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Bearing-Only Formation Control with Limited Visual Sensing: Two Agent Case ⋆

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Abstract: This work solves the bearing-only formation control problem for two agents with limited field-of-view sensing. We propose a bearing-only control strategy for both the position and heading of each agent that guarantees the desired formation is obtained from almost all initial conditions, and ensures that the agents always remain inside the field-of-view of the sensor. We support our analysis with simulations and explore an extension for the 3- and 4-agent case.

Keywords: multi-agent systems, formation control, visual sensing

1. INTRODUCTION

Formation control can be considered one of the basic tasks for a multi-agent system. The goal is to design a distributed controller for each agent that drives the ensemble to some desired geometric pattern. Inspired by birds or fish that move in certain shapes, formations of multi-agent systems are used in underwater exploration, surveillance, and deployment in space Eren et al. (2003); Fidan et al. (2007), among others.

The survey paper on multi-agent formation control, Oh et al. (2015), provides a high-level classification of different formation control strategies. They include position-based, displacement-based, distance-based, and bearing-based approaches, each depending on the sensing mediums available to the agents. Distance-based approaches were extensively studied in Anderson et al. (2008), Krick et al. (2008), Oh and Ahn (2014) and Zelazo et al. (2015). Bearing-based formation control has become more popular in recent years since visual sensing can be used to extract bearing information. Compared to range sensors, cameras are cheaper, lighter, and require less power. Estimating relative position information from bearings has been studied in Zelazo et al. (2014), whereas Zhao and Zelazo (2016) used bearing information directly to stabilize formations for an arbitrary number of agents. In both articles, it is assumed that the visual sensors can cover the entire surroundings to extract relative angles to its neighbors. However, this assumption is not realistic in applications using ground robots or UAVs. Usually cameras are only able to record a bounded area defined by its field-of-view (FOV). This leads to state-dependent sensing graphs where the neighbors are not static - they depend both on the position and orientation of the sensing agent, and the FOV constraints of the sensor.

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Motivated by this real-world problem, a number of approaches have been considered in the literature. Formation control over directed sensing graphs have been studied in Hendrickx et al. (2007) for distance constrained formations, and in Zhao and Zelazo (2015); Trinh et al. (2018, 2016) for bearing formations. The limited FOV problem has been studied in other multi-agent problems. The authors of Asadi et al. (2016) study consensus and containment with limited FOV. In the work of Dias et al. (2016) a limited FOV is introduced but the sensing graph remains static and range information is needed.

In this work, we aim to directly address the formation control problem using bearing sensors with limited FOV constraints. Our starting point is the bearing-only formation control strategy proposed by Zhao and Zelazo (2016), which we augment by introducing the state-dependent FOV constrained bearing measurement. We assume the sensor is mounted rigidly to the body frame of the robot, and we also propose a control for the heading of the robot, corresponding to the pointing direction of the sensor In this preliminary work we focus on the two-agent case. Our main contributions can be stated as follows:

i) We propose a novel controller for the heading direction based only on sensed bearing measurements. This controller guarantees that once an agent enters the FOV of the sensor, it will remain inside for the remainder of the trajectory.
ii) We provide a complete characterization of the different equilibrium configurations attainable by the two agent case and show that if at least one agent is initially sensed, then the desired formation is a stable equilibrium.

We also demonstrate the results with a number of simulation examples. While this work focuses on the two-agent case, we also provide simulation examples for the three and four agent case indicating the promise of this approach for larger formations.

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This paper is organized as follows. In Section 2 we introduce the dynamics of the agents and present the sensing model. Section 3 reviews the bearing-only controller and extends it to the FOV constrained case; here we also propose a controller for the heading direction. In Section 4 we characterize four different initial condition sets that are possible under the FOV constraints and then prove stability of our proposed control law. We support our analysis with some numerical simulations in Section 5. Concluding remarks are given in Section 6.

2. SYSTEM AND SENSING MODEL

In this section, we introduce the multi-agent system model with limited FOV bearing sensing. For this preliminary work, we focus on the $n = 2$ agent case. Each agent is described by its position vector, $p_i(t) \in \mathbb{R}^2$, and its heading, $\psi_i(t) \in \mathcal{S}^1$, for $i = 1, 2$.[1] The dynamics of the agents are modeled as single integrators,

$$\dot{\chi}_i(t) = \begin{bmatrix} \dot{p}_i(t) \\ \dot{\psi}_i(t) \end{bmatrix} = \begin{bmatrix} u_i(t) \\ \omega_i(t) \end{bmatrix}, \quad (1)$$

where $u_i(t) \in \mathbb{R}^2$ controls the linear velocity of the agents, and $\omega_i(t) \in \mathbb{R}$ the rotation rate. The complete state vector follows as $\chi(t) = \begin{bmatrix} \chi_1(t)^T & \chi_2(t)^T \end{bmatrix}^T$. From here on, we will neglect the time dependency of our signals where it is obvious, but include it when necessary.

