

On two-degrees-of-freedom agreement protocols ★

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Abstract: We propose a distributed two-degrees-of-freedom (2DOF) architecture for driving autonomous, possibly heterogeneous, agents to agreement. The scheme mirrors classical servo structures, separating local feedback from network filtering. This separation enables independent network-filter design for prescribed noise attenuation and allows controller heterogeneity to reject local disturbances, including disturbances exciting unstable agreement poles – which is known to be impossible via standard diffusive couplings. The potential of the framework is illustrated via two numerical examples.

Keywords: Multi-agent systems, Distributed control, Servo control.

1. INTRODUCTION

We study multi-agent systems (MAS) comprised of ν possibly heterogeneous agents interacting over a communication network represented by an undirected graph $\mathcal{G}$. The goal is to achieve asymptotic *agreement*, in the sense that the difference between all measured outputs, y_i , goes to zero. If each agent has information only about a subset of other agents, dubbed *neighbours*, then agreement may be achieved via variations of the celebrated consensus protocol (Olfati-Saber et al., 2007; Ren and Beard, 2008; Mesbahi and Egerstedt, 2010; Bullo, 2024). Originally proposed for homogeneous agents and static gains, variations of the consensus protocol were later proposed for dynamic gains (Ren and Atkins, 2007; Seo et al., 2009; Li et al., 2010; Yu et al., 2013) and heterogeneous agents (Wieland et al., 2011).

Despite its ubiquitousness in the literature, it is widely acknowledged that consensus-like protocols behave poorly when affected by external signals. Measurement noise significantly degrades performance (Zelazo and Mesbahi, 2011, §III-A), while disturbance and uncertainties can hardly be attenuated even by dynamic (Li et al., 2010; Ding, 2015) or non-linear (Mou et al., 2016) controllers. This can be attributed, in part, to the inherent internal instability of the architecture if the agents themselves are unstable (Barkai et al., 2023). In particular, it is well known that any additive white Gaussian noise (AWGN) may result in unbounded variance of the agreement variable if the controllers are LTI (Solo and Piggott, 2016). This can be countered by time-varying, asymptotically vanishing gains (Li and Zhang, 2009; Huang and Manton, 2009; Cheng et al., 2011), which can significantly harm the response to persistent disturbances at the agent level.

We argue that these shortcomings can be, in part, attributed to the consensus structure. In a sense, the consensus protocol may be viewed as a distributed incarnation of the classical unity-feedback control architecture, where the controller acts only

on the mismatch between a reference and regulated signals, see Section 2 for details. In this paper we propose a different protocol, which is inspired by the more flexible two-degrees-of-freedom (2DOF) architecture (Lang and Ham, 1955), where signals from qualitatively different measurement systems are processed differently. We show that the proposed architecture has the potential to counter two well known shortcomings of consensus-like protocols, namely attenuating noise and persistent disturbances. In particular, the proposed controller completely decouples the noise response from the local dynamics, and naturally accommodates heterogeneity between the agents. Two examples illustrate how these properties can be exploited to easily reject persistent disturbances as well as attenuate the noise's effect in the agreement direction. The paper is organized as follows. Section 2 contains necessary background on servo control, as well as reframing of the consensus protocol within this context. Section 3 builds on the tracking perspective and introduces the 2DOF agreement protocol, its agreement, and overall structure. Section 4 contains an illustrative example showcasing the potential upside of the architecture.

Notations The sets of all non-negative integers are denoted as $\mathbb{Z}_+$ and $\mathbb{N}_\nu := \{i \in \mathbb{Z} \mid 1 \leq i \leq \nu\}$. Given a set $S \subset \mathbb{Z}$, its cardinality is denoted as $|S|$. The sets of real and complex numbers are denoted by $\mathbb{R}$ and $\mathbb{C}$ respectively. By e_i we understand the i th standard basis vector in $\mathbb{R}^\nu$ and by $\mathbf{1}_\nu$, or simply $\mathbf{1}$ when the dimension is clear from the context, the all-ones vector from $\mathbb{R}^\nu$. The complex-conjugate transpose of a matrix M is denoted by M' . The notation $\text{diag}\{M_i\}$ stands for a block-diagonal matrix with diagonal elements M_i . The image (range) and kernel (null) spaces of a matrix M are notated $\text{Im } M$ and $\text{ker } M$, respectively. Given two matrices (vectors) M and N , $M \otimes N$ denotes their Kronecker product, while $\text{spec } M$ refers to the set of eigenvalues of M .

