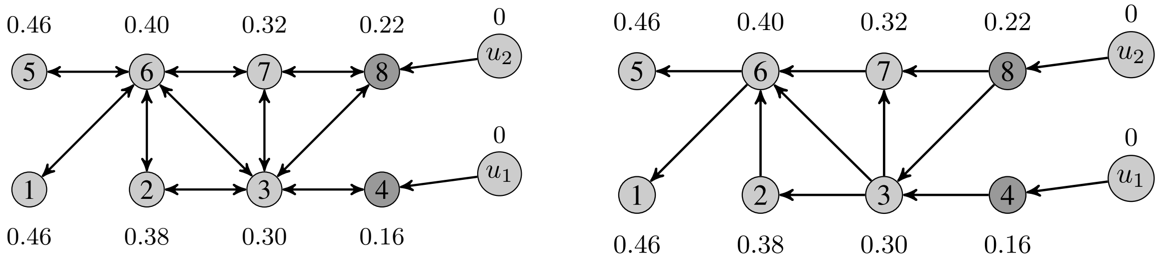
for all $i, j \in \mathcal{V}$ and some norm $\|\cdot\|$ on $\mathbb{R}^d$.

II.C "Following the slower neighbor" (FSN) networks

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$ be a graph modeling a SAN with input matrix B. Denote with $\lambda_i^{L_B}$ the eigenvalues of $L_B(\mathcal{G})$ computed as in (3), such that $\lambda_1^{L_B} \leq \cdots \leq \lambda_n^{L_B}$. It can be shown [1] that $\lambda_1^{L_B} > 0$ holds for any given connected and undirected SAN. Moreover, the computation of each eigenvector $\nu_i^{L_B}$ associated to $\lambda_i^{L_B}$, allows to obtain the so-called *following the slower neighbor* network $\bar{\mathcal{G}}$ from $\mathcal{G}$. This process is simply carried out by (i) labeling node i with the i-th entry of $\nu_1^{L_B}$, that is $[\nu_1^{L_B}]_i$; (ii) selecting the orientation for each edge (i, j) in $\mathcal{G}$ such that the arrow of the respective oriented edge in $\bar{\mathcal{G}}$ points toward the node with highest entry $[\nu_1^{L_B}]_k$, $k \in \{i, j\}$. More formally, the definition of a FSN network follows.

Definition 1 (FSN network)**.** *$\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{W}})$ is the FSN network corresponding to SAN $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$ if $\bar{\mathcal{V}} = \mathcal{V}$, $\bar{\mathcal{E}} \subseteq \mathcal{E}$ and $[\bar{W}]_{ij} = \bar{w}_{ij}$ is such that $\bar{w}_{ij} = w_{ij}$, if $[\nu_1^{L_B}]_i > [\nu_1^{L_B}]_j$; $\bar{w}_{ij} = 0$, otherwise.*

Fig. 3 provides an example of FSN network construction.



(a) Given network $\mathcal{G}$ (b) Corresponding FSN network $\bar{\mathcal{G}}$

Figure 3: From a SAN (a) to the corresponding FSN network (b). The first eigenvector of $L_B(\mathcal{G})$ is given by $(\nu_1^{L_B})^\top = \begin{bmatrix} 0.46 & 0.38 & 0.30 & 0.16 & 0.46 & 0.40 & 0.32 & 0.22 \end{bmatrix}^\top$.

II.D Known results on FSN networks from DNS

In the following lines, two theoretical results [2,3] are presented, which show a fundamental *leader-follower reachability property* and the relation between eigenvector $\nu_1^{L_B}$ and a new quantity called *relative tempo*, representing a speed comparison between couples of given nodes.

Definition 2 (Agent reachability in FSN networks)**.** *Given a FSN network $\bar{\mathcal{G}}$, an agent $i \in \bar{\mathcal{V}}$ is reachable from an agent $j \in \bar{\mathcal{V}}$, if there exists a direct path in $\bar{\mathcal{G}}$ connecting j to i or $i = j$.*

Theorem II.1 (LF-reachability)**.** *Let $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{W})$ be the FSN network of a SAN on connected network $\mathcal{G} = (\mathcal{V}, \mathcal{E}, W)$. Then each agent $i \in \bar{\mathcal{V}}$ is reachable from an agent $l \in \bar{\mathcal{V}}$ associated to some external input u_l in $\bar{\mathcal{G}}$.*

Definition 3 (Relative tempo)**.** *Given agents $i, j \in \mathcal{V}$ and their speed ratio $g_{ij}(t) := \|\dot{x}_i(t)\|_2 / \|\dot{x}_j(t)\|_2$, $\tau_{ij} = \lim_{t\to\infty} g_{ij}(t)$ denotes the relative tempo between agents $i \in \mathcal{V}$ and $j \in \mathcal{V}$.*

Theorem II.2 (Relation between relative tempo and $\nu_1^{L_B}$)**.** *Let us assign $\boldsymbol{v}_1(L_B)_{ij} := \boldsymbol{v}_1(L_B)_i / \boldsymbol{v}_1(L_B)_j$. Then it holds that $\tau_{ij} = \boldsymbol{v}_1(L_B)_{ij}$.*

III Conclusions

In this work, we have seen that the formalism of SANs yields a suitable setup to construct an interesting problem "Network Structure Identification". In particular, it has emerged that DNS-based techniques seem to be key for the identification of leader nodes in SANs. At the light of this observation, a computational strategy has been devised to this aim and partially tested. Nonetheless, identifying all leaders in a SAN still looks very challenging and further studies are required to fully comprehend how to design smart ways to perform such a task.

Acknowledgments

This research project was supported by Rafael Advanced Defence Systen.

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