Each agent is equipped with a sensor that is able to measure the relative bearing (in a common reference frame) to the other agent. We assume the sensor is mounted such that it points in the direction of the agent heading, ψ_i. The unit bearing vector between agent 1 and 2, is defined as

$$g_{12} := \frac{p_2 - p_1}{\|p_2 - p_1\|} = \frac{z_{12}}{d_{12}}. \quad (2)$$

The initial distance is denoted as $\bar{d}_{12} = d_{12}(0)$.

As the bearing vector is expressed in a global reference frame, it follows that $g_{12} = -g_{21}$. Denote by δ_{ψ_i} the angle between the facing direction of agent i and the angle of the sensed bearing g_{ij}. That is,

$$|\delta_{\psi_i}| = \cos^{-1}\left([\cos(\psi_i) \ \sin(\psi_i)]\, g_{ij}\right).$$

The sensor is also characterized by a FOV constraint. We denote the FOV of the sensor by the angle $\bar{\gamma}$. Thus, an agent is able to sense its neighbor if and only if $|\delta_{\psi_i}| < \bar{\gamma}/2$, i.e., when the neighbor is inside the FOV of the sensor. Also note that the angle of the bearing g_{ij} with respect to the (world frame) x-axis is defined as

$$\alpha_{ij} = \tan^{-1}\left(\frac{[g_{ij}]_y}{[g_{ij}]_x}\right) = \cos^{-1}([g_{ij}]_x) = \sin^{-1}([g_{ij}]_y). \quad (3)$$

Note that $\alpha_{21} = \alpha_{12} \pm \pi$ defines the angle of g_{21} at the position of agent two. Therefore, $\delta_{\psi_i} = \alpha_{ij} - \psi_i$. The sign of δ_{ψ_i} indicates if an agent is on the right- or on the left-side with respect to the facing direction. These notations are illustrated in Figure 1.

We introduce an indicator function for each agent that indicates if a neighbor can be sensed or not,

[1] Here, $\mathcal{S}^1$ denotes the 1-dimensional manifold on the unit circle.

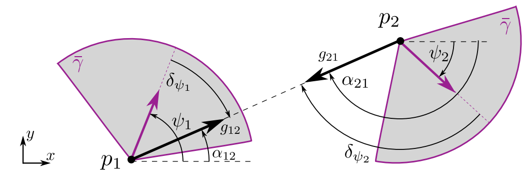


Fig. 1. Two agent configuration with FOV constrained bearing sensing note that $|\delta_{\psi_2}| > \bar{\gamma}/2$.

$$w_i(t) = \begin{cases} 1 \text{ if } |\delta_{\psi_i}(t)| < \dfrac{\bar{\gamma}}{2} \\ 0 \text{ else.} \end{cases} \quad (4)$$

The weights for both agents can be compactly written in matrix form as

$$W(t) := \begin{bmatrix} w_1(t) & 0 \\ 0 & w_2(t) \end{bmatrix}. \quad (5)$$

In the next section, we will provide a brief overview of a bearing-only formation control strategy proposed in Zhao and Zelazo (2016), and propose an extension to this law to cope with the limited FOV sensor constraints.

3. CONTROLLER FOR THE TWO AGENT CASE

Zhao and Zelazo (2016) proposed a bearing-only formation control strategy for a team of n agents modeled by integrator dynamics. The desired formation shape is specified by the constant unit bearing vectors g_{ij}^* (the angle of the desired bearing follows as $\alpha_{ij}^* = \tan^{-1}\left([g_{ij}^*]_y/[g_{ij}^*]_x\right)$). For $n = 2$, the controller has the form

$$\dot{p}_i = -P_{g_{ij}} g_{ij}^*, \; i = 1, 2. \quad (6)$$

Here, $P_{g_{ij}} := (I_2 - g_{ij} g_{ij}^T)$ is a projection matrix. The projection matrix is idempotent and therefor $P_{g_{ij}} = P_{g_{ij}}^2$ holds, and $P_{g_{ij}}(\pm g_{ij}) = 0$. This control strategy was proven in Zhao and Zelazo (2016) to almost globally stabilize the formation.

When the FOV constraints of the bearing sensor are considered, the dynamics described in (6) become

$$\dot{p} = u = \begin{bmatrix} -w_1 P_{g_{12}} g_{12}^* \\ w_2 P_{g_{12}} g_{12}^* \end{bmatrix} = (WH \otimes I_2) P_{g_{12}} g_{12}^*, \quad (7)$$

where $H = [-1 \ 1]^T$, and $W(t)$ is the indicator matrix defined in (5). Note that $\dot{p}_i = 0$ when $g_{12} = \pm g_{12}^*$ or when $w_i = 0$. It is thus clear that if the agents do not also control their facing direction, it may be they will not converge to the desired formation.

In the two agent case, a natural approach for controlling the facing direction is to align the facing direction with the bearing measurement, ensuring it is inside the FOV of the sensor. That is, we would like to design a control for the facing direction that drives δ_{ψ_i} to zero.

With this setup, we state the FOV constrained bearing formation control problem below.