A simple undirected graph, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of order ν , consists of a set of nodes $\mathcal{V} = \{v_1, \dots, v_\nu\}$, and a set of n_e edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. An edge, e_{ij} , in the network, is the unordered pair of nodes (v_i, v_j) indicating bidirectional information flow between node i and node j . A path between two nodes v_i and v_j is a sequence of edges leading from v_i to v_j . The *neighbourhood*

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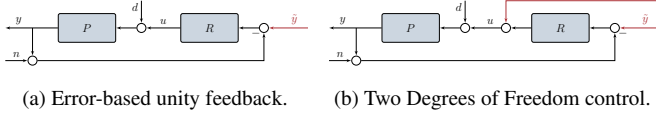


Fig. 1. Classical servo-regulation control architectures.

of node v_i is the set of all nodes v_j such that $(v_i, v_j) \in \mathcal{E}$, and is denoted by $\mathcal{N}_i$. The degree of a vertex is the cardinality of its neighbourhood set. The adjacency matrix, $A_{\mathcal{G}} \in \mathbb{R}^{v \times v}$ is a symmetric matrix which encodes the adjacency relationship in the graph. The degree matrix of a graph, $D_{\mathcal{G}}$, is a diagonal matrix containing the vertices degrees on its diagonal, and the graph Laplacian is a symmetric square matrix defined by $L_{\mathcal{G}} = D_{\mathcal{G}} - A_{\mathcal{G}}$. An undirected graph is said to be *connected* if there is a path between each pair of nodes in the graph. For a connected undirected graph the matrix $A_{\mathcal{G}}^* := D_{\mathcal{G}}^{-1}A_{\mathcal{G}}$, called the *normalized adjacency matrix*, is well defined and diagonalizable.

2. BACKGROUND

In this section we review concepts from classical servo control, namely unity-feedback and 2DOF control structures. We then revisit the consensus structure and show its similarities to error-based tracking control, paving the way to a 2DOF counterpart.

2.1 Classical control architectures

Consider a linear time-invariant (LTI) plant P with reference signal $\tilde{y}$, load disturbance d , measured output y , and additive measurement noise n . The goal is to design a controller that simultaneously tracks the reference $\tilde{y}$ and attenuates the effects of n and d . The most common approach is via error-based control in a unity feedback setup as depicted in Fig. 1(a) where R is the controller. The closed-loop system for this design is given by

$$y = (I + PR)^{-1}(PR\tilde{y} - PRn + Pd) := T\tilde{y} - Tn + T_d d.$$

Contingent on the spectra of $\tilde{y}$, n and d , designing a single R to attenuate n and d while ensuring that $T\tilde{y} \approx 1$ may be a non-trivial task. One classical solution to the servo problem is through a two degrees-of-freedom (2DOF) architecture, using separate controllers for outputs and reference signals. Indeed, variations of 2DOF architectures, first introduced more than 70 years ago (Lang and Ham, 1955), have been extensively studied (Qiu and Zhou, 2010, Sec. 2.9). The term “two-degrees-of-freedom control” is used to refer to several slightly different control architectures. Here we consider the architecture shown in Fig. 1(b), which can completely decouple the disturbance and tracking design. This is accomplished by designing the control law in the following fashion. First, design a feedback controller R to attenuate n and d , disregarding the reference tracking. Then, the signal $\tilde{u}$ is designed to achieve the tracking behavior in an open-loop fashion, i.e., satisfying the *consistency condition*

$$\tilde{y} = P\tilde{u}. \quad (1)$$

If the signals $\tilde{y}$ and $\tilde{u}$ are bounded and R is stabilizing the system will be stable. Moreover, when (1) holds, then the controlled variables satisfy

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} \tilde{y} \\ \tilde{u} \end{bmatrix} + \begin{bmatrix} I \\ R \end{bmatrix} (I - PR)^{-1} Pd + \begin{bmatrix} P \\ I \end{bmatrix} R(I - PR)^{-1} n,$$

and the command response is independent of the feedback controller, R , in the nominal case.

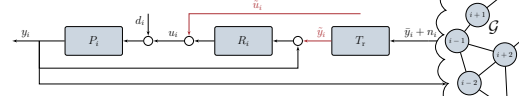


Fig. 2. A Two-Degrees-of-Freedom agreement protocol.

