Asynchronous sampled-data synchronization with small communications delays

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***Index Terms*— Sampled-data systems, network control systems, time delays.**

I. Abstract

The study of multi-agent systems (MAS) has been a central tenant in a multitude of scientific and engineering disciplines over the past decades, with applications including biology, optimization, and robotics. Perhaps the most widely known control problem in MAS is the consensus [1], [2] or synchronization [3], [4] problem, in which the agents must *agree* on some common steady-state trajectory. The crux of this problem is that the agents must come to agreement distributively with limited communication, limitations which may be divided into two categories: spatial and temporal.

Spatially constrained communication reflects the requirement that each agent must generate its control law based on partial information. Each agent is only privy to information from a subset of the group, its *neighbours* [5], [6]. These neighbourhoods can be time-varying and induce a *graph* structure on the overall system, which has been widely exploited to design control laws [7]. While spatial constraints restrict the *identity* of interacting agents, temporal constraints restrict the communication *time*. These constraints encompass both sampled communication and various forms of time-delays, both staples of networked systems.

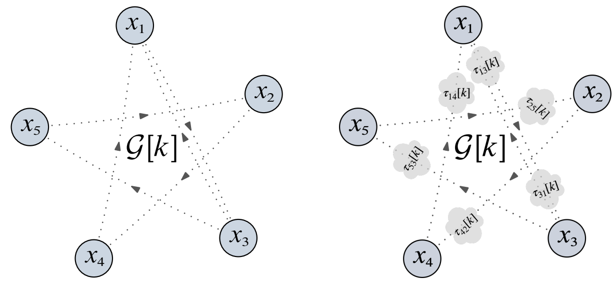
In practice systems must cope with both types of constraints. Moreover, common assumptions such as continuous communication, static graphs, or periodic synchronous sampling [8]–[10] might not be realistic. This cumulative burden makes even consensus of simple integrators challenging [11], often leading to conservative, robustness-based, approaches [12], [13] even without delays. To the best of our knowledge, achieving synchronization under asynchronous communication and heterogeneous transmission delays is still an open problem.

We propose a sampled-data control protocol that guarantees asymptotic synchronization for switching graphs, asynchronous sampling, and time-varying, heterogeneous transmission delays. Other than standard stabilizability assumption we require only persistent connectivity of the graphs induced by the sampling sequence, which can be asynchronous and unknown. As for the delays, we simply assume that the incoming information is time stamped and the delays are small. More precisely,

$\mathcal{A}_1$: let $\tau_{ij}[k]$ denote the transmission delay from agent j to agent i at time instance $t = s_k$, then we assume that

$$s_k + \tau_{ij}[k] < s_{k+1}, \quad \forall i, j \in \mathbb{N}_\nu,\ k \in \mathbb{Z}_+.$$

The control structure is an extension of the emulation-based hybrid controller proposed in [14], with an added internal predictor. Synchronization is made possible by leveraging the hybrid nature of the original controller, which couples the agents only when information arrives. As a result the delays are akin to a known mismatch between the real state and the delay-free one, which is then compensated by a predictor.



(a) Standard. (b) With delays.

Fig. 1. Sampled-data multi-agent control architectures.

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