Problem 1. Given a desired bearing g_{12}^*, an initial formation $\chi(0)$, and the limited FOV $\bar{\gamma}$, find the control inputs u_i and ω_i such that $\delta_{\psi_i} \to 0$ and $g_{12} \to g_{12}^*$ as $t \to \infty$.

To solve this problem, we propose the following controller for the facing direction to augment the bearing-only formation control in (7),

$$\omega_i = w_i(\underbrace{\kappa\left([g_{12}]_x[g_{12}^*]_y - [g_{12}]_y[g_{12}^*]_x\right)}_{v(\kappa,g_{12})} + \delta_{\psi_i}), \tag{8}$$

for some scalar $\kappa > 0$ and w_i the FOV indicator function (4). This controller is decoupled from the position controller (7) and uses only local information that can be obtained by a visual sensor. Furthermore we can bound bound the term $v(\kappa, g_{12})$, which will be useful in our subsequent analysis.

Proposition 2. The control term $v(\kappa, g_{12})$ is bounded by $|\kappa|$, that is, $v(\kappa, g_{12}) \leq \kappa$.

Proof. Since g_{12} and g_{12}^* are unit vectors $[g_{12}]_x^2 + [g_{12}]_y^2 = 1$ holds. Therefore $([g_{12}]_x[g_{12}^*]_y - [g_{12}]_y[g_{12}^*]_x) \leq 1$, and we conclude $v(\kappa, g_{12}) \leq \kappa$. ■

The complete dynamics of the closed loop system can now be written as

$$\dot{\chi} = \begin{bmatrix} u \\ \omega \end{bmatrix} = \begin{bmatrix} (WH \otimes I_2)P_{g_{12}}g_{12}^* \\ W\left(v(\kappa, g_{12})\mathbf{1} - \delta_\psi\right) \end{bmatrix}, \tag{9}$$

where $\delta_\psi = [\delta_{\psi_1} \ \ \delta_{\psi_2}]^T$ and $\mathbf{1} = [1 \ 1]^T$.

In the next section we describe the different equilibrium configurations of the system and prove their stability.

4. STABILITY ANALSYIS

The proposed control strategy presented in (9) naturally will depend on the initial conditions of the agents. Indeed, if the facing direction of both agents are such that neither are in the sensor's FOV, then both agents will remain stationary, and the objective can not be met. In this direction, we identify four different possible initial conditions, characterized by the value of the indicator function $w_i(0)$, that lead to different behaviors of the system. For each set of initial conditions, we provide a complete stability and convergence analysis of the closed-loop system. Qualitatively, the trajectories of the agents can produce four behaviors of the indicator function: i) both agents never sense eachother, ii) only one agent senses the other, iii) an agent enters or leaves the FOV during the trajectory, and iv) both agents sense eachother.

4.1 No Sensing: $w_1(0) = w_2(0) = 0$

Formally, the first case is stated as $|\delta_{\psi_i}(0)| > \bar{\gamma}/2, \ i = 1, 2$ which means that the visual sensor cannot extract any information of its neighbor, and both weights are zero. Thus, we have that $u = 0$ and $\omega = 0$. It follows that the equilibrium point is the initial condition, and the agents simply do not move.

This clearly degenerate case motivates the inclusion of the following assumption on our dynamics, which ensures that at least one agent is inside the FOV of the other.

Assumption 3. The initial condition $\chi(0)$ is such that $w_1(0) + w_2(0) \geq 1$.

4.2 Complete Sensing: $w_1(0) = w_2(0) = 1$

In the second case both agents can sense each other and $|\delta_{\psi_i}(0)| < \bar{\gamma}/2, \ i = 1, 2$. Controller (7) becomes the bearing only control law introduced in Zhao and Zelazo (2016). This has been shown to be stable when $\bar{\gamma} = 2\pi$. In the limited FOV setup, however, the facing direction has to change to ensure the agents remain inside the FOV. The first result shows that the controller (8) guarantees that the facing error, δ_{ψ_i} is bounded by $\bar{d}_{12}^{-1}$.

Proposition 4. If the indicator function $w_i(0) = 1, \ i = 1, 2$, controller (8) guarantees that the facing error stays bounded, such that $|\delta_{\psi_i}(t)| \leq 1/\bar{d}_{12}$ holds as $t \to \infty$.

Proof. First, observe that $\cos(\alpha_{12}) = [1 \ 0]\, g_{12}$. Therefore,

$$\begin{aligned}
&\frac{d}{dt}\cos(\alpha_{12}) = -\dot{\alpha}_{12}\sin(\alpha_{12}) = [1 \ 0]\, \dot{g}_{12} \\
&= [1 \ 0]\, \frac{P_{g_{12}}}{\bar{d}_{12}}(H \otimes I_2)^T \dot{p} = \frac{1}{\bar{d}_{12}}\, [1 \ 0]\left((w_1 + w_2)I_2\right)P_{g_{12}}g_{12}^* \\
&= \frac{(w_1 + w_2)[g_{12}]_y}{\bar{d}_{12}}\left([g_{12}]_y[g_{12}^*]_x - [g_{12}]_x[g_{12}^*]_y\right).
\end{aligned}$$