Remark 2.1. The consistency condition (1) essentially requires the inversion of the plant. When this is not possible, $\tilde{y}$ is usually chosen as a filtered version of the real reference, $\tilde{y} = T_r r$, such that $P^{-1}T_r$ is realizable and stable. This is standard practice and will also be the case in the sequel. ∇

2.2 Consensus protocol revisited

To see how servo problems relate to agreement, consider a group of agents each with m inputs and p outputs

$$\Sigma_i : y_i = P_i(u_i + d_i) + y_{0,i}, \quad \text{for all } i \in \mathbb{N}_v \quad (2)$$

where P_i are given LTI models, u_i is a control input, d_i is a disturbance input, y_i is a measured regulated output, and $y_{0,i}$ is an initial condition response of the agent. The agent’s goal is to reach output *agreement* in the sense that

$$\lim_{t \rightarrow \infty} \|y_i(t) - y_j(t)\| = 0, \quad \forall i, j \in \mathbb{N}_v, \quad (3)$$

In particular, when $\lim_{t \rightarrow \infty} y_i(t) = \text{const}$, the problem is referred to as *consensus* and is considered non-trivial if the outputs do not converge to zero for some initial conditions. If y_i converges to a non-constant regular, e.g. periodic, signal, then the problem is referred to as *synchronization* (Scardovi and Sepulchre, 2009). The agents are controlled by a generalized consensus protocol given by

$$u_i = k_i F \sum_{j \in \mathcal{N}_i} (y_j - y_i), \quad \forall i \in \mathbb{N}_v, \quad (4)$$

where $\mathcal{N}_i \subset \mathbb{N}_v \setminus \{i\}$ is the set of neighbours, some gains $k_i > 0$, and filter F , which are design parameters. It is not unreasonable to assume that measurements coming from neighboring agents are imperfect, e.g. corrupted by additive noise. In this case

$$u_i = k_i F \sum_{j \in \mathcal{N}_i} (y_j - y_i + n_{ij}), \quad (5)$$

for some noise signals n_{ij} . A slightly different outlook on this structure can be obtained by rewriting (5) as

$$u_i = -k_i F |\mathcal{N}_i| (y_i - \bar{y}_i + n_i) \quad (5')$$

where

$$\bar{y}_i := \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} y_j \quad \text{and} \quad n_i := \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} n_{ij}, \quad (6)$$

sums up the measured outputs and noise from all measurement channels of Σ_i . This form is reminiscent of a servo problem in unity feedback where only the error is supplied to the controller (Qiu and Zhou, 2010, Sec. 1.3). In this perspective, $k_i |\mathcal{N}_i|$ is the local feedback gain and $\bar{y}_i$, which is the average of measured neighbors, is the “reference” signal.

A similar viewpoint was first proposed in (Fax and Murray, 2004, §III.A), and indeed agreement is achieved iff the underlying graph is connected and all the agents simultaneously solve this tracking problem. It is reasonable to assume that, as in servo regulation, a 2DOF architecture might be a viable alternative to the consensus protocol and may simplify the design.

3. A 2DOF AGREEMENT PROTOCOL

Motivated by the parallels to tracking problems, we wish to derive a 2DOF variant of the consensus protocol, as illustrated

in Fig. 2. The main difference between servo-regulation and agreement problems is that the latter lacks a well-defined reference signal. Consequently, there are various ways to select $\bar{y}$. It can be done in an open-loop way, with the agents exchanging controller variables and agreeing on their common rendezvous point or trajectory. This would lead to a control structure akin to that of (Wieland et al., 2011), where the agents exchange the state of some common internal model. Alternatively, it can be done in a closed-loop way, where $\bar{y}$ is generated using only the measured neighbors outputs.

Consider the latter approach, which can be motivated by the classic consensus protocol. To this end, assume that $F = 1$ and $n_i = 0$, then the aggregate form of (5') reads

$$u = -(KD_{\mathcal{G}} \otimes I)(y - \bar{y}), \quad \text{where } \bar{y} := (A_{\mathcal{G}}^* \otimes I_p)y,$$

and $K := \text{diag}\{k_i\}$. This is the tracking representation of the consensus protocol with $\bar{y} = (A_{\mathcal{G}}^* \otimes I_p)y$ as the reference signal. It is tempting to picking $\bar{y} = y$ as the global reference since it is naturally distributed according to the graph structure. However, a standing assumption is that the noise, n , is generated at the network level. Hence, we assume it is additive to $\bar{y}$. This implies that a good model for the required behavior is a *filtered* version of $\bar{y}$, i.e.,

$$\tilde{y} = (I \otimes T_r)((A_{\mathcal{G}}^* \otimes I_p)y + n), \quad (7)$$

where T_r is the additional degree of freedom and n represents additive noise induced by the network as discussed in §2.2. Note that we assume a uniform filter T_r for all the agents (even with heterogeneous dynamics).