Since $\sin(\alpha_{12}) = [g_{12}]_y$ we obtain

$$\dot{\alpha}_{12} = \dot{\alpha}_{21} = \frac{w_1 + w_2}{\bar{d}_{12}}\left([g_{12}]_x[g_{12}^*]_y - [g_{12}]_y[g_{12}^*]_x\right). \tag{10}$$

Note that $\alpha_{12} = \alpha_{21} \pm \pi$, and therefore $\dot{\alpha}_{12} = \dot{\alpha}_{21}$.[2] In the case $w_1(0) = w_2(0) = 1$, the dynamics of $\dot{\delta}_{\psi_i}, \ i = 1, 2$ become

$$\begin{aligned}
\dot{\delta}_{\psi_i} &= \dot{\alpha}_{ij} - \dot{\psi}_i = \frac{1}{\bar{d}_{12}}\left([g_{12}]_x[g_{12}^*]_y - [g_{12}]_y[g_{12}^*]_x\right) - \delta_{\psi_i} \\
&= v(1/\bar{d}_{12}, g_{12}) - \delta_{\psi_i}.
\end{aligned}$$

Here we choose $\kappa = 1/\bar{d}_{12}$ such that we can subtract the terms. By using Proposition 2 we conclude that $v(1/\bar{d}_{12}, g_{12}) \in [-1/\bar{d}_{12}, \ 1/\bar{d}_{12}]$. In a next step we look at the solution of the differential equation

$$\delta_{\psi_i}(t) = e^{-t}\delta_{\psi_i}(t_0) + \int_{t_0}^{t} e^{-(t-\tau)}v(1/\bar{d}_{12}, g_{12}(\tau))d\tau$$

which satisfies the bound

$$|\delta_{\psi_i}(t)| \leq e^{-t}\left[|\delta_{\psi_1}(t_0)| - \frac{1}{\bar{d}_{12}}\right] + \frac{1}{\bar{d}_{12}}.$$

It follows that the bearing error does not exceed $\pm 1/\bar{d}_{12}$. ■

Proposition 4 can be used to establish a relationship between the initial distance of the agents and the required FOV of the sensor to ensure that if $w_1(0) = w_2(0) = 1$ that the agents will not leave the FOV of their neighbors under the trajectories of the system.

Corollary 5. If $\bar{\gamma}/2 > 1/\bar{d}_{12}$ and $w_1(0) = w_2(0) = 1$, then under control law (9), $w_1(t) = w_2(t) = 1$ for all $t \geq 0$.

Note that if $\bar{d}_{12} \leq 1/\pi$ the sensor has to cover 360 degrees. Corollary 5 leads to another Assumption on the system that will guarantee agents will not leave the FOV of their neighbor once sensed.

Assumption 6. The sensor FOV satisfies $\bar{\gamma}/2 > 1/\bar{d}_{12}$.

We now examine the equilibrium point of the closed loop (9) under Assumption 6. The equilibrium point of controller (7) is given as $g_{12} = \pm g_{12}^*$, as shown by the following,

$$\begin{aligned}
&\dot{p}_i = 0 = (I_2H \otimes I_2)P_{g_{12}}g_{12}^* \\
&= (g_{12}^*)^T\left(I_2H \otimes I_2\right)^T\left(I_2H \otimes I_2\right)P_{g_{12}}g_{12}^* = (g_{12}^*)^T P_{g_{12}}g_{12}^*.
\end{aligned}$$

[2] For more details on the dynamics on $\dot{g}_{12}$, we refer to Trinh et al. (2018).

Note that from the properties of $P_{g_{12}}$ it follows that $(g^*_{12})^T P_{g_{12}} g^*_{12} = g^T_{12} P_{g^*_{12}} g_{12}$. Here we used *Lemma 8* from Zhao and Zelazo (2016) and since the kernel of the projection matrix contains the bearing measurement, it follows that Null $\left(P_{g^*_{12}}\right) = \text{span}\left\{\pm g^*_{12}\right\}$ and therefore we can conclude that $\dot{p} = 0$ only holds if $g_{12} = \pm g^*_{12}$. Note that if $g_{12} = \pm g^*_{12}$ holds then the bearing dynamics become zero, which leads to $v(1/\bar{d}_{12}, g_{12}) = 0$. From Proposition 4 we then conclude that δ_{ψ_i}, $i = 1, 2$ will exponentially converge to zero under control law (8).

In order to show the stability of the equilibrium point we introduce $\delta_g = g_{12} - g^*_{12}$ as the bearing error. Then we show that $\delta_g = -2g^*_{12}$ is unstable and $\delta_g = 0$ is stable, which refers to $g_{12} = g^*_{12}$ and $g_{12} = -g^*_{12}$ respectively. First, we note that $|\delta_{\psi_i}(0)| < \bar{\gamma}/2,\ i = 1, 2$ holds whenever the facing direction belongs to the intervals below,

$$\mathcal{B}_1 := \left[\alpha_{12}(0) - \frac{\bar{\gamma}}{2}, \alpha_{12}(0) + \frac{\bar{\gamma}}{2}\right], \mathcal{B}_2\bar{\gamma} := \left[\alpha_{21}(0) - \frac{\bar{\gamma}}{2}, \alpha_{21}(0) + \frac{\bar{\gamma}}{2}\right].$$

Theorem 4.1. Under Assumption 6 and initial conditions satisfying $\psi_1(0) \in \mathcal{B}_1$ and $\psi_2(0) \in \mathcal{B}_2$, then $\delta_g \to 0$ and $\delta_\psi \to 0$ for almost all initial configurations $p_i(0) \in \mathbb{R}^2$, $i = 1, 2$, except for the point corresponding to $g_{12}(0) = -g^*_{12}$.

Proof. First we show that $\delta_g = -2g^*_{12}$ is an unstable equilibrium point for controller (7). Consider the dynamics of the bearing error, δ_g,

$$\begin{aligned}\dot{\delta}_g = f(\delta_g) = \dot{g}_{12} &= \frac{P_{g_{12}}}{d_{12}} \left(H \otimes I_2\right)^T \left(I_2 H \otimes I_2\right) P_{g_{12}} g^*_{12} \\ &= \frac{2}{d_{12}} P_{g_{12}} g^*_{12}.\end{aligned}$$

It can be verified that the Jacobian $A = \partial f(\delta_g)/\partial \delta_g$ is

$$\begin{aligned}A|_{\delta_g = -2g^*_{12}} &= \frac{4}{d_{12}} \left((g^*_{12})^T g^*_{12} I_2 + g^*_{12}(g^*_{12})^T\right) \\ &= \frac{4}{d_{12}} (I_2 + g^*_{12}(g^*_{12})^T) \succeq 0 \qquad (11)\end{aligned}$$

The last line of (11) is true since g^*_{12} is a unit vector with length one. The Jacobian is positive semi-definite and therefore we conclude that $g_{12} = -g^*_{12}$ corresponds to an unstable equilibrium point

Now we will show that $g_{12} = g^*_{12}$ is a stable equilibrium point which will be reached from every initial condition except the unstable equilibrium point ($g_{12} = -g^*_{12}$). We define the Lyapunov function $V_1 := 1/2(\delta_g^T \delta_g + \delta^2_{\psi_1} + \delta^2_{\psi_2})$ which is positive semi-definite and zero only at $\delta_g = 0, \delta_{\psi_1} = \delta_{\psi_2} = 0$. Its time derivate follows as

$$\begin{aligned}\dot{V}_1 d_{12} = &-\delta^2_{\psi_1} + \delta_{\psi_1}\left([g_{12}]_x [g^*_{12}]_y - [g_{12}]_y [g^*_{12}]_x\right) \\ &- \left([g_{12}]_x [g^*_{12}]_y - [g_{12}]_y [g^*_{12}]_x\right)^2 \\ &- \delta^2_{\psi_2} + \delta_{\psi_2}\left([g_{12}]_x [g^*_{12}]_y - [g_{12}]_y [g^*_{12}]_x\right) \\ &- \left([g_{12}]_x [g^*_{12}]_y - [g_{12}]_y [g^*_{12}]_x\right)^2 \le 0\end{aligned}$$

The last line is true since it is an elliptic paraboloid that is smaller then zero everywhere but the origin, concluding the proof. ■

We showed that if both neighbors see each other initially and Assumption 6 is fulfilled, then the desired bearing will be reached and the proposed facing controller (8) guarantees that the connection does not get lost during the movement of the agents.

4.3 Partial Sensing: $w_1(0) = 1, w_2(t) = 0, t \ge 0$

Without loss of generality, we will consider the case where agent one can sense its neighbor, but is outside the FOV of agent two (i.e., $|\delta_{\psi_1}(0)| < \bar{\gamma}/2$ and $|\delta_{\psi_2}(0)| > \bar{\gamma}/2$, see Figure 1). The equilibrium point of (9) now clearly depends on the facing direction of agent two. We now focus on the scenario where agent one can achieve the desired formation without ever entering the FOV of agent two. We will define initial conditions for $\psi_2(0)$ that ensures this behavior. Before defining the interval explicitly, we first analyse the movement of agent one assuming $|\delta_{\psi_2}(t)| > \bar{\gamma}/2$ holds (and also $w_2(t) = 0$) for all $t \ge 0$.

The dynamics of agent two will be zero and agent one will move on a static circle around agent two.

Lemma 4.2. If $w_1(0) = 1$ and $w_2(t) = 0$ for all $t \ge 0$, agent one evolves on a circle with radius $r = \bar{d}_{12}$ and center $c = p_2(0)$.