Now we are ready to introduce the *2DOF consensus protocol*, as seen in Fig. 2, and its dependencies on design parameters R_i and T_r , dynamics P_i , and the graph $\mathcal{G}$.

Proposition 3.1. Let $P := \text{diag}\{P_i\}$ and $R := \text{diag}\{R_i\}$ denote the aggregate plants and local controllers respectively. The 2DOF consensus protocol is given by

$$u = Ry + (I_{\nu m} - RP)\tilde{u}, \quad (8a)$$

where $\tilde{y}$ is as in (7) and $\tilde{u}$ satisfies that consistency relation (1). If $\mathcal{G}$ is undirected and connected then the resulting closed loop dynamics are

$$y = U^{-1} \text{diag}\{\hat{S}_i\}U (T_d d + (I \otimes T_r)n + S y_0), \quad (8b)$$

with

$$\hat{S}_i := (I_p - \alpha_i T_r)^{-1}, \quad S := (I_{\nu p} - PR)^{-1}, \quad T_d := SP. \quad (8c)$$

Furthermore, $U \in \mathbb{R}^{\nu \times \nu}$ is such that $UD_{\mathcal{G}}^{-1/2}$ is unitary, satisfying $UA_{\mathcal{G}}^*U^{-1} = \text{diag}\{\alpha_i\}$.

Note that at the agent level (8a) is given by

$$u_i = R_i y_i + (I_m - R_i P_i)P_i^{-1}T_r(\bar{y}_i + n_i)$$

which has two distinct components: a *decentralized* component $R_i y_i$, and a *distributed* component driven by $\bar{y}_i$. In the output equation, this translates to a series interconnection of a decentralized component and a distributed component. Interestingly, unlike 1DOF consensus protocols the closed-loop dynamics in (8b) explicitly separates the network and local dynamics. The network component depends only on T_r , which is uniform across agents, while the local dynamics captured by S and T_d are decentralized by construction. This inherent separation naturally accommodates agent heterogeneity provided that their network component, T_r , is homogeneous. The following theorem formalizes these observations.

Theorem 3.2. Consider heterogeneous agents driven only by initial conditions, interacting over an undirected and connected

graph $\mathcal{G}$, and controlled by (8b). If each local controller R_i stabilizes its corresponding plant P_i , then the agents reach asymptotic agreement if and only if

$$\hat{S}_i := (I_p - \alpha_i T_r)^{-1} \in H_{\infty}, \quad \forall \alpha_i \in \text{spec } A_{\mathcal{G}}^* \setminus \{1\}$$

and $\hat{S}_1 = (I_p - T_r)^{-1}$ has all poles in the closed left half-plane.

Theorem 3.2 provides clear conditions for agreement but does not explicitly specify the resulting agreement trajectory. Since S is stable, the trajectory is determined solely by the unstable poles of $\hat{S}_1$. Hence, T_r must be designed to both solve a simultaneous stabilization problem against the eigenvalues of $A_{\mathcal{G}}^*$ and satisfy certain interpolation constraints.

Still, pole cancellations can occur in the series interconnection $U^{-1} \text{diag}\{\hat{S}_i\}US$, altering the agreement trajectory. Such cancellations, however, are outside the feedback loop and thus do not jeopardize stability. The following proposition provides a simple necessary condition for these cancellations.

Proposition 3.3. Let R , P , T_r be finite-dimensional systems, and denote by p_i the imaginary-axis poles of $\hat{S}_1$. If p_i is not a pole of $U^{-1} \text{diag}\{\hat{S}_i\}US$, then it must be a zero of S_i for all i .

Proposition 3.3 has an important implication for robustness. In consensus-like protocols, the resulting agreement trajectory is generally vulnerable to persistent disturbances in the agreement direction, as these disturbances excite common unstable poles. Yet intentionally introducing heterogeneity in local controllers can exploit the cancellation properties described above to improve robustness against such disturbances without altering the agreement trajectory. This insight follows a conjuncture made in the concluding remarks of (Barkai et al., 2023).