Proof. First, observe that the distance between the agents remains invariant along the trajectories of (7). Indeed, $d^2_{12} = z^T_{12} z_{12}$ and $\frac{d}{dt} d^2_{12} = 2z^T_{12} \dot{z}_{12} = -2z^T_{12} P_{g_{12}} g^*_{12} = 0$, since $z_{12} \in \text{Null}(P_{g_{12}})$. Furthermore, p_2 is stationary if $w_2 = 0$ and we conclude that it is the center of a circular movement with radius $r = d_{12}$. Due to the invariance of the distance, $d_{12}(t) = \bar{d}_{12}$ holds for all $t \ge 0$. ■

In a next step, we define the direction in which agent one moves on the circle. This is important to formalize the interval for $\psi_2(0)$ that ensures $w_2(t) = 0$ holds. The rotation direction is indicated by the sign of the angle between g_{12} and g^*_{12} which we define as $\alpha_{\delta_g} = \alpha^*_{12} - \alpha_{12}$. The directions follow as

$$\begin{cases}\alpha_{\delta_g} > 0, & p_1 \text{ moves clockwise} \\ \alpha_{\delta_g} < 0, & p_1 \text{ moves counter-clockwise.}\end{cases} \qquad (12)$$

Note that $\alpha_{\delta_g} = 0$ only holds if $g_{12} = g^*_{12}$, which is the desired equilibrium point. We define two intervals for which $|\delta_{\psi_2}(t)| > \bar{\gamma}/2$ holds for all $t \ge 0$, depending on the direction that agent one moves,

$$\mathcal{M}_L := \left[\alpha_{12}(0) + \frac{\bar{\gamma}}{2}, \alpha^*_{12} - \frac{\bar{\gamma}}{2}\right], \mathcal{M}_U := \left[\alpha^*_{12} + \frac{\bar{\gamma}}{2}, \alpha_{12}(0) - \frac{\bar{\gamma}}{2}\right].$$

Now we will provide a stability proof that fist shows that the facing controller (8) guarantees that agent two will always be inside the FOV of agent one. In a second step we analyse the intervals $\mathcal{M}_L$ and $\mathcal{M}_U$ and show that if the facing direction of agent two is in one of the sets (depending on the sign of α_{δ_g}), agent two will never be able to track agent one. Finally we show that agent one is able to reach the objective without agent two moving.

Theorem 4.3. Under Assumption 3 and initial conditions satisfying $\psi_2(0) \in \mathcal{M}_L$ if $\alpha_{\delta_g} < 0$, or $\psi_2(0) \in \mathcal{M}_U$ if $\alpha_{\delta_g} > 0$, then $\delta_g \to 0$, $\delta_{\psi_1} \to 0$, and $\delta_{\psi_2} = \alpha^*_{21} - \psi_2(0)$ for almost all initial configurations $p_i(0) \in \mathbb{R}^2$, $i = 1, 2$, except for the point corresponding to $g_{12}(0) = -g^*_{12}$.

Proof. First we will show that the sets $\mathcal{M}_L$ and $\mathcal{M}_U$ are invariant under the dynamics of the system (9). To do so we look at the upper and lower bound of δ_g. We then define the region in which it is possible for agent two to track agent one and then conclude that every facing direction of agent two outside those bounds will never be able to track agent one. We introduce a Lyapunov function

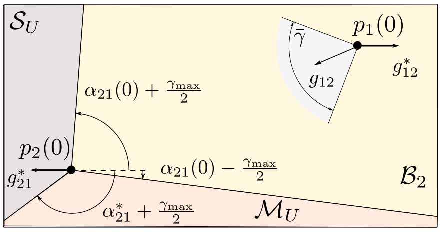


Fig. 2. When $\psi_1 \in \mathcal{B}_1$ holds, there are three possible trajectories depending on the facing of agent two, marked by three different colors.

$V_2 = 1/2\delta_g^T\delta_g$ which is positive definite everywhere but zero. Its time derivate follows as

$$\dot{V}_2 = \delta_g^T\dot{\delta}_g = -\left(g_{12}^*\right)^T \frac{P_{g_{12}}}{d_{12}} g_{12}^* \leq 0. \qquad (13)$$

Now we see that $0 \leq |\delta_g(t)| < |\delta_g(0)|$ holds. Then we add $\pm\bar{\gamma}/2$ to the upper and lower bound of the bearing error, since agent two is able to track its neighbors in the area $\psi_2 \pm \bar{\gamma}/2$. The sign depends on the moving direction of agent one and therefor on α_{δ_g}. For $\alpha_{\delta_g} > 0$ we get the bounds of

$$\alpha_{21}(t) \pm \frac{\bar{\gamma}}{2} \in \left(\tan^{-1}\left(\frac{[g_{21}(0)]_y}{[g_{21}(0)]_x}\right) - \frac{\bar{\gamma}}{2}, \tan^{-1}\left(\frac{[g_{21}]_y^*}{[g_{21}]_x^*}\right) + \frac{\bar{\gamma}}{2}\right),$$

and if $\alpha_{\delta_g} < 0$ we get

$$\alpha_{21}(t) \pm \frac{\bar{\gamma}}{2} \in \left(\tan^{-1}\left(\frac{[g_{21}]_y^*}{[g_{21}]_x^*}\right) - \frac{\bar{\gamma}}{2}, \tan^{-1}\left(\frac{[g_{21}(0)]_y}{[g_{21}(0)]_x}\right) + \frac{\bar{\gamma}}{2}\right).$$

If ψ_2 is inside these bounds then it could sense $g_{12}(t)$, and if not then $|\delta_{\psi_2}(t)| > \bar{\gamma}/2$ holds for all $t \geq 0$. We conclude that if $\alpha_{\delta_g} < 0$, $\mathcal{M}_L$ is invariant, and if $\alpha_{\delta_g} > 0$, $\mathcal{M}_U$ is invariant, both under the dynamics of the closed loop (9).

An illustration of the angles can be seen in Figure 2. Since $w_2 \equiv 0$ we get $\psi_2(t) = \psi_2(0)$. From Theorem 4.1 we conclude that $\delta_g = 0$ is a stable equilibrium point for controller (7) and $\delta_g = -2g_{12}^*$ is unstable. Now we are ready to show that controller (9) reaches the desired bearing and that the facing direction of agent one aligns with it.

First we show that $w_1(t) = 1$ for all $t \geq 0$ by introducing $V_3 = 1/2\delta_{\psi_1}^2$, which is positive definite everywhere but zero, from

$$\dot{V}_3 = \delta_{\psi_1}\dot{\delta}_{\psi_1} = \delta_{\psi_1}\left(\dot{\alpha}_{12} - \dot{\psi}_1\right) = -\delta_{\psi_1}^2. \qquad (14)$$

We conclude $|\delta_{\psi_1}(t)| < |\delta_{\psi_1}(0)|$ and $\delta_{\psi_1} \to 0$ as $t \to \infty$. We choose $\kappa = 1/d_{12}$ such that we can eliminate $\dot{\alpha}_{12}$ dynamics. In (13) we showed that $\dot{V}_2 \leq 0$ and therefor conclude that $\delta_g \to 0$ as $t \to \infty$. ■

Note that in this case Assumption 6 is not required since the dynamics of g_{12} change slower if only one agent moves.

4.4 Partial Sensing: $w_1(0) = 1, w_2(0) = 0$ and $w_2(t) = 1$ for $t > T$.

Now we look at the case where agent one moves inside the FOV of agent two such that $|\delta_{\psi_2}(T)| \leq \bar{\gamma}/2$ for some time $T > t$, but $|\delta_{\psi_2}(0)| > \bar{\gamma}/2$ initially. First only agent one moves and once the trajectory enters the FOV of agent two the indicator function w_2 becomes active.

As before we first define an interval for the facing direction of agent two. We further define a switching point s which specifies the point where agent one enters the FOV of agent two. In a last step we show that the desired bearing will be reached after agent one hits the switching point. The intervals for the facing direction depend on α_{δ_g} defined in (12),

$$\mathcal{S}_L := \left[\alpha_{21}^* - \frac{\bar{\gamma}}{2}, \alpha_{21}(0) - \frac{\bar{\gamma}}{2}\right], \ \mathcal{S}_U := \left[\alpha_{21}(0) + \frac{\bar{\gamma}}{2}, \alpha_{21}^* + \frac{\bar{\gamma}}{2}\right].$$

Theorem 4.4. Under Assumptions 3 and 6 and initial conditions satisfying $\psi_2(0) \in \mathcal{S}_L$ if $\alpha_{\delta_g} < 0$ or $\psi_2(0) \in \mathcal{S}_U$ if $\alpha_{\delta_g} > 0$, then $\delta_g \to 0$ and $\delta_{\psi_1} = \delta_{\psi_2} \to 0$ from almost all initial configurations $p_i(0) \in \mathbb{R}^2, i = 1, 2$, except for the point corresponding to $g_{12}(0) = -g_{12}^*$.

Proof. Agent two is only able to track agent one if it satisfies the intervals defined in Theorem 4.3, from there we remove the part where $|\delta_{\psi_2}(0)| \leq \bar{\gamma}/2$ holds and get $\mathcal{S}_L$ and $\mathcal{S}_U$. If the facing direction is in that region the trajectory of agent one will come into the FOV of agent two, before the desired bearing is reached.

In Theorem 4.1 we showed that $g_{12} = -g_{12}^*$ is a unstable equilibrium point of controller (7) and therefor undesired.

Now we can define a switching point and show that controller (7) will converge first to that point. Furthermore w_2 jumps from zero to one when agent one reaches the point, then both agents can see each other and converge to the desired formation, as shown in Theorem 4.1. We define the switching point as

$$s = \begin{bmatrix} \cos\left(\psi_2(0) - \text{sgn}(\alpha_{\delta_g})\frac{\bar{\gamma}}{2}\right) \\ \sin\left(\psi_2(0) - \text{sgn}(\alpha_{\delta_g})\frac{\bar{\gamma}}{2}\right) \end{bmatrix} - g_{12}^*, \qquad (15)$$

in terms of bearing error. In Theorem 4.3 we showed that controller (8) guarantees that agent one keeps track of agent two. Since