4. NUMERICAL EXAMPLE

Consider a group of $\nu = 5$ integrator agents, $P_i = 1/s$, attempting to reach static consensus. The agents interact over the undirected whose spectral properties are given by $\text{spec } A_{\mathcal{G}}^* = \left\{ \frac{\pm\sqrt{33}-3}{12}, -0.5, 0, 1 \right\}$. We compare two architectures: (i) classic 1DOF consensus (5) with $F = 1$, and (ii) the 2DOF protocol (8). The controllers are tuned to achieve similar nominal performance as measured by settling time. Then, we show how we can improve the behavior of the agreement mode under non-nominal conditions. To this end, a 1DOF consensus protocol with $K = kI_{\nu}$, for $k = 2.65$ yields a settling time of $t_s \approx 1.433[s]$ for the given graph. When driven by additive white noise this design results in Wiener process with a linearly increasing drift with a slope of $k/\nu = 0.53$. Moreover, it can be shown that the steady-state variance of the disagreements equals $\|T_n\|_2^2 = 0.9858$.

4.1 Design of a network filter

A notable property of dynamics (8b) is that the network filter can be designed almost independently of the local dynamics. Moreover, T_r and the graph completely determine the response to noise. For SISO agents we can even design *generic* network filters that will ensure consensus for any connected graph and arbitrary agents. For example, consider a general third order network filter

$$T_r(s) = \frac{\omega_n^2}{(\tau s + 1)(s^2 + 2\zeta\omega_n s + \omega_n^2)},$$

resulting in noise response

$$\hat{S}_i(s)T_r(s) = \frac{\omega_n^2}{\tau s^3 + (2\zeta\omega_n\tau + 1)s^2 + (\tau\omega_n^2 + 2\zeta\omega_n)s + \omega_n^2(1 - \alpha_i)}.$$

Clearly $\hat{S}_i(s)$ has a single pole at the origin for all possible parameters, resulting in consensus assuming stability of $\hat{S}_i$ and no cancellations with the local loop. It can be shown that any ζ , ω_n and τ satisfying $2\zeta\omega_n > 2\tau$ will ensure stability for every $\alpha_i \in [-1, 1)$.

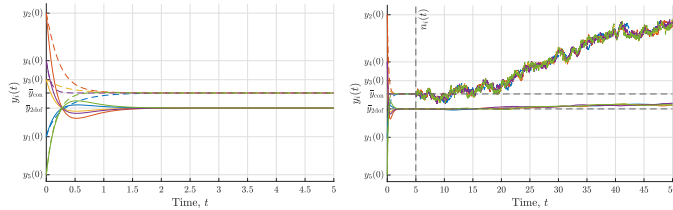
As for performance, a known issue of noisy consensus-based systems is that their agreement mode evolves as a Wiener process (Solo and Piggott, 2016). Minimizing the slope of the Wiener process' drift amounts to minimizing the squared H_2 norm of $s\hat{S}_i(s)T_r(s)$, which for this simple case can be calculated analytically and is given by

$$\|s\hat{S}_i(s)T_r(s)\|_2^2 = \frac{\omega_n^3}{(2\omega_n\tau + 2\zeta)(2\omega_n\tau\zeta + 1)}.$$

Combined with the stability constraint $\zeta\omega_n > \tau$ this is a non-linear minimization problem. A heuristic minimization strategy would be to keep ω_n small and τ large, while selecting ζ to enforce the stability constraint. This, however, could lead to slow poles and a dominant zero at $1/\tau$ in $\hat{S}_i(s)$ which would impact the nominal convergence rate. After some trial and error, the parameters $\omega_n = 3$, $\tau = 5$ and $\zeta = 2$ together with the uniform local controller

$$R_0(s) = -\frac{7.586s + 16}{s + 0.4143}$$

resulted in a nominal settling time of $t_s \approx 1.432[s]$ which is comparable to the standard protocol.



(a) Evolution of the outputs under nominal conditions. (b) Evolution of the outputs w/ additive white measurement noise.

Fig. 3. Simulations of the control designs for the example (solid: 2DOF design, dashed: standard consensus).

Fig. 3(a) shows the nominal behavior of both designs, which indeed have comparable settling time. Note that the designs converge to different consensus points, classical consensus to the average of initial conditions and the 2DOF to some weighted average which also depends on T_r and S . Fig. 3(b) shows the same setups, now with white noise at each agent as in (6) applied at $t = 5[s]$. Since both designs have a pole at the origin for the agreement mode, both behave as a Wiener process with linearly diverging variance. However, the 2DOF design has noticeably smaller drift compared to the system controlled by classic consensus. In fact, denoting the nominal consensus values by $\bar{y}_{2dof}$ and $\bar{y}_{con}$, after 60 seconds we have errors of $\|y_{2dof}(60) - \bar{y}_{2dof}\|_2 = 0.64$ and $\|y_{con}(60) - \bar{y}_{con}\|_2 = 9.1698$, respectively.

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