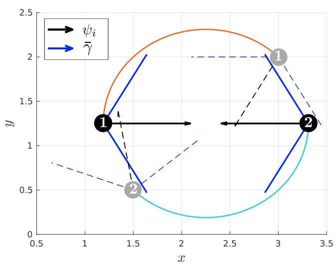
$$0 \leq s < \delta_g(0) \qquad (16)$$

holds, we have to show that δ_g decreases constantly. The Lyapunov function V_2 is positive-semidefinite, radially unbounded and zero only at the equilibrium point. The time derivative defined in (13) is negative semi-definite and zero only at the equilibrium point. Now we have shown that the bearing error constantly decreases and since (16) holds we guaranteed that agent one will hit the switching point s in finite time $t > T$. Once s is reached the stability of the equilibrium point follows as in Theorem 4.1. ■

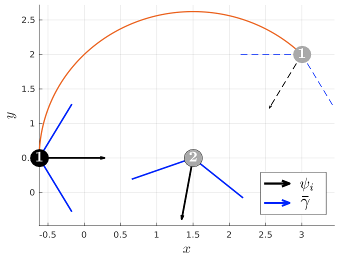
We showed now that $g_{12} = g_{12}^*$ will be stabilized if Assumption 3 and 6 hold and $g_{12}(0) \neq -g_{12}^*$. The introduced controller for the facing direction (8) therefor gives a criteria on the required FOV (Corollary 5) and uses only information locally available on each agent. In the next section we will provide some simulation results.

5. SIMULATIONS

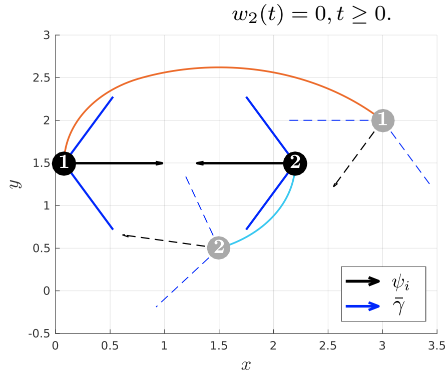
The control law (9) is applied for different initial facing directions but static positions in the two agent case. The results are shown in Figure 3a-3c. The simulation in Figure 3c shows the case where agent one moves into the FOV of agent two. The initial position is set to $p(0) = [3\ 2\ 1.5\ 0.5]^T$, $\psi(0)$ is different in all four cases, the desired



(a) Complete sensing: $w_1(0) = w_2(0) = 1$.

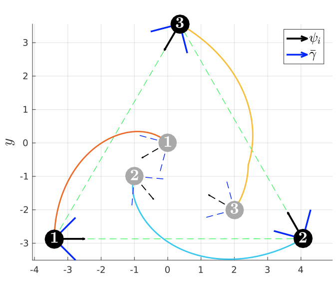


(b) Partial sensing: $w_1(0) = 1$, $w_2(t) = 0, t \geq 0$.

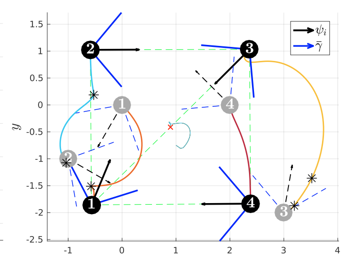


(c) Partial sensing: $w_1(0) = 1$, $w_2(t) = 1, t \geq T$.

Fig. 3. Simulation results for $n = 2$ agent cases.



(a) Three agents where the desired bearing is the triangle.



(b) Four agent case where the desired bearing is a rigid square.

Fig. 4. Simulation results for $n = 3, 4$ agent cases.

bearing is $g_{12}^* = [1\ 0]^T$. While the analysis in this work focused exclusively on the two agent case, we demonstrate in Figures 4a and 4b that the proposed strategy may also work for $n > 2$ agents. The main modification relates to the facing direction control. In the three agent case the desired facing direction was chosen to be the closest neighbor, while in the four agent case we choose it to be in the middle of all neighbors that can be sensed. The stars on the trajectories in Figure 4b ($*$) indicates when the number of sensed agents changes. The FOV is set to $\bar{\gamma} = 90°$ and $\bar{\gamma} = 100°$ for the three and four agent case respectively. In both examples, the agents successfully attain the desired formations, even while the number of sensed agents changes along the trajectories. The desired formation was set to $g_3^* = \begin{bmatrix}1 & 0 & -1/2 & \sqrt{3}/2 & -1/2 & -\sqrt{3}/2\end{bmatrix}^T$ and $g_4^* = \begin{bmatrix}0 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & \sqrt{2}/2 & \sqrt{2}/2\end{bmatrix}^T$, for the three and four agent case respectively.

6. CONCLUSION

In this work, we solved the bearing-only formation control problem with limited field-of-view sensing constraint for the two agent case. To achieve this, we implemented a bearing-only control for the heading of the agent that ensures agents remain inside the FOV once they enter. We provided a complete analysis of the resulting system showing the approach can stabilize the desired formation from almost all initial conditions of the positions. We validated the work with simulation examples, and demonstrated that this idea may be extended to larger formations. The formal analysis of $n > 2$ formations is the subject of future work, in addition to considering more realistic robot models, such as unicycle dynamics